1993-94 [3]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1993-94

Statistical Inference I
Semestral-I Examinations

Date : 15.11.1993 Maximum Marks : 100 Time : 3 Hours

Note : Answer any five questions. All questions carry equal marks.

1.(a) $X_1, X_2, \ldots, X_n$ are iid random variables with $N(\sigma, \sigma^2)$ distribution, where $\sigma > 0$ is the unknown parameter. Prove that $\left( \sum_1^n X_i, \sum_1^n X_i^2 \right)$ is a minimal sufficient statistic.

(b) $X_1, X_2, \ldots, X_n$ are iid random variables with Cauchy density

$f_\theta(x) = \frac{1}{\pi} \frac{1}{1+(x-\theta)^2}$ where $\theta \in \mathbb{R}$ is the unknown parameter.

Prove that the set of order statistics based on $X_1, X_2, \ldots X_n$ is minimal sufficient.

[10+10]

2. Let $X_1, X_2, \ldots, X_n$ be iid observations with the common density

$$f_\theta(x) \begin{cases} = e^{-(x-\theta)} & \text{for } x > \theta \\ = 0 & \text{for } x \leq \theta \end{cases}$$

where $\theta \in \mathbb{R}$ is the unknown parameter.

(a) Show that $T(X_1, X_2, \ldots, X_n) = \text{Min}\{X_1, X_2, \ldots X_n\}$ is sufficient. Is it complete ? Justify briefly. Show that $T(X_1, \ldots X_n)$ and $X_1 - X_2$ are independently distributed.

(b) Find the UMVUE of $g(\theta)$ where

$$g(\theta) = P_\theta[X_1 > 0] = \int_{\max(0,\theta)}^{\infty} e^{-(x-\theta)}\, dx.$$

[(3+5+4)+8]

3. Let $X_1, X_2, \ldots, X_n$ be iid observations from $N(\theta, 1)$ distribution, $\theta \in \mathbb{R}$. What is the Cramér-Rao lower bound for the variance of an unbiased estimator of a parametric function $g(\theta)$ ? Use this bound to prove that the sample mean $\overline{X}$ is an admissible estimator for $\theta$ with squarred error loss.

[5+15]

4.(a) Consider the problem of estimating $\theta$ on the basis of an observation $X$ from the Poisson distribution with mean $\theta$, $\theta > 0$. Let $\theta$ have the prior distribution given by th density

$$\pi(\theta) \begin{cases} = \frac{2^p}{\Gamma(p)} e^{-2\theta} \theta^{p-1} & \text{for } \theta > 0 \\ = 0 & \text{otherwise} \end{cases}$$

Obtain the Bayes estimator of $\theta$ with squarred error loss.

(b) Show that the correlation coefficient between the UMVUE $T$ of an unbiasedly estimable parametric function $h(\theta)$ and any other unbiased estimator $T_1$ of $h(\theta)$ is $\sqrt{\frac{Var_\theta(T)}{Var_\theta(T_1)}}$. [12+8]

5.(a) State carefully the Neyman-Pearson lemma. Prove that the power $\beta$ of the MP level $\alpha$ test for testing a simple hypothesis against a simple alternative is greater than or equal to $\alpha$. What can you say about strict inequality ?

(b) For testing $H : \theta = 1$ against $K : \theta = 2$ on the basis of observation $X$ from the probability distribution with density

$$f_\theta(x) \begin{cases} = \frac{e^{-x} x^{\theta-1}}{\Gamma(\theta)} & \text{for } x > 0 \\ = 0 & \text{for } x \leq 0 \end{cases}$$

Obtain the MP level $\alpha$ test. Show that this test is also UMP level $\alpha$ against the alternative $K' : \theta > 1$. Determine and roughly sketch the risk set $R$ for the testing proble H vs. K.

$$R = \{ (E_1\phi(X), E_2\phi(X)) : \phi \text{ is a test function}$$

[(5+2+1)+(4+4+4)]

6. Define an unbiased test. Determine the UMP unbiased test of level $\alpha$ for testing $H : \sigma^2 = \sigma_0^2$ against the alternative $K : \sigma^2 \neq \sigma_0^2$ on the basis of n imdependent observations $X_1, X_2, \ldots, X_n$ from $N(0, \sigma^2)$ distribution. [5+15]

7.(a) Let $X_1, X_2, \ldots X_n$ be iid observations from the uniform di tribution on $(0, \theta)$, $\theta > 0$. Show that the test $\phi$ given by

$$\phi(x_1, \ldots x_n) = 1 \text{ if either } \max_i x_i < \theta_0 \alpha^{1/n} \text{ or } \max_i x_i$$
$$= 0 \quad \text{otherwise}$$

is UMP level $\alpha$ for testing $H : \theta = \theta_0$ against $K : \theta \neq \theta_0$.

(b) Write down the expression for the power function of the UMP level $\alpha = 0.05$ test for testing $H: \mu = 0$ against $K: \mu > 0$ on the basis of n independent observation from $N(\mu, 1)$. Determine the smallest sample size n such that the power of this test against the alternative $\mu=1$ is at least 0.95 (up 5% point of N(0,1) is 1.645). [12+(2+6)]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1993 - 94

SEMESTERAL - I EXAMINATION

Sample Surveys

Date: 19.11.1993 Maximum Marks: 100 Time: 3 hours.

Note: Attempt ALL questions.

1.(a) Distinguish between sampling and nonsampling errors in a survey. List down the various sources of nonsampling errors and discuss briefly the methods of assessment and control of these errors.

(b) A survey was conducted in a village consisting of 675 households by covering a sample of 50 households selected using a simple random sampling without replacement scheme to estimate the average monthly expenditure on postage. The estimate was found to be Rs.4.20 with a standard error of 0.47. Using this information, determine the sample size needed to estimate the same characteristic in a neighbouring village on the basis of a sample to be selected by simple random sampling with replacement scheme such that the length of the confidence interval at 95% confidence level is 20% of the true value. State clearly the assumptions involved in finding out the sample size.

[(3+6+5) + (7+2)] = [23]

2.(a) Explain with illustrations what you understand by the terms 'inclusion probability of a unit $U_i$' and 'joint inclusion probability of the pair $(U_i, U_j)$, $i \neq j$' for a sampling design.

(b) Define the estimator $\hat{Y}_{HT}$ of the population total $Y = \sum_{1}^{n} Y_i$ of a characteristic y taking values $Y_i$ on the units $U_i$, $i = 1, 2, \ldots N$ based on a given sampling design with positive inclusion probabilities as proposed by Horvitz and Thompson. Establish its unbiasedness and give an expression for its variance.

(c) Write down the unbiased estimator of variance of $\hat{Y}_{HT}$ as suggested by Yates and Grundy for a fixed sample size n with positive joint inclusion probabilities.

contd..... 2/-

(d) Show that the above variance estimator in (c) is uniformly nonnegative if the sample is selected by the Midzuno - Sen scheme.

[(2+2) + (2+3) + 4+8)] = [21]

3. As a consultant on 'Sampling Techniques' indicate the suggestions you would offer and the advice you would give on the following problems (precise mathematical statements will gain more credit for your answer):

(a) A school teacher having planned to estimate the average consumption of sweets in a week for all the children of her school, got hold of a complete list of (in alphabetical order) all the 1217 school children and decided to take a sample of size 120. Using this list the teacher had selected a 'linear systematic sample' with the random start 5. She then computed the sample average of the number of sweets consumed during the week as an estimate for the whole school.- Her problem is to know if the estimate is valid and if not, how a valid estimate would be obtained. She is also interested in estimating the sampling error of the estimate from her sample.

(b) The principal of a college wishes to estimate the 'average no. of study hours' of students. He believes that score in 'the previous test' is a good indicator of the study hours and collects data for a random sample of students on 'study hours' as well as 'previous score' from records. The principal is now interested in an 'efficient' method of estimation.

(c) An agriculturalist plans to select a sample of 9 plots from 3 groups of plots containing 20, 30 and 40 plots each. While his assistant allocated the sample plots in the proportion 2 : 3 : 4, he thought that an equal allocation of 3 plots each would be 'optimum'. He is now interested in having a justification for his claim.

(6+6+6) = [18]

4. A sample survey was conducted to estimate the total yield of paddy in a district. A stratified two-stage sampling design was adopted with villages as first stage units and plots within them as second stage units. From each stratum 4 villages were selected with probabilities proportional to

contd..... 3/-

area and with replacement and 4 plots were selected from each selected village with equal probability and without replacement. The data on yield for the sample plots together with information on selection probabilities are given in the table below:

| stratum | sample village | inverse of probability of selection | total no. of plots | yield of sample plots 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| I | 1 | 440.21 | 28 | 104 | 182 | 148 | 87 |
| | 2 | 660.43 | 14 | 108 | 64 | 132 | 156 |
| | 3 | 31.50 | 240 | 100 | 115 | 50 | 172 |
| | 4 | 113.38 | 76 | 346 | 350 | 157 | 119 |
| II | 1 | 21.00 | 256 | 124 | 111 | 135 | 216 |
| | 2 | 16.80 | 288 | 123 | 177 | 106 | 138 |
| | 3 | 24.76 | 222 | 264 | 78 | 144 | 55 |
| | 4 | 49.99 | 69 | 300 | 114 | 68 | 111 |
| III | 1 | 67.68 | 189 | 110 | 281 | 120 | 114 |
| | 2 | 339.14 | 42 | 80 | 61 | 118 | 124 |
| | 3 | 100.00 | 134 | 121 | 212 | 174 | 106 |
| | 4 | 68.07 | 161 | 243 | 116 | 314 | 129 |

Total number of plots in stratum I, II, III are 8423, 6355 and 12853 respectively. Using the above data,

(i) Obtain an unbiased estimate of the total yield of paddy in the district.

(ii) Obtain an unbiased estimate of the sampling variance of the above estimator.

(iii) Also compare the efficiency of the above design with that of unistage simple random sampling with replacement of 16 plots in each stratum.

(iv) Verify whether the design is a self-weighting design.

(10+12+13+3) = [38]

---

:bcc:

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1993-94
Linear Statistical Models
Semestral-I Examination.

Date : 22.11.1993 Maximum Marks : 100 Time : 4 Hours

Note : Attempt all the questions.

1. Consider the models

   (i) $(Y, X_1\beta_1 + X_2\beta_2 + \sigma^2 I)$

   and (ii) $(R'Y, R'X_2\beta_2, \sigma^2 I)$

   where $(R, R')$ is a rank factorization of $I-P_{X_1}$.

   ($P_{X_1}$ is the orthogonal projector projecting vectors into $\mathcal{C}(X_1)$).

   (a) Show that $p'\beta_2$ is estimable under model (i) if and only if it is estimable under model (ii).

   (b) Let $p'\beta_2$ be estimable under model (i). Show that its BLUE's under model (i) and model (ii) coincide. Show also that the BLUE has the same variance under both the models.

   (c) Show that the conventional unbiased estimator of $\sigma^2$ (based on residual sum of squares) is the same under both the models (i) and (ii).

   (d) Assume further that Y has a multivariate normal distribution. Prove or disprove the statement :
   " The conventional (likelihood ratio) statistic for testing $H : A\beta_2 = \zeta$ against $K : A\beta_2 \neq \zeta$ (where $A\beta_2$ is estimable) remains the same under both the models and has the same null distribution under both the models."

   [20]

2. Consider the model

   $$y_{ijk} = \mu + \alpha_i + \beta_{ij} + \epsilon_{ijk},$$

   $$K = 1,\ldots,r_{ij} \; ; \; j=1,\ldots,q_i \text{ and } i = 1,\ldots,p$$

   where $\epsilon_{ijk}$'s are iid $N(0, \sigma^2)$.

   Obtain test statistics and derive their null distributions for each of the following testing problems :

p.t.o.

(a) $H_1: \beta_{i1} = \beta_{i2} = \ldots = \beta_{iq_i}$ , $i = 1,\ldots,p$
against $K_1$ : negation of $H_1$

(b) $H_2$: There are no differences among $\Theta$'s
where $\Theta_i = \alpha_i + \frac{1}{q_i} \sum_{j=1}^{q_i} \beta_{ij}$, $i = 1,\ldots,p$.

against $K_2$ : negation of $H_2$.
Simplify your results as far as practicable. [15]

3. Prove or disprove each of the following statements :

(a) If we include one more observation $y_{n+1}$ with mean $x'_{n+1}\beta$ and variance $\sigma^2$ to $(Y, X\beta, \sigma^2 I)$ where $y_{n+1}$ is uncorrelated with Y, then the residual sum of squares will not decrease but the conventional estimate of $\sigma^2$ may decrease.

(b) In two-way classified data with single observation per cell and with one covariate,

$$y_{ij} = \mu + \alpha_i + \beta_j + \gamma x_{ij} + \varepsilon_{ij}$$

($\varepsilon_{ij}$'s are iid with mean 0 and variance $\sigma^2$)
all contrasts among $\beta_j$'s are always estimable.

(c) If one is interested in obtaining BLUE's of estimable functions of $\beta$ and in obtaining the usual unbiased estimator of $\sigma^2$, deleting one observation from $(Y, X\beta, \sigma^2 I)$ is equivalent to considering $(Y, X\beta + u\gamma, \sigma^2 I)$ where u is a suitably chosen column vector.

(d) If we have a two-way classified data (say with the classifications : A classes and B classes) with unequal number of observations per cell, the unadjusted sum of squares and the adjusted sum of squares for differences among A classes can never be equal.
(Assume that there is no interaction between A classes and B classes.) [20]

4. In order to estimate the parameters in the Cobb-Douglas Production function, 15 observations were taken :
$(Q_t, K_t, L_t)$, $t = 1,\ldots,15$ where $Q_t$ = index of output in the $t^{th}$ period

$K_t$ = index of capital in the $t^{th}$ period

and $L_t$ = index of labour in the $t^{th}$ period

The following is a summary of the data obtained.

$\Sigma(\log Q_i)^2 = 313.5576$; $\frac{1}{15}\Sigma \log Q_i = 4.57127$

$\Sigma(\log K_i)^2 = 309.8788$; $\frac{1}{15}\Sigma \log K_i = 4.54317$

$\Sigma(\log L_i) = 316.9182$; $\frac{1}{15}\Sigma \log L_i = 4.59601$

$\Sigma \log Q_i \log K_i = 311.665$

$\Sigma \log Q_i \log L_i = 313.201$

$\Sigma \log Q_i \log L_i = 313.299$.

Assume the model

$$Q_t = \alpha K_t^{\beta_1} L_t^{\beta_2} e^{u_t}$$

with suitable assumptions on the error term $e^{u_t}$ (to be specified by you).

(a) Estimate $\alpha, \beta_1$ and $\beta_2$ using the method of least squares and compute their standard errors.

(b) Test $H : \beta_1 + \beta_2 = 1$ against $K : \beta_1 + \beta_2 < 1$.

(c) Based on the data, comment on the usefulness of K and L to predict Q using the multiplicative model.

[27]

5. The following table gives the yield of Latex under uniform conditions (X) in 1991 and under treatment (Y) in 1992 for each of 9 rubber plants. Three treatments A,B,C were find out each treatment being applied to 3 plants chosen at random. From the yield data, examine whether there are any treatment differences eliminating variability in the normal yield rates.(State your assumptions clearly).

Yields

| Treatment | | | | |
|---|---|---|---|---|
| A | Y | 16 | 17 | 12 |
| | X | 15 | 5 | 12 |
| B | Y | 23 | 18 | 16 |
| | X | 6 | 15 | 18 |
| C | Y | 22 | 25 | 29 |
| | X | 15 | 10 | 24 |

[18]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year: 1993-94

Economics III
Semestral-I Examination

Date : 24.11.1993 Maximum Marks : 100 Time : 3 Hours.

Note : Answer any four of the following questions. Figures in the margin indicate full marks.

1. Suppose the government raises investment expenditure by Rs.2000 crores by disinvesting in the shares of public sector enterprises by exactly the same amount. How would this affect the fiscal deficit, the revenue deficit and the current account deficit in the balance of payments ?

[25]

2. What is seniorage (or inflation tax) ? [5]

Explain whether there is any limit to the use of seniorage as a means of financing government expenditure.

[20]

3. Do you agree with the view that devaluation improves the balance of trade of a developing country, but only at the expenses of inflation and lower capacity utilization ? Give reasons for your answer.

[25]

4. Consider an economy where production is limited by the shortage of imported inputs. Examine the efficacy of a fully fluctuating exchange rate system in raising domestic absorption and containing the inflationary pressure.

[25]

5. In an economy all interest rates are administered and there is credit rationing. What will be the effects of

(a) an increase in all interest rates by the same percentage points;

(b) an increase in only the interest rates on bank deposits; and

(c) a decrease in the Statutory Liquidity Ratio ?

[25]

6. Examine, in terms of an explicit model, the implications of the withdrawal of food subsidy in a dual economy.

[25]

[1993-94 B24(b)]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III YEAR: 1993-94
SEMESTRAL-I BACKPAPER EXAMINATION
SAMPLE SURVEYS

Date: 31.12.93 Maximum Marks: 100 Time: 3 Hours

Note: Attempt all questions.

1.(a) Distinguish between sampling and non-sampling errors in a survey. List down various sources of non-sampling errors and discuss briefly the methods of assessment and control of these errors.

(b) Let the population size be 3 and the sample size be 2 and let $s_1 = \{U_1, U_2\}$, $s_2 = \{U_1, U_3\}$ and $s_3 = \{U_2, U_3\}$. Consider sampling design given by $p(s_i) = 1/3$ for $i = 1, 2, 3$. Define the estimator $t(s)$ by

$$t(s) = \begin{cases} (y_1+y_2)/2 & \text{, if } s_1 \text{ occurs} \\ (y_1/2)+(2y_3/3) & \text{if } s_2 \text{ occurs} \\ (y_2/2)+(y_3/3) & \text{if } s_3 \text{ occurs} \end{cases}$$

and let $\bar{y}$ be the usual sample mean.

Show that $\bar{y}$ and $t(s)$ are both unbiased for $\bar{Y}$ and that there exist populations $(Y_1, Y_2, Y_3)$ for which $V(t) > V(\bar{y})$. What does this example illustrate? (3+6+5)+(6+3)=23

2.(a) Define 'Sampling Design' and 'Sampling Scheme'. Explain with illustrations what you understand by the terms 'inclusion probability of a Unit' and 'joint inclusion probability of a pair of Units'.

(b) Define the estimator $\hat{Y}_{HT}$ for the populations total Y of a characteristic y based on a given sampling design with positive inclusion probabilities as suggested by Horvitz and Thompson and establish its unbiasedness. Give an expression for the variance of $\hat{Y}_{HT}$.

(c) Write down the unbiased estimator of variance of $\hat{Y}_{HT}$ as suggested by Yates and Grundy indicating when this is applicable.

(d) When a a sample is selected with selection probabilities for units at each draw equal to $p_i$ and with replacement, write down the corresponding Horvitz-Thompson estimator.

(2+2+2+2)+(2+3)+5+3=(21)

p.t.o.

3. As a consultant on 'Sampling Teachniques' indicate the suggesti[illegible] you would offer and the advice you give on the following proble[illegible] (preceise mathematical statements will fetch you more marks):

(a) A school has 1217 pupils from whom the headmaster took a sample of 120 pupils by a linear systematic sample with a random start 5 using an alphabetical frame. He has than computed the average height based on the sample and wanted to know if this is a 'valid' estimate' of the average height for this school; if not he is interested in getting a 'valid' estimate together with its estimated error.

(b) A District Health Officer wants to estimate the number of discharges from hospitals in his district during a month. He tried to obtain a list of all hospitals but could not do so. On the basis of an 'efficient' sampling technique he wishes to obtain his estimate and an estimated sampling error.

(c) A survey team selected 50 households from a village consisting of 625 households by simple random sampling without replaceme[illegible] and estimated the monthly expenditure on 'panmasala' to be Rs.4.20 with a standard error of 0.47. They wish to do the same survey in a neighbouring village, but this time using srs with replacement scheme. They also would like to have a sample size which would ensure that the length of the confidence interval at 95% level is 20% of the true value.

(6+6+6=18)

4. The table below presents the summary of data for all the 112 villages in a certain tehsil in WestBengal. The villages are stratifie[illegible] by size of their agricultural area into 3 strata, as shown in col. (2) of the table. The number $N_i$ of villages in different strata is given in col.(3). Col.(4) gives the number of villages $n_i$ selected from the i th stratum and in col.(5) the sampling design used in the stratum is given. The yield of jute for the selected villages and the auxiliary information where available are given in col.(6) of the table:

contd. ....3/-

| Stratum | Size of the villages in acres of agricultural area | $N_i$ | $n_i$ | Sampling design | Yield jute $y_{ij}$ | Auxiliary information on agricultural area $x_{ij}$ |
|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | | (6) |
| I | ≤40 | 51 | 6 | Probability Proportional to agricultural area with replacement | 75<br>101<br>5<br>78<br>79<br>45 | 25<br>39<br>2<br>26<br>26<br>16 |
| II | >40 and ≤80 | 38 | 4 | circular systematic sampling with two independent random starts | srs sample -1<br>247<br>238<br>359<br>125 | sus sample 2<br>256<br>214<br>368<br>141 |
| III | >80 | 23 | 2 | Probability proportional to agricultural area without replacement | 427<br>326 | 147<br>101 |

t is also known that the total agricultural area of villages in the first, econd and third stratum are 1268,3443 and 3112 acres respectively.

a) Estimate the total yield of jute for the tehsil

b) Obtain an estimate of the sampling error of your estimator in (a) above

c) If a survey were planned next year how do you propose to allocate a total sample of size 12 to the strata? (14+18+6=38)

[1993-94 | 37A(b)]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS) III YEAR 1993-94
SEMESTRAL-I BACKPAPER EXAMINATION
PHYSICS-I

Date: 3.1.94 Maximum Marks: 100 Time: 3 Hours

Note: Given: Charge of an electron: $1.6\times10^{-19}$C $=4.8\times10^{-10}$ Stat-coul(esu)

Rest mass of an electron: $9.1\times10^{-31}$kg

velocity of light: $3\times10^{8}$m/s

Planck's constant: $6.63\times10^{-34}$ J-s

Mev = $1.6\times10^{-13}$J

1 amu=931 Mev

Symbols have their usual meanings.

Answer both Group A and Group B.

## GROUP A (Total marks = 40)

Answer all questions.

1. Write short notes on: i) constraint of motion, ii) force of constraint, iii) generalized coordinates, iv) degrees of freedom for a mechanical system. $(4\times2\frac{1}{2})$ = [10]

2.(a) Derive Lagrange's equations of motion from Hamilton's principle.

(b) Write down the Lagrange's equation of motion for a one dimensional harmonic oscillator. (7+3)= [10]

3.(a) What is infinitesimal contact transformation? Justify the following statement: ''The motion of the system may be regarded as a succession of infinitesimal contact transformation where Hamiltonian plays the role of the generator of the system.''

(b) Define Poisson bracket of two functions with respect to the canonical variables (q,p).
Show that Poisson bracket of $q_i, p_j$ with respect to the canonical variables q,p is $\delta_{ij}$. (6+2+2)= [10]

4. Derive Hamilton's canonical equations of motion. Compare these equations with Lagrange's equations of motion regarding the solution of a mechanical problem.
Define Lagrange's bracket of two functions with respect to the canonical variables (q,p). (6+2+2)= [10]

## GROUP-B (Total marks=60)

Answer all questions.

1.(a) A rod of rest length 1m is moving longitudinally on a smooth table with velocity 0.8 c relative to the table. A circular hole of rest diameter 1m. lies in its path. i) What is the diameter of the hole as seen by the rod? ii) What is the length of the rod as seen by the hole? Does the rod fall into the hole? [6]

p.t.o.

(b) A 0.5 MeV electron moves at right angles to a magnetic field in a path whose radius of curvature is 2 cm. What is the magnetic induction B? By what factor does the effective mass of the electron exceed its rest mass? [6]
=12

2.(a) An object moves with speed u at an angle θ to the X-axis in system S. A second system S' moves with speed v relative to S along X. What speed v' and angle θ' will the object appear to have to an observer in S'? [7]

(b) Give the wavelength shift, if any, in the Doppler effect for the sodium $D_2$ line (5890 °A) emmitted from a source moving in a circle with constant speed 0. C measured by an observer fixed at the centre of the circle. [5]
= [12]

3.(a) Show that the Kelvin-Planck and the Clauseies statements of the second law of thermodynamics are equivalent. [7]

(b) Air, initially at a temperature of 15°C, is allowed to expand adiabatically until its volume is doubled.
What is its new temperature? If the pressure was originally atmospheric, what would be the final pressure? [5]
= [12]

4.(a) Two identical bodies of constant heat capacity at temperatures $T_1$ and $T_2$ are used as reservairs for a heat engine. If the bodies remain at constant pressure, and undergo no change of phase, show that the amount of work obtainiable is

$W = C_p \,(T_1+T_2-2T_f)$ where $T_f$ is the final temperature. [6]

(b) Give a brief description of Carnot cycle. Find an expression for the efficiency of the cycle. [6]
= [12]

5.(a) Explain Bragg's theory of reflection of X-rays by crystals. [6

(b) Distinguish between nuclear fusion and nuclear fusion. Why does $U^{235}$, and not $U^{238}$ undergo fusion with thermal neutrons? [6]
[12]

# INDIAN STATISTICAL INSTITUTE
## B.STAT.(HONS.) III YEAR: 1993-94
## SEMESTRAL-I EXAMINATION
## PHYSICS-I

Date: 25.11.93 | Total Marks: 100 | Time: 3 Hours

Note: Given: Charge of an electron: $1.6\times10^{-19}$ C $= 4.8\times10^{-10}$ stat-coul.(esu)

Rest mass of an electron: $9.1\times10^{-31}$ kg.

velocity of light : $3\times10^{8}$ m/S

Planck's constant : $6.63\times10^{-34}$ J-S.

1 MeV $= 1.6\times10^{-13}$ J

1 amu $=$ 931 MeV

Symbols have their usual meanings.

Answer both Group A and Group B.

### GROUP A (Total marks = 35)

Answer question No.1 and any two from the rest.

1.(a) "For studying mechanics of a system of particles Lagrangian formulation is much more convenient than Newtonian formulation" - Explain it with one reason which you consider most significant. (4)

(b) What is constrained motion? What are the main classifications of constraints? Explain with examples. (2+3)

(c) In what way is Hamiltonian formalism different from Lagrangian formalism? Explain, mentioning corresponding spaces. (3)

(d) What is Hamiltonian of a system, and in what conditions Hamiltonian will be equal to total energy? (3)

=15

2. What do you mean by forces of constraints? Deduce Lagrange's equations of motion from D'Alemberts' principle. Mention categorically how the motion with constraints have been managed? (2+6+2) = 10

3. What is the main purpose of canonical (contact) transformation? Describe Hamilton - Jacobi method for solving the motion of a mechanical system. (3+7) = 10

4.(a) A circular hoop rolling down an inclined plane without slipping. Prove that the frictional force of constraint is $1/2$ Mg Sin $\phi$, where M is the mass of the hoop, g is the acceleration due to gravity and $\phi$ is the angle of inclination of the plane to the horizontal.

p.t.o.

4.(b) A heavy particle hanging from a fixed point by a light flexible and inextensible string and made to oscillate in a vertical plane. Determine its equation of motion by Hamilton's canonical equation of motion, and, assuming amplitude of the motion being small enough, find the period of oscillation. (5+5)=10

GROUP B (Total marks = 65)

Questions 1,2 and 3 are compulsory. Answer any three from the rest.

1.(a) What is the proper time interval between the occurrence of two events if in some inertial frame the events are separated by $5\times10^8$ m and occurs 1.5 S apart? (3)

(b) Suppose that a particle moves parallel to the X-X' axis, that v = 25,000 mi/hr and $u_x' = 25,000$ mi/hr. What percent error is made in using the Galilean rather than the Lorentz equation to calculate $u_x$? Speed of light is $6.7\times10^8$ mi/hr. (3)

(c) Show that the relativistic acceleration transformation equation may be written as

$$a_x' = \frac{a_x(1-v^2/c^2)^{3/2}}{(1-u_x v/c^2)^3} \qquad (4)$$

=10

2.(a) A body of rest mass $m_0$ travelling initially at a speed 0.6 c makes a completely inelastic collision with an identical body initially at rest. What is the speed of the resulting single body? What is its rest mass? [4]

(b) Show that $p = \frac{\sqrt{K^2+2m_0c^2K}}{c}$ (3)

where p and K denote momentum and kinetic energy respectively. =7

3.(a) Is a straight world line between two events in Minkowski space shorter or longer proper time than a curved world line connecting these same events? Explain. (12)

OR

(b) Show explicitly that two successive Lorentz transformations at right angles ($v_1$ along X' and $v_2$ along Y') do not commute. Show further, that in whatever order they are applied, the result is not the same as that of a single Lorentz transformation with a velocity $\bar{v} = \hat{i}\, v_1 + \hat{j}\, v_2$. (12)

contd. ....3/

4.(a) Show that the first TdS equation may be written as

$$TdS = C_v dT + \beta T/_k \, dV \quad (3)$$

(b) One mole of an ideal gas is expanded from $T, p_1, V_1$ to $T, p_2, V_2$ in two stages - in the first stage pressure changes from $p_1$ to $p'$ and volume changes from $V_1$ to $V'$, and in the second stage, the corresponding changes are $p' \longrightarrow p_2$ and $V' \longrightarrow V_2$. For what value of $p'$ does the above work attain its maximum and what is the maximum value of work? (5)

(c) Two bodies of equal and constant thermal capacity C and at absolute temperatures $T_1$ and $T_2$ are allowed to attain the same temperature by being placed in direct thermal contact. Show that the entropy change of the universe is

$$C \, l_n \frac{T_1 + T_2}{4T_1T_2} \quad (4)$$

=12

5.(a) Show that for 1 gm-mole of real gas (obeying van der Waal equation) and undergoing adialatio change,

$$T(v-b)^{R/C_v} = \text{constant.} \quad (5)$$

(b) Starting with the 1st T-dS equation, write down an expression for the variation of $C_v$ with volume.
If the equation of state of a system is given by $\frac{PV}{RT} = 1 + B/V$,

where $B = f(T)$, show that

$$C_v = \frac{-RT}{V} \frac{d^2}{dT^2}(TB) + (C_v)_o$$

where $(C_v)_o$ represents the value of $C_v$ when V is very large.

(2+5)
=12

6.(a) Calculate the energy generated in MeV when 0.1 Kg of $L_i^7$ is converted to $H_e^4$ by proton bombardment. Given: masses of $L_i^7$, $H_e^4$ and $H^1$ in amu are —— 7.0183, 4.0040, 1.0081 respectively. (5)

(b) Discuss the Bethe carbon - nitrogen cycle for the production of stellar energy. (7)

=12

contd. ...4/

7.(a) Show that the angular frequency of a charge moving in a uniform magnetic field is given by

$$\omega = \left(\frac{qB}{m}\right)\sqrt{1-u^2/c^2} \text{ relativestically.}$$

Compare this with the classical result upon which some cyclotron designs are based. (6)

(b) A sample of characoal from an archeological site is found to have about 1/4 the $C^{14}$ activity per gram of carbon found in living wood. How old is the archeological site? Take $\lambda$ for $C^{14}$ as $1{\cdot}24 \times 10^{-4}\ yr^{-1}$. (6)

=12

---

1993-94 | 254

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR: 1993-94
SEMESTRAL-I EXAMINATION
ANTHROPOLOGY

Date: 26.11.93 Maximum Marks: 100 Time: 3 Hours

Note: Use separate answerscript for Group A and Group B.
Answer all questions from Group A, and answer any five questions from Group B.

GROUP A

1. Define human biology and bring out its difference from classical physical anthropology. [20]

OR

Enumarate briefly the major changes that took place in anatomical characteristics of man due to assumption of erect posture.

2. State briefly the salient features of Darwin's theory of evolution. In what way it differs from Lamarckism? [20]

3.(i) Does the uniqueness of anthropology as a discipline lie in its holistic approach or subdisciplinary specialisation? [2]

(ii) Homeostasis essentially involves maintenance of a stable internal environment of an organism. True or false? [2]

(iii) Preferential choice of mate in the South Indian Hindu society is generally made on the basis of (A) Homogamy, (B) Hypergamy (C) Uncle-niece or Cousin relationship, (D) Social status. [2]

(iv) Univariate analysis is more useful in adoptational studies and multivariate analysis is more useful in population comparison. True or false? [2]

(v) "Fecundity" represents the actual event of child bearing (i.e., total number of live births) while "Fertility" implies total potential capacity for child bearing. True or false? [2]

GROUP B

1.(a) State briefly the Mendel's laws of inheritance. [4]

(b) A husband and his wife are with normal skin pigmentation. Their first child is an albino (autosomal recessive trait). Give genotypes of the parents and the albino child. What is the chance of their second and third child of being an albino? [6]

p.t.o.

2. Give an account of normal human chromosomes with special reference to their number, structure and classification. [10]

3. State briefly with suitable example the criteria of inheritance of traits due to sex linked dominant and sex linked recessive genes. [10]

4. What will be the blood types of children in the following parental combinations?
   (a) M x N (b) MN x N (c) N x MN (d) O x AB (e) A x B. [10]

5. (a) Define Hardy-Weinberg principle. [2]
   (b) Assume a population in which the blood group genes O, A and B are in the proportions .6, .3 and .1. If marriages occur at random, what will be the frequencies of persons with the four blood groups? [4]
   (c) In United States the frequency of the 'O' blood group is .67 and Rh positive is .66. What is the frequency of persons who are group 'O' and Rh positive, assuming a long history of random mating? [4]

6. Write short notes on any <u>two</u> of the following:
   (a) Autosomal recessive traits in man [5]
   (b) ABO blood groups [5]
   (c) Twins [5]
   (d) Kleinefelter's Syndrome [5]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR:1993-94
SEMESTRAL-I EXAMINATION
DIFFERENTIAL EQUATIONS

Date:26.11.93 Maximum Marks:100 Time: 3 Hours

NOTE: Answer any FIVE questions.

1.(a) Obtain the general solution of the ODE's [3+3+4]

(i) $\frac{dy}{dx} - y \tan x = \sec x$ (ii) $(y+x)\frac{dy}{dx} - y+x = 0$

(iii) $(1+y^2e^{2x})\frac{dy}{dx} + y = 0$ (You may put $y=ue^{-x}$).

(b)(i) Solve the ODE [5+5]

$$\frac{dy}{dx} + y = e^{2x}\cos 3x.$$

(ii) Find the function $f(x)$ which satisfies

$$f(x) = 2 + \int_1^x f(t)dt.$$

2.(a) Verify that $y = e^{x^2}$ is a solution of the ODE [10]

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2-2)y = 0.$$

Hence or otherwise find its general solution.

(b) Given that one solution of the ODE [10]

$$x^2\frac{d^2y}{dx^2} - 2y = 0$$

is $y_1(x) = x^2$, find the general solution of the ODE

$$x^2\frac{d^2y}{dx^2} - 2y = 2x-1$$

3.(a) Find two linearly independent solutions (in powers of x) of the ODE [10]

$$\frac{d^2y}{dx^2} + x^3\frac{dy}{dx} + x^2y = 0$$

(b) Find the general solution of the ODE [10]

$$\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 1$$

in the neighbourhood of $x = 0$.

p.t.o.

4. For the ODE

$$x \frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + \alpha y = 0 \quad (\alpha \text{ is a constant}),$$ [4+10+6]

(i) show that the equation has a regular singular point at x=0,

(ii) find a solution $\varphi$ of the form

$$\varphi(x) = x^{\rho} \sum_{n=0}^{\infty} c_n x^n,$$

(iii) show that if $\alpha = n$, a non-negative integer, there is a a polynomial solution of degree n.

5.(a) Let $J_\nu(x) = \sum_{0}^{\infty} \frac{(-1)^m (x/2)^{2m+\nu}}{m!\,\Gamma(m+\nu+1)} \quad (\nu \geq 0).$ [6+6]

(i) Show that $J_\nu(x)$ is a solution of the ODE

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \nu^2)\, y = 0$$

(ii) $\frac{d}{dx}(x^{-\nu} J_\nu(x)) = -x^{-\nu} J_{\nu+1}(x)$

(b) Prove that the problem [8]

$$\frac{dy}{dx} = 3y^{2/3} \text{ with } y(0)=0$$

has infinitely many solutions. Discuss what condition is violated here for the existence of a unique solution to the problem.

6.(a) Compute the first four successive Picard iterates for the ODE [10

$$\frac{dy}{dx} = x^2+y^2, \quad y(0)=0.$$

(b) A material point moves from the point A(1,0) to the point B(2,1) with velocity v = x in the xy-plane. Find its path so that the time taken is minimum. [10]

7.(a) Solve the PDE [10]

$$2y(u-3)u_x + (2x-u)u_y = (2x-3)y$$

Given that u=0 on $x^2+y^2 = 2x$.

(b) Reduce the PDE [10]

$$y^2u_{xx} - 2xy\,u_{xy} + x^2\,u_{yy} = \frac{y^2}{x}\,u_x + \frac{x^2}{y}\,u_y$$

into its canonical form and hence find its general solution.

---

1993-94 | 318

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year: 1993-94
SEMESTRAL II EXAMINATION

Statistical Inference II

Date: 25.4.1994 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: Answer ALL the questions.

1. Describe the concept of Pitman's asymptotic relative efficiency. [10]

2. Describe the Kolmogorov-Smirnov test for goodness of fit. Show that the one-sample Kolmogorov-Smirnov statistics $D_n^+$ and $D_n^-$ are identically distributed (assume the underlying distribution to be continuous). (6+9) = [15]

3. Consider the two-sample problem where the populations sampled may be assumed to be normal. Suppose that only the ranks of the observations are available, the original observations having been lost. How would you test the hypothesis that the two populations are identical against a location alternative ? [6]

EITHER

4. Consider the paired sample problem. Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be a random sample of n pairs.

   (a) Construct an upper confidence bound for the median $\theta$ of the difference X-Y on the basis of the Wilcoxon signed rank statistic.

   (b) How do you use this confidence bound to test the hypothesis $H_0: \theta = 0$ against an appropriate one-sided alternative ? (18+6) = [24]

(a) Suppose we have two independent random samples from two populations with continuous and strictly increasing d.f.'s F and G. What is the level - $\alpha$ test based on the Mann-Whitney U-statistic for the null hypothesis $H_0$: F = G against the one-sided alternative H : $F(x) \geq G(x)$ for all x, $F(x) > G(x)$ for some x?

Show that the same test is also a level - $\alpha$ unbiased test for the null hypothesis $H_0'$ : $F(x) \leq G(x)$ for all x against the same alternative.

contd..... 2/-

(b) Consider the shift model $G(x) = F(x-\theta)$ for all $x$, in a two-sample location problem where $F$ and $G$ are d.f.'s for the two populations and $\theta$ is the amount of shift.

Show that the Hodges-Lehmann estimator $\hat{\theta}$ is distributed symmetrically about $\theta$ if the distribution $F$ is symmetric about some point. (13+11) = [24]

State the fundamental identity of sequential analysis and using this obtain approximate expressions for the O.C. and A.S.N. function of the SPRT. [20]

[illegible]

2. Let $X_1, X_2, \ldots,$ be i.i.d. $N(\mu, \sigma^2)$ where both $\mu$ and $\sigma^2$ are unknown. Describe Stein's double sampling procedure and using this obtain the following:

(a) A bounded length confidence interval for $\mu$ with a given confidence level $(1-\alpha)$.

(b) An unbiased estimator for $\mu$ for which the variance is bounded by some preassigned number not depending on $\sigma$.

(c) A test for $H_0: \mu = \mu_0$ against $H_1: \mu = \mu_1$ $(\mu_1 > \mu_0)$ for which the error probabilities are less than or equal to two pre-assigned numbers $\alpha$ and $\beta$ respectively.

(5+6+5+9) = [25]

OR

3. (a) Let $X_1, X_2, \ldots$ be i.i.d. $\sim B\,in(1,\theta)$, $0 < \theta < 1$. Using a sequential procedure obtain an unbiased estimator of $\frac{1}{\theta}$ which attains the corresponding Cramer-Rao lower bound. Justify your answer.

(b) Suppose we want to find a good unbiased estimate of $\theta$ on the basis of i.i.d. observations from a $N(\theta,1)$ population. Compare the best fixed sample size procedure with the sequential procedures.

(c) Write a short note (with illustrative examples) on the need for a sequential procedure.

(12+5+8) = [25]

---

:bcc:

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1993-94
SEMESTRAL II EXAMINATION

1993-94 | 223

# Introduction to Stochastic Process

Date: 29.4.1994 | Maximum Marks: 100 | Time: $3\frac{1}{2}$ hours

Note: All Markov Chains (MC) considered in this question paper are homogeneous in time. The whole question paper carries 115 marks. The maximum you can score is 100. Answer as much as you can.

1. Consider a Markov Chain $X_n$, $n = 0, 1, 2, \ldots$ with state space $S = \{0, 1, 2, \ldots\}$ and transition probabilities given by

$$p_{i\,i+1} = \frac{1}{i+1}, \quad p_{i0} = \frac{i}{i+1} \qquad i = 0, 1, 2, \ldots$$

Determine whether this MC is transient, null-recurrent or positive recurrent. [15]

2. Consider a discrete time MC with states $0, 1, 2, \ldots, N$ and transition probabilities given by

$$p_{ij} = \begin{cases} \beta_i & \text{if } j = i-1 \\ \alpha_i & \text{if } j = i+1 \\ 1-\alpha_i-\beta_i & \text{if } j = i \\ 0 & \text{if } |j-i| > 1 \end{cases}$$

Suppose that $\alpha_0 = \beta_0 = \alpha_N = \beta_N = 0$ and all the other $\alpha_i$'s and $\beta_i$'s are positive. Find the absorption probabilities $\tau_k$ at 0 given that $X_0 = k$, $k = 1, 2, \ldots, N-1$. [15]

(a) For a discrete time MC $X_n$, $n = 0, 1, 2, \ldots$ prove that for all $i, j$ in S

$$f_{ij}^{*} = \lim_{N \to \infty} \frac{\sum_{n=1}^{N} p_{ij}^{(n)}}{\sum_{n=0}^{N} p_{jj}^{(n)}}$$

where $f_{ij}^{*} = P\{X_n = j \text{ for some } n \geq 1 \mid X_0 = i\}$ and $p_{ij}^{(n)}$'s are the n-step transition probabilities.

(b) Define an essential state. Show that for a finite MC a state is transient if and only if it is not essential.

contd..... 2/-

(c) Define the period of a state. Let $P = ((p_{ij}))$ be the transition matrix of an irreducible MC and suppose that $P$ is idempotent (i.e., $P^2 = P$). Prove that $p_{ij} = p_{jj}$ for all $i, j$ in $S$ and the MC is aperiodic (Hint: consider the limits of $\frac{1}{n+1}\sum_{0}^{n} p_{ij}^{(\nu)}$ ).

[15 + (2+4) + (3+6)] = [30]

4. A person who owns r umbrellas distributes them between home and office according to the following routine. If it is raining upon departure from either place, an event that has probability p, say, then an umbrella is carried to the other location, if available at the location of the departure. If it is not raining, an umbrella is not carried. Let $X_n$ denote the number of umbrellas available at whatever place the person happens to be departing from on the nth trip, $n = 0, 1, 2, \ldots$ .

(a) Determine the transition probability matrix and the stationary initial probability distribution (steady-state) for this MC.

(b) Let $0 < \alpha < 1$. What is the minimum number of umbrellas that the person should own so that the probability of getting wet under the steady state distribution is at most $\alpha$ against a given value of (climate) p ? What is the minimum number r which works against all p, $0 < p < 1$, for the probability $\alpha$ ?

[(5+10) + 5] = [20]

5. Suppose d identical machines are subject to failures and repairs. Times - to - failure are independent exponential random variables with parameter $\beta$. Repair times are independent (also independent of failure times) exponential with parameter $\alpha$. Assume that there is only one repairman. Let $X_t$, $t \geq 0$ denote the number of machines that are in working condition at time t. This can be modelled as a birth and death process. Find the infinitesimal parameters and determine the stationary initial probability distribution.

[15]

6. Consider a pure death process $X_t$, $t \in [0, \infty)$ on $S = \{0, 1, 2, \ldots\}$ with death rates $\beta_i = i\beta$, $i = 0, 1, 2, \ldots$

(a) Write down the forward differential equations.

(b) Find $P_{ii}(t)$.

(c) Solve for $P_{ij}(t)$ in terms of $P_{ij+1}(t)$

(d) Find $P_{i\,j-1}(t)$. $P_{cc-1}(t)$

(e) Show that $P_{ij}(t) = \binom{i}{j}(e^{-\beta t})^{j}(1-e^{-\beta t})^{i-j}$ $\quad 0 \leq j \leq i$

(4+2+3+3+8) = [20]

:bcc:

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1993-94
SEMESTRAL II EXAMINATION

Design of Experiments

Date: 2.5.1994 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: Answer any FOUR questions from Group A.
Marks are indicated within brackets.

GROUP - A

1.(a) Three treatments A, B, C are to be compared using a completely randomized design where A, B, C are replicated $r_1$ times, $r_2$ times and $r_3$ times respectively. If $\frac{1}{3}[V(\hat{\tau}_A - \hat{\tau}_B) + V(\hat{\tau}_A - \hat{\tau}_C) + V(\hat{\tau}_B - \hat{\tau}_C)]$ is to be minimized how would you choose $r_1$, $r_2$ and $r_3$ given that (i) $r_1 + r_2 + r_3 = 21$ and (ii) $r_1 + r_2 + r_3 = 23$ ?

(b) Obtain the efficiency of a latin square design relative to (i) completely randomized design and (ii) randomized block design ignoring column classification.

(7+6+4+6) = [23]

2.(a) (i) Show that for an equireplicate block design with equal block sizes

$$V(\underline{\ell}'\hat{\underline{\tau}}) \geq \frac{(\underline{\ell}'\underline{\ell})}{r}\sigma^2$$

where r is the replication number and $\underline{\ell}'\underline{1} = 0$.

(ii) Characterize the design for which equality holds in case (i) for all $\underline{\ell}'\underline{\tau}$ with $\underline{\ell}'\underline{1} = 0$.

(b) Show that the maximum number of m.o.l.s of order v is v-1. Construct a complete set of m.o.l.s of order 4.

(4+5+4+10) = [23]

. (i) Show that under missing plot situation,

$$\min_{\underline{w}} SSE_{\Omega^*(\underline{w})} = SSE_{\Omega^*(\underline{w}^*)} = SSE_{\Omega_e}$$

where the symbols have their usual significance.

contd..... 2/-

(ii) Show that $\underline{u}$ is unique iff

$$R\begin{bmatrix} X_e \\ \cdots \\ X_u \end{bmatrix} = R[X_e].$$

(16+7) = [23]

4. What is a split-plot design ? Write an appropriate linear model for a split-plot design in r randomized blocks with p main treatments and q sub-treatments and indicate its analysis of variance explaining clearly how the various sums of squares are computed. Give also the table of various comparisons and their estimated standard errors.

(2+3+14+4) = [23]

5.(a) Identify all the confounded interaction components in one replication of a $3^5$ experiment in blocks of size $3^2$, of which the following constitute a block:

(2 0 1 2 2)
(1 1 2 2 2)
(0 0 2 0 0)
(2 1 0 0 0)
(1 2 1 0 0)
(1 0 0 1 1)
(0 2 0 2 2)
(0 1 1 1 1)
(2 2 2 1 1)

(b) Construct the key blocks for a balanced confounded $2^5$ design in blocks of size $2^3$, saving main effects and two factor interactions. Also, for each replication, indicate how the other blocks can be formed from the key blocks.

(9+14) = [23]

GROUP - B

6. Practical Records. [8]

---

:bcc:

[1993-94] 268

# INDIAN STATISTICAL INSTITUTE
## B.STAT.(HONS.) II YEAR
## ANTHROPOLOGY
## SEMESTRAL-II EXAMINATION

Date: 6.5.94 Maximum Marks: 100 Time: 3 Hours

## GROUP A

1. Explain what do you understand by the discipline Anthropology. Discuss its relationship with Sociology and human genetics. [5+5+5]

OR

State embryological, palaeontological and anatomical evidences for biological evolution. [5+5+5]

2. Write seven distinctive characteristic features of the order Primate and ascertain the position of man in the major Primate taxa. [7+3]

3. Write notes on any three of the followings: [15]
(a) Life table (b) Neolithic culture (c) Hypoxia (d) Adaptive radiation (e) Family (f) Genetic drift

4. Fill up the gaps

(i) Racial classification of mankind was first made by ____________

(ii) Marriage of a woman with more than one man is designated as ____________

(iii) Racial classification of Indian population was first made by ____________

(iv) Total reproductive performance of an organism is known as ____________

(v) Tree climbing habit of the anthropoid apes is known as ____________

(vi) Orang utan belongs to the family ____________

(vii) The earliest stone tools are called ____________

(viii) Aurignacian is the name of an industry of ____________ period

(ix) Science of classification of animals is called ____________

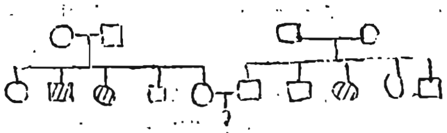
(x) Rate of change of biological evolution is very slow whereas for cultural evolution it is ____________.

p.t.o.

## GROUP B

Noté: Answer any five questions from group B.

1. Give an account of Mendel's Laws of inheritance and illustrate with suitable experiments. [10]
2. State briefly with suitable example the criteria of inheritance of traits due to autosomal dominant and autosomal recessive genes. [10]
3. (a) Define Hardy-Weinberg principle. [4]
   (b) Assume a population in which the blood group genes O, A and B are in the propertions. .5, .3 and .2, if marriages occur at random, what will be the frequencies of persons with four blood groups? [6]
4. (a) Following meiosis, each sperm or egg cell contains how many chromosomes? [2]
   (b) In meiosis, how many sperm cells are produced from a single male cell? [2]
   (c) Can mutation be induced? [2]
   (d) What type of offspring could be produced between a taster and non-taster parents? [2]
   (e) What are the possible chromosomal constituents of Down's Syndrome? [2]
5. (a) A man with blood group A had parents with blood groups O and A. This man marries a woman with blood group B but who had parents with blood group AB and O. Workout the possible blood groups which the children of the couple can have. [3]
   (b) Laura and Michael (II-5 and II-6) meet while attending a lecture concerning a rare human metabolic disease that is inherited as an autosomal recessive trait. They both have a sib who has this disease. They fall in love but are apprehensive about having children. What is the chance of their first child. [3]



contd. .....3.

5.(c) A normal woman whose father was colour-blind (sex-linked recessive) marries a normal man. Show the expected genotype of the children and its proportion. [4]

6. Write short short notes on any five of the following:

(a) Alleles, (b) Autosome, (c) Barr body, (d) Rh incompatibility, (e) Y-linked genes (f) sex-chromosome. [10]

----------

# INDIAN STATISTICAL INSTITUTE
## B.Stat. (Hons.) III Year: 1993-94
## SEMESTRAL II EXAMINATION

### Economics IV

Date: 6.5.1994 Maximum Marks: 100 Time: 3 hours

Note: This question paper carries 125 marks.
You can answer any part of any question.
The maximum that you can score is 100.
Marks allotted to each question are given within brackets.

1.(a) Describe the Breusch-Pagan test for testing for the presence of heteroscedasticity among the disturbances of a regression model. Explain in what ways this test could be considered to be more general than the other standard tests like the Goldfeld-Quandt test or the Glesjer test.

(b) Consider a system of two seemingly unrelated regressions in which the regressors for the second equation are a subset of the regressors of the first equation. Show that the ordinary least squares (OLS) estimator of $\beta_2$ (the vector of coefficients of the second equation) is the same as the seemingly unrelated regression estimator of $\beta_2$.

$(\overline{10+4}+11) = [25]$

2.(a) "A significant DW test statistic value does not necessarily mean that there is an autocorrelation problem." Explain clearly the implications of this statement. Hence, indicate the different solutions to the problem of "autocorrelation" in a regression model.

(b) Describe briefly the different methods of estimation that are available for estimating a linear regression model with an AR(1) disturbance term.

(13+12) = [25]

3.(a) Consider the regression model $y_i = \beta_1 x_{1i} + \beta_2 x_{2i} + \epsilon_i$, $i = 1, 2, \ldots, n$, where $x_1$ is not observed. The observed values are given by $\tilde{x}_{1i} = x_{1i} + u_i$, where u is uncorrelated with $x_1, x_2$ and $\epsilon$. Suppose we now drop $x_2$ from the equation and run an OLS regression of y on $\tilde{x}_1$ to obtain an estimator $\hat{\beta}_1$ of $\beta_1$. Show that $\text{plim}\ \hat{\beta}_1 = \beta_1 - \gamma\beta_1 + \beta_2 b_{21}$, where $\gamma = \text{var}(u)/\text{var}(\tilde{x}_1)$ and $b_{21}$ is the regression coefficient from a regression of $x_2$ on $\tilde{x}_1$.

contd..... 2/-

(b) In the context of instrumental variable (IV) method of estimation, state the assumptions that the instrumental variables must satisfy and then prove that the IV method would always produce consistent estimators of the regression coefficients.

(c) Use a simple errors-in-variables model to show how lagged values can be used as instruments for consistent estimation of the model.

(9+9+7) = [25]

4.(a) Consider the following regression equation:

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \varepsilon_t .$$

This model was estimated by OLS procedure, based on 26 observations. The results are given below.

$$\hat{y}_t = 2 + \underset{(1.9)}{3.5x_{1t}} - \underset{(2.2)}{0.7x_{2t}} + \underset{(1.5)}{2.0x_{3t}}, \quad R^2 = 0.982,$$

where t-ratios are given in parentheses. The same model was estimated with the restriction $\beta_1 = \beta_2$, and the following results were obtained:

$$\hat{y}_t = 1.5 + \underset{(2.7)}{3(x_{1t} + x_{2t})} - \underset{(2.4)}{0.6x_{3t}}, \quad R^2 = 0.876.$$

(i) Test the significance of the restriction $\beta_1 = \beta_2$. State also the assumptions under which this test is valid.

(ii) Suppose that $x_2$ is dropped from the equation. Would $R^2$ as well as $\bar{R}^2$ rise or fall ? Explain.

(b) Write a short note on the method of ridge regression.

(7+7+11) = [25]

5.(a) Derive the rank condition for identifiability of an equation in a simultaneous equations system when the prior informations are in the form of linear homogeneous restrictions on the structural coefficients.

(b) Consider the following simultaneous equations model

$$y_1 = \alpha y_2 + \delta x + \varepsilon_1$$
$$y_2 = \beta y_1 + \gamma x + \varepsilon_2 ,$$

where x is exogenous and the error terms $\varepsilon_1$ and $\varepsilon_2$ have mean zero and are uncorrelated.

(i) Write down the equations expressing the reduced form coefficients in terms of structural parameters.

(ii) Show that if $\gamma = 0$, then $\beta$ can be identified. Are the parameters $\alpha$ and $\delta$ identified in this case ? Explain.

:bcc:

(12+5+8) = [25]

INDIAN STATISTICAL INSTITUTE

203, Barrackpore Trunk Road

Calcutta 700 035

---

B.Stat Yr.III Second Semestral Examination Physics II

Full marks 100 Time 3 Hrs. Date: 6.5.94

---

Instructions : Answer Groups A and B in separate answer booklets.

Given:

1 eV = $1.6\times10^{-19}$ Joule

charge of an electron e = $1.6\times10^{-19}$ coulomb = $4.8\times10^{-10}$ cgs(esu)

rest mass of an electron $m_e = 9.1\times10^{-31}$ kg

rest mass of a proton $m_p = 1836\ m_e$

Planck's constant = $6.6\times10^{-34}$ J-s

velocity of light c = $3\times10^{8}$ m/s

nuclear radius ~ $10^{-14}$ m

Symbols have their usual meanings.

---

## GROUP A ( Full marks = 60 )

Answer all questions.

1a. When 400 eV electrons are diffracted by a crystal, the angular diffraction pattern is identical with that produced by X rays of wavelength 0.01 Å . Calculate Planck's constant.

b. An intense beam of electrons is diffracted through a fine hole in a screen A so as to produce a diffraction pattern on a sensitive screen B on the far side of A. Each electron received on the screen B produces an observable flash of light. Will the diffraction pattern persist if the beam intensity is reduced to a very low value ? Explain.

P.T.O.

c. Show, on the basis of de Broglie's hypothesis, that it is impossible to find the exact location of an electron in an atom.

(6+6+6)=18

2a. At time t = 0, the state of a free particle is specified by a wave function $\psi(x,0) = Ae^{-(x^2/a^2)+ik_0x}$. (i) Determine A and the region where the particle is localized. (ii) Determine the probability current density j .

b. A quantum particle, free to move on a straight line, has state function $\psi(x) = (1/2a)^{1/2}$ , $|x| \le a$

$\psi(x) = 0$ , $|x| > a$ .

Show that the relative probability of finding the particle with momentum $\pi\hbar/2a$ and zero is $4/\pi^2$.

(10+10)=20

3a. Show that $\left[ x , [ x , H ] \right] = -\hbar^2/m$ .

b. Write down Dirac's relativistic wave equation for a single particle of mass m. What are its advantages over Klein-Gordon equation ?

c. Write down the energy eigen values of Dirac equation. How do you interpret the results ?

d. Would you expect spin angular momentum to commute with a function of x and p ? Why ? Name a particle with spin 1/2 and another with spin zero.

e. Would you consider the spherical polar coordinate $\phi$ as a proper dynamical variable in the quantum mechanical context ? Give reasons for your answer.

(8+4+4+4+2)=22

GROUP B (Full marks = 40 )

---

Answer question 1 and any two from the rest.

1. Short answer type. Attempt all.

a. State the basic postulates of general ensemble theory in quantum statistical mechanics. [2+2] P.T.O.

b. Write down the expressions for the statistical distribution of particles $n_r$ in the $r^{th}$ state for BE and FD systems. Explain physically the effect on $\bar{n}_r$ of a BE system if the chemical potential $\mu$ > energy level of a single particle state $\varepsilon_r$ for all r. 4

c. Justify the statement physically:
"As the temperature T → 0, the lowest energy of a FD system is greater than the lowest energy of a BE system." 4

d. The logarithm of a grand canonical partition function ln Z has a linear relationship with the volume V when V is large. With this assumption show that for an ideal gas the equation of state becomes $\bar{p}V = kT \ln Z$. 4

2. In an isolated statistical system, when mechanical interaction takes place, the number of microstates within a given energy range E to E + $\delta$E is given by $\Omega(E,x)$ when x is some external parameter of the system. If x changes due to the mechanical interaction and $\bar{X}$ be the generalised force conjugate to x, show that

$$\bar{X} = 1/\beta \cdot (\partial \ln \Omega(E,x) / \partial x)$$ 12

3. Consider an ideal gas consisting of N distinguishable particles obeying classical statistics. Suppose that energy-momentum relationship of a single particle is $\varepsilon = pc$ when c is a constant. Show that the canonical partition function of the system is
$Z(N,V,T) = (8\pi V\ (kT/ch)^3)^N$.
Also find out the Helmholtz free energy. [10 + 2]

4. Show that when the density of a gas consisting of particles with mass m is sufficiently low and its temperature is sufficiently high so that the condition mean deBroglie wavelength ($\lambda_{av}$) << mean distance between particles ($r_{av}$) is satisfied, one can use MB statistics as a good approximation irrespective of whether the particles obey Fermi or Bose statistics. 12

INDIAN STATISTICAL INSTITUTE
203, Barrackpore Trunk Road
Calcutta 700 035

---

B.Stat Yr.III — Second Semestral Examination — Physics II
(Back Paper)

Full marks 100 — Time 3 Hrs. — Date: 26.6.94

---

Instructions : Answer Groups A and B in separate answer booklets.

Given:

1 eV = $1.6\times10^{-19}$ Joule

charge of an electron e = $1.6\times10^{-19}$ coulomb = $4.8\times10^{-10}$ cgs(esu)

rest mass of an electron $m_e = 9.1\times10^{-31}$ kg

rest mass of a proton $m_p = 1836\ m_e$

Planck's constant=$6.6\times10^{-34}$ J-s

velocity of light c = $3\times10^{8}$ m/s

nuclear radius ~ $10^{-14}$ m

Symbols have their usual meanings.

---

## GROUP A ( Full marks - 60 )

Answer all questions.

1a. Describe briefly an experimental evidence that demonstrates the wave-like properties of moving electrons.

b. Calculate the glancing angle at which electrons of energy 100 eV must be incident on the lattice planes of a metal crystal in order to give a strong Bragg reflection in the first order. Given lattice spacing = 2.15 Å. (10+10)=20

2a. A particle is in a one dimensional potential well $0 < x < a$, for which V=0 inside the well and V = $\infty$ outside. Solve the time-independent Schrödinger equation for this case.

b. Find the matrix element $\langle n' | x^2 | n\rangle$ , using

$x = (\hbar/2m\omega)^{1/2} (a + a^{+})$ and $p = i(m\hbar\omega/2)^{1/2} (a^{+} - a)$, $a$ and $a^{+}$ being the destruction and creation operators respectively.

(10+8)=18

3a. Consider a one-dimensional harmonic oscillator. Using Heisenberg's equation of motion, calculate the time dependence of $a$ and $a^{+}$. Hence obtain the time dependence of $\hat{x}$ and $\hat{p}$.

b. Show that $[L_i, L^2]=0$, where $L_i$ denotes the $i^{th}$ component of the angular momentum operator.

c. Write down Dirac's relativistic wave equation for a single particle of mass m.

(10+8+4)=22

## GROUP B (Full marks = 40)

Answer question 1 and any two from the rest.

1. Consider a system consisting of two particles, each of which can be in any one of the three quantum states of respective energies $0, \varepsilon$ and $2\varepsilon$. The system is in contact with a heat reservoir of temperature $T=(k\beta)^{-1}$.

a. Derive an expression for the partition function Z if the particles obey classical MB statisticsand are considered distinguishable.

b. Derive Z if the particles obey BE and FD statistics.

[4+6+6]=16

2. If a classical statistical system obeys canonical distribution, find out the expression for generalised force in the presence of mechanical interaction in terms of the partition function. Obtain also the equation of state for an ideal gas treating the system as a canonical ensemble.

[6+6]=12

3a. There are many, [illegible] two systems A, B, C......... which are almost independent [illegible] other. Suppose that they interact with each other weakly, so that they can be regarded as a compound system A+B+C+................. Show that the canonical partition function $Z_{A+B+C+........}$ and the Free energy $F_{A+B+C+........}$ are given by $Z_{A+B+C+........} = Z_A Z_B Z_C$ ................ and

$$F_{A+B+C+........} = F_A F_B F_C \quad ................$$

respectively where $Z_A, Z_B, Z_C$,................ are the partition functions of the individual systems.

b. Explain the physical significance of the chemical potential $\mu$ in a grand canonical ensemble. [8+4]=12

4. A simple harmonic one-dimensional oscillator has energy levels given by $E_n = (n+1/2)\hbar\omega$, where $\omega$ is the characteristic frquency of the oscillator and where the quantum number n can assume the possible integral values n=0,1,2,.......... Suppose that such an oscillator is in thermal contact with a heat reservoir at temperature T low enough so that $kT/\hbar\omega << 1$.

a. Find the ratio of the probability of the oscilltor being in the first excited state to the probability of its being in the ground state.

b. Assuming that only the ground state and first excited state are appreciably occupied, find the mean energy of the oscillator as a function of the temperature T. [6+6]=12

---

INDIAN STATISTICAL INSTITUTE

203, Barrackpore Trunk Road

Calcutta 700 035

---

B.Stat Yr.III — Second Semestral Examination — Physics II

(Back Paper)

Full marks 100 — Time 3 Hrs. — Date: 26.6.94

---

Instructions : Answer Groups A and B in separate answer booklets.

Given:

$1\ eV = 1.6\times10^{-19}$ Joule

charge of an electron $e = 1.6\times10^{-19}$ coulomb $= 4.8\times10^{-10}$ cgs(esu)

rest mass of an electron $m_e = 9.1\times10^{-31}$ kg

rest mass of a proton $m_p = 1836\ m_e$

Planck's constant $= 6.6\times10^{-34}$ J-s

velocity of light $c = 3\times10^{8}$ m/s

nuclear radius $\sim 10^{-14}$ m

Symbols have their usual meanings.

---

## GROUP A ( Full marks = 60 )

Answer all questions.

1a. Describe briefly an experimental evidence that demonstrates the wave-like properties of moving electrons.

b. Calculate the glancing angle at which electrons of energy 100 eV must be incident on the lattice planes of a metal crystal in order to give a strong Bragg reflection in the first order. Given lattice spacing = 2.15 $\mathring{A}$. (10+10)=20

2a. A particle is in a one dimensional potential well $0 \leq x \leq a$, for which V=0 inside the well and $V = \infty$ outside. Solve the time-independent Schrodinger equation for this case.

b. Find the matrix element $\langle n' | x^2 | n \rangle$, using each other weakly, so that they can be regarded as a compound system A+B+C+................ Show that the canonical partition function $Z_{A+B+C+........}$ and the Free energy $F_{A+B+C+........}$ are given by $Z_{A+B+C+........} = Z_A Z_B Z_C$ ................ and

$$F_{A+B+C+........} = F_A F_B F_C \quad ................$$

respectively where $Z_A, Z_B, Z_C,$................ are the partition functions of the individual systems.

b. Explain the physical significance of the chemical potential $\mu$ in a grand canonical ensemble. [8+4]=12

4. A simple harmonic one-dimensional oscillator has energy levels given by $E_n = (n+1/2)\hbar\omega$, where $\omega$ is the characteristic frquency of the oscillator and where the quantum number n can assume the possible integral values n=0,1,2,.......... Suppose that sucn an oscillator is in thermal contact with a heat reservoir at temperature T low enough so that $kT/\hbar\omega << 1$.

a. Find the ratio of the probability of the oscilltor being in the first excited state to the probability of its being in the ground state.

b. Assuming that only the ground state and first excited state are appreciably occupied, find the mean energy of the oscillator as a function of the temperature T. [6+6]=12

1993-94 328(b)

# INDIAN STATISTICAL INSTITUTE
## B.STAT.(HONS.) III YEAR
## SEMESTRAL II BACK-PAPER EXAMINATION
## INTRODUCTION TO STOCHASTIC PROCESS

Date: 22.6.94 Maximum Marks: 100 Time: $3\frac{1}{2}$ Hours

Note: All Markov Chains (MC) considered in this paper have stationary transition Probabilities. The whole question paper carries 115 marks. The maximum you can score is 100. Answer as much as you can.

1. N black balls and N white balls are placed in 2 urns so that each urn contains N balls. At each step one ball is selected at random from each urn and the two selected balls are interchanged. The state of the system at the nth stage is the number of white balls in the first urn, n = 0, 1,2,.... .

   (a) Find the transition matrix of this Markov Chain. Is the MC irreducible?

   (b) Find the stationary initial probability distribution.

   (c) What is the period of the state 0 ? What are the limit points of the sequence $\{p_{01}^{(n)}\}$ ? [5+8+(2+5)]

2. Let $X_n$, n=0,1,2,... be a MC with state space $S=\{0,1,2,..\ N\}$ and transition probabilities given by

$$p_{i\,i+1} = \frac{i}{N}(1-p) \qquad i = 0,1,2,\ldots N-1$$

$$p_{ii} = \frac{i}{N}p + \frac{(N-i)}{N}(1-p) \qquad i = 1,2,\ldots N-1$$

$$p_{i\,i-1} = \frac{N-i}{N}p \qquad i = 1,2,\ldots N$$

$$p_{00} = p_{NN} = 1 \quad , \qquad 0<p<1.$$

   (a) Find the absorption probabilities at 0 starting from i , i = 1,2,.. N-1.

   (b) Find $e_i$, i=1,2,... N-1 where $e_i$ is the expected time before being absorbed at 0 or N starting from i. [10+1

3. Let $X_n$ be a MC on the state space S.

   (a) Let i, j be two states in S such that $f^*_{ij}=f^*_{ji}=1$ where $f^*_{rs}$ denotes the probability of ever reaching s starting from r. Show that i,j are recurrent states.

3. (b) Let $(\pi_i)$ $i \in S$ be a stationary initial distribution for $\{X_n\}$ and let $i,j$ be 2 states such that $\pi_i > 0$, and $i \rightsquigarrow j$. Show that $\pi_j > 0$.

(c) Define the period of a state. Show that for any state $i$, the period $d_i$ is equal to the greatest common divisor of all $n \geq 1$ such that $f_{ii}^{(n)} > 0$. [6+6+(2+6)]

4. For a Poisson process $X_t$, $t \geq 0$ with rate $\lambda$, let $W_r$ denote the random time of occurrence of the rth Poisson event. Let $0 \leq s \leq t$ and $1 \leq r \leq n$.

Show that

$$P(W_1 \leq s \mid X_t = n) = 1 - \left(1 - \frac{s}{t}\right)^n$$

and more generally

$$P\left[W_r \leq s \mid X_t = n\right] = \sum_{k=r}^{n} \binom{n}{k} \left(\frac{s}{t}\right)^k \left(1 - \frac{s}{t}\right)^{n-k}.$$

[1

5. Consider a birth and death process $X_t$, $t \in [0,\infty)$ on $S = \{0,1,2\}$ with birth rates $\alpha_i$, death rates $\beta_i$ where

$$\alpha_2 = \beta_0 = 0, \quad \alpha_0 = \beta_2 = a, \quad \alpha_1 = \beta_1 = b$$

(a) Write down the forward equations

(b) Find $P_{00}(t), P_{01}(t)$, and $P_{02}(t) = 1 - P_{00}(t) - P_{01}(t)$ using the forward equations. [5+15]

6. A telephone exchange has $m$ channels. Calls arrive in the manner of a Poisson process with rate $\lambda$; they are accepted if there is an empty channel, otherwise they are lost (no waiting line is formed). The duration of each call is exponential with parameter $\mu$. Let $X_t$ denote the number of busy channels at time $t$. Find the birth and death rates for the process $X_t$. Find also the stationary distribution. [1