2nd Class Test
PHYSICS
B. Stat - III yr
First Semester.

Full Marks: 40 Time: 1 hour.

1. (a) What is an equation of state?
   (b) Write down (do not deduce) Van der Waal's equation of state.
   (c) What is critical coefficient? Compute its values for (i) an ideal gas, (ii) a Van der Waal gas.

2+2+2+1+3 [=10]

2 (a) Establish the relation $TV^{\gamma-1}$ = constant for an adiabatic transformation of an ideal gas.
   (b) Draw (do not describe) neatly in an indicator diagram a Carnot cycle. Shih (no deduction) the work done per cycle and hence find the thermal efficiency of a Carnot engine.

6+3+2+3 = [14]

3. (a) What is Boyle temperature? Show that Boyle temperature is related to the critical temperature T_c by $T_B = \frac{27}{5} T_c$, (T_B = Boyle temp.)

   (b) Compute the value of the second virial coefficient for a van der Waal gas.

   (c) A person claims to have invented an engine which operating between 327°C and 27°C is capable of having an efficiency 55%. Comment on his claim.

   (d) What are the different ways of increasing the efficiency of a Carnot engine? Which one should you advocate and why?

4 x 4 = [16]

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INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year : 1992-93

Sample Surveys

CLASS TEST I

Date : 11.8.92 Maximum Marks : 45 Time : 2 Hours.

Note : Answer ANY THREE questions. Marks allotted to each question are given within parentheses.

1. A simple random sample of size 3 is drawn from a population of size N with replacement. Show that the probabilities that the sample consists of 1,2 and 3 distinct units are

$$P_1 = \frac{1}{N^2}, \quad P_2 = \frac{3(N-1)}{N^2}, \quad P_3 = \frac{(N-1)(N-2)}{N^2}.$$

As an estimate of $\bar{Y}$ if we take $\bar{y}'$, the unweighted mean over the distinct units in the sample, show that the average variance of $\bar{y}'$ is

$$V(\bar{y}') = \frac{(2N-1)(N-1)}{6N^2} S^2.$$

Hence show that $V(\bar{y}') < V(\bar{y})$ where $\bar{y}$ is the ordinary mean of the 3 observations in the sample. (5+7+3) = [15]

2. A simple random sample of size $n = n_1+n_2$ with mean $\bar{y}$ is drawn from a finite population and a simple random subsample of size n_1 is drawn from it with mean $\bar{y}_1$.

Show that (a) $V(\bar{y}_1 - \bar{y}_2) = S^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)$, where $\bar{y}_2$ is the mean of the remaining n_2 units in the sample,

(b) $V(\bar{y}_1 - \bar{y}) = S^2 \left(\frac{1}{n_1} - \frac{1}{n} \right)$,

and (c) $\mathrm{Cov}(\bar{y}, \bar{y}_1 - \bar{y}) = 0$.

Repeated sampling implies repetition of the drawing of both the sample and the subsample.

(5+5+5) = [15]

3. A company intends to interview a simple random sample of employees who have been with it for more than 10 years. The company has Rs.2000/- to spend, and each interview costs Rs.20/-. There is no separate list of employee with more than 10 years of service, but a list can be compiled from the

p.t.o.

files at a cost of Rs.400/ The company can either (a) compile the list and interview a simple random sample drawn from the eligible employees or (b) draw a simple random sample of all employees, interviewing only those eligible. The cost of rejecting those not eligible in the sample is assumed negligible.

Show that for estimating a total over the population of eligible employees, plan (a) gives a smaller variance than plan (b) only if $C_j < 2\sqrt{Q_j}$, where C_j is the coefficient of variation of the item among eligible employees and Q_j is the proportion of noneligibles in the company. You may ignore the fpc.

[15]

4.(a) Obtain the expressions for the probabilities of inclusion of a particular unit and a pair of units in a sample of size n drawn according to PPSWR.

(b) While selecting a PPSWR sample according to Lahiri's method find the expected number of draws required to select a particular unit.

(c) Obtain an unbiased estimator of $\frac{1}{n}\sum_{i=1}^{N} Z_i^2/p_i$ based on a sample drawn according to PPSWR with p_i's as normed size measures and $Z_i = y_i - Y.p_i$, $Y = \sum_{i=1}^{N} y_i$.

(5+5+5)=[15]

5. In a finite bivariate population of N units, the means and and standard deviations of the variables x and y are $\bar{X}$, $\bar{Y}$ and σ_x, σ_y respectively and the correlation coefficient b between x and y is ρ. Derive the correlation coefficient $\rho(\bar{x}, \bar{y})$ between the sample means $\bar{x}$ and $\bar{y}$ based on the same sample of n units selected with SRSWOR. Also find $\rho(\bar{x}+\bar{y}, \bar{x}-\bar{y})$.

(10+ 5) =[15]

INDIAN STATISTICAL INSTITUTE

203 . T ROAD

CALCUTTA 700 035

B.Stat Year III Physics class test I August 17,1992.

Full Marks 40 Time 1 hr. 40 min.

1a) Classify the following according as they are i) scleronomic or rheonomic ii) holonomic or non holonomic iii) conservative or non conservative.

A particle constrained to move along a line under the influence of a force which is inversely proportional to the square of its distance from a fixed point and a damping force proportional to the square of the instantaneous speed.

b) The point of suspension (assumed massless) of a simple pendulum executes simple harmonic motion along the vertical line. Solve the problem by Lagrange's equation. (3+7)=10

2) Suppose that due to a change of the mass of the sun, due to radiation, the potential energy of a planet is written

$$-[GM_0 m (1 - \alpha t)]/r.$$

(G = Gravitational constant, M_0 = mass of the sun, m = mass of the planet).

What is the Hamiltonian ? Do you expect it to be a constant of motion ? (8+2)=10

3a) Show that if the Hamiltonian and a quantity F are constants of motion, then $\partial F/\partial t$ must also be a constant of motion.

b) The transformation equations between two sets of coordinates are

$$Q = \log (1 + q^{1/2}\cos p)$$

$$P = 2(1 + q^{1/2}\cos p)q^{1/2}\sin p$$

What will be the generating function for this transformation ?.

(7+5)=12

4) A uniformly heavy flexible string is fixed at two ends. Find the equation of the curve describing the form of the string.
(Assume the configuration of the string as that corresponding to the minimum potential energy). 8

INDIAN STATISTICAL INSTITUTE

B.Stat(Hons) II & III year: 1992-93

Class test I

Introduction to Sociology and Sociometry

Date: 18.8.92 Full Marks: 50

Note: Answer Question 4 and any one from the rest.
Figures in the margin indicate marks.

1. Discuss how sociological theory is relevant for planning a statistical investigation. 25

2. Caste can be interpreted by different sociological approaches-structural, functional, cultural and Marxist. Which of these do you support most? Briefly state the reasons. 25

3. Critically review the content of any piece of sociological work/study or the approach of any sociologist who has studied Indian society. 25

4. Write briefly and precisely on any two of the following:- $12\frac{1}{2} \times 2 = 25$

   a) Stratification in sociology and statistics.

   b) Sanskritization.

   c) Relation between class and caste.

   d) Social status and power.

CLASS TEST I –B . STAT.

DIFFERENTIAL EQUATIONS

Date: 4 Aug. 1992 time:2 hrs. max. marks: 50

ANSWER AS MANY AS YOU CAN. THE MAXIMUM YOU CAN SCORE IS 50.

1 (a) Suppose $f(x, y)$ is a continuous function that satisfies the Lipschitz condition

$$| f(x, y_1) - f(x, y_2) | \leq k | y_1 - y_2 |$$

on the strip $[a , b] \times \mathbb{R}$. *Let* (x_0 , y_0) be any point of the strip. Show that the IVP (initial value problem)

$$dy/dx = f(x, y) , y(x_0) = y_0$$

is equivalent to an integral equation.

Now , obtain a sequence of successivive approximations (by Picard's method) and explain how this sequence leads to a unique solution of the initial value problem. (No detail proof required). (7)

(b) Show that the following IVP

$$dy/dx = y^{1/3}, y(0) = 0,$$

has more than one solution on $[0 , 1]$.

Does this contradict Picard-Lindelöf theorem (1(a) above) ? If not, why not? (6)

2 (i) Find the general solution of

$$y' = (x + y + 4) / (x + y - 6). \quad (5)$$

(ii) Show that the following D. E.

$$(xy - 1) + (x^2 - xy) dy/dx = 0$$

is not exact. Find an integrating factor and then solve. (7)

(iii) Find the shape of the curved mirror such that light from the origin will be reflected in a beam of rays parallel to the y-axis.(Consider only the vertical section of the mirror through the origin). (8)

3 (i)Show that the set of solutions (on an interval I) of the linear differential equation

$$D^n y + P_1(x) D^{n-1} y + \cdots + P_n(x) y = 0$$

forms an n-dimensional vector space over $\mathbb{R}$, where P_i 's are continuous functions on I . (You may use the necesary existence-uniqueness theorem). (7)

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B.Stat. (Hons.) III Year : 1992-93

Sample Surveys

CLASS TEST II

Date :7.9.92 Maximum Marks : 75 Time : 2 Hours.

Note : You can answer as many questions as possible. The maximum you can score is 75. Marks allotted to each question are given within parentheses.

1.(a) If in a sample of three units, drawn with PPSWR, only two units are distinct, show that the estimators

(i) $\frac{1}{3}\left(\frac{y_1}{p_1}+\frac{y_2}{p_2}+\frac{y_1+y_2}{p_1+p_2}\right)$ and (ii) $\frac{y_1}{1-(1-p_1)^3}+\frac{y_2}{1-(1-p_2)^3}$

are unbiased for the population total Y. If the size measure used for selection is approximately proportional to y, state, giving reasons, which of the two estimators you would prefer.

(5+5+2)=[12]

(b) Suggest a sampling scheme for which the selection probability of a sample is proportional to the aggregate size measure of the units in the sample. Obtain the expressions for the probabilities of inclusion of a particular unit and a pair of units in a sample of size n drawn according to this scheme.

(6+3+4)=[13]

2. A population of N units is divided at random into n groups with N_i units in the ith group, $i = 1,2,..,n$ and then one unit is selected with PPS from each of the random groups. Suggest an unbiased estimator of the population total. Derive the variance of the proposed estimator. Show that it attains a minimum value when the random groups are of equal sizes.

Compare this minimum variance with the variance of the Hansen-Hurwitz estimator in PPSWR sampling. Also suggest an unbiased estimator of the variance of the estimator.

(3+7+4+4+7)=[25]

3. A population of N units is divided into two strata of sizes N_1 and N_2 units and samples of n_1 and n_2 units are selected from each of the two strata with SRSWR. Show that the efficiency of the estimator of $\bar{Y}$ in this case compared to that of optimum allocation of the total sample size (n_1+n_2) is not less

p.t.o.

than $4t/(1+t)^2$, where $t = n_1 n_2' / n_2 n_1'$, n_1' and n_2' being the optimum allocation to the two strata. [25]

4.(a) A sampler proposes to take a stratified random sample. He expects that his field costs will be of the form $\Sigma c_h n_h$. His advance estimates of relevant quantities for the two strata are as follows :

Stratum	W_h	S_h	C_h
1	0.4	10	$1
2	0.6	20	$9

(i) Find the values of $\frac{n_1}{n}$ and $\frac{n_2}{n}$ that minimize the total field cost for a given value of $V(\bar{y}_{st})$.

(ii) Find the sample size required, under this optimum allocation, to make $V(\bar{y}_{st}) = 1$. Ignore the fpc.

(iii) How much will the total field cost be ? (4+4+4)=[12]

(b) After the sample is taken, the sampler finds that his field costs were actually $2 per unit in stratum 1 and $12 in stratum 2

(i) How much greater is the field cost than anticipated ?

(ii) If he had known the correct field costs in advance, could he have attained $V(\bar{y}_{st}) = 1$ for the original estimated field cost in (iii) above, or would it have been necessary to increase the field cost to reduce $V(\bar{y}_{st})$ to 1 ? What is the minimum field (6+7)=[13]

5. In a stratification with two strata, the values of the W_h and S_h are as follows :-

Stratum	W_h	S_h
1	0.8	2
2	0.2	4

Compute the sample sizes n_1 , n_2 in the two strata needed to satisfy the following conditions. Each case requires a separate computation. (Ignore the fpc.).

(a) The standard error of the estimated population mean $\bar{y}_{st}$ is to be minimized when the total sample size is 2400.

(b) The standard error of the estimated mean of each stratum is to be 0.1.

(c) The standard error of the difference between the two estimated stratum means is to be 0.1 again minimizing the total size of sample.

(7+8+10)=[25]

INDIAN STATISTICAL INSTITUTE

B.Stat (Hons) II & III Year: 1992 - 93

Class Test III

Introduction to Sociology and Sociometry

Date: 13.10.92 Full marks: 50

(Questions carry equal marks)

1. What are the sources of sociological data? How can you classify them? Discuss briefly the advantages and limitations of any two sources in the context of sociological research.

Or

"Observation is a primary tool of scientific enquiry". Critically evaluate the statement with an illustration.

2. Prepare a village census for collecting demographic, economic, housing, educational and social characteristics of household members.

Or

What is a research design? Prepare a research design for studying any social problem in a multi-ethnic village.

$(10 + 10) = [20]$

P. T. O.

where y_t is the value of the series in year t .

INDIAN STATISTICAL INSTITUTE

B. Stat (Hons) III Year : 1992 – 93

Sample Surveys

CLASS TEST III

Date : 14.10.92 Maximum Marks : 100 Time : 2 Hours

Note : The paper carries 140 marks. The maximum you can score is 100. Marks allotted to each question are given within parentheses.

1. Find the bias in $\hat{Y}_R = \frac{1}{n}\sum_{i=1}^{n} \frac{y_i}{x_i} . X$ as an estimate of the population total Y of the variable (y) under enquiry and also find an unbiased estimate where X is the population total of the auxiliary variable (x) and $(x_1, y_1), \ldots, (x_N, y_N)$ are the values of the auxiliary variable and the variable under enquiry.

$(10 + 10) = [20]$

Suggest a sampling scheme under which the ratio estimator becomes unbiased. Obtain the variance of the ratio estimator under the sampling scheme. Also suggest an unbiased estimator of the variance. Is it always non-negative?

$(5 + 7 + 7 + 1) = [20]$

Obtain an exact expression for the bias of the ratio estimator of the population ratio. Also obtain an upper bound to the ratio of the bias to the standard error of the estimator.

$(10 + 10) = [20]$

P.T.O.

[25]

where y_t is the value of the series in year t.

4. Obtain an approximation to the bias and the variance of the ratio estimator of the population total based on SRSWOR valid in large samples. Also obtain a sample estimate of the variance. How does the estimator compare with the mean per unit estimator of the population total based on SRSWOR? $(5+5+5+5) = [20]$

5. In selecting a linear systematic sample of size n from a population of size N show that the sample mean is an unbiased estimator of the population mean when N is a multiple of n. When N is not a multiple of n, (i) can you revise the sampling scheme so that the sample mean still continues to be an unbiased estimator of the population mean? (ii) can you think of an alternative unbiased estimator?
$(6+7+7) = [20]$

6. Obtain the variance of the sample mean based on a linear systematic sample of size n when the sampling interval k satisfies $N = nk$. Express the variance in terms of the intra-class correlation coefficient. Obtain the efficiency of the systematic sampling as compared to that of (i) SRSWR and (ii) SRSWOR.

7. Is it possible to estimate unbiasedly the sampling variance of the estimate in case of systematic sampling? If not, explain the reasons. Explain the use of interpenetrating subsamples in systematic sampling in providing an unbiased estimator of the variance. Can you mention a special case of circular systematic sampling where it is possible to estimate unbiasedly the variance of the estimate? $(6+6+4+4) = [20]$

$(8+7+5) = [20]$

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year:1992-93

STATISTICS COMPREHENSIVE

Class Test 1

Date: 5 February 1993 Maximum Marks: 100 Time: 100 minutes

Figures in brackets [ ] indicate marks allotted to the questions.

1. Consider the family of distributions $\mathcal{P} = \{P_N : N \geq 1\}$, where

$$P_N\{X = k\} = \begin{cases} \frac{1}{N} & \text{for } k = 1, 2, \ldots, N \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $\mathcal{P}$ is complete. (You may use induction on N.)

(b) Fix a positive integer n. Consider

$$\phi_0(k) = \begin{cases} 0 & \text{for } k = 1, 2, \ldots, n-1, n+2, \ldots \\ a & \text{for } k = n \\ -a & \text{for } k = n+1. \end{cases}$$

and obtain $E_N(\phi_0(X))$.

(c) Hence or otherwise show that $\mathcal{P} - \{P_n\}$ is not complete.

(d) Obtain the minimum variance unbiased estimator (MVUE) (based on X) of N for the family $\mathcal{P}$.

(e) Show that $\phi_1(X)$ is MVUE of N for the family $\mathcal{P} - \{P_n\}$, where

$$\phi_1(k) = \begin{cases} 2n & \text{for } k = n, n+1 \\ 2k-1 & \text{for } k \text{ any positive integer other than } n \text{ and } n+1. \end{cases}$$

[An unbiased estimator is MVUE iff it is uncorrelated with every unbiased zero function with finite variance.]

[40]

PLEASE TURN OVER

where y_t is the value of the series in year t.

2. An experiment is to be conducted to determine the difference between blood pressures in the left and right brachial arteries. For this experiment 100 persons are available. Consider the following two choices of design of the experiment:

   - *Design 1*: The 100 persons are divided into two groups each of 50 at random. In the first group, left blood pressure is observed and in the second group, right blood pressure is observed.
   - *Design 2*: On each of the 100 persons both left and right blood pressures are observed.

   Assume that the order of sides is immaterial. State any other assumptions that you make. Formulate a suitable model for each design.

   Formulating appropriate criterion for comparing the two designs in this case, investigate which design is better. [25]

3. An economist is interested in fitting a multiple linear regression of Y on X_1, X_2, X_3, X_4 by the method of least squares. He knows from economic theory that the regression coefficient of X_4 in this multiple linear regression is -3. He has data on 50 units on these variables Y, X_1, X_2, X_3, X_4. He also has a computer program which would fit a multiple linear regression by the method of least squares and compute relevant statistics thereof. Explain, with suitable mathematical explanations how this program could be used in this situation. [20]

4. Write a short account (not more than 2 pages) of the contributions of P.C.Mahalanobis to the methodology of sample surveys. [15]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year:1992-93
STATISTICS COMPREHENSIVE
Class Test 2

Date: 5 March 1993 Maximum Marks: 100 Time: 100 minutes

Figures in brackets [ ] indicate marks allotted to the questions.

1. Let $\Sigma_{a,c} = aI + 1c' + c1'$, where 1 and c are column vectors of appropriate dimensions and a is a scalar. (This structure is called the Baldessari structure.)

   (a) Obtain necessary and sufficient conditions on a and c so that Σ is a dispersion matrix. (Hint: Obtain eigenvalues of $1c' + c1'$. Note that non-null eigenvalues of AB and BA are the same.) [10]

   (b) Consider $(Y, X\beta, \sigma^2\Sigma_{a,c})$ where $\Sigma_{a,c}$ is a dispersion matrix and $1 \in \mathcal{C}(X)$. Show that the BLUE of every estimable parametric function remains the same under $(Y, X\beta, \sigma^2 I)$ and $(Y, X\beta, \sigma^2\Sigma_{a,c})$ if and only if $c \in \mathcal{C}(X)$. [15]

   (c) Consider one-way classified data with

   $$y_{ij} = \mu + \alpha_i + \epsilon_{ij}, j = 1, 2, \ldots, n_i, i = 1, 2, \ldots, k$$

   $$\text{where } \epsilon_{ij} \sim \mathcal{N}(0, \bullet) \text{ for all } i, j.$$

   Consider two models: Model I where all observations y_{ij}'s are uncorrelated and Model II where the observations have the dispersion matrix $\Sigma_{a,c}$. Consider test of the hypothesis H_0: $\alpha_1 = \alpha_2 = \cdots = \alpha_k$ against H_1: negation of H_0. Show that the likelihood ratio tests for the above hypothesis remain the same under models I and II. [15]

PLEASE TURN OVER

where y_t is the value of the series in year t .

2. In a genetic study of alcoholic cirrhosis, data on the genetic marker *acid phosphatase* (ACP) were collected on random samples of 157 patients and 1086 healthy individuals. The ACP trait is determined at a single locus with three codominant alleles. Data obtained are as follows:

Phenotype	Controls	Cirrhotic patients
AA	81	5
AB	420	66
AC	22	15
BB	508	65
BC	54	5
CC	1	1

(a) Examine if the two groups are in Hardy-Weinberg equilibrium. Estimate the gene frequencies (i.e., allele frequencies) of each group under such an equilibrium. [10]

(b) Examine if the cirrhotic patients have a different phenotypic distribution from the normal. [5]

(c) Examine if the cirrhotic patients have a different gene frequency distribution from the normal. [10]

(d) Do you think that the Goodman-Kruskal Λ is a relevant measure of association between phenotype and disease-status in this case? Anyway, compute such a measure. [5]

3. Explain clearly the concept of *regression to the mean*, with suitable examples of the phenomenon. Also, demonstrate the phenomenon with a mathematical formulation and derivation. [15]

4. Answer each of the following questions in not more than two pages.

(a) Describe how 'Student' (Gosset) obtained the t distribution by 'simulation'. If you were to simulate the t-distribution today, explain how you would do it. [8]

(b) Describe Fisher's contributions to the 'Design of Experiments'. [7]

INDIAN STATISTICAL INSTITUTE

B III 1991-92

3rd Class Test on Indian Economics

HOME ASSIGNMENT

Max. Marks 100 Date 30 October 1991

The table below gives groupwise index numbers of industrial production in India (base 1970 = 100) during the period 1960-84. The groups are : basic goods, capital goods (including motor cars), intermediate goods, consumer goods (excluding motor cars).

Year	Basic goods	Capital goods	Intermediate goods	Consumer goods	ALL INDUSTRY
(Weight)	(32.28)	(15.74)	(20.95)	(31.03)	(100.00)
1960	45.1	44.5	63.0	64.6	55.3
1961	50.8	52.5	66.6	68.9	60.4
1962	57.8	68.1	71.5	69.8	66.2
1963	66.1	75.7	77.4	71.4	71.7
1964	68.6	91.7	83.2	76.7	77.9
1965	74.3	108.7	88.2	82.4	85.1
1966	78.0	93.5	86.1	84.8	84.4
1967	79.6	91.4	87.9	81.2	83.7
1968	87.8	93.6	93.3	85.3	89.1
1969	95.6	95.3	97.2	93.9	95.4
1970	100.0	100.0	100.0	100.0	100.0
1971	104.6	105.6	104.4	103.5	104.2
1972	113.0	106.3	111.2	108.2	110.2
1973	109.5	123.4	114.2	107.6	112.0
1974	113.8	128.7	112.3	109.8	114.3
1975	129.0	128.1	113.7	108.1	119.7
1976	147.5	142.1	122.2	119.0	132.4
1977	155.0	150.3	125.8	126.3	139.4
1978	162.4	154.9	135.9	139.3	147.8
1979	166.2	158.5	139.6	136.1	149.5
1980	164.6	165.8	140.7	136.7	150.4
1981	188.6	179.2	145.9	147.4	164.6
1982	203.6	178.2	148.6	156.1	172.0
1983	214.8	186.0	160.3	156.6	179.7
1984	240.1	204.3	163.0	157.8	194.2

(a) Classify the above period into five subperiods : 1960-65, 1966-70, 1971-75, 1976-80, 1981-84 and compute the average annual rate of growth of each series for each subperiod .

(b) fit the following trend curve to each of the following series, viz., basic goods, consumer goods, all industry :

$$\log y_t = \alpha + \beta t + \gamma t^2$$

where y_t is the value of the series in year t .

INDIAN STATISTICAL INSTITUTE

203 B.T. ROAD

CALCUTTA 700 035

B III PHYSICS CLASS TEST 3 NOVEMBER 04, 1992

FULL MARKS 50 TIME 1 HOUR 40 MIN

GIVEN: velocity of light $c = 3 \times 10^8$ m/s.

1 light year = 10^{16} m. (approx.)

$1^oA = 10^{-10}$m.

rest mass of an electron $m_0 = 9.1 \times 10^{-31}$ kg.

charge of an electron $e = 1.6 \times 10^{-19}$ coulombs.

CALCULATORS MAY BE USED .

Symbols have their usual meanings.

Use separate answer-scripts for Group A and Group B. Answer both the groups

GROUP A (Total Marks 33)

1 a) State the postulates of Special Theory of Relativity. 3

b) An S' observer observes two events occur at the same place but are separated in time. An S observer declares that two events occur at different places. Comment. 3

c) We have stressed the utility of relativity at high speeds. Relativity is also useful in cosmology, where great distances and long time intervals are involved.Show, from the form of the Lorentz transformation equations, why this is so. 3

P.T.O.

d) The radius of our galaxy is 3×10^4 light years. Can a person, in principle, travel from the center to the edge of our galaxy in a normal lifetime? What constant velocity would he need to make the trip in 30 years proper time ? 3

e) Give the wavelength shift in relativistic Doppler effect for the 6563°A H_α line emitted by a star receding from the earth with a relative velocity $10^{-1}c$. 3

(question 1 total 15)

2a) Show explicitly that two successive Lorentz transformation at right angles (v_1 along x' and v_2 along y') do not commute . 6

b) Is it true that a particle that has energy must also have momentum ? What if the particle has no rest mass ? 2

c) If photons have a speed c in one reference frame, can they be found at rest in any other frame ? 2

(question 2 total 10)

3a) Find the energy equivalent to the rest mass of the electron. 3

b) An electron acquires kinetic energy by falling through an electrostatic potential difference of 10^4 volts starting from rest in a uniform electric field. Find its mass at the end of the acceleration. 5

(question 3 total 8)

Group B.

1(a) Prove the following relations :

(i) $C_p - C_v = \dfrac{R\left(p + \frac{a}{V_1^2}\right)}{p - \frac{a}{V_1^2} + \frac{2ab}{V_1^3}}$ (ii) $\dfrac{C_p}{C_v} = \dfrac{E_s}{E_T}$

the symbols have their usual meanings

(b) Show that an absolute scale of temperature follows from the efficiency relation of a 2 reversible heat engine.

[5+5+7] =