INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1992-93
Linear Statistical Models
Semestral-I Examination

Date : 16.11.1992 Maximum Marks : 100 Time : 4 Hours

Attempt questins 7 and 8 and any four of the rest.

1. Consider the model $(Y, X\beta, \sigma^2 I)$ where β is subject to linear restrictions $A\beta = 0$. $(A \neq 0)$. Show that $\ell'Y$ is a linear unbiased estimator of $p'\beta$ if and only if $X'\ell - p \in \mathcal{C}(A')$. Hence or otherwise show that for $p'\beta$ to be estimable it is sufficient but not necessary that $p \in \mathcal{C}(X')$. [15]

2. Consider two models with the same (known) design matrix but different dispersion matrices viz. $(Y, X\beta, \sigma^2 I)$ and $(Y, X\beta, \sigma^2\Sigma)$. Assume further that Σ is a known positive definite (p.d.) matrix. Obtain a necessary and sufficient condition for every estimable linear parametric function to have the same BLUE under both the above models. Let $\Sigma_c = I + lc' + cl'$ where l is a vector of 1's and c is a vector such that Σ_c is p.d. (Such a structure is called Baldessari structure). Let the first column of X be l. Show that every estimable linear parametric function has the same BLUE under $(Y, X\beta, \sigma^2 I)$ and $(Y, X\beta, \sigma^2\Sigma_c)$ if and only if $c \in \mathcal{C}(X)$. [15]

3. Consider $(Y, X\beta, \sigma^2 I)$ where $X = \begin{pmatrix} X_1 \\ x_n' \end{pmatrix}$ and $Y = \begin{pmatrix} Y_1 \\ y_n \end{pmatrix}$ with $x_n \notin \mathcal{C}(X_1')$. Show that BLUE of $X_1\beta$ and the conventional estimator of σ^2 remain the same under both the models. Hence or otherwise examine the following statement. 'For a model $(Y, X\beta, \sigma^2 I)$ the analysis with deletion of an observation can be achieved by including one more parameter into the model in a suitable manner and analysing the resultant model'. (Here, by analysis we mean obtaining BLUE's of estimable parametric functions and estimating σ^2). [15]

4. Examine whether the following is true in principle. Comment whether you agree with it fully considering all aspects.

" It is good enough if one knows how to analyse the model $(Y, X\beta, \sigma^2 I)$ where X is of full column rank as even the most general linear model $(Y, X\beta, \sigma^2\Sigma)$ with linear restrictions

p.t.o.

$A\beta = \xi$ where X and Σ could be rank deficient can be reduced to a model $(U, W\Theta, \sigma^2 I)$ by a suitable transformation when W is of full column rank".

[15]

5. Consider a 3-way cross-classified data with equal number s(≥ 2) of observations per cell. Let there be p A-classes, q B-classes and r C-classes.
   (a) Write down the model with interactions.
   (b) Write down the analysis of variance table for the model in (a).
   (c) Explain how you interpret various tests of main effects contrasts depending upon the results of tests for interactions.

[15]

6. The problem is to determine the true weights $\theta_1, \theta_2, .., \theta_p$ of p objects in an unbiased Chemical Balance using N weighing operations.
   (a) Formulate the problem in the set-up of a Linear Model Model and obtain BLUE of θ_i's, under appropriate assumptions (to be clearly stated by you).
   (b) Show that $V(\hat{\theta}_i) \geq \sigma^2/N$ where σ^2 = constant error variance. Examine the cases of "=" for one i and also for all i=1,2,..,p.
   (c) Examine the role of Hadamard matrices in this context.
   (d) Define the best design as one which provides "=" in (b) above for all i=1,2,..,p.
   Examine when the best design exists. Provide the best designs for N=8,p=5 and N=12,p=5.
   (e) Examine how far the above results are valid when the balance is biased.

[(2+1)+(1+1)+2+(1+2+2)+3 = 15]

7. In order to relate the annual cost of operating live stock auction markets to the quantity and mixture of live stock sold through the respective markets, the following cost and volume data were assembled for 19 markets for a calendar year. Let Y, X_1, X_2, X_3, X_4 denote the annual cost of operating an auction market, the number of cattle sold, the number of calves sold, the number of hogs sold and the number of sheep sold respectively.

Contd.....

Market	No. sold($X10^3$) Cattle	Calves	Hogs	Sheep	Cost ($X £10^3$)
1.	3.437	5.791	3.268	10.649	27.698
2.	12.801	4.558	5.751	14.375	57.634
3.	6.186	6.223	15.175	2.811	47.172
4.	11.685	3.212	0.639	0.694	49.295
5.	5.733	3.220	0.534	2.052	24.115
6.	3.021	4.348	0.839	2.356	33.612
7.	1.609	0.634	0.318	2.209	9.512
8.	2.339	1.895	0.610	0.605	14.755
9.	1.025	0.834	0.734	2.825	10.570
10.	2.936	1.419	0.331	0.231	15.394
11.	5.049	4.195	1.539	1.957	27.843
12.	1.693	3.602	0.837	1.582	17.717
13.	1.187	2.679	0.459	18.837	20.253
14.	9.730	3.951	3.780	0.524	37.465
15.	14.325	4.300	10.781	36.863	101.334
16.	7.737	9.043	1.394	1.524	47.427
17.	7.538	4.538	2.565	5.109	35.944
18.	10.211	4.994	3.081	3.681	46.945
19.	8.697	3.005	1.378	3.338	46.890

The following computations are available for the above data.

Corrected sum of squares and products matrix (S)$X10^6$ where S is given below.

	X_1	X_2	X_3	X_4	Y
X_1	315.995	61.922	136.656	267.706	1442.187
X_2		71.324	59.283	31.460	387.724
X_3			276.590	291.198	994.148
X_4				1462.073	2322.290
Y					8467.80

Contd.....

Let S_1 denote the corrected sum of squares and products matrix of X_1, X_2, X_3 and X_4. Some elements of $10^{-6} \times S_1^{-1}$ are available as given below :

	X_1	X_2	X_3	X_4
X_1	0.004698	-0.002923	-0.001082	-0.000582
X_2		*	*	-0.000825
X_3			*	-0.000890
X_4				0.000950

(a) Fit a multiple linear regression $\beta_0+\beta_1X_1+\beta_2X_2+\beta_3X_3+\beta_4X_4$ of Y on X_1, X_2, X_3 and X_4.

(b) Test whether X_1, X_2, X_3 and X_4 are useful for predicting Y.

(c) Is the number of hogs sold useful for predicting the annual cost of operating the market in the presence of X_1, X_2 and X_4 ?

(d) Is it fair to interpret the intercept estimate $(\hat{\beta}_0)$ as an estimate of the fixed cost of operating the market ? Give reasons for your answer.

(e) Explain the steps involved in computing the modified estimates of $\beta_0, \beta_1, \beta_2, \beta_3$ and β_4 when observation corresponding to the 15th market is dropped making full use of the computations already available to you. You need not do the actual numerical computations. [5x5 = 25]

8. An industrial engineer conducted an experiment on the eye-focus time (insec). He was interested in the effect of distance of the object from the eye on the focus time. Four different distances were of interest. He had five objects available for the experiment. To study also the differences among individuals, he conducted the experiment in a randomised block design. But the observation corresponding to 3rd individual for the distance of 8 ft. is missing. The remaining data are given below.

Distance (ft)	Subject 1	2	3	4	5
4	10	6	6	6	6
6	7	6	6	1	6
8	5	3	*	2	5
10	6	4	4	2	3

Analyse the data (stating your assumption) and comment on your findings. [15]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1992-93
Sample Surveys
Semestral-I Examination

Date : 18.11.1992 Maximum Marks : 100 Time : $3\frac{1}{2}$ Hours

The paper carries 140 marks. The maximum you may attempt is 120 and the maximum you may score is 100. The marks allotted to each question are given within parentheses. The symbols used are as-usual.

1. From an SRS of n elements a random sub-sample of n_1 elements is duplicated and added to the original sample. Show that the mean based on $(n+n_1)$ elements is an unbiased estimator of the corresponding population mean and its variance is greater than the variance of the mean based on n elements by an approximate factor

$$F = n(n+3n_1)/(n+n_1)^2. \qquad (10+10) = [20]$$

2.(a) A sampler has two strata with relative sizes W_1, W_2. He believes that S_1, S_2 can be taken as equal but thinks that c_2 may be between $2c_1$ and $4c_1$. He would prefer to use proportional allocation but does not wish to incur a substantial increase in variance compared with optimum allocation. For a given cost $C = c_1n_1 + c_2n_2$, ignoring the fpc, show that

$$\frac{V_{prop}(\bar{y}_{st})}{V_{opt}(\bar{y}_{st})} = \frac{W_1c_1 + W_2c_2}{(W_1\sqrt{c_1} + W_2\sqrt{c_2})^2}.$$

If $W_1 = W_2$, compute the relative increases in variances from using proportional allocation when $c_2/c_1 = 2, 4$.

(b) With two strata, for stratified simple random sampling, a sampler would like to have $n_1 = n_2$ for administrative convenience, instead of using the values given by the Neyman optimum allocation. If $V(\bar{y}_{st})$, $V_{opt}(\bar{y}_{st})$ denote the variances for the cases (i) $n_1 = n_2$ and (ii) the Neyman optimum allocations, respectively, show that the fractional increase in variance is given by

$$\frac{V(\bar{y}_{st}) - V_{opt}(\bar{y}_{st})}{V_{opt}(\bar{y}_{st})} = \left(\frac{r-1}{r+1}\right)^2$$

where " $r = \frac{n_1}{n_2}$ as given by Neyman allocation" .

(10+10)=[20]

p.t.o.

3.(a) Describe Rao-Hartley-Cochran sampling strategy for estimation of a finite population total.

(b) Show that for a sample drawn according to above scheme

$$E\left[\sum_{i=1}^{n}\left(\frac{y_{it}}{p_{it}} - Y_i\right)^2\right] = f_1 \sum_{i=1}^{N}\frac{y_i^2}{p_i} + f_2 \sum_{i=1}^{N} y_i^2$$

where $f_1 = \frac{N-n}{n(N-1)}$, $f_2 = \frac{N(n-1)}{n(N-1)}$.

Assume the random groups to be formed are of equal sizes.

(5+15)=[20]

4. Suppose the units in a population are grouped on the basis of the equality of their sizes and that each group has at least n units. Thus a sample of n units is chosen with PPSWR from the whole population and repeated units are replaced by units selected by SRSWOR method from the respective groups. Suggest an unbiased estimator of the population total Y and derive its sampling variance. Also obtain an unbiased variance estimator. How does the sampling variance in this case compare with that in sampling with PPSWR ?

(4+7+7+2)=[20]

5. Obtain a necessary form of a non-negative quadratic unbiased estimator of the mean square error of a homogeneous linear estimator of a finite population total and use it to suggest an unbiased estimator of the variance of the ratio estimator based on Lahiri-Midzuno-Sen sampling scheme. Derive a condition under which this variance estimator is non-negative.

(10+8+2)=[20]

6. In order to unbiasedly estimate a finite population total suggest a suitable two-stage sampling scheme for which the procedure of variance estimation does not require unbiased variance estimators for estimators of first stage unit totals based on sampling scheme adopted at the second stage. Also suggest unbiased estimators of the between and within components of the variance of the estimator of the finite population total in this case.

(8+12)=[20]

Contd.....

7. To estimate the total number of words (Θ) in an English dictionary, letters out of 26 alphabets were selected in 10 draws with PPSWR, size being the number of pages devoted to an alphabet and for each selected alphabet, two pages were selected by SRSWOR method. The relevant sample data are given in the following Table.

Serial No.	Sample alphabet	No. of pages devoted	No. of words in sampled page 1	2
1	S	131	34	27
2	C	97	27	26
3	N	21	44	38
4	S	131	24	29
5	F	43	25	32
6	J	7	42	48
7	U	18	24	21
8	P	85	53	24
9	A	49	47	55
10	D	54	38	57

(Total number of pages in the dictionary is 980)

(a) Estimate unbiasedly Θ and obtain an estimate of its variance.

(b) Estimate also the efficiency of the above method of sampling compared to that of drawing 20 pages from the dictionary by SRSWR method.

(10+10) = [20]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1992-93
Physics I
Semestral-I Examination

Date : 20.11.1992 Maximum Marks : 100 Time : 3 Hours

Answer Group A and Group B in separate answerscript.

GROUP A Max.Marks : 60 (50+Assignments 10)

GIVEN : velocity of light $c = 3X10^{8}$ m/s.
charge of an electron $e = 1.6X10^{-19}$ coulomb
$= 4.8X10^{-10}$ stat-coul (esu)
rest mass of an electron $m_0 = 9.1X10^{-31}$ kg

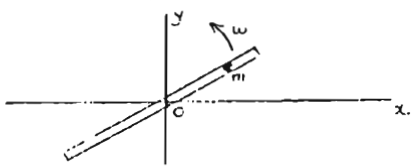
rest mass of a proton = 1836 m_0.
1 BeV = 10^9 eV = 10^9 X 1.6 X 10^{-19} J
1 $^{o}A = 10^{-10}$ m.
1 weber/m^2 = 1 tesla = 10^4 gauss
Plancks' constant $h = 6.63 X 10^{-34}$ J-S.
Symbols have their usual meanings.

Answer any FIVE questions : (10x5 = 50)

1.(a) Classify the following according as it is (i) scleronomic or rheonomic (ii) holonomic or non-holonomic (iii) Conservative or non conservative :

A cylinder rolling down a rough inclined plane of angle α without slipping. (2)

(b) A particle of mass m is constrained to move inside a thin hollow frictionless tube (see figure), which is rotating with constant angular velocity ω in a horizontal xy plane about a fixed vertical axis through O. Using Lagrange's equations, describe the motion.



(8)

p.t.o.

2.(a) Write down the Principle of Least Action in the special case when there are no external forces acting on the system. Give an example. (4)

(b) Prove that the transformation $P = \frac{1}{2}(p^2+q^2)$ and $Q = \tan^{-1}(q/p)$ is canonical. (6)

3.(a) A particle of mass m moves in a field whose potential is $V = -(K \cos\theta) / r^2$.

What is the Hamiltonian of the system ? (4)

(b) Consider the uniform motion of a free particle of mass m. There exists a constant of motion $F = x - \frac{pt}{m}$.

Show by direct computation that $\frac{\partial F}{\partial t}$ agrees with $[H, F]$. (6)

4.(a) In addition to time-like and space-like intervals , we might talk about light-like intervals. What value would τ or σ have for such intervals ? (3)

(b) (x,y,z,t) can be put as a four-vector. Are there any other physical quantities that emerge as a four-vector ? (2)

(c) A body of rest mass m_0 travelling initially at 0.6c makes a completely inelastic collision with an identical body initially at rest. What is the speed of the single resulting body ? What is its rest mass ? (5)

5.(a) " Just as observers in different places have different spatial perspectives of the universe, so observers with different velocities have different temporal perspectives [G.J.Whitrow, The Natural Philosophy of Time, 1963]. Comments invited. (2)

(b) An electron moves in the positive X-direction in frame S at a speed v = 0.8c (i) What are its momentum and energy in frame S ?

(ii) Frame S' moves to the right at a speed 0.6c wrt S. Find the momentum and energy of the electron in this frame. (5)

(c) Compute the effective mass for an X-ray photon with $\lambda = 1$ Å. (3)

Contd....

6.(a) Show that the electromagnetic wave equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 1/c^2 \frac{\partial^2\phi}{\partial t^2}$$

is not invariant under Galilean transformation. (2)

(b) A positive kaon K^+ has a rest mass 494 MeV/c^2. A proton has a rest mass of 938 MeV/c^2. If kaon has a total energy equal to the proton rest energy, the speed of the kaon is
(i) 0.25c (ii) 0.40c (iii) 0.55c (iv) 0.70c
(v) 0.85c (vi) none of these. (4)

(c) Give the wavelength shift, δ if any, in the Doppler effect for sodium D_2 line (5890 Å) emitted from a source moving in a circle with constant speed 0.1c measured by an observer fixed at the centre of the circle. (4)

GROUP B Max.Marks : 40

Answer Q.3 and any two of the rest.

1.(a) State the second law of thermodynamics in terms of entropy.

(b) Assuming that the change in entropy in a reversible cyclical process is zero, obtain the expression for the thermal efficiency of a Carnot engine.

(c) State (do not prove) the Carnot's theorem. What is its significance ?

(d) M gramme of water at T^oK is mixed adiabatically and isobarically with an equal quantity of water at T'^oK. Show that the net change in entropy of the universe is given by

$$2MC_p \ln \frac{(T+T')/2}{\sqrt{TT'}}$$

where C_p is the specific heat of water at constant pressure.

2+3+(3+2)+5 = [15]

Contd.....

2.(a) The general expression for the equation of state is $f(P,V,T) = 0$. Show that

$$\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1$$

(b) Use the Maxwell's relation(s) to prove that

$$C_P - C_V = T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P$$

(c) Show that for a Van der Waal gas

$$C_P - C_V = \frac{R\left(P+\frac{a}{V^2}\right)}{P - \frac{a}{V^2} + \frac{2ab}{V^3}},$$

the symbols having their usual significance.

5+5+5 = [15]

3.(a) Immediately on explosion of an atom bomb, the ball of fire produced had a radius of 100 m and a temperature of 10^5K. What will be the approximate temperature when the ball expands adiabatically to 1000 m radius ? The ratio of the two specific heats, $\gamma = 1.66$.

(b) A reversible engine converts one-sixth of the heat input into work. If the temperature of the sink is reduced by 62°C, its efficiency is doubled. Find the temperatures of the source and the sink.

5+5 = [10]

4.(a) Show that the work done by a perfect gas undergoing adiabatic change from pressure P_i and volume V_i to pressure P_f and volume V_f is

$$(P_iV_i - P_fV_f)/(\gamma-1)$$

where γ = ratio of two specific heats C_P and C_V.

(b) What is Boyle temperature ? Find its value in terms of a,b and R, the symbols having their usual meanings.

(c) Show that an absolute scale of temperature follows from the efficiency relation of a reversible heat engine.

5+(2+3)+5 = [15]

INDIAN STATISTICAL INSTITUTE
B. Stat.(Hons.) III Year:1992-93
Economics III
Semestral-I Examination

Date : 20.11.1992 Maximum Marks : 100 Time : 3 Hours

Answer Five questions taking Three from Group A and Two from Group B. Use separate booklets for answering each Group. Each question carries 20 marks.

Group A

1.(a) Do you think that the process of vertical disintegration taking place in the production structure of Indian engineering industries in the recent decades leads to external economies enjoyed by the oligopolistic firms in the above mentioned industries ? Elaborate your answer.

(b) Show that the production cost reduction due to economies of scale resulting from access to larger share of the domestic market accrued to oligopolists in Indian industries does not necessarily result in the expansion of the export market. Illustrate your answer through an assessment of Govt. of India policies in recent decades liberalising import in the intermediate and capital goods sector while practicing high protection to the final goods sectors.

(c) 'Indian large firms prematurely diversify into multi-products in search of monopoly profits on the import substitution cycle'. Elaborate.

2. Make an assessment of Mahalanobis Four sector model as an investment planning model for India.

or

Discuss the structure of the macromodel constituting an essential part of the Fifth Five Year Plan of India.

3.(a) In the Mathur-Hashim location model the same I-O table for India has been assumed applicable for the regions also. What justification for the above has been given by Mathur-Hashim ?

p.t.o.

3.(b) On the application of the Mathur-Hashim location model for India, the results suggest that the choice of some of the locations of Petroleum refineries at Western region of India conforms to efficiency. Do you think that the above result on intuitive considerations looks absurd ? If the above result is really absurd, do you think that it is due to some defect either in the formulation of the model or may be due to wrong data base only. Is it possible to obtain such results due to some simplifying assumptions in respect of the framework (empirical) of analysis ? Clarify your answer.

4.(a) Consider an economy having the level and some components of economic activity at a point of time as follows : GNP is Rs.200, private consumption is Rs.160, public consumption is Re.10, there is no foreign trade or household savings, and the government's consumption is matched by its net tax receipts from households. Now, answer the following:

(a) What is the household income ?

(b) What items (if any) do appear on the credit side of t capital account ?

(c) If the households' consumption at Rs.160, absorbed the whole of their incomes (so that they pai no taxes or saved), and if the government still covered all its consumption by tax receipts, where could these taxes come from, and what would be the composition of the debit side of the production account ?

(b) Let, I = Total investment, C_n = Private consumption
S_n = Household Savings,
Y_n = Households' income (wages and profit)
C_g = Govt. consumption, T_n = Household taxes,
T_f = Firms' taxes, S_g = Govt. surplus (or deficit it negative)
Firms make no savings or tax payments, $C_g = 25$,
After spending on private consumption, households distribute their income as follows : $S_n = 5$, $T_n = 35$
What is I ?

Contd.....

GROUP 3

5. Develop a model of rural - urban migration where the urban sector consists of one formal and one informal subsectors. In the formal sub-sector, the real wage rate is fixed; in the informal sub-sector it is determined by demand and supply. Capital stocks in the urban sectors and land in the rural sector are all given.

   Find out the effects of wage subsidy given to the formal and informal sectors in terms of the model

6. Analyse the effects of free international trade on the growth rate of a small developing economy. How would you modify your analysis for a large economy ?

7. In a rural credit market , show that the collateral price may, in general, be undervalued. How does this phenomenan compare with the lender's risk hypothesis ?

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year:1992-93
Statistical Inference I
Semestral-I Examination

Date : 23.11.1992 Maximum Marks : 100 Time : 3 Hours

The paper contains 120 points. The maximum you can score is 100.

1.(a) Suppose that $X_1,..,X_n$ are i.i.d $U(0,\theta)$, $\theta>0$. Let $\phi(X_1,..,X_n)$ be a test for $H_0: \theta = 1$ Vs. $H_1: \theta=2$. Find the optimal ϕ^* which minimizes (type I error + type II error) among all possible ϕ. Is it a Neyman-Pearson test for suitable size ? Justify.

(b) Let $X \sim P_{0i}(\theta)$, $Y \sim P_{0i}(\mu)$ be independent. We want to test $H_0: \theta = \mu$ against $H_1: \theta > \mu$. Consider all tests $\phi(X,Y)$ satisfying; $E_{\theta=\mu}[\phi(X,Y)|X+Y=t] \leq \alpha$ for all t. Does there exist A UMP test for H_0 in this class ? Find it out if your answer is yes and justify.

[20]

2.(a) Define the consistency of tests.

(b) Let $X_1,X_2,..,X_n$ be i.i.d P_θ. Suppose $P_{\theta_1} \neq P_{\theta_0}$. If for each n there exists a size α MP test of $H_0: \theta = \theta_0$ Vs. $H_1 : \theta = \theta_1$, show that the sequence of these tests is consistent.

(c) Let $X_1,..,X_n$ be i.i.d $N(\theta,\sigma^2)$. Find out $\lim_{n\to\infty} P_{\frac{\Delta}{\sqrt{n}}}\{T_n \geq t_{n-1}(\alpha)\}$, where T_n is the usual t-test for $H_0: \theta = 0$ Vs. $H_1 : \theta > 0$, and $\Delta > 0$ is a constant.

[5+10+10=25]

3. Let (X_i,Y_i), $1 \leq i \leq n$ be i.i.d $N_2(\mu_1,\mu_2, \sigma_1^2, \sigma_2^2, \rho)$. We want to obtain a confidence interval for $\Delta = \mu_2 - \mu_1$. Let C_{1n} denote the confidence interval based on T_{1n} (the usual t-statistic based on $Y_i - X_i$, $1 \leq i \leq n$) and $C_{2n} = (\bar{Y} - \bar{X} \pm t_{n-2}\ (1 - \frac{\alpha}{2})\ \frac{\sqrt{2}s_2}{\sqrt{n}})$

where, $s_2^2 = \frac{1}{n-1}[\sum_1^n (X_i-\bar{X})^2 + \sum_1^n (Y_i - \bar{Y})^2]$.

p.t.o.

(a) Show that in large samples

$$P[\Delta \in C_{2n}] > 1-\alpha \quad \text{if } \rho > 0.$$

(b) Let $|C_{in}|$ denote the respective lengths of the confidence intervals for $i = 1,2$. Show that

$$\sqrt{n}\,|C_{1n}| \to 2\left(\sqrt{\sigma_1^2+\sigma_2^2-2\rho\sigma_1\sigma_2}\right) z\left(1-\frac{\alpha}{2}\right)$$

and $\sqrt{n}\,|C_{2n}| \to 2\sqrt{\sigma_1^2+\sigma_2^2}\; z\left(1-\frac{1}{2}\alpha\right)$

in probability. Comment on the statistical significance of the above fact. Here $z_{1-\alpha/2}$ is the upper $100\alpha/2\%$ point of standard normal distribution.

[25]

4. Let $(N_1,..,N_k) \sim \text{Mult}(n, p_1,..,p_k)$ where $p_i = p_i(\theta)$, $1 \le i \le k$ are smooth functions. Let h be a smooth function such that $h(p_1(\theta),..,p_k(\theta)) = q(\theta)$.

(a) Find the Crammer-Rao lower bound for an unbiased estimate of $q(\theta)$.

(b) Show that the asymptotic variance of $h\left(\frac{N_1}{n}, .., \frac{N_k}{n}\right)$ obtained by Δ-method is always greater than or equal to the C-R lower bound.

(c) Obtain one Bhattacharya bound for unbiased estimate of $q(\theta)$ where $q(\theta)$ is a quadratic form of $(p_1,..,p_k)$.

[10+15+15=40]

5. Show that the family of Hypergeometric distributions, $H(N\theta, N, n)$, $\theta = \frac{j}{N}$, $0 \le j \le N$ is complete.

[10]

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year: 1992-93

Differential Equations

Semestral-I Examination

Date : 25.11.1992 Maximum Marks : 100 Time : $3\frac{1}{2}$ Hours

Answer as many as you can. The maximum you can score is 100.

1.(a) You know that Picard's method can be extended to a system of first order differential equations. Consider the following system of first order linear equations

$$\left.\begin{aligned}\frac{dy}{dx} &= p_1(x)y+q_1(x)z + r_1(x)\\ \frac{dz}{dx} &= p_2(x)y+q_2(x)z + r_2(x)\end{aligned}\right\} *$$

where p_i, q_i, r_i are continuous function on $[a,b]$. Apply Picard's theorem to show that solutions of * (on $[a,b]$) form a vector space of dimension 2.
(Note that a solution of * is a pair of functions).

(b) Consider the following

$$\left.\begin{aligned}\frac{dx}{dt} &= a_1x + b_1y\\ \frac{dy}{dt} &= a_2x + b_2y\end{aligned}\right\} \quad (**)$$

Suppose $x = Ae^{\lambda t}$; $y = Be^{\lambda t}$ is a non-trivial solution of (**). Show that $|C - \lambda I| = 0$, where $C = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$ and I is the 2x2 identity matrix.

Hence find the general solution of

$$\left\{\begin{aligned}\frac{dx}{dt} &= x+y\\ \frac{dy}{dt} &= 4x-2y.\end{aligned}\right.$$

[6+7+7]

2.(a) Consider the equation

$y' = p(x) + q(x)y + r(x)y^2$.

Show that if $y_1(x)$ is a particular solution, then the general solution has the form

$y(x) = y_1(x) + z(x)$,

where $z(x)$ is the general solution of the Bernoulli equation $z' - [q(x)+ 2r(x)y_1(x)]z = r(x)z^2$.

p.t.o.

Find the general solution of

$$y' = y/x + x^3y^2 - x^5.$$ [7+8]

(b) An integral curve $y=u(x)$ of the differential equation $y'' - 3y' - 4y = 0$ intersects an integral curve $y=v(x)$ of the differential equation $y'' + 4y' - 5y = 0$ at the origin. Determine the functions u, v if the two curves have equal slopes at the origin and if

$$\lim_{x\to\infty} \frac{[v(x)]^4}{u(x)} = 5/6$$ [12]

3. One solution of the equation

$$y'' + p(x)y' + q(x)y = 0$$

is $(1+x)^2$ and the Wronskian of any two solutions is constant. Find the general solution of

$$y'' + p(x)y' + q(x)y = 1+x.$$ [13]

4. Find the general solution of Chebyschev's equation

$(1-x^2)y'' - xy' + p^2y = 0, \quad p \geq 0,$

near $x = 1$ in terms of the hypergeometric functions. Show that the only solution where derivatives are bounded near $x=1$ are constant multiples of $F(p,-p,1/2, \frac{1-x}{2})$.

Conclude that the only polynomial solutions are constant multiples of $T_n(x) \stackrel{\text{def}}{=} F(n,-n,1/2, \frac{1-x}{2})$,

where $p = n \geq 0$ is an integer.

Show that $T_n(x)$ is the polynomial for which $T_n(\cos\theta) = \cos n\theta$.

[6+7+7]

5. Show that

$$J_p(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(x/2)^{2n+p}}{n!\,(p+n)!}$$

is a solution of $x^2y'' + xy' + (x^2-p^2)y = 0$,

where p is a non-negative integer. [10]

p.t.o.

6.(a) Consider the right circular cone $z^2=a^2(x^2 + y^2)$, $z\geq 0$. This cone can be represented parametrically by the equation $x = r\cos(\theta\sqrt{1+a^2})/\sqrt{1+a^2}$;

$y = r \sin(\theta\sqrt{1+a^2})/\sqrt{1+a^2}$; $z = ar/\sqrt{1+a^2}$. If the cone is cut along a generator and flattened into a plane, then one can show that any point on the cone represented as above will have polar coordinates (r,θ) (with suitable choice of axes). Prove that any geodesic on the cone becomes a straight line on the flattered cone.

(b) If the curve $y = g(z)$ is revolved about z - axis, the the resulting surface of revolution has $x^2+y^2= g(z)^2$ as its equation. A convenient parametric representation of this is given by $x = g(z) \cos\theta$, $y=g(z)\sin\theta$, $z = z$, where θ is the polar angle in the xy plane . Show that a geodesic $\theta = \theta(z)$ on this surface has

$$\theta = C_1 \int \frac{\sqrt{1+[g'(z)]^2}}{g(z)\sqrt{g(z)^2-C_1^2}} + C_2$$ as its equation.

What will be the equation if the surface of revolution is a right circular cylinder ?

[12+13]

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$y = r \sin(\theta\sqrt{1+a^2})/\sqrt{1+a^2}$; $z = ar/\sqrt{1+a^2}$. If the cone is cut along a generator and flattened into a plane, then one can show that any point on the cone represented as above will have polar coordinates (r,θ) (with suitable choice of axes). Prove that any geodesic on the cone becomes a straight line on the flattered cone.

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What will be the equation if the surface of revolution is a right circular cylinder ?

[12+13]

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INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year : 1992-93

SEMESTRAL II EXAMINATION

Intro. to Stochastic Processes

Date: 26.4.1993 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL questions. The maximum you can score is 100.

1. N black balls and N white balls are placed in 2 urns so that each urn contains N balls. At each stage one ball is selected at random with equal probabilities from each urn and the 2 selected balls are interchanged. The state of the system X_n, at the nth stage is the number of white balls in the first urn $n = 0, 1, 2, \ldots$

   (a) Find the transition matrix of this MC.

   (b) Find a stationary initial probability distribution.

   (c) Is the MC irreducible ? What is the period of the state 0 ? What are the limit points of the sequence $\left\{p_{01}^{(n)}, n \geq 1\right\}$.

   (5+5+10) = [20]

2. Let X_n $n \geq 0$ be a discrete parameter homogeneous MC with state space S. Let f_{ij}^* be the probability of ever visiting j starting from i and g_{ij} be the probability of visiting j infinitely many times starting from i. Show that $g_{ij} = f_{ij}^*$ or 0 according as j is recurrent or not. Define an essential state. Show that a recurrent state is essential.

   [8+4+8) = [20]

3. In a branching process the number of offsprings per individual has a binomial distribution with parameters 2, p, $(0 < p < 1)$. Starting with a single individual calculate

   (a) the extinction probability,

   (b) the probability that the population becomes extinct for the first time in the 3rd generation.

   (5+5) = [10]

Suppose that cars enter a one-way infinite highway according to a Poisson process with rate λ. The ith car to enter chooses a velocity V_i and travels at this velocity. Assume that V_i's are independent positive random variables having a common

contd..... 2/-

continuous distribution function F. Let $t > 0$ be fixed and let N denote the number of cars that are located in the interval (a,b) (i.e., at some distance d, $a < d < b$ from the starting point) on the highway at time t. Assume that no time is lost when one car overtakes another car. Show that N follows Poisson distribution with mean

$$\lambda \int_0^t (F(\frac{b}{t-s}) - F(\frac{a}{t-s}))\, ds . \qquad [20]$$

5. A telephone exchange has m channels. Calls arrive in the pattern of a Poisson process with rate λ ; they are accepted if there is an empty channel otherwise they are lost. The duration of each call is a random variable with exponential $\varepsilon(\mu)$ - distribution. The life-times of separate calls are independent random variables. Let X_t, $t \geq 0$ denote the number of busy channels at time t. Find the infinitisimal parameters of the X_t process. Find the stationary initial distribution.

(5+10) = [15]

6. Consider a birth and death process having 3 states 0, 1 and 2 and birth and death rates $\alpha_0 = 1$, $\alpha_1 = 2$, $\alpha_2 = 0$

$\beta_0 = 0$, $\beta_1 = 2$, $\beta_2 = 1$

(a) Write down the Forward equations.

(b) Find $P_{0j}(t)$, $j = 0, 1, 2$.

(5+20) = [25]

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B.Stat.(Hons.) III Year:1992-93

STATISTICS COMPREHENSIVE

SEMESTRAL EXAMINATION

Date: 28 April 1993 Maximum Marks:100 Time:$3\frac{1}{2}$ hours

NOTES, BOOKS, ETC., MAY BE USED, BUT NOT TRANSFERRED BETWEEN CANDIDATES.

Figures in brackets [ ] indicate marks allotted to the questions.

The paper carries 125 marks. You may attempt any part of any question. However, the maximum you can score is 100.

1. Given below are data on the results of an insulin experiment with four dosage levels (treatments) A, B, C, D carried out on four rabbits on each of four occasions (phases) according to the Latin Square design shown in Table 1. The percentage fall in blood sugar is given in Table 2. The •'s in Table 2 indicate missing observations. Examine whether all the dosage levels are equally effective.

Table 1: Design

Phase	Rabbit			
	1	2	3	4
1	C	A	B	D
2	B	D	C	A
3	A	C	D	B
4	D	B	A	C

Table 2: Percentage fall in blood sugar

Phase	Rabbit			
	1	2	3	4
1	32.7	11.2	23.2	48.1
2	26.2	•	28.9	18.7
3	-4.0	14.0	27.5	•
4	•	16.5	21.2	40.2

[35]

PLEASE TURN OVER

$c = \min(c_1, m)$.

contd..... 2/-

2. In a capture-recapture study of a *Microtus pennsylvanicus* (meadow voles) population, conducted in Laurel, Maryland, U.S.A., during a 5-day period in 1981, 125 voles were captured, marked and released in the first period; then in the second period again 125 voles were captured. The following observations were made:
n_{1m}: number of males caught and marked in the first period (days 1 & 2) = 58;
n_{2m}: number of males caught in the second period (days 3, 4 & 5) = 66;
m_{2m}: number of marked males caught in second period (days 3, 4 & 5) = 46;
n_{1f}: number of females caught and marked in the first period (days 1 & 2) = 67;
n_{2f}: number of females caught in the second period (days 3, 4 & 5) = 59;
m_{2f}: number of marked females caught in second period (days 3, 4 & 5) = 54;
The gender ratio (females : males) for the population is known to be 1.07. Stating your assumptions clearly, formulating reasonable models for the data and using suitably or not using the known gender ratio as you see fit, obtain estimates of the population size of the voles in this area. [35]

3. (a) Suppose you have observations on ten brother-sister pairs of their Intelligence Quotients (IQ). Suppose you want to test using these data if boys and girls are equally intelligent. For this, describe, with details

   i. a classical method. [5]

   ii. a modern computer-intensive assumption-free method using a suitable randomisation or a resampling technique. Explain how this may be an improvement over the classical method. [15]

   (b) Suppose instead of ten brother-sister pairs, you have 10 boys and 10 girls. Describe in what way your randomisation or resampling procedure will change and why. [10]

CONTINUED

4. Explain the phenomenon of 'regression to the mean' that led to the term *regression*. Give a theoretical support to this phenomenon. [15]

5. Describe (in about two pages) the contributions of *either* Fisher *or* Mahalanobis to the development of Statistics. [10]

$c = \min(t_1, m)$.

contd..... 2/-

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1992-93

SEMESTRAL II EXAMINATION

Statistical Inference II

Date: 3.5.1993 Maximum Marks: 50 Time: $3\frac{1}{2}$ hours

Note: The part I consists of 23 points. The maximum is 20. The part II has 35 points. The maximum is 30.
Total credit: 50.

I. Sequential

1. X_i's are i.i.d. Uniform $(\theta - \frac{1}{2}, \theta + \frac{1}{2})$. Consider $H_0: \theta = 0$ vs $H_1: \theta = \frac{1}{2}$.

(a) Suppose you want to test with $\alpha = \beta = 0.05$. If you use Wald approximations to boundaries, what would be the true error probabilities and expected sample size under H_0, H_1 of the corresponding SPRT ?

(b) Find the sample size of the MP test with α, β equal to the true error probabilities of the SPRT in 1.(a).

[(3+3) + 3] = [9]

2. X_i's are i.i.d., taking values + 1 and -1 with probabilities p and q respecties p and q respectively, $0 < p < 1$, $q = 1-p$. Fix $b < 0 < a$, and let $S_n = X_1 + \ldots + X_n$, and τ = 1st n such that $S_n \geq a$ or $S_n \leq b$.

Let $\phi(t) = E(e^{X_1 t})$. Show that

$$E\left[\frac{e^{tS_\tau}}{(\phi(t))^\tau}\right] = 1$$ and hence find $P(S_\tau = a)$.

(4+2) = [6]

3. Let X_i's be i.i.d. taking values 1 and 0 with probabilities p and q = 1-p respectively. Fix a positive integer m = 10, let

$$\tau_1 = 1 \text{ st } n \text{ s.t. } X_1 + \ldots + X_n = 1$$

and

$$\tau = \min(\tau_1, m).$$

Consider the sequential test of H_0: $p = 0.1$ vs H_1: $p = 0.2$ which observes $X_1, \ldots, X_\tau$, rejects H_0 if $X_\tau = 1$ and accepts if $X_\tau = 0$. Can this test be exhibited as an SPRT with boundaries $B < 1 < A$? [5]

4. (Bonus). Let X_i's be i.i.d. with exponential density $\frac{1}{\theta} e^{-x/\theta}$. Consider a SPRT for H_0: $\theta = \frac{1}{2}$ vs H_1: $\theta = 2$, with boundaries $B < 1 < A$. Let Z_i's be defined as in class, τ be the stopping time of the SPRT, and $S_\tau = Z_1 + \ldots + Z_\tau$. Can you calculate (as exactly as possible)

$$E_\theta (S_\tau - \log A \mid S_i \geq \log A, \quad \tau = m).$$

Where m is a fixed positive integer? [3]

II. Nonparametrics

5. Consider the two sample location shift (Δ) problem with m and n as the sample sizes for the first and the second sample respectively.

(a) Describe the rank tests as a subclass of permutation tests for this problem.

(b) Suppose the first population is $N(0,1)$ and the second population is $N(\Delta, 1)$. Show that the permutation distribution is MLR in a suitable statistic T. Hence obtain the UMP test for H_0: $\Delta = 0$ vs H_1: $\Delta > 0$.

(c) Assume now that the populations are symmetric about their medians and that the median of the first population is 0. How would you extend the permutation principle in this case?

(2+5+4) = [11]

6.(a) Define the finite sample (replacement) breakdown point of an estimator of location based on $X = (X_1, \ldots, X_n)$.

(b) What is the breakdown point of the L_1-median, i.e., θ obtained by minimizing $\sum_1^n \|X_i - \theta\|$ where $X_1, \ldots, X_n \in R^d$, $d \geq 1$.

(c) Let the functional T be defined on all distributions with support $(0, \infty)$ with a strictly positive density on $(0, \infty)$ as

$$T(F) = \frac{1}{\alpha} \int_0^{F^{-1}(\alpha)} x \, dF(x) \quad \text{for some } 0 < \alpha < 1.$$

Compute the influence function of T.

(2+4+6) = [12]

7. Let W^+ be the Wilcoxon signed rank statistic for testing $H_o: \theta = \theta_o$. Show that

$$W^+ \overset{d}{=} \sum_{1}^{n} V_j$$

where $P(V_j = 0) = P(V_j = j) = \frac{1}{2}$, $1 \leq j \leq n$, with $V_1, \ldots, V_n$ stochastically independent. [5]

8. It is claimed that the effect of advertizements is more pronounced among teenagers than older people. To test the hypothesis, 8 matched pairs of teenagers and 6 matched pairs of people above thirtly were randomly chosen. In each pair one randomly chosen person was asked to rate a product in a continuous scale (0 - 100) before seeing the advertizement and the other rated it after watching the advertizement. The data is as follows:

	Teenagers								Thirty-plus					
Before	5	60	40	20	50	60	50	40	45	25	55	70	50	50
After	50	65	40	45	50	70	45	45	50	15	55	60	50	55

Test the claim using a suitable statistic with $\alpha = 0.05$. [7]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year: 1992-93
SEMESTRAL II EXAMINATION

Design of Experiments

Date: 5.5.1993 Maximum Marks: 100 Time: 3 hours

Note: Answer any FOUR questions. Marks allotted to each question are given within parentheses.

1. Discuss the advantages and disadvantages of a split-plot design.

A split-plot design with p whole-plot treatments and q sub-plot treatments is arranged in a p x p Latin square. Give the complete analysis of the design on a sub-plot basis.

Also give the standard errors for various types of treatment comparisons.

(4+13+8) = [25]

2.(a) Arrange a 2^3 experiment in three 4 x 4 squares such that the second order interaction is confounded with the columns of each of the three squares, while each first order interaction is partially confounded with the rows of one square only.

(b) Obtain a balanced layout of a $(2^4, 2^2)$ design partially confounding two-factor and three-factor interactions.

(12+13) = [25]

3.(a) Below is given an incomplete Key-block of a 2^5 factorial experiment conducted in four 8-plot blocks:

Incomplete Key-block: (1) sn spk skmd ... ...

Search out the other 4 treatment combinations for the Key-block and also the confounded interactions.

Also give the treatment combinations of the other 3 blocks.

(b) Consider a 3^3 factorial experiment conducted in r replicates involving 3 factors A, B and C each at the levels 0, 1 and 2. Discuss how would you estimate the linear and quadratic components of the main effects and interactions. Also indicate how would you test for their significance. Derive expressions for the estimated standard errors of the linear and quadratic components of the main effects.

(10+15) = [25]

p.t.o.

4. Data relating to a pair of related characters x and y obtained under 3 different conditions are given below. Assuming linear regression in each case test if the regression slopes are same.

I		II		III	
y	x	y	x	y	x
1314	42.1	1246	41.1	1130	40.3
1590	41.0	1411	39.4	1033	40.4
1339	41.1	1232	40.2	1029	40.1
1551	40.1	1368	39.6	1035	40.3
1553	41.0	1316	41.2	1056	39.9

[25]

5. The following table gives the plan and yields in an agricultural experiment with 3 factors (A, B, C) each at 3 levels (0, 1, 2). The experiment was arranged in two complete replications each containing 9 3-plot blocks.

Block	Replicate I			Replicate II		
1	(000)	(111)	(222)	(000)	(112)	(221)
	47.8	62.7	54.2	39.4	53.6	47.6
2	(021)	(102)	(210)	(101)	(210)	(022)
	61.4	63.0	47.1	51.8	33.5	51.9
3	(120)	(201)	(012)	(011)	(202)	(120)
	47.2	46.5	70.9	46.6	59.9	32.4
4	(001)	(112)	(220)	(100)	(212)	(021)
	55.7	69.3	35.2	36.4	61.4	49.4
5	(022)	(100)	(211)	(201)	(010)	(122)
	76.0	47.8	48.2	36.6	33.3	51.4
6	(121)	(202)	(010)	(111)	(002)	(220)
	46.5	64.1	44.6	44.2	53.6	31.2
7	(002)	(110)	(221)	(200)	(012)	(121)
	70.5	37.1	56.2	33.2	56.0	44.4
8	(020)	(101)	(212)	(001)	(110)	(222)
	35.4	62.5	72.2	49.4	32.0	56.6
9	(122)	(200)	(011)	(211)	(102)	(020)
	65.4	38.0	59.7	45.3	63.1	25.7

contd..... 3

The 3 digits within brackets denote the level of the 3 factors A, B, C in order. The lower figures denote the yield.

(a) Determine the confounded effects in each replicate and analyze the data fully.

(b) The levels of the factor C are known to correspond to three quantities of manures, the increment in quantity from level 0 to 1 being same as that from level 1 to 2. If the main effect of C comes out to be significant test the significance of its linear and quadratic components and interpret the results.

(18+7) = [25]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year:1992-93
SEMESTRAL II EXAMINATION
Economics IV

Date: 7.5.93 Maximum Marks:100 Time: 3 Hours

Note: This question paper carries 125 marks. You can answer any part of any question. But the maximum that you can score is 100. Marks allotted to each question are given within parentheses.

1. Examine whether each of the following statements is true or false or uncertain. Give brief but pointed explanations in support of your answers.

   (a) The residuals from a regression of y on x are uncorrelated with x but are correlated with y.

   (b) Suppose that the coefficient associated with a variable in a regression equation is significantly different from zero at 20% level of significance. If we drop this variable from the regression, both R^2 and $\bar{R}^2$ would necessarily decrease.

   (c) Least squares technique when applied to time series data yield biased estimate of the regression coefficients because many economic time series are autocorrelated.

   (d) The Durbin-Watson test for autocorrelation is not applicable if the errors are heteroscedastic.

   (e) In a simultaneous equation system it cannot be determined from the data which variables should be treated as exogenous and which as endogenous. (5x5=25)

2. The following estimated equation was obtained by OLS regression using quarterly data from 1958 till 1976 (both the first quarter of 1958 and the last quarter of 1976 are included):

$$y_t = 2.20 + 0.104x_{t1} - 3.43x_{t2} + 0.34x_{t3}$$
$$\quad (3.4) \quad (0.005) \quad (2.2) \quad (0.15)$$

The standard errors are given in parentheses. The explained sum of squares was 109.6, and the residual sum of squares 18.48

   (a) Test the significance of each of the slope coefficients.

   (b) Calculate the coefficient of determination R^2 as also the adjusted R^2 (i.e., $\bar{R}^2$).

   (c) When three seasonal dummy variables were added and the equation was reestimated, the explained sum of squares rose to 114.8. Test for the presence of seasonality. (9+5+6=20)

p.t.o.

3.(a) Consider the following linear regression model.

$$y_i = \beta x_i + u_i \quad , \quad i = 1,2,\ldots, n \ ;$$

$E(u_i) = 0$ for all i, $E(u_i^2) = \sigma^2 x_i^2$, $E(u_i u_j) = 0$

for all $i \neq j$, and $\sum_{i=1}^{n} x_i^2 = n$.

Prove that the usual (OLS) formula for the estimated variance of the OLS estimator of β yields a downward biased estimator of the true variance.

(b) Obtain the Zellner's seemingly unrelated regression equations (SURE) estimator of the regression coefficients in a set of equations. State also the assumptions underlying the SURE models. Further, find conditions under which this estimator reduces to the usual OLS estimator obtained from each equation separately in the set of equations. (8+17=25)

4.(a) Describe how BLUS residuals are obtained. Also indicate the usefulness of these residuals for testing the presence of autocorrelation among the disturbances in a linear regression model.

(b) Critically examine the performance of the different estimators available for estimating linear regression models with AR(1) disturbances. (Detailed derivations are <u>not</u> required) (13+17=30)

5.(a) Obtain the rank condition for identifiability of an equation in a simultaneous equation system where the prior information is embodied in linear homogeneous restrictions on the structural coefficients.

(b) Consider the following two-equation system.

$$\beta_{11} y_{1t} + \beta_{12} y_{2t} + \gamma_{11} x_{1t} + \gamma_{12} x_{2t} = u_{1t}$$

$$\beta_{21} y_{1t} + \beta_{22} y_{2t} + \gamma_{21} x_{1t} + \gamma_{22} x_{2t} = u_{2t} \ ; \ t = 1,2, \ldots, n,$$

where the symbols have their usual meaning.
Suppose the following restrictions on the structural coefficients are imposed for identifiability:

$$\gamma_{11} = 0, \ \gamma_{12} = 0, \ \gamma_{21} + \beta_{21} = 0 \text{ and } \gamma_{22} = 0.$$

Examine the identifiability status of the two equations.

(12+13=25)

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II and III Year : 1992-93

SEMESTRAL II EXAMINATION

Anthropology

Date: 7.5.1993 Maximum Marks: 100 Time: 3 hours

Note: Use separate answerscript for Group A and Group B.

GROUP - A

1. Outline the main points of Darwin's theory of Evolution. How do you distinguish between Lamarkism and Darwinism ?

[20]

2. What are the major stresses on man at high altitude ? What are their major effects on human biological traits ?

[20]

3. What do population pyramids having the following shapes suggest:

(a) triangular with broad base,

(b) cylindrical or,
Roughly resembling an inverted triangle ? [10]

Or

What are the salient characteristics of

(i) a stable population.

(ii) a stationary population ?

[10]

GROUP - B

1. Define Mendel's first law of inheritance. [2]

(a) "A backcross is a mating between a heterozygote and a homozygote" — This form of mating is known as a backcross to the recessive/dominant. Justify your answer with the help of two examples.

[14]

(b) Explain the result of a mating between two heterozygotes taster (Tt). What is the name of this form of mating ? What are the ratio of dominant/recessive characters in the offspring ?

[14]

p.t.o.

2. (a) Define Mendel's second law of inheritance. [2]

(a) What kinds of offsprings will be produced by a mating between a taster, roller with genotype TTR_r and a non-taster, non roller (genotype ttrr) ?

[8]

3. (a) What is Hardy-Weinberg principle ? [3]

(b) If 75% of a population was of the dominant phenotype (A-), then 25% would have the recessive phenotype (aa). Prove that the population is in Hardy-Weinberg equilibrium.

[7]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II and III Year: 1992-93
SEMESTRAL II EXAMINATION

Biology I

Date: 7.5.1993 Maximum Marks: 90 Time: 3 hours

Note: Use separate answerscript for each group.

GROUP - A

1. Answer any two of the following:

   (a) Classify human gland cells according to their shape, layers of cells and mode of secretion.

   (b) Give a comparative account of the sporophytes of Riccia, Marchantia, Anthoceros and Funaria with diagrams.

   (c) Explain Calvin-Basshman Cycle to elucidate the path of carbon in photosynthesis.

   (10 x 2) = [20]

2. Write short notes on the following (Any four):

   (a) Essential amino acids;

   (b) Economic importance of the family Gramineae (Poaceae);

   (c) Benthan and Hooker's system of classification;

   (d) Transaminases and Isomerases;

   (e) Plasma proteins;

   (f) Phytohormones. (4 x 5) = [20]

3. Practical Examination held on 31 March 1993. [10]

GROUP - B

Note: Answer Question no.1 and any TWO from the rest.

1. Distinguish between (any FOUR):

   (i) Fibres and sclereids; (ii) Vessels and tracheids;
   (iii) Phellogen and Phellem; (iv) Anthophore and androphore;

contd..... 2/-

(v) Axile placentation and Parietal placentation;
(vi) Monocot stem and dicot stem; (vii) Sieve plate and perforation plate; (viii) Uniparous cyme and multiparous cyme.

(4 x 4) = [16]

2. What is tissue ? Describe with illustrations the different types of complex permanent tissues found in angiosperms.

(2+10) = [12]

3. Justify the following statements (any THREE):

(i) In the first step of glycolysis, where glucose is converted to fructose - 1, 6 diphosphate, there is no energy gain.

(ii) Acetyl COA is the 'connecting link' between glycolysis and the Krebs cycle.

(iii) Complete oxidation of glucose to CO_2 and H_2O will show that there is a net gain of 38 ATP.

(iv) A variety of micororganisms performs a different pathway for energy release from that of the higher organisms.

(3 x 4) = [12]

4. What is meristem ? Classify them according to their position in the plant body and state their functions.

(2+6+4) = [12]

5. What is root ? Describe with illustrations the different regions of a root. What are the morphological characteristics of the root ?

(2+5+5) = [12]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1992-93
SEMESTRAL II EXAMINATION
Physics II

Date: 7.5.1993 Maximum Marks: 100 Time: 3 hours

Note: Use separate answerscript for each group.

GROUP - A Max. Marks: 60

Answer any THREE questions.

1. Consider a system distributed over its accessible states r in accordance with an arbitrary probability distribution P_r and and let its entropy be defined by the relation $S = -K\Sigma P_r \ln P_r$ with the normalization condition $\Sigma P_r = 1$. The probability distribution for the canonical ensemble is denoted as $P_r^{(0)}$ and its entropy S_0. These two systems have the same mean energy. Now using the inequality $\ln X \leq X-1$, show that $S_0 \geq S$.

[20]

2. Define Helmholtz free energy. Write down its relationship with enthalpy and gibbs free energy. From the fundamental thermodynamic relation derive

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

(Symbols have their usual meaning).

(4+4+12) = [20]

3. Quantum mechanically, a harmonic oscillator has its energy levels given by $\epsilon = \hbar\omega(n+\frac{1}{2})$ with $n = 0, 1, 2 \ldots$.

A system of such oscillators obey Bose-Einstein statistics. Find the partition function and the mean energy.

(12+8) = [20]

4. Discuss briefly the motion of an electron in one dimension according to band theory. What is the physical significance of negative effective mass ? Calculate the number of possible electrons present in each completely filled energy band.

(5+5+10) = [20]

(a) State and prove Block theorem. Obtain the total number of vibrational modes of a continuous medium within the frequency interval ν to $\nu + d\nu$.

(b) Briefly develop Debye's T^3 law.

(5+5+10) = [20]

p.t.o.

GROUP - B

Max. Marks: 40

Note: Answer any TWO questions. Give all the necessary steps.

1.(a) State de-Broglie's hypothesis. A free particle is moving with velocity v. Show that its phase velocity is greater than c, the velocity of light. Find its group velocity.

(2+4+4) = [10]

(b) Apply the the Bohr-Sommer field quantization rules (the angular momentum $J = \frac{n.h}{2\pi}$) to determine the energy levels of the circular orbits in a hydrogen atom.

[10]

2.(a) Show that $\frac{d\langle x\rangle}{dt} = \frac{\langle p_x \rangle}{m}$, the symbols having their usual meanings. [10]

(b) A particle (one dimensional) moves in the potential given by

$$V = 0 \quad , \quad -a < x < a$$

$$= \infty \qquad |x| \geq a .$$

Determine the wave functions every where. Also find the energy eigen values.

(5+5). = [10]

3.(a) Prove that $Axe^{-\frac{1}{2}\alpha^2 x^2}$ where A is a constant is an eigen state for the Hamiltonian

$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}Kx^2$ if $\alpha^4 = \frac{mK}{\hbar^2}$. Find the eigen value corresponding to this eigen state.

(6+2) = [8]

(b) If L_x, L_y, L_z be the components of the angular momentum operator $\vec{L} = \vec{r} \times \vec{p}$.

Prove that in quantum mechanics

$$[L_x , L_y] = i L_z , \quad [L_y , L_z] = i L_x$$

$$[L_z , L_x] = i L_y .$$

Hence show that if ψ_{lm} be an eigen function of L_z with eigen value m then $(L_x + i L_y)\psi_{lm}$ is also an eigen functi of L_z.

(8+4) = [12]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1992-93

BACKPAPER SEMESTRAL II EXAMINATION

Introduction to Stochastic Processes

Date: 28.6.1993 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL question. The maximum you can score is 100.

1. X_n, $n = 0, 1, 2, \ldots$ is a homogeneous Markov Chain with state space S and transition matrix P.

   (a) Define positive recurrence of a state. Show that positive recurrence is a class property.

   (b) Show that there are no null recurrent state when S is finite.

   (c) Let $S = 1, 2, \ldots t, t+1, \ldots d$ where $1, 2, \ldots t$ is the set of all the transient states. Show that $(I-Q)$ is a non-singular matrix where I is the $t \times t$ identity matrix and $Q = ((p_{ij}))_{\substack{i=1,2,\ldots t \\ j=1,2,\ldots t}}$.

   [(4+6) + 10+10] = [30]

2. A particle moves according to a Markov Chain on the state space $S = 1, 2, \ldots c, c+1, \ldots c+d$ where c and d are positive integers. Starting from any one of the first c states the particle jumps to a state chosen at random and with equal probability from the last d states; starting from any one of the last d states the particle jumps in one transition to a state chosen uniformly from the first c states.

   (a) Write down the transition matrix of this MC. Show that the chain is irreducible.

   (b) Find the stationary initial distribution.

   [(3+3) + 9] = [15]

3. Consider the following Random Walk Chain X_n, $n = 0, 1, \ldots$ where $S = 0, 1, 2, \ldots d$.

   $p_{i,i+1} = p$, $p_{i,i-1} = 1-p$, $i = 1, 2, \ldots d-1$, $(0 < p < 1)$.

   $p_{oo} = p_{dd} = 1$.

   Calculate $e(k) = E(\text{Time taken before being absorbed into the states o or d} \mid X_o = k)$

   $k = 1, 2, \ldots d-1$.

   [10]

p.t.o.

4. Suppose that every man in a certain society has exactly 3 children each of which has independent probability $\frac{1}{2}$ of being a boy and $\frac{1}{2}$ of being a girl. Find the probability that the male live of a given man eventually become extinct. [10]

5. (a) For a Poisson process X_t, $t \geq 0$ let W_r be the random time of occurrance of the rth poisson event. Let $0 \leq s \leq t$ and $1 \leq r \leq n$. Show that

$$P\ W_r \leq s \mid X_t = n = \sum_{k=r}^{n} \binom{n}{k} \left(\frac{s}{t}\right)^k \left(1 - \frac{s}{t}\right)^{n-k}.$$

(b) A continuous time Markov Chain has 2 states 0 and 1. The waiting time in states 0 and 1 are exponential $\varepsilon(\alpha)$ and $\varepsilon(\beta)$ respectively ($\alpha > 0$, $\beta > 0$). Write down the Forward equations for this MC. Compute $P_{00}(t)$ the probability of being in state 0 at time t starting from the state 0 at time 0. Let $\alpha = \beta$ and let N(t) denote the number of times the syste has changed states. Find the probability distribution of N(t).

[7 + (4+10+4)] = [25]

6. Consider M/M/k queue where the arrival process is Poisson with rate λ, the service distribution is exponential $\varepsilon(\mu)$ and there are k service counters, X_t $t \geq 0$ number of customers in the syste at time t. Find the infiniticimal parameters of this MC. Find $\lim_{t \to \infty} P_{ij}(t)$, where $P_{ij}(t) = P(X_t = j \mid X_0 = i)$.

(5+10) = [15]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year: 1992-93
BACKPAPER SEMESTRAL II EXAMINATION
Design of Experiments

Date: 28.6.1992 Maximum Marks: 100 Time: 3 hours.

Note: Answer any FOUR questions. Marks allotted to each question are given within parentheses.

1.(a) In a split-plot design, with p levels of the whole-plot treatment and q levels of the sub-plot treatment, in r replicates, discuss how would you test for the difference between two whole-plot treatment means at the same or different levels of the sub-plot treatment. Also indicate how would you decide on the d.f. of the test statistic to be used. Show that a good approximation to the correct d.f. is given by

$$\frac{[MSE_I + (q-1)MSE_{II}]^2}{\frac{[MSE_I]^2}{(r-1)(p-1)} + \frac{[(q-1)MSE_{II}]^2}{p(q-1)(r-1)}}$$

where MSE_I and MSE_{II} represent the whole-plot error MS and the sub-plot error MS respectively.

(b) Disregarding the difference in number of d.f. show that the efficiency of the split-plot design relative to RCBD on the sub-plot treatment comparison and the whole-plot X sub-plot interaction is

$$\frac{(p-1)MSE_I + p(q-1)MSE_{II}}{(pq-1)MSE_{II}}.$$

(15+10) = [25]

2.(a) Five factors each at two levels are required to be tested and to control heterogeneity it is desired that each block should contain only 8 plots. Obtain a balanced arrangement in 5 replications such that the main effects and first order interactions remain unconfounded while $\frac{1}{5}$ of the information on each of the second and third order interactions is lost.

(b) Sixteen treatment combinations representing four factors, each at two levels, are arranged in four blocks of four plots each.

contd..... 2/-

The intra-block subgroup is given below:

(1), abc, abd, cd.

Determine the confounded effects and give the treatment combinations of the other blocks.

(20+5) = [25]

3.(a) Construct a balanced 3^3 design in blocks of 9 plots in 4 replications partially confounding second order interactions.

(b) The following design was used to test 9 rations fed to rats. The ration numbers are as follows:

Replication - 1

Block	1	1	4	7
"	2	3	6	9
"	3	2	5	8

Replication - 2

Block	1	7	8	9
"	2	1	2	3
"	3	4	5	6

Replication - 3

Block	1	1	9	5
"	2	8	4	3
"	3	6	2	7

Replication - 4

Block	1	5	7	3
"	2	1	6	8
"	3	9	2	4

Identify the above design and the confounded effects, if any.

(20+5) = [25]

4. In the experiment described below four materials were tested in each of four runs on a machine with four different positio The letters A to D refer to the four materials. The layout o the experiment is given below where the figures denote the loss in weight in a run of standard length:

Run	Position in Machine 4	2	1	3
2	A(251)	B(241)	D(227)	C(229)
3	D(234)	C(273)	A(274)	B(226)
	C(235)	D(236)	B(218)	A(268)
4	B(195)	A(270)	C(230)	D(225)

(a) Analyse the data and comment.

(b) If the variation due to the different positions of the machin is ignored will you modify your conclusion ?

(15+10) = [25]

5. The following table contains the plan and yields of a 2^4 field experiment on beans. The factors were Dung (D), Nitrochalk (N), Superphosphate (P) and Muriate of potash (K):

Replicate I

Block 1	p	k	d	npk	dnk	dnp	dpk	n
	45	55	53	36	41	48	55	42
Block 2	dp	nk	dk	pk	dnpk	(1)	dn	np
	50	44	43	51	44	58	41	50

Replicate II

Block 1	npk	d	p	dnk	n	dnp	k	dpk
	43	42	39	34	47	52	50	44
Block 2	nk	dp	(1)	np	pk	dk	dnpk	dn
	43	52	57	39	56	52	54	42

Identify the confounded effect, if any and analyse the data.

[25]

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