INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) III YEAR: 1991-92
SEMESTRAL-II BACKPAPER EXAMINATION
DESIGN OF EXPERIMENTS

Date: 29.6.92 Maximum Marks: 100 Time: 3 Hours

Note: Answer Question No.1 and any three out of the rest. Marks alloted to a question are indicated in brackets [ ] at the end. Submit your Practical Records to the instructor on or before the date of exam. They carry 10 marks.

1. The following table gives the yields (lbs/plot) of ear corn (y) and the number of plants (x) for four varieties A, B, C, D in 3 randomised blocks. Analyse the data

Blocks		Treatments A	B	C	D
I	x:	18	16	18	14
	y:	8.6	7.5	6.7	6.5
II	x:	14	16	15	16
	y:	6.5	7.6	8.2	8.3
III	x:	16	15	15	17
	y:	7.1	7.5	5.7	7.5

and draw conclusions about the significance of corn yield differences of the four varieties. [21]

2.(a) Develop the analysis of variance of a latin square design for v treatments

(b) What are Bonferroni's method and Tukey's method of multiple comparisons? Develop the expressions for the confidence intervals for elementary treatment contrasts in a latin square design. [13+10=23]

3.(a) Define the main effects and interactions of a 2^n factorial experiment, and show that they represent 2^n-1 mutually orthogonal treatment contrasts.

(b) Construct the key block of a replication of a confounded design for a 2^8 experiment in blocks of size 2^4, where the following independent effects are to be confounded:

ABCE, ABDF, AEFG, ABGH.

Write down the other factorial effects confounded in the design and indicate how the other blocks of this replication can be obtained.

3.(c) Show that in a 2^n factorial experiment in blocks of 2^k plots the maximum number of factors, that can be accommodated without confounding main effects, two-factor interactions and 3-factor interactions, is 2^{k-1}.

[10+8+5=23]

4.(a) Give a balanced confounding scheme of a 3^3 factorial experiment in block of 3^2 plots. Construct the independent treatment combinations of all the key blocks of this design.

(b) Describe the analysis of variance for the suggested design.

(c) Describe the generalized Yates' algorithm for the design if one is interested in single d.f. components.

[8+10+5 = 23]

5.(a) What is a strip-plot design? When would you recommend the use of such designs? Give an example.

(b) Describe an appropriate model for the design. By using an appropriate orthogonal transformation show how its ANOVA splits into three ANOVA's on three different sets of variables having three different variances to be worked out. Hence give the ANOVA of a strip-plot design.

(c) Obtain the variances of various comparisons of treatment means and get their unbiased estimates. [3+15+5 = 23

INDIAN STATISTICAL INSTITUTE
B.STAT. (HONS.) III YEAR: 1991-92
SEMESTRAL II BACKPAPER EXAMINATION
STATISTICS COMPREHENSIVE

Date: 24.6.92 Maximum Marks: 100 Time: 4 hours.

Note: Answer all questions.

1. Let $X_1, X_2, \ldots, X_n$ be iid $N(\mu, 1)$ random variables. Construct a confidence interval for μ of width 0.5 and maximum possible confidence coefficient. Given α, find the smallest size n for which this confidence interval has a confidence coefficient $\geq 1-\alpha$. [12]

2. Compute an attainable lower bound of the variance of an unbiased estimator of θ based on n iid observations from $U(0,\theta)$ distribution. [12]

3. Let $X_1, X_2, \ldots, X_n$ be iid with

$$P[X_1 = r] = \frac{1}{N_2 - N_1},\ r = N_1+1, N_1+2, \ldots, N_2,$$

$N_1 < N_2$ are unknown integers.

Find suitable estimators of $N_2 - N_1$, N_2, and N_1 and state some properties of the estimators. [12]

4. The girl was happy because she was incorrect only in one math problem in a test. It was ascertained that there were either 12 or 15 problems in the test. The girl usually scores around 90% in her math tests. Estimate the number of questions in the test by the method of maximum likelihood stating necessary assumptions. [12]

5. (a) In most out-puts of computer packages, p-values are printed. How are these used for inference?

(b) When the normal equations do not have a unique solution, can these have exactly 4 solutions?

(c) In the Gauss-Markoff set-up in linear models, how do you estimate σ^2, the common variance of the error components?

(d) n iid observations from a $N(\mu,1)$ populations are available. Compute the power of the MP test for $H_0: \mu=0$ against $H_1: \mu=1$.

(e) X is Binomial (n,p) and Y is Binomial (m,p). Is X+Y Binomial (m+n,p)? Justify. [2x5=10]

p.t.o.

6. Ichthyosis vulgaris is a skin disease. It is of interest to study high frequency (hf) electrical conductance of skin affected by the disease. In a study 32 subjects of both sexes -16 patient of this disease and 16 normal ones - were used.

A small drop of distilled water was applied at the site of disease for 30 seconds by means of a dropper, blotted on immediately and the conductance was measured. The conductance values thereafter were recorded at 15 seconds intervals for $\frac{1}{2}$ minute. Data are given below.

It is of interest to examine the regression of conductance on time and find out if there are disease and sex effects on the regression.

(a) Work out the contrast that describes the linear regression of conductance on time (at the three points 0,15,30 seconds

(b) Examine if there are sex and disease effects on the linear regression.

I. Ichthyosis vulgaris cases					II. Normal cases				
Sl. No.	Sex	Value of conductance($\mu\Omega^{-1}$) at time in seconds initial	15	30	Sl. No.	Sex	Value of conductance($\mu\Omega^{-1}$) at time in seconds initial	15	30
1	M	191	89	41	1	M	237	143	66
2	M	149	5	6	2	M	435	127	78
3	F	273	60	39	3	F	192	43	5
4	F	102	9	2	4	F	247	79	17
5	F	294	88	6	5	F	264	112	16
6	F	264	40	8	6	F	130	18	14
7	F	282	54	10	7	F	147	15	7
8	M	127	41	6	8	M	216	88	15
9	F	114	18	6	9	F	345	68	11
10	M	98	13	2	10	M	401	260	87
11	M	333	153	33	11	M	254	54	27
12	M	338	19	8	12	M	300	125	39
13	M	148	47	39	13	M	227	100	37
14	F	380	340	52	14	F	493	267	18
15	M	424	308	132	15	M	234	31	21
16	F	352	128	16	16	F	247	61	19

[16+26=42

INDIAN STATISTICAL INSTITUTE
B.STAT(HONS.) III YEAR: 1991-92
SEMESTRAL-II BACKPAPER EXAMINATION
PHYSICS-II

Date: 24.6.92 | Maximum Marks: 100 | Time: 3 Hours

GROUP A

Total marks (12x5) = 60

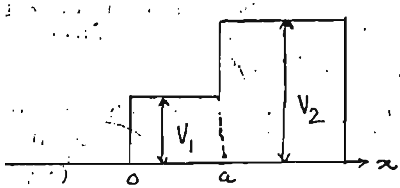
1.(a) Electron beams are accelerated to potentials 40KV. Estimate the wavelength of the electrons. Given:

$h = 6.6 \times 10^{-34}$ joule-sec., $m_e = 9.04 \times 10^{-31}$ Kg.,

and 1 eV = 1.6×10^{-19} joules.

(b) In a series of experiments on the determination of the mass of the ω^0 particle, the results showed a variation of $\pm$ 20 m_e, where m_e denotes electron mass. What can you say about the life time of these particles?

(c) Consider a two step potential as shown in the figure.



Write down the Schrodinger equation for the system.

(4+5+3)=12

2.(a) Consider a wave function given by $\psi(x)$ = A Sin kx. What values of momentum p would you obtain from its measurements in the state $\psi(x)$?

(b) Given $L_+ = L_x + iL_y$ and $L_- = L_x - iL_y$, show that $[L_+, L_-] = 2\hbar L_z$ (Symbols have their usual meanings).

(c) Show that, $e^{ipa/\hbar} \; x \; e^{-ipa/\hbar} = x+a$ (4+4+4)=12

p.t.o.

3.(a) Find the matrix element $\langle n' / x / n \rangle$ using

$$x = \left(\frac{\hbar}{2mw}\right)^{\frac{1}{2}} (a + a^{+})$$ where a^{+} and a are the creation and destruction operators respectively.

(b) Write down Heisenberg's equation of motion. What is the basic difference between this picture and the Schrodinger picture? [6+(2+4)]=12

4.(a) Give the essential features of the energy bands in a solid crystal.

(b) From the point of view of the band theory of solids, distinguish between metals, semi conductors and insulators. (6+6)=12

5.(a) Give the unit cell specification of an orthorhombic crystal system.

(b) Calculate $C_p - C_v$ per mole of sodium at room temperature, if at this temperature the compressibility of sodium is 12.3×10^{-12} cm^2/dyne, and linear coefficient of expansion is 6.22×10^{-5} cm/^{o}c.

(c) What are the main assumptions of Einstein's theory of specific heats? What are the drawbacks of the theory?

(2+4+6)=12

GROUP B

Full Marks:40

Note: Answer Q.1 and any two of the rest.

1. Establish the following relation, the symbols having their usual significance

(a) $T = (\partial U/\partial S)_v$ (b) $p = -(\partial F/\partial V)_T$

(c) $\ln(x!) = x \ln x - x$, when $x \to$ very large. Suppose that there are three cells in phase space: 1,2 and 3. Let $N = 30$; $N_1=N_2=N_3$ and $w_1=2j$, $w_2=4j$ and $w_3=6j$. If $\delta N_3=$ find δN_1 and δN_2 such that $\delta N=0$, $\delta U=0$. (2+2+4+2=[10

contd. ...3/

2. Show, from Maxwell-Boltzmann statistics, that once the partition function Z of the system is evaluated, all the thermodynamic properties of the system can be evaluated.

   Derive the barometric equation applying MB-statistics. (8+7)=[15]

3. Write down the relation connecting entropy and probability.

   Compute the thermodynamic probability of Bose's statistics.

   Why is the condition equation $\delta N=0$ not applicable to an assembly of photon gas enclosed in a container ?
   What is its implication on the distribution function ?

   (2+7+3+3)=[15]

4. Write down the distribution function of FD-statistics.
   Employ FD-statistics to derive Richardson's equation for thermionic emission. (2+13)=[15]

INDIAN STATISTICAL INSTITUTE
B.STAT. III YEAR: 1991-92
SEMESTRAL II EXAMINATION
PHYSICS II

Date: 8.5.92 Maximum Marks: 100 Time: 3 Hours

GROUP A

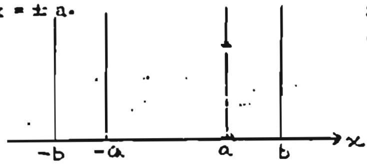
TOTAL MARKS [(11x5)+5 (assignments)]=60

Answer any FIVE questions.

1.(a) Yukawa had suggested the existence of a massive meson to explain nuclear force. Using the uncertainty principle, derive a relation between the range of the force and the mass of the meson.

(b) Calculate the flux associated with the wave function $\psi(x) = U(x)\,e^{ikx}$, where $U(x)$ is real.

(c) A particle is in the ground state of a box with sides $x = \pm a$. Suddenly, the sides of the box are moved to $x = \pm b$. What is the probability that the particle will be in the ground state of the new potential? (4+3+4)=11



2.(a) Show that the expectation value of momentum operator is real.

(b) The Hamiltonian operator for a one dimensional harmonic oscillator is given by

$$H = \hbar\omega(a^{+}a + \tfrac{1}{2}),$$

where a and a^{+} are the destruction and creation operators respectively.

Show that there is a lower bound of energy of the harmonic oscillator. What is it called?

(c) In Cockroft - Walton experiment, the following reaction takes place,

$$Li_3^7 + H_1^1 \longrightarrow (Be_4^8) \longrightarrow 2He_2^4 + Q$$

even with protons whose energy is much less than the potential barrier of the lithium nucleus. How is this possible? Give a brief and qualitative answer. (4+4+3) = 11

p.t.o.

3. The Hamiltonian $H = p^2/2\mu + V(x)$ has a set of eigen kets $|k\rangle$ with eigen values E_k.

(a) Verify that $[x, [x, H]] = -\hbar^2/\mu$

(b) If $|l\rangle$ is any ket having a discrete eigen value, show that

$$\sum_{l} (E_k - E_l) |\langle k| x |l\rangle|^2 = \frac{-\hbar^2}{2\mu}.$$

(4+7) = 11

4.(a) By introducing a suitable limiting process in

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} \, dk,$$

show that, $\delta(x) = \frac{1}{\pi} \lim_{\varepsilon \to 0+} \frac{\varepsilon}{x^2 + \varepsilon^2}$.

(b) In describing the change with time of a physical system in the Schrodinger picture, which of the following is correct?

i) the dynamical variables are constant in time
ii) state vectors are constant in time
iii) state vectors vary with time
iv) dynamical variables vary with time

(c) Considering the momentum operator p, show that expectation values satisfy the equations of motion similar to classical mechanics. (5+2+4) = 11

5.(a) Consider a monochromatic X-ray beam falling on a one-dimensional row of atoms with interatomic distance 'a'. What kind of pattern do you expect the diffracted beams to form?

(b) Consider an infinite array of equidistant mass points (each of mass m) with near-neighbour interactions only. Assuming that Hooke's law is obeyed, show that there exists a maximum frequency which can be propagated through the chain. (3+8) = 1[illegible]

contd. ...3/·

6.(a) Consider a linear crystal of length L. Calculate the number of electrons in a completely filled energy band. (Use reduced wave vector $-\pi/a \leq k \leq \pi/a$).

(b) Consider a crystal where the 1st. Brillouin zone contains only one electron, initially in a state k. An external electric field F is now applied to the crystal. Discuss the motion of the electron. What will be the value of the effective mass of the electron? Show how will it vary within the band. (5+6) = 11

GROUP B

Full Marks:40

Answer Q.1 and any two of the rest.

1. Write down monatomic ideal gas equation. Derive the same by the application of Maxwell-Boltzmann statistics. 2+10=[12]
2. Write down Planck's law of radiation. Derive it by applying Bose's statistics. Hence show that the total energy emitted by a black body is proportional to the fourth power of its absolute temperature. 2+8+4= [14]
3. What are Pauli's exclusion principle and Heisenberg's uncertainty relation? What are their implications on Fermi-Dirac statistics? Derive an expression for the thermodynamic probability of Fermi statistics. 4+6+4 = [14]
4. What are bosons and fermions? Give one example of each. With usual significance of the symbols, show that according to F.D. statistics

$$p_{mo} = \left(\frac{3Nh^3}{8\pi V}\right)^{\frac{1}{3}}$$

where p_{mo} is the maximum momentum of electrons at absolute zero and hence show that the momentum space is uniformly populated within the momentum sphere of radius p_{mo} at 0^oK and there are no phase points outside the sphere. 2+2+7+3=[14]

INDIAN STATISTICAL INSTITUTE
B.STAT. III YEAR: 1991-92
SEMESTRAL II EXAMINATION
ECONOMICS IV

Date: 8.5.92 | Maximum Marks: 100 | Time: 3 Hours

Note: Answer any four questions. All questions carry equal marks.

1.(a) How would you obtain a confidence interval for the expected value of Y corresponding to a given value of $X = X_0$ (not observed in the sample) in the linear structure

$$Y_i = \alpha+\beta X_i+\mu_i \quad (i=1,2,\ldots,n)$$

(b) Suppose you have estimated a linear regression equation of Y on $X_2, \ldots, X_k$ on the basis of n sample observations. Describe how you would test the hypothesis that a new observation

$$(Y_{n+1}, X_{2\,\overline{n+1}}, \ldots, X_{k\,\overline{n+1}})$$

has come from the same structure as that presumed to have generated the n sample observations. [15+10]

2. Describe, with the help of an example, the usefulness of Dummy variables in linear regression models for analysing economic data. Explain, in this context, how one can take into account 'interaction' between two qualitative explanatory variables by the use of Dummy variables. [18+7]

3. Consider the following linear regression model

$$Y_i = \beta x_i+u_i \quad , \quad (i=1,2,\ldots,n)$$

where x_i is nonstochastic and the u_i's are serially uncorrelated disturbances with zero mean and variance σ^2.

Suppose the observations are arranged in m groups; the i-th group contains $n_i (\Sigma n_i = n)$ observations of y and x and the group means are denoted by $\bar{y}_i$ and $\bar{x}_i$ $(i=1,2,\ldots,m)$. Suppose that only the data on the number of observations (n_i) and the group means $(\bar{y}_i, \bar{x}_i)$ are available.

(a) Find the best linear unbiased estimator (BLUE) of β and its variance.

Now suppose that the ungrouped data (y_i, x_i), $(i = 1,2,\ldots, n)$ are also available. Then

(b) Show that the BLUE of β that may be obtained from the ungrouped data is more efficient than the estimator in (a).

p.t.o.

3.(c) Examine if the R^2 computed from the regression on grouped data is necessarily lower than that obtained from the regression based on individual observations. [10+10+5]

4. Describe the Durbin-Watson (DW) test for detecting the presence of autocorrelation among the disturbances in a linear regression model. Discuss the applicability of the test when lagged value of the dependent variable is present among the explanatory variables. What would you do in such a situation? [10+10+5]

5. Describe the errors-in-variables model commonly adopted in econometric regression analysis. Show that the least squares estimator of the slope parameter in a two-variable linear regression model with error in both the variables will be an underestimate even if the sample size is large. Derive the maximum likelihood (ML) estimator of the regression coefficient in the two-variable errors-in-variables model stating all the assumptions clearly. [2+8+5]

6.(a) Explain clearly the "identification problem" in the context of linear simultaneous equations model.

(b) Consider the following three equation model

$$y_{1t} = \beta_{12}y_{2t} + \nu_{11}x_{1t} + \epsilon_{1t}$$

$$y_{2t} = \beta_{21}y_{1t} + \beta_{23}y_{3t} + \nu_{21}x_{1t} + \nu_{22}x_{2t} + \epsilon_{2t}$$

$$y_{3t} = \beta_{32}y_{2t} + \nu_{32}x_{2t} + \epsilon_{3t}$$

where the y's are the endogenous variables and the x's denote the predetermined variables; the β's and ν's represent the structural form parameters, ε's are the disturbances and 't' denote time.

Examine the identifiability of each of the equations. [14+1

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INDIAN STATISTICAL INSTITUTE
B.STAT. III YEAR: 1991-92
SEMESTRAL II EXAMINATION
STATISTICS : COMPREHENSIVE

Date: 6.5.92 Maximum Marks: 50+50=100 Time: 4 Hours

GROUP A

Answer any 5 questions. [5x10=50]

1.(a) n iid $N(\mu,1)$ observations are available. Compute the power of the MP test for $H_0: \mu=0$ against $H_1: \mu=1$.

(b) In a certain problem, one of the normal equations is

$$6\beta_1+3\beta_2+3\beta_3=35.$$

Is $2\beta_1+\beta_2+\beta_3$ estimable? Justify. If it is estimable, find its BLUE.

(c) X is Binomial (n,p) and Y is Binomial (m,p). Is X + Y Binomial (n+m,p) ? Justify.

(d) Use CLT to compute $P[X \leq \chi^2(25,0.95)]$, where X has a Chi-square distribution with 25 d.f. and $\chi^2(25,0.95)$ is the upper 5% point of a Chi-square distribution with 25 d.f.

(e) A box contains 75 red and 75 green objects. Objects are drawn randomly from the box one at a time without replacement. Let $X_i=1$ if the colour of the object at the ith draw is red and zero otherwise. Compute regression of X_{150} on X_{149}. [5x2=10]

2. Consider the usual linear model $Y=X\beta+\epsilon$, where $\epsilon \sim N_n(0,\sigma^2 I_n)$ and rank $(X_{n\times p}) = r < p$. Write an expression for the MLE of σ^2 and show that it is unique. [10]

3. Let X be $N(\mu,1)$. Consider a test with the critical region of the form $X < C_1$ or $X > C_2$ for testing $H_0: \mu=0$ against $H_1: \mu\neq 0$. Obtain condition(s) on C_1 and C_2 so that the test is unbiased. [10]

4. Draw one observation from a bivariate normal distribution with mean $\begin{pmatrix}-2\\5\end{pmatrix}$ and covariance matrix $\begin{pmatrix}2 & -1\\-1 & 4\end{pmatrix}$, using a random number table and a table of normal probabilities. [10]

p.t.o.

5. Compute the distribution of the sample variance in a sample from a Bernoulli distribution.

6. Each farmer uses his own variety and a newly introduced improved variety, in two independent trials. It is proposed to analyse the data using the one-way classified model

$$Y_{ij} = \mu_i + \epsilon_{ij},\ i=1,2,\ j=1,2,\ldots,J.$$

Only the effect of farmer's own variety μ_2 is treated as $N(\mu, \sigma^2)$, and is independent of ϵ_{ij}'s . How would you test $\mu_1 = \mu$?

7. The performance P of students organizing integration in Class Tests conducted immediately after integration was found to be negatively correlated with the number of hours H spent for integration activities during the month prior to integrat… It was also evidenced that the number of hours S spent on stud… during the same one month period was positively correlated wit… P, but S and H were negatively correlated. The correlations computed on the basis of 31 students were

$$r_{PH} = -0.619,\ r_{PS} = 0.410,\ r_{HS} = -0.239$$

Analyse the data suitably to bring-out the effect of " Integration" on performance.

contd. ...

GROUP B

Answer any one question. [1x50] = [50]

1. The conventional method of measuring blood pressure by Sphygmomanometer can not be used for persons suffering from artari "pulseless diseases" . There is an alternative method using an instrument called pulse oximeter.

In a study intended to compare the systolic blood pressure read by these two instruments, blood pressure of 20 volunteer men were measured by both the instruments. Results are as follows.

(a) Formulating the statistical problem clearly, analyse the data to answer the question whether oximeter reading can be used as a substitute for sphygmomanometer reading.

Sl. no.	Systolic Oximeter	Blood pressure by sphygmomanometer	Sl. No.	Systolic Oximeter	Blood pressure by sphygmomanometer
1	100	95	11	118	115
2	102	122	12	115	115
3	105	120	13	120	120
4	103	110	14	130	130
5	105	104	15	132	131
6	107	110	16	132	131
7	110	109	17	135	134
8	112	120	18	140	138
9	112	115	19	140	141
10	115	115	20	145	148

(b) Comment on the design of the study-its inadequaries, limitations, etc. - and suggest any possible improvements you would make on the design, if the study is to be made on a fairly small scale.

[30+20=50]

contd. ...4/

2. MIOH, Bonhooghly treats patients of a rare disease. Five among such patients were selected in such a way that they were from five large disjoint geographical areas around MIOH. Starting from the household of each patient, all persons in a cluster of about 200 households around the household of each patient were screened for the disease giving the following data

Patient	# of households in the cluster	# of persons in the cluster	# of newly identi positive cases
1	212	1187	29
2	193	992	29
3	166	825	31
4	207	1146	32
5	201	1083	36
Total	979	5233	157

With aims to estimate the incidence rate of the disease, its s.e., the pattern of incidence over geographical areas and other related issues, analyse the data after clearly formulating the relevant statistical problems.

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1991-92

SEMESTRAL-II EXAMINATION

Design of Experiments

Date: 4.5.1992 Maximum Marks: 100 Time: 3 hours

Note: Answer Question No.1, and any THREE out of the rest. Marks allotted to a question are indicated in brackets [ ] at the end. Submit your Practical Records, if not already done, to the Course Instructor, on or before 30.4.1992. These records carry 10 marks.

1. In an experiment on wheat, three dates of transplanting d_1, d_2 and d_3 on whole plots and two rates of seedling s_1 and s_2 on subplots were tested in four randomised blocks. Grain yields in kg. per plot alongwith the exact layout plan are given below.

Replication I	d_1		d_3		d_2	
	s_2	s_1	s_1	s_2	s_2	s_1
	6.5	7.2	8.2	8.5	8.7	8.2
Replication II	d_1		d_2		d_3	
	s_1	s_2	s_1	s_2	s_2	s_1
	7.7	8.2	7.5	8.5	7.0	6.0
Replication III	d_3		d_1		d_2	
	s_2	s_1	s_1	s_2	s_2	s_1
	7.0	6.2	8.7	8.5	8.0	8.0
Replication IV	d_2		d_3		d_1	
	s_1	s_2	s_1	s_2	s_2	s_1
	8.7	9.2	9.0	10.0	8.0	8.5

Analyse the data and draw conclusions. [21]

2. Starting with two appropriate linear models and their assumptions, show how the analysis of covariance can be carried out easily using the results of the corresponding analyses of variance. Illustrate your results with respect to a latin square design with one concomitant variable.

(15+8) = [23]

3.(a) Show that at most v-1 mutually orthogonal latin-squares (MOLS) of order V exist. Give a method of construction of a complete set of MOLS of order v with an illustration, when v is a prime number or a prime power.

(b) Describe Bonferroni's, Scheffe's, and Tuke's methods of multiple comparisons with necessary derivations.

((3+5)+15) = [23]

4.(a) Explain confoundring with an example. Give a balanced confounding scheme for a 2^5 factorial experiment in blocks of size 2^3. Construct the key blocks of all the replications of your design.

(b) Describe the analysis of variance of the above design indicating clearly how the various sums of scquares are to be computed.

((5+5+5)+8) = [23]

5.(a) An experiment to study the effects of irrigation (I) at thre levels was originally planned in four randomised blocks. Subsequently it was decided to include in the study two more factors, namely N and P, the nitrogeneous and phosphatic fertilizers, each at three levels. Consequently a split plot design was thought appropriate, with more importance attached to the factors N and P. Give a rough sketch of the layout plan for the suggested design.

(b) Starting with an appropriate model for this design, describe its analysis of variance, showing clearly the expressions f the various sums of squares involved. Also derive the variances of various treatment mean comparisons, and give their unbiased estimators.

(3 + (12+8)) = [2

:bcc:

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INDIAN STATISTICAL INSTITUTE
B.STAT. III YEAR: 1991-92
SEMESTRAL-II EXAMINATION
STATISTICAL INFERENCE II

Date: 30.4.92 | Maximum Marks: 100 | Time: $3\frac{1}{2}$ Hours

Note: Answer all questions.

1. Suppose $X_1, X_2, \ldots$ are i.i.d. random variables and H_0 and H_1 are two simple hypotheses concerning $(X_1, X_2, \ldots)$. Then the SPRT for testing H_0 against H_1 terminates with probability one under both H_0 and H_1. Prove this statement only under H_1. [9]

2. Let $X_1, X_2, \ldots$ be i.i.d. $N(\mu, \sigma^2)$ where both μ and σ^2 are unknown. Consider the setup of Stein's double sampling procedure.

   (a) Find a bounded length confidence interval for μ with a given confidence coefficient $(1-\alpha)$. [8]

   (b) Suppose we want to test $H_0: \mu = \mu_0$ vs $H_1: \mu = \mu_1$ $(\mu_1 > \mu_0)$. Show that the test procedure that rejects H_0 if

   $$\bar{X}_n > \frac{t_{\alpha, m-1}\mu_1 + t_{\beta, m-1}\,\mu_0}{t_{\alpha, m-1} + t_{\beta, m-1}}$$

   has probability of type I error $\leq \alpha$ and probability of type II error $\leq \beta$ if the stopping time n is properly chosen. Here m denotes the 1st sample size and $t_{\alpha, r}$ is the upper α-point of t_r. [8]

3. Suppose $X_1, X_2, \ldots$ are i.i.d. $B(1, \theta)$, $0 < \theta < 1$.

   (a) Suggest a sequential procedure to obtain an unbiased estimator of $\frac{1}{\theta}$ which attains the corresponding Cramer-Rao lower bound.

   (b) Show that with a fixed sample size, $\frac{1}{\theta}$ is not unbiasedly estimable. [7+3=10]

4. Consider the one-sample location problem. Assume that the common distribution of the observations $X_1, X_2, \ldots, X_n$ is symmetric about the unknown median θ.

   (a) What is the Wilcoxon signed rank statistic? Express this as a sum of indicator functions involving the averages $X_i + X_j/2$, $i \leq j$ and hence construct a confidence interval for θ based on this statistic. [14]

CR

p.t.o.

(b) Assuming the above (as in (a)) representation of Wilcoxon signed rank statistic find its asymptotic distribution under the hypothesis that $\theta=0$. You may use the results on U-statistics (to be stated clearly if used.) [·

5. Show that the Mann-Whitney U-test is unbiased for general one-sided alternatives. Assume that the distribution functions are strictly increasing. [1(

6. Explain the concept of Pitman's asymptotic relative efficiency. [7

7. In each of the following problems state the model, the null hypothesis and the alternative and suggest a suitable non-parametric test.

(a) Consider the following data where the y's are exam. scores for students who pre-enrolled in a course and the x's are similar scores who did not pre-enroll. Can you conclude that the pre-enrolled students did significantly better than the others?

x	73	68	82	62	75	66
y	86	81	91	76	84	73

(b) The following table gives the estimated value of θ, the ratio of the mass of the earth to that of the moon, obtained from seven different spacecraft.

Spacecraft	θ
Mariner 2	81.3001
Mariner 4	81.3015
Mariner 5	81.3006
Mariner 6	81.3011
Mariner 7	81.2997
Pioneer 6	81.3005
Pioneer 7	81.3021.

In this compatible with the previous Ranger space craft finding on the basis of which scientists had considered the value of θ to be approximately 81.3035?

contd. 3/-

7.(c) The following are the weights in pounds, before and after, of 8 persons who stayed on a certain reducing diet for four weeks:

Before	147.0	183.5	232.1	161.6	197.5	206.3	177.0	215.4
After	137.9	176.2	219.0	163.8	193.5	201.4	180.6	203.2

Test whether the weight reducing diet is effective. [15]

8. Carry out analysis for any one of the problems of Question 7. [7]
(Use tables of exact distributions and 5% level).

9. Assignments. [12]

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INDIAN STATISTICAL INSTITUTE
B.STAT. III YEAR: 1991-92
SEMESTRAL II EXAMINATION
INTRODUCTION TO STOCHASTIC PROCESSES

27.4.92 Maximum Marks: 100 Time: $3\frac{1}{2}$ Hours

Note: The paper carries 120 marks. The maximum one can score is 100. Marks are indicated within brackets.

1. Let a fair die having six faces be tossed repeatedly with X_i defined to be the number appearing up on the ith toss.

   Define $S_n = \sum_{i=1}^{n} X_i$. For any given k, let

   $$n_k = \min\{n > 0 : S_n \geq k\}. \text{ Define } D_k = S_{n_k} - k,\ k \geq 0.$$

   Consider the stochastic process $\{D_k, k \geq 0\}$.

   (a) Determine the state space of the process.

   (b) Show that the process is a homogeneous Markov Chain and calculate the transition probabilities. [8+(6+6)=20]

2. Let $\{X_n, n \geq 0\}$ be a homogeneous Markov Chain having countable state space I and one step transition matrix $P = ((p_{ij}))$. Call a probability distribution π on I a <u>stationary distribution</u> for the Markov Chain if

   $$\pi(j) = \sum_{i \in I} \pi(i)\, p_{ij} \text{ for each } j \in I.$$

   Prove that if the initial distribution of the chain happens to be stationary, then the distribution of X_n is independent n.

   Let π be a stationary distribution of a Markov Chain. Show that if $\pi(i) > 0$ and i leads to j then $\pi(j) > 0$.

   The transition matrix of a Markov Chain is called <u>doubly</u> stochastic if $\sum_{i \in I} p_{ij} = 1$ for all $j \in I$.

   What is the stationary distribution of an irreducible, finite Markov Chain having d states and a doubly stochastic transition matrix? [6+8+10=24]

p.t.o.

3. Consider a homogeneous Markov Chain on $I = \{0,1,2,\ldots,d\}$ satisfying $\sum_{j=0}^{d} j p_{ij} = i$, $i = 0,1,\ldots,d$.

Show that (i) $E\{X_{n+1} \mid X_0 = i_0, X_1 = i_1, \ldots, X_n = i\} = i$.

(ii) 0 and d are necessarily absorbing states.

(iii) if all the states other than 0 and d are transient, each stationary distribution of the chain is of the form

$\pi_a = (1-a)\pi_0 + a\pi_1$, where $0 \leq a \leq 1$ and

$$\pi_0(i) = \begin{cases} 1 & \text{if } i = 0 \\ 0 & \text{if } i \neq 0 \end{cases}, \qquad \pi_1(i) = \begin{cases} 1 & \text{if } i = d \\ 0 & \text{if } i \neq d \end{cases}$$

(iv) for any $i \in I$, $p_{i0}^{(n)} = P\{\tau_0 \leq n \mid X_0 = i\}$ for all $n \geq 1$

where $\tau_0 = \inf\{n \geq 1 : X_n = 0\}$

$= \infty$ if no such n exists. [5+5+8+4=22]

4.(a) Consider a Markov Chain on the non-negative integers such that, starting from i the chain goes either to state i+1 with probability p, $0 < p < 1$, or to state 0 with probabil[ity] 1-p.

(i) Show that the chain is irreducible.

(ii) Does the following system of equations

$$\sum_{j=1}^{\infty} p_{ij} x_j = x_i, \qquad i = 1,2,3,\ldots$$
$$0 \leq x_i \leq 1$$

admit a non-zero solution?

(iii) Does the answer to (ii) above help you to decide whether the chain is recurrent or transient? State clearly the result you are likely to use.

(b) Let $\{X_n, n \geq 0\}$ be a homogeneous Markov Chain. Show that

(i) if j is a transient state, $\sum_{n=0}^{\infty} p_{ij}^{(n)} \leq \sum_{n=0}^{\infty} p_{jj}^{(n)}$ for every state i.

(ii) $P\{X_0 = i_0 \mid X_1 = i_1, X_2 = i_2, \ldots, X_n = i_n\} = P\{X_0 = i_0 \mid X_1 = i_1\}$

whenever left hand side is defined. [(2+8+4)+(6+4)=

contd. 3/-

5. Let $\{X(t), t \geq 0\}$ be a continuous time parameter Markov Process with the state space I, a countable subset of integer. and having stationary transition probabilities $p_{ij}(t)$.

(a) Assuming that $p_{ij}(t)$ is continuous at zero, prove that $p_{ij}(t)$ is uniformly continuous on $[0, \infty)$.

(b) Define the infinitesimal generators of the process. Define a stable state and an instantaneous state. When do you say that the Markov process is conservative?

(c) When is the Markov Process $\{X(t) : t \geq 0\}$ called a Poisson Process with parameter $\lambda (> 0)$? Determine the infinitesimal generators of the Poisson process. Is this process conservative? [6+(2+2+2+4)+6+6+2=30]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year: 1991-92
SEMESTRAL-I BACKPAPER EXAMINATION

Sample Surveys

Date: 2.1.1992 Maximum Marks: 100 Time: 3 hours

Note: Attempt ALL questions. The paper carries 110 marks but the maximum you can score is 100. The marks allotted are given at the end of each question.

1. Prove or disprove the following statements:

   (i) For SRSWOR designs, the sample mean is BLUE in the T_1 - class of linear unbiased estimators for population mean.

   (ii) Yates - Grundy unbiased estimator for the variance of the H - T estimator of a finite population total is always non-negative.

   (iii) For estimating the population mean $\bar{Y}$ of a variable y in a bivariate population in which $\bar{X}$ is known and

   $$3C_y \le 2C_x \quad \text{and} \quad \rho_{yx} \le 0.7$$

   the ratio estimator is better than the sample mean $\bar{y}$.

   (8+8+4) = [20]

2. (i) Define combined and separate regression estimators, for population mean $\bar{Y}$, in stratified simple random sampling.

   (ii) Derive optimum allocation for the use of separate regression estimator and find the conditions under which it reduces to

   (a) proportional allocation (b) equal allocation.

   (15+10) = [25]

3. (i) Suggest three different two stage sampling strategies, ~~based on~~ for estimating the population mean per ssu.

   (ii) Derive expression for the variance of the corresponding estimator for any one of the proposed strategies.

   (10+15) = [25]

p.t.o.

4. (i) The quantity (in kg.) of fish sold in Bonhooghly market in a particular week was found to be

10.5 ; 15.1 ; 20.5 ; 12.5 ; 18.7 ; 22.4 ; 30.2

Draw a circular systematic sample of 3 days and estimate the total fish sold during the week on the basis of data for sampled days.

(ii) The following are the Durga Puga subscriptions made by 15 randomly selected households, out of 75 in a street in a locality, during 1991. The corresponding subscriptions made by them in 1990 are given in bracket ( ).

250 (200); 20 (15); 50 (30); 100 (75); 30 (20)
50 (40) ; 25 (20); 40 (25); 200 (150); 60 (50)
60 (50) ; 50 (30); 50 (40); 75 (60) ; 50 (30).

It is known that on the average subscription given by the street hhs during 1990 was Rs.50/- per hh. Estimate the total Puja subscriptions, made by the street hhs, in 1991 using

(a) estimator based on sample mean

(b) ratio estimator

(c) regression estimator.

Obtain the estimated relative efficiencies of the estimator in (b) and (c) over that in (a). You may use large sample approximations.

(5+20) = [25]

5. A sample of n units is selected, out of N units in the population, using PPXWR sampling. If the population satisfies

$$y_i = \alpha + \beta x_i \qquad i = 1, \ldots, N$$

find condition under which the strategy $T_P \equiv$ (PPXWR, usual unbiased estimator) is superior to the strategy $T_O \equiv$ (SRSWR, sample mean) for estimating the population mean $\bar{Y}$.

[15]

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1991-92	354(b)

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year: 1991-92
SEMESTRAL-I BACKPAPER EXAMINATION

Physics I

Date: 1.1.1992 Maximum Marks: 60 Time: 2 hours

Note: GIVEN: Rest mass of an electron: 9.11×10^{-31} kg.
Charge of an electron: 1.6×10^{-19} coulomb
Planck's constant h: 6.63×10^{-34} joule sec.
Speed of light in free space $c = 3 \times 10^{8}$ m/s
1 eV = 1.6×10^{-19} joule.
Symbols have their usual meanings.

1.(a) Write down the equation of constraint for a simple pendulum.

(b) Two masses m_1 and m_2 are tied to the ends of an inextensible string passing over a fixed pulley at the edge of a smooth table. Set up Lagrange's equations.

(3+7) = [10]

2.(a) Write down Hamilton's Canonical equations.

(b) The Hamiltonian of a system is given by,

$$H = \frac{1}{2}(p_1^2 q_1^4 + p_2^2 q_1^2 - 2\alpha q_1)$$

where α is a constant.

Write down the expression for the conserved quantity.

(c) State the principle of least action for a system where the Hamiltonian is conserved.

(3+3+4) = [10]

3.(a) Considering a generating function of the type $F_2 = \sum_i q_i P_i$, show that F_2 generates an identity transformation.

(b) Express the temporal evolution of any variable $\alpha(q, p, t)$ in terms of the Poisson brackets.

From this show that if the Hamiltonian H does not involve time explicitly, then H will be a conserved quantity.

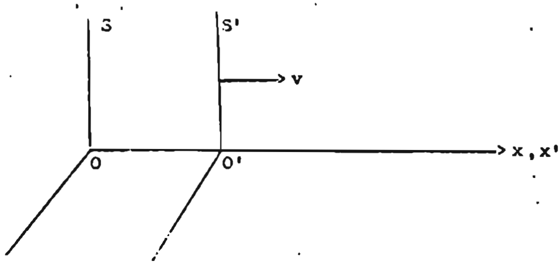
p.t.o.

(c) An infinitesimal canonical transformation leaves the Hamiltonian invariant. What can you say about the generating function of such a transformation ?

[4 + (2+2) + 2] = [10]

4.(a) State the postulates of Special Relativity Theory.

(b) The inertial frame S' moves with a velocity v with respect to the inertial frame S, as shown in the figure.



Write down the Lorentz transformation equations.

(c) Is it true that two events which occur at the same place and at the same time for one observer will be simultaneous for all observers ?

(4+4+2) = [10]

5.(a) If the proper life time of a μ-meson is 2.3×10^{-6} s, what average distance would it travel in vacuum before decaying, as measured in the reference frames in which its velocity is 0.60 c ?

(b) Compute the effective mass of a photon of wavelength 1 $\overset{\circ}{A}$.

(5+5) = [10]

6.(a) Show that the relativistic expression for kinetic energy K is given by $K = (m - m_o) c^2$.

(You may start with $K = \int_{u=o}^{u} \vec{F} \cdot \vec{dl}$)

(b) How much energy is released in the explosion of a fission bomb containing 3.0 kg of fissionable material. Assume that 0.1 % of the rest mass is converted to released energy.

(6+4) = [10]

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1991-92	314(b)

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year: 1991-92
SEMESTRAL-I BACKPAPER EXAMINATION

Linear Statistical Models

Date: 31.12.1991 Maximum Marks: 100 Time: 3 hours

Note: Answer <u>ALL</u> questions.

Consider the following model:

$$Y_{nx1} = X\beta + \varepsilon,$$

components of ε are i.i.d. with zero mean and unknown variance σ^2. X_{nxp} is a known matrix and β_{px1} is a vector of unknown constants, unless stated otherwise.

1. Discuss with practical examples how samples are drawn for simulation of the above model when

   (a) X is known and β is known but fixed.

   (b) Elements of X are either zero or one and β is known but fixed.

   (c) Elements of X are either zero or one and some components of β are random and other components are fixed but unknown.

   (3x3) = [9]

2. (a) Let the rank of X be full. State and prove Gauss-Markoff theorem.

   (b) Show that there does not exit LUE of atleast one component of β when X is not of full rank.

   (12+4) = [16]

3. State and prove a necessary and sufficient condition for the estimability of $l'\beta$.

   [10]

4. Let $l'\beta$ be estimable. Write an expression for the BLUE of $l'\beta$, show that it is unique and it is uncorrelated with every error function.

   [10]

p.t.o.

5. Consider the model under the following specific set-up :

$$Y_{ij} = \mu + \tau_i + \xi_j + \epsilon_{ij}, \quad i = 1,2,\ldots, I,$$
$$j = 1,2,\ldots, J, \text{ except when } i = 1 \text{ and } j = 1.$$

(a) Write the normal equations.

(b) Show that $\tau_i - \tau_{i'}$ is estimable.

(c) Find the BLUE of $\tau_2 - \tau_3$ and another of its estimator other than the BLUE by inspection.

(4+1+5) = [10]

6. Showing necessary computations, carry-out ANOVA for testing $H_0: \beta_1 = \beta_2 = 0$, when $n = 12$, $Y'Y = 48$, and the normal equation $X'X\beta = X'Y$ are

$$2\beta_1 = 4, \quad 3\beta_2 = 3, \quad \beta_3 = 5.$$

[10]

7. (a) Write the lay-out of ANACOVA for a two-way classified data with one covariate.

(b) Rewrite the lay-out when ANACOVA is used for handling one missing observation.

(10+5) = [15]

8. Show that the hypothesis of no three-factor effect for a 3-way r x s x t table under log-linear model set-up is equivalent to

$$\frac{m_{ijk}\, m_{rsk}}{m_{rjk}\, m_{isk}} = \frac{m_{ijt}\, m_{rst}}{m_{rjt}\, m_{ist}}$$

for all i, j, k, where the expected (i, j, k)th cell frequency is m_{ijk}.

[20]

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1991-92 | 374(b)

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1991-92
SEMESTRAL-I BACKPAPER EXAMINATION

Differential Equations

Date: 3?.12.1991 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL Questions.

1.(a). Let A be a constant n x n matrix, B and C constant n-dimensional vectors. Prove that the solution of the system

$$Y'(t) = AY(t) + C, \quad Y(a) = B,$$

on $(-\infty, \infty)$ is given by the formula

$$Y(x) = e^{(x-a)A}B + \left(\int_0^{x-a} e^{uA}\,du\right)C.$$ [10]

(b) If A is non-singular, prove that the integral in part (a) has the value $\left\{e^{(x-a)A} - I\right\}A^{-1}$. [5]

(c) Compute Y(x) explicitly when

$$A = \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad B = \begin{bmatrix} b \\ c \end{bmatrix}, \quad a = 0$$ [5]

2. Consider the initial-value problem

$$y' + y = 2e^x \text{ with } y = 1 \text{ when } x = 0.$$

(a) Find the exact solution Y of this problem. [8]

(b) Apply the method of successive approximation, starting with the initial guess $Y_0(x) = 1$. Determine $Y_n(x)$ explicitly and show that

$$\lim_{n \to \infty} Y_n(x) = Y(x)$$

for all real x. [12]

3. By changing the independent variable to $t = e^x$, transform the equation

$$(1-e^x)y'' + \frac{1}{2}y' + e^x y = 0$$

to the hypergeometric type and write down the general solutio near the singular point x = 0.

[20]

p.t.o.

4. (i) Use the convolution theorem for Laplace transforms to find the inverse Laplace transform of $s/\{(s-1)(s^2+1)\}$. [10]

(ii) Find the function $f(t)$ given that

$$f(t) = t^2 - \int_0^t e^u f(t-u)\,du.$$ [10]

5. Consider the differential equation $u'' + q(x)u = 0$.

(a) If $u(x)$ is a <u>non-trivial</u> solution, prove that $u(x)$ has at most one zero if $q(x) < 0$. [8]

(b) On the other hand, if $q(x) > 0$ for all $x > 0$, find a sufficient condition on $q(x)$ which ensures that $u(x)$ has infinitely many zeros on the positive x-axis. Prove your result. [12]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1991-92
SEMESTRAL-I BACKPAPER EXAMINATION

Statistical Inference I

Date: 27.12.1991 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL questions. This is an open note examination. You may consult your notes but books are NOT allowed. 15 points are reserved for numerical assignments.

1. Let $X_1, X_2, \ldots, X_n$ be i.i.d. observations with a common $N(\theta,1)$ distribution, where $\theta \in (-\infty, \infty)$ is an unknown parameter. Obtain the uniformly minimum variance unbiased estimate for $\exp(\theta^2)$. Provide an explicit expression for the estimate and an adequate justification for your answer.

[20]

2. Let $X_1, X_2, \ldots, X_n$ be i.i.d. observations with a common Poisson distribution with parameter $\theta \in (0, \infty)$. Is $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, which is the UMVUE of θ, an admissible estimate of θ w.r.t. the squared error loss (justify your answer) ? Is $\bar{X}$ minimax w.r.t. the squared error loss (justify your answer) ?

[20]

3. Let $(Y_1, X_1), (Y_2, X_2), \ldots, (Y_n, X_n)$ be i.i.d. observations with a common bivariate normal distribution such that $E(X_i) = \mu$, $E(Y_i) = \nu$, $Var(X_i) = \sigma^2$, $Var(Y_i) = \gamma^2$ and $Corr(X_i, Y_i) = \rho$ (all parameters unknown). Consider competing hypotheses $H_0 : \rho = 0$ and $H_A : \rho \neq 0$. Derive explicitly the likelihood ratio test statistic and the rejection region (with a preassigned level) for H_0 after a suitable simplification of the test statistic. What can you say about the distribution of the likehood ratio test statistic if H_0 is true ?

[25]

4. Let $X_1, X_2, \ldots, X_n$ be i.i.d. observations each with a uniform distribution on $(0,\theta)$. Based on these observations, obtain a uniformly most accurate confidence interval for θ with coverage probability 95%. What happens to this confidence interval as the sample size n grows to infinity (justify your answer) ?

[20]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II and III Year: 1991-92
SEMESTRAL - I EXAMINATION

Anthropology

Date: 27.11.1991 Maximum Marks: 50 Time: 3 hours

Note: Answer ALL questions.

1. Define adaptation. What is the difference between adaptation and acclimatization ? Is the elevation of haemoglobin content in the sojourners' blood an example of adaptation or acclimatization ?

2. (i) What do population pyramids having the following shapes suggest ?
   (a) triangular with broad base
   (b) cylindrical, and
   (c) roughly resembling an inverted triangle ?

   (ii) What are the salient characteristics of
   (a) a stable population, and
   (b) a stationary population ?

3. (i) The best definition of Anthropology is
   (a) the study of human social evolution
   (b) the study of human biological evolution
   (c) the study of primate evolution
   (d) the study of human variation, biological and cultural, in time and space.

   (ii) Human races are
   (a) subspacies
   (b) divisions of mankind
   (c) linguistic groups
   (d) religious groups.

contd..... 2/-

(iii) Java man is classified under

(a) Homo erectus

(b) Homo neandertal

(c) Homo sapiens

(d) Homo habilis

(iv) Homo sapiens is solely characterized by

(a) sagittal crest

(b) brachial locomotion

(c) erect posture

(d) canine tooth

(v) Fecundity is defined in terms of

(a) ability to conceive

(b) number of live born offspring

(c) number of surviving offspring

(d) sperm count.

4. The ability to taste PTC depends on a dominant gene, T, inability to taste the substance on its recessive allele, t. Persons with genotypes TT or Tt are called tasters, persons with the genotype tt, nontasters.

(a) If a Tt x Tt mating results in four children, what is the chance that all four will be tasters ? That all four will be nontasters ?

(b) If a Tt x tt mating results in four children, what is the chance that all four will be tasters ? That all four will be nontasters ?

Illustrate your answer with diagrams.

5. Write notes on

(a) Homozygote, heterozygote and hemizygote

(b) Dominant, recessive and codominant.

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year: 1991-92

SEMESTRAL - I EXAMINATION

Differential Equations

Date: 28.11.1991 Maximum Marks: 100 Time: 3 hours

Note: Answer questions 1 and 2, and any <u>TWO</u> of the remaining.

1. Solve the initial-value problem

$$Y'(t) = AY(t) + C(t), \; Y(0) = B$$

on the interval $(-\infty, \infty)$ where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 3 & -1 \\ 2 & 1 & 3 \end{bmatrix}, \quad C(t) = \begin{bmatrix} e^{2t} \\ 0 \\ te^{2t} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad Y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix}.$$

[20]

2.(a) Consider the differential equation $Y'(x) = F(x,Y)$ where F is a function defined on a set S in $\mathbb{R}^{n+1}$ given by

$$S = \left\{ (x,Y) : |x-a| \le h, \; \|Y-B\| \le k \right\}.$$

By placing suitable restrictions on F, prove that there exists an open interval $I = (a-c, a+c)$, where c is to be chosen appropriately, such that there is one and only one n-dimensional function Y defined on I with $Y(a) = B$, $(x, Y(x)) \in S$ and

$$Y'(x) = F(x, Y(x)) \text{ for each } x \in I.$$

[10]

(b) Let f be defined on the rectangle $R = [-1,1] \times [-1,1]$ as follows:

$$f(x,y) = \begin{cases} 0 & \text{if } x = 0 \\ 2\frac{y}{x} & \text{if } x \neq 0 \text{ and } |y| \le x^2 \\ 2x & \text{if } x \neq 0 \text{ and } y > x^2 \\ -2x & \text{if } x \neq 0 \text{ and } y < -x^2 \end{cases}$$

contd..... 2/-

(i) Prove that $|f(x,y)| \leq 2 \ \forall \ (x,y) \in R$.

(ii) Show that f does not satisfy a Lipschitz condition on R.

(iii) For each constant C satisfying $|C| \leq 1$, show that $y = Cx^2$ is a solution of the initial-value problem $y' = f(x,y)$ with $y = 0$ when $x = 0$. Show also that the graphs of these solutions over $(-1,1)$ lie in R.

(iv) Apply the (Picard) method of successive approximation to this initial-value problem, starting with the initial guess $Y_0(x) = 0$. Determine $Y_n(x)$ and show that the approximations converge to a solution of the problem on the interval $(-1,1)$.

(i) Repeat (iv), starting with initial guess $Y_0)x) = x$. Determine $Y_n(x)$ and show that the approximating functions converge to a solution different from any of those given in (iii).

[20]

3. Consider the differential equation

$$x(1-x)y'' + [p-(p+2)x]y' - py = 0,$$

where p is a constant.

(a) If p is not an integer, find the general solution near $x = 0$ in terms of hypergeometric functions.

(b) Write the general solution found in (a) in terms of elementary functions.

(c) When $p = 1$, the solution in (b) is no longer the general solution. Find the general solution in this case.

[25]

4. (i) Show that (in the usual notation for Laplace transforms and Bessel functions), $L[J_0(t)](s) = \dfrac{1}{\sqrt{1+s^2}}$ and deduce the Laplace transforms of the functions

$$J_1(t), \ tJ_1(t), \ t^{-1} J_1(t).$$

(ii) Show that

$$\int_0^t J_0(t-u)\, J_1(u) \frac{du}{u} = J_1(t).$$

contd..... 3/-

(iii) Evaluate

$$\int_0^t J_0(t-u)\,J_1(u)\,u \quad du .$$

[25]

5.(a) Defining the polynomials $P_n(x)$ as

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n ,$$

prove that $y = P_n(x)$ satisfies the differential equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0.$$

(b) By writing $(x^2-1)^n = (x-1)^n(x+1)^n$, or otherwise, prove that

$$P_n(\cos 2\theta) = \sum_{r=0}^{n} (-1)^r \binom{n}{r}^2 \cos^{2n-2r}\theta \ \sin^{2r}\theta ,$$

where $\binom{n}{2}$ denotes the binomial coefficient $n!/\,(n-r)!\ r!$.

Deduce that

$$P_n(\tfrac{1}{2}) = \frac{1}{2^n} \sum_{r=0}^{n} (-1)^r \binom{n}{r}^2 3^{n-r} .$$

[25]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year: 1991-92
SEMESTRAL - I EXAMINATION

Economics III

Date: 26.11.1991 Maximum Marks: 100 Time: 3 hours

Note: Answer ANY FOUR questions. All questions carry equal marks. Marks allotted to different parts (if any) of a question are shown in brackets [ ].

1.(a) What is meant by the 'head-count ratio' (HCR) measure of poverty ?

(b) An enamination of annual data for India over a number of years reveals that the HCR for rural India has varied inversely with the per capita production of foodgrains, but directly with the relative price of foodgrains (price of foodgrains relative to that of manufactures).

Could you develop a theoretical model which would lead to such a result ? If so, outline the basic structure of such a model and derive its implications as far as HCR in rural India is concerned.

[25]

2. Critically discuss some of the major explanations which have been suggested for the so-called industrial stagnation of the Indian economy after the mid-sixties.

[25]

3.(a) What is meant by the concept of 'equally distributed equivalent' income ? How is it computed ?

(b) Suppose you are asked to compare the absolute levels of living of two given states in India in a given year, where the absolute level of living in a state is measured by the average per capita total expenditure (PCE) in that state.

Would the ratio of nominal average PCE's for the two states serve the purpose ? If not, what adjustments are required to be made to this ratio ?

contd..... 2/-

(c) The table below gives some data for the rural areas of selected states in India for the year 1973-74. Compare the level of living of each state with that of West Bengal for this year:

Item / State :	West Bengal	Punjab	U.P.	Tamil-nadu	Maha-rashtra
1. Nominal average PCE (Rs.)	47.50	75.51	51.32	47.74	52.27
2. Index of consumer price differential⁺ (base ; West Bengal)	100	131.2	114.5	117.7	112.1
3. Lorenz ratio of nominal PCE's	0.296	0.270	0.236	0.269	0.263

⁺base : the price level in West Bengal = 100.

(9+8+8) = [25]

4. Discuss the salient features of Mohalanobis approach paper for the Second Plan of India. Obtain an assesment of the above in the light of the relevant literature.

(12+13) = [25]

5. Write short notes on any two of the following:

(a) Economic backwardness as a quasistable equilibrium.

(b) Block angularity in the economic structure of India.

(c) Inverse relation between marketed surplus and prices of foodgrains.

(d) New Agricultural Strategy (Indian Agriculture).

$(12\frac{1}{2}+12\frac{1}{2})$ = [25]

6.(a) Discuss briefly the Minhas-Vaidyanathan decomposition scheme to determine the components of growth of crop output in India for the period 1951-54 to 1958-61. Comment on the above in the light of some of the important empirical findings.

(10+5) = [15]

(b) Discuss any suitable framework to characterise the pattern of interyear changes in the balance of trade of India.

[10]

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SEMESTRAL - I EXAMINATION

Physics I

Date: 25.11.1991 Maximum Marks: 100 Time: 3 hours

Note: Use separate answerscript for each group.

GIVEN: Rest mass of an electron: 9.11×10^{-31} kg
Charge of an electron: 1.6×10^{-19} coulomb
Planck's constant $h = 6.63 \times 10^{-34}$ joule-sec
Speed of light in free space, $c = 3 \times 10^{8}$ m/S
$1 \text{ eV} = 1.6 \times 10^{-19}$ joule

Symbols have their usual meanings.

GROUP - A

[50 + 10 (Assignments)] = [60]

Note: Answer question no.1 and any FOUR of the rest.

1. Give brief answers to the following: (7 x 2) = [14]

(a) Could a mechanical experiment be performed in a given reference frame which would reveal information about the acceleration of that frame relative to an inertial one ?

(b) If photons have a speed c in one reference frame, can they be found at rest in any other inertial frame ?

(c) What is the speed (in terms of c) of an electron whose kinetic energy equals its rest energy ?

(d) When a source of light and an observer are moving away from one another, the observed frequency ν is given by

$$\nu = \nu' \sqrt{\frac{c - v}{c + v}} .$$

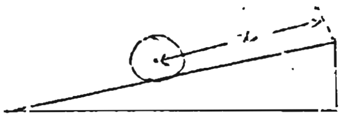
This is longitudinal Doppler effect in relativity.

contd..... 2/-

A rocketship is receding from earth at a speed of 0.2c. A light in the rocketship appears blue to passengers on the ship. What colour would it appear to an observer on the earth ?

(e) Suppose the motion of a pendulum is not restricted to a plane. Such a pendulum is called a spherical pendulum. Write the equation of constraint.

(f) Give a set of generalized coordinates needed to completely specify the motion of a circular cylinder rolling down an inclined plane. (Refer to the figure).



(g) A particle is moving on a very long frictionless wire which rotates with constant angular speed about a horizontal axis. Is the system i) scleronomic or rheonomic ii) holonomic or non-holonomic iii) conservative or non-conservative ?

2.(a) The point of suspension of a simple pendulum is attached to a moving lift which falls with an acceleration f. Find the Lagrangian function and the equation of motion.

(b) The Lagrangian of a system is

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2) - V(r)$$

Write down the expression for the conserved quantity.

(6+3) = [9]

3. A particle moves in the x-y plane under the influence of a central force depending only on its distance from the origin.

(a) Set up the Hamiltonian for the system.

(b) Write Hamilton's equations of motion.

(5+4) = [9]

4.(a) Considering a generating function of the first kind, given by,

$$F_1 = \sum_k q_k Q_k$$

contd..... 3/.

show that the transformation in effect interchanges the momenta and coordinates.

(b) The angular momentum vector is given by $\bar{L} = \bar{r} \times \bar{p}$.

Show that $[L_x , L_y] = L_z$.

[ ] denote the Poisson brackets.

(4+5) = [9]

5.(a) Show that $m_0 = \frac{\sqrt{E^2 - p^2 c^2}}{c^2}$

(You may start with $m = \frac{m_0}{\sqrt{1 - u^2/c^2}}$).

(b) The average life time of μ-mesons at rest is 2.3×10^{-6} s. A laboratory measurement on μ-mesons yields an average life time of 6.9×10^{-6} s.

(i) What is the speed of the mesons in the laboratory ?

(ii) The rest mass of a μ-meson is 207 m_e (m_e —> rest mass of an electron). What is the effective mass of such a meson when moving at this speed ?

(iii) What is its momentum ?

(4+5) = [9]

6.(a) The relativistic expression for acceleration $\bar{a}$ of a single particle under the influence of a force $\bar{F}$ is given by,

$$\bar{a} = \frac{\bar{F}}{m} - \frac{\bar{u}}{mc^2} (\bar{F} \cdot \bar{u})$$

which shows that in general, the acceleration is <u>not</u> parallel to the force.

Can you cite two cases, giving examples, where acceleration <u>is</u> parallel to the force ?

(b) Consider a radioactive nucleus moving with uniform velocity 0.05 c relative to the laboratory. The nucleus decays by emitting an electron with a speed 0.8 c along the direction of motion. Find the velocity (magnitude and direction) of the electron in the lab frame.

(5+4) = [9]

p.t.o.

GROUP - B Max. Marks: 40

Note: Answer question no.1 and any TWO of the rest.

1.(a) Establish the radioactive decay equation.

(b) Comment on the statement - "The law of radioactive decay is a statistical law."

(c) A large amount of radioactive material of half-life 20 days got spread in a room making the level of radiation 40 times the permissible level of normal occupancy. After how many days, the room would be safe for occupation ?

(4+4+4) = [12]

2.(a) Derive the relation TV^{r-1} = constant, for an adiabatic transformation of a perfect gas; the symbols having their usual significance.

(b) Immediately on explosion of an atom bomb, the ball of fire produced had a radius of 100 m and a temperature 10^5 degrees Kelvin. What will be the approximate temperature when the ball expands adiabatically to 1000 m radius ? $\gamma = 1.66$.

(8+6) = [14]

3.(a) Derive, from entropy consideration, the expression for the thermal efficiency of a Carnot engine.

(b) Show that no irreversible engine can be more efficient than a reversible one, working between the same two temperature limits.

(c) Equal masses of water, each of mass m, at absolute temperatures T_1 and T_2 respectively are mixed adiabatically and isobarically. Show that the entropy change of the universe is

$$2mc_p L_n \frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}$$

and it is positive.

(3+6+4+1) = [14]

p.t.o.

4.(a) Define the critical constants of a gas and correlate them with van der Waal's constants.

(b) With usual meanings of the symbols. Prove that

$$E_S / E_T = C_P / C_V .$$

(c) State the second law of thermodynamics in terms of entropy.

(3+5+4+2) = [14]

SEMESTRAL (I) EXAMINATION (1991–92)
B-Stat (Hons.) 3-rd Year
Statistical Inference (I)

Answer all questions. This is an open note examination. You may consult your notes but books are NOT *allowed. 15 points are reserved for numerical assignments.*

Date: 22.11.1991

Maximum marks : 100 Duration : 3 hours

(1) A coin with unknown probability $p \in (0,1)$ of turning head will be tossed until a head occurs, and let X be the total number of tosses that will be required. What will be the uniformly minimum variance unbiased estimate for $\sqrt{p}$ based on the observed value of X ? Provide an explicit expression for the estimate and an adequate justification for your answer.

[20 points]

(2) Consider a linear regression set up with $Y_i = \alpha + \beta X_i + e_i$, where $X_i = \dfrac{i}{n+1}$, $1 \le i \le n$ and the e_i's are independent with e_i having $N(0, X_i^2)$ distribution. Obtain the maximum likelihood estimates for the parameters $\alpha, \beta \in (-\infty, \infty)$ based on $(Y_1, X_1), (Y_2, X_2), \ldots, (Y_n, X_n)$. Are these estimates unbiased (justify your answer) ?

[15 points]

(3) Consider an observation X with probability density function $f(x)$, where $-\infty < x < \infty$. Two competing hypotheses concerning $f(x)$ are H_0 : $f(x) = \dfrac{1}{\sqrt{\pi}} \exp(-x^2)$ and $H_A : f(x) = \dfrac{1}{\pi(1+x^2)}$. Derive explicitly the most powerful test for H_0 against H_A with level 5% based on X. Compute the power of the most powerful test.

[25 points]

(4) Let $X_1, X_2, \ldots, X_n$ be i.i.d. observations with a common $N(\theta, 1)$ distribution. Assume that the parameter θ has a prior distribution that is normal with zero mean and variance $= \tau^2$ (known). Compute the shortest length Bayesian credible interval for θ with credibility (i.e. posterior probability) 95%. What happens to this Bayesian credible interval as the prior standard deviation τ tends to infinity ? Interpret the result.

[25 points]

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p.t.o.

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1991 - 92
SEMESTRAL - I EXAMINATION

Sample Surveys

Date: 20.11.1991 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 115 marks but the maximum you can score is 100 marks. The marks allotted are specified at the end of each question.

Attempt ALL questions.

1. (i) Define the Horvitz-Thompson estimator for a finite population mean and show that it is BLUE in the T_2-class of linear unbiased estimators.

(ii) Derive the expression for its variance due to Yates and Grundy for a fixed effective size sampling design.

(5+15) = [20]

2. Let d_1 and d_2 be unbiased estimators for parameters ϕ_1 and ϕ_2 respectively based on a sampling design. Give estimators for $R = \phi_1 / \phi_2$ and $P = \phi_1 \phi_2$ and derive approximate expressions for their MSEs ignoring third and higher order moments.

[15]

3. (i) Define ratio, product, difference and regression estimators for population mean, based on the following :

(a) SRSWOR

(b) VPSWR

(c) Stratified simple random sampling.

(ii) Find conditions under which combined and separate ratio estimators are better than the customary unbiased estimator, for population mean, in stratified simple random sampling.

(15+10) = [25]

p.t.o.

4. A finite population consists of N FSU's, the ith FSU consisting of M_i SSUs. Consider aparameter $\phi(y) = \frac{1}{N}\sum_{i=1}^{N} W_i \bar{Y}_i$ where $\bar{Y}_i$ is the mean of ith FSU and W_i's are known constants.

(i) Describe the problem of estimating the parameters Y, $\bar{Y}$, $\bar{\bar{Y}}$ and $\bar{Y}^*$ as that of estimating $\phi(y)$ (the notations having usual significance).

(ii) Give an unbiased estimator of $\phi(y)$ when FSUs are selected according to a sampling design with inclusion probabilities $\{\pi_i\}_{i=1}^{N}$ and SSU's according to SRSWOR.

(iii) Derive expression for the variance of the estimator in (ii).

(5+5+15) = [25]

5. A rural block was divided in to 12 clusters consisting of 10 villages each. A two-stage sample with SRSWOR at each stage was selected. The following table gives the member of house-holds (hhs) below 'poverty line' for sampled villages The figures in braces ( ) are total no. of house-holds of these villages:

Sampled clusters	Number of hhs below 'poverty line' for sampled villages		
1	40 (125);	15 (35);	45 (85)
2	60 (150);	55 (75);	45 (65)
3	50 (80) ;	45 (120);	40 (90)

(i) Give estimates for average number of hhs below poverty line (a) per village (b) per cluster.

(ii) Give estimate for percentage of hhs below poverty line in the block.

(iii) Obtain estimated relative standard errors of the estimators in (i).

(5+5+5) = [15]

6. Describe Hansen-Hurwitz unbiased sampling strategy for estimating a finite popuIation mean in the presence of non-response and derive expression for the variance of the estimator used. [15]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1991-92
SEMESTRAL - I EXAMINATION

Linear Statistical Models

Date: 18.11.1991 Maximum Marks: 100 Time: 3 hours

Note: Answer any FOUR questions from Group A and any ONE question from Group B. All questions carry equal marks.

GROUP A

1. Assume, in the usual set-up $Y = X\beta + \varepsilon$, that the components of ε are independently normally distributed with the same unknown variance σ^2, and the rank of $X: n \times p$ is p. Partition X and $\beta: p \times 1$ so that $X\beta = X_{(1)}\beta_{(1)} + X_{(2)}\beta_{(2)}$.

   Derive = LR test procedure for testing
   $H_0 : \beta_{(1)} = 0$, $\beta_{(2)}$ unspecified, against all alternatives.

2. In the usual set-up $Y = X\beta + \varepsilon$, let $\lambda'\hat{\beta}$ be an estimator of $\lambda'\beta$. Characterise the vector space V so that if λ belongs to V then $\lambda'\hat{\beta}$ is the BLUE of $\lambda'\beta$, for any solution $\hat{\beta}$ of the normal equation.

3. Clearly stating the ANACOVA technique for handling data with missing values, carry-out computations to test the equality of effects due to row classification.

		Col. no. 1	2	3	4
row no.	1	56	39	60	missing
	2	50	36	50	41
	3	62	42	58	47

4. Derive the MLE of β in the relationship $y = \alpha + \beta x$, on the basis of n pairs of independent observations (X_i, Y_i), $i = 1, 2, \ldots, n$, such that

contd..... 2/-

$$X_i = x_i + \delta_i, \quad \delta_i \sim N(0, \sigma^2)$$
$$Y_i = y_i + \varepsilon_i, \quad \varepsilon_i \sim N(0, \lambda\sigma^2).$$

Assume independence of ε_i and δ_i, and λ to be known.

5. Derive the MLE of γ for the standard two-way classified data with one covariate ω,

$$Y_{ij} = \mu + \tau_i + \beta_j + \gamma(\omega_{ij} - \omega_{oo}/IJ) + \varepsilon_{ij},$$

$i = 1, 2, \ldots, I, \; j = 1, 2, \ldots, I.$

6. Based on n pairs of observations (x_i, Y_i), so that $x_i = a + ih$, compare usual polynomial regression and regression using orthogonal polynomials. You may restrict your comparisons with an interest to test $\beta = 0$ in the set up $Y_i = \alpha + \beta x_i + \varepsilon_i$ with usual assumptions.

7. Reparametrize, giving exact relationships between parameters of the two models, the following model to a full-rank model

$$Y_{ij} = \mu + \tau_i + \varepsilon_{ij}, \quad i = 1, 2; \quad j = 1, 2, 3,$$

as an illustration of the technique of reparametrization.

GROUP B

1. Consider a 4-way I X J X K X L table and log-linear hierarchical models. In each of the following cases, write the minimal sufficient configurations, check if direct ML estimation of cell frequencies is possible, and give ML estimate of all frequencies when direct estimation is possible:

   (a) All 3-factor effects and any three 2-factor effects are absent.

   (b) All 3-factor effects and any four 2-factor effects are absent.

2. Show that, for a 3-way I X J X K table, a variable is collapsible with respect to the interaction between the other two variables if and only if it is at least conditionally independent of one of the other two given the third.

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