INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) III YEAR: 1996-97
SEMESTRAL-I EXAMINATION
DIFFERENTIAL EQUATIONS

Date: 15.11.96 Maximum Marks:100 Time: 3 hrs. 45 min.

Note: You may answer all the questions. But the maximum you can score is 100.

1.(a) When do you say that a function $f(x,y)$ is homogenous of degree K. Show that the variables in the equation $y' = f(x,y)$ can be seperated when $f(x,y)$ is homogenous of degree zero. (4)

(b) Solve the equation $xy'' - y' = 3x^2$. (3)

(c) Show, using the result of (a) that the equation

$y' = F\left(\frac{ax+by+c}{dx+ey+f}\right)$ can be transformed into an equation whose variables are seperable. Here a,b,c,d,e,f are constants. (10)

(d) Under what conditions on $M(x,y)$ and $N(x,y)$ does the equation $M(x,y)\,dx + N(x,y)\,dy = 0$ have an integrating factor which is a function of x alone. Prove your result. (6)

(a) A pendulum consisting of a bob of mass m at the end of a rod of negligible mass and length a is pulled aside through an angle α and released. Show that the period of oscillation T is given by

$$T = 4\sqrt{\frac{a}{2g}} \int_0^{\alpha} \frac{\theta}{\sqrt{\cos\theta - \cos\alpha}}$$

Here of g the acceleration due to gravity. (6)

(b) Find the shape assumed by a flexible chain of constant linear density, suspended between two points and hanging under its own weight. (7)

(c) Find the curve passing through the point (3,4) and such that its normal at any point coincides in direction with the radius vector to that point from the origin. (4)

.(a) State wether the following statement is true or false and justify your answer: The general solution of $x^2y'' - 2xy' + 2y = 0$ on $[-1,1]$ is given by $y = c_1x + c_2x^2$. (4)

(b) Define a regular singular point for a second order homogenous linear differential equation and explain (briefly) how you can obtain a power series solution of the equation

$y'' + P(x)y' + Q(x)y = 0$ in the neighbourhood of a regular singuler point. (6)

p.t.o.

INDIAN STATISTICAL INSTITUTE
B.STAT. (HONS.) III YEAR: 1996-97
SEMESTRAL-I EXAMINATION
SOCIOLOGY

ite: 15.11.96 Maximum Marks: 100 Time: 3 Hours

Note: Answer question 8 and any <u>four</u> of the following. Questions carry equal marks.

What is a social group? Discuss the characteristics of primary and secondary groups.

Discuss Weber's methodology with particular reference to his concept of social action.

Explain briefly the contributions of N.K. Bose to contemporary problems of Indian society.

What is structural-functional approach in academic sociology? How does it differ from evolutionary and culturological approaches? Give examples.

A,B,C,D,E,F,G,H,I,J, and K are students of the same class. But to do their studies, all do not help each other. The class teacher observed who goes to whom, and finds the following:

A goes to D,G,H,I and K. B does not go to A,C,D,E,F,G,H but goes to others, while C goes to A,B,D,K but not to others. K goes to A,C only but not to others. G,H,I and J have formed a clique and go to each other, but not to any other. Some others, however, come for help to some of them. D does not go to any one. E goes to B,D,F,K and lastly, F goes to B,E.

(a) Draw a diagram showing the above, (b) How many pairs of students help each other? (c) The class teacher wants to use this number for comparison. How should he do it?—Dividing by total number of all possible pairs out of eleven or, maximum possible number of pairs of students helping each other using the above as data? Give your arguments and obtain the standardized value.

A researcher wants to study why "child-marriage" is still practised in the villages. He can do this study by case study method or a standard survey with structured questionnaire. He is in a dilemma. Which of these two will you recommend and why?

Suppose, you want to study the impact of T.V. on village life. Illustrate how you may use what you have learnt from sociological theories in formulating questions in this regard.

Write short notes on (Any two)

(a) Sanskritization

(b) Organic solidarity

(c) Participant - observation

(d) Isolate.

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) III YEAR : 1996
SEMESTRAL-I BACKPAPER EXAMINATION
LINEAR STATISTICAL MODELS

Date:2.1.1997 Maximum Marks:100 Time: 3 Hours

Note:The paper carries 115 points. All questions are compulsory.

1. Let $X_1,\ldots,X_n$ and θ be i.i.d $N(0,1)$ random variables. Define

$$Y_i = \frac{i}{(n+1)}\theta + X_i, \quad 1 \le i \le n.$$

Find out the exact distribution of $S_n = \sum_{i=1}^{n}(Y_i-\bar{Y})^2$. Statement of all relevant results is needed. [15]

2. Show that the matrix given by

$$\begin{bmatrix} 0.65 & 0.45 & 0.05 & -0.15 \\ 0.45 & 0.35 & 0.15 & 0.05 \\ 0.05 & 0.15 & 0.35 & 0.45 \\ -0.15 & 0.05 & 0.45 & 0.65 \end{bmatrix}$$

is an orthogonal projection matrix. Find out the subspace, $\mathcal{V}$, on which the projection operates. Also, obtain the new projection matrix onto the subspace given by $\mathcal{V} + \langle(1,-1,-1,1)'\rangle$. [10+10=20]

3. Consider an interaction model of the following form:

$$y_{ijk} = \mu + a_i - a_j + t_{ijk}\eta_{ij} + \varepsilon_{ijk}, \quad \text{for } i > j,$$

$$y_{ijk} = \mu + t_{ijk}\eta_{ij} + \varepsilon_{ijk} \quad \text{for } i = j,$$

where $1 \le j \le i \le 2$ and $1 \le k \le 2$. The constants t_{ijk} are given constants.

(a) Write down the column space of the design matrix.

(b) Are the elementary contrasts namely, $(a_i - a_j)$, estimable under this model? If so, obtain their BLUEs.

(c) Obtain the F-statistic for the hypothesis $H_0: \eta_{11} = \ldots = \eta_{rr}$, $\eta_{ij} = 0$ for $i \ne j$.

(d) Write down a suitable ANOVA decomposition for testing the equality of the treatment effects nested in the hypothesis (b). [5+5+10+10=30]

p.t.o.

4. A consumer organisation studied the effect of age of automobile owner on size of cash offer on a used car by utilising 7 persons in each of three age groups (young, middle, elderly) who acted as the owner of the used car. A medium price, three year old car was slected for the experiment, and the 'owners' solicited cash offer for this car from 21 dealers selected at random from the dealers in that region.

i	j: 1	2	3	4	5	6	7
1 young	23	25	21	22	21	22	20
2 middle	28	27	27	29	26	29	27
3 elderly	23	20	25	21	22	23	21

(a) Obtain the fitted values.

(b) Obtain the residuals.

(c) Construct the ANOVA table.

(d) What appears to be the nature of relationship between age of owner and mean cash offer? [10+5+10+5=]

5. Consider the non-linear interaction model given by

$$Y_{ijk}=\theta+\alpha_i+\beta_j+\eta\alpha_i\beta_j+\varepsilon_{ijk},$$

where $1 \le i \le r$, $1 \le j \le b$, $1 \le k \le m$.

(a) Suggest a suitable iterative algorithm for computing the least squares estimates of the model parameters. Give only the heuristics why you think this algorithm might work.

(b) Suggest a F-type test statistic for testing $H_0:\eta=0$. [20]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) III YEAR:1996-97
SEMESTRAL-I BACKPAPER EXAMINATION
ECONOMICS-III

Date: 3.1.97 Maximum Marks: 60 Time: 3 Hours

Note: Answer all questions

1. What is the efficiency wage hypothesis? Show how involuntary unemployment may exist if the efficiency of a warker is related to his income. (15)

2. Show that the seasonality in agricultural price is related to the market structure and storage cost. Calculate the social welfare (which is equal to consumers' plus producers' surplus) maximizing seasonality if the storage cost is zero. (15)

GROUP-B

3. Critically examine how the various economic policies adopted in India since 1991-92 differ from those pursued earlier. (15)

4. Write a note on the debate between farm size and productivity in the context of India's agriculture. (15)

INDIAN STATISTICAL INSTITUTE
B.STAT. (HONS.) III YEAR: 1996-97
SEMESTRAL-I BACKPAPER EXAMINATION
DIFFERENTIAL EQUATIONS

Date: 3.1.97 Maximum Marks: 60 Time: 3 Hours

1. (a) For a body of Mass m, starting from rest at time zero and falling freely under gravity, prove the principle of conservation of energy. (6)

   (b) What is the differential equation for the motion of a body falling under gravity and for which air resistance is proportional to velocity. (3)

2. (a) State necessary and sufficient condition for the equation $M(x,y)dx + N(x,y)dy = 0$ to be exact. Prove your result. (7)

   (b) Solve the equation $(x+y)dx - (x-y)dy = 0$ (4)

3. (a) Suppose y_1 and y_2 are two solutions of the equation $y''+P(x)y' + Q(x)y = 0$ on the interval $[a,b]$. Show that their wronskian is either identically zero or never zero on $[a,b]$. (5)

   (b) The equations of motion of a particle moving according to Newton's law of gravitation is in polar co-ordinates given by

   $$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = \frac{-k}{r^2} \; ; \; r^2\frac{d\theta}{dt} = h.$$

   Obtain the equation of the trajectory of the particle, with a suitably chosen polar axis. (7)

   (c) Suppose $u(x)$ is a non-trivial solution of the equation $u''+q(x)\,u = 0$ on $[a,b]$, where $q(x)<0$. $\forall\ x \in [a,b]$. Then u has at most one zero. Prove this statement. (7)

4. Let x_0 be an ordinary point of the differential equation

   $y''+P(x)y'+Q(x)y = 0$ and let a_0 and a_1 be arbitrary constants. Show that this equation has a unique solution $y(x)$, which is analytic at x_0 and satisfies $y(x_0) = a_0$ and $y'(x_0) = a_1$. (7)

5. (a) Express $\cos n\theta$ as a function $T_n(\cos\theta)$ of $\cos\theta$, where $T_n(x)$ is a polynomial. (3)

   (b) Show that $T_n(x) + T_{n-2}(x) = 2x\,T_{n-1}(x)$. (3)

p.t.o.

5. (c) Show that $T_n(x)$ satisfies the differential equation

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + n^2y = 0 .$$

(d) Let $P_n(x)$ denote the n^{th} Legendre polynomial.

Show that $\int_{-1}^{1} P_m(x)\, P_n(x)\, dx = 0$ if $m \neq n$.

6. (a) Show that $f(x,y) = y^{1/2}$ does not satisfy a Lipschitz condition on the rectangle $|x| \leq 1$, $0 \leq y \leq 1$.

(b) Solve the system of equations

$$\frac{dy}{dx} = z, \quad \frac{dz}{dx} = -y, \quad y(0) = 1, \quad z(0) = 0.$$

INDIAN STATISTICAL INSTITUTE
B.STAT III (1996-97)
SEMESTER 2 EXAMINATION
STOCHASTIC PROCESSES

DATE: 2 MAY 1997 | **MAXIMUM MARKS :100** | **TIME 3 HOURS**

Note: You may answer as many questions as you like. The whole question paper carries 115 marks but the maximum you can score is 100. All Markov chains considered in this paper are homogeneous in time.

1. Consider the MC $\{X_n, n = 0, 1, 2, ...\}$ with the state space $S = \{0, 1, 2, 3, 4, 5\}$ and the transition matrix P given by

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{3}{8} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & 0 \\ 0 & \frac{1}{8} & 0 & 0 & \frac{7}{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(a) Determine all the recurrent and the transient classes.
(b) Find the absorption probability at 0, starting from each of the transient states.
(c) Find the following limits :

$$\lim_{n\to\infty} p_{14}^{(n)}$$
$$\lim_{n\to\infty} p_{21}^{(n)}$$
$$\lim_{n\to\infty} p_{23}^{(n)}$$

[5 + 8 + 12]

2. Let $\{X_n, n = 0, 1, 2, ...\}$ be a MC with state space S. Let $f_{ij}^{(*)}$ be the probability of ever reaching j from i and let g_{ij} be the probability of visiting j infinitely often starting from i, for i, j in S.

(a) Show that $\lim_{N\to\infty} \frac{\sum_{n=1}^{N} p_{ij}^{(n)}}{\sum_{n=1}^{N} p_{jj}^{(n)}} = f_{ij}^{(*)}$

(b) Show that for i, j belonging to the same recurrent class $g_{ij} = 1$.

[10 + 10]

3. Consider a MC $\{X_n, n = 0, 1, 2, ...\}$ on the state space $S = \{0, 1, 2, ..., d\}$ and the transition probabilities given by :

$$p_{i,i-1} = \frac{i}{d}, \; p_{i,i+1} = 1 - \frac{i}{d} \text{ and } p_{ij} = 0 \text{ otherwise.}$$

Find the stationary initial distribution.

[10]

4. (a) Define an essential state. Show that in a finite MC a state is transient if and only if it is not essential.

P.T.O

(b) Define the period of a state. Show that a finite irreducible MC is aperiodic if and only if there exists an integer $n \geq 1$ such that $p_{i,j}^{(n)} > 0$ for all i, j.

[5 + 10]

5. Let $\{X_t, t \geq 0\}$ be a Poisson process with intensity parameter λ, $(\lambda > 0)$. Suppose that each arrival for the Poisson process $\{X_t, t \geq 0\}$ is registered with probability p, $(0 < p < 1)$ independent of other arrivals. Let $\{Y_t, t \geq 0\}$ be the process of registered arrivals. Prove that $\{Y_t, t \geq 0\}$ is a Poisson process with rate λp.

[10]

6. There are 2 external P&T lines connected with the NE20M telephone exchange of the ISI campus. P&T calls arrive at this exchange according to a Poisson process at the rate of 10 calls per hour. Any incoming call is accepted if at least one line is free, otherwise (i.e. if both lines are busy) the call is lost. The durations of the calls are iid exponential random variables with mean 3 minutes and are independent of the incoming Poisson process. Let $\{X_t, t \geq 0\}$ denote the number of busy lines at time t, $t \geq 0$.
(a) Set this up as a birth and death process and find the infinitesimal parameters.
(b) Find the long run (steady state) probability that both lines are busy.

[6+9]

7. Write down the Forward differential equations for the continuous time Markov process $\{X_t, t \geq 0\}$ with state space $S = \{0, 1, 2\}$ and the infinitesimal parameters given by

$$Q = \begin{pmatrix} -1 & 1 & 0 \\ 2 & -4 & 2 \\ 0 & 1 & -1 \end{pmatrix}$$

Solve these equations to find $P_{00}(t)$, $P_{01}(t)$, $P_{02}(t) = 1 - P_{00}(t) - P_{01}(t)$ explicitly.

[5+15]

INDIAN STATISTICAL INSTITUTE
Final Examination: Semester II, (1996-97)
Inference-II
B-III

Date 5.5.97 | Maximum marks: 50 | Time 3 hours

1. (a) Let $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ be i.i.d. samples from two continuous distributions with densities f and g, respectively, where $g(x) = f(x - \delta)$, f being the density of $U[0,1]$, and $0 \leq \delta \leq 1$. Express $p_1 = P[X_1 < Y_1]$ and $p_2 = P[X_1 < Y_1, X_1 < Y_2]$ in terms of δ and find

$$\lim_{\delta \to 0} \frac{Var(U_{12}|\delta)}{Var(U_{12}|\delta = 0)},$$

where $U_{12} = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} I_{(X_i < Y_j)}$.

(b) Define efficacy. Find the efficacy of the Wilcoxon test to test $H_0 : \delta = 0$ vs $H_1 : \delta > 0$, [for f in part (a)], and compute the Pitman efficiency of the Wilcoxon test w.r.t. the t-test. [5+5]

2. (a) Twenty five (n) observations $X_1, \ldots, X_{25}$ are to be taken from an unknown density f. Define the kernel density estimator of $f(2.5)$ based on a uniform kernel $U(-\frac{1}{2}, \frac{1}{2})$ and bandwidth $h = 0.5$. Find the exact bias of this estimator when $f(x) = \frac{2}{25}x \times I_{0<x<5}$.

(b) [bonus question] Without redoing the computation in part (a), [provide some intuitive argument] indicate the changes in the bias of the estimator in part (a), for each of the following three alterations: (i) sample size n is changed to 5; (ii) n unchanged, but h is increased to 1; (iii) n, h unchanged, but consider estimating $f(2)$. [5+4]

3. A procedure is supposed to choose treatment X and treatment Y in a random fashion. If the records in the past year show the following order of assignment of treatments,

$$XXXYXXXXYYYXYXYYYXYYYXYYXXXXYY$$

$$XXXXXXXXXXXYYYYYYYXYYYXXXXXYYY$$

Compute the p-value approximately for the run-test. [5]

4. The following fifteen sample observations are obtained from a continuous distribution.
.257, .414, .814, .853, .889, .302, .687, .940, .583, .497, .458, .771, .882, .813, .940

(a) Using an appropriate nonparametric procedure, test at 7% level of significance if the 40th percentile of the population is different from 0.40.

(b) Use the Signed Rank test (at 5% level) to test $H_0 : \mu(\text{median}) = 0.5$ vs $H_1 : \mu > 0.5$. [5+5]

5. Consider a standard 'Secretary's problem' with n candidates, where a candidate may be chosen only at the time of completion of his/her interview. Show that if the objective is only to maximize the probability of selecting the best candidate, then the optimal strategy is to interview at least r candidates and then choose the first candidate who is at least as good as any of the previous candidates, where r is the smallest integer satisfying

$$\sum_{i=r}^{n-1} \frac{1}{i} \leq 1,$$

and is approximately given by n/e. [10]

6. Two methods of producing items need to be compared. Mutually independent experiments are being performed in pairs. Let $S_m^{(i)}$ be the number of non-defective items in the first m experiments with method i ($i = 1, 2$). The process is stopped when $S_m^{(1)} - S_m^{(2)}$ is $\leq -r$ or $\geq s$. Find the explicit expressions for (i) the probability that the method 1 will be declared better and (ii) the expected number of pairs to be examined. [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year: 1996-97
SEMESTRAL-II EXAMINATION

Biology II

Date: 9.5.1997 Maximum Marks: 100 Time: 3 hours

PART I

1. Justify the following statement (any three) : (3 x 5) = [15]

(a) $$r = \frac{2.303\,(\log_{10} w - \log_{10} w_o)}{(t_2 - t_1)}$$

when r = The growth rate
w = Final dry weight
w_o = Initial dry weight
t_1 = Initial time
t_2 = Final time.

(b) The dam phase is much more essential to synthesis of 'florigen'.

(c) Structural variation of Stomata may resist the entry of pathogenic organism.

(d) Different histological changes occur in the host plant which prevent the further spread of the pathogen.

2.(a) How the accumulation of phenolic substance in the infected host cause protection against pathogen ? [7]

(b) Write in brief note on factors affecting enzyme activity. [8]

3. Give a brief description about the classification of enzyme. [10]

PART II

4. Write in brief the different cultural practices for direct seeded and transplanted rainfed rice. [10]

5. What are the differences between manures and fertilizers ? Find out the amount of FYM, Urea, Single super phosphate and KCl for rice if the recommendation of nutrients are

contd..... 2/-

120 kg N + 80 kg P_2O_5 + 60 kg K_2O. 50% of the recommended nitrogen should be given through FYM.

(5+10) = [15]

6. What are the different yield attributing characters of rice ? What would be the yield of rice grain if:

   (a) The average reproductive tiller/m^2 - 260.

   (b) Average panicle length - 12 cm.

   (c) Average numbers of grain/panicle - 150

   (d) Unfilled grain - 15%

   (e) Test weight - 22 g.

[10]

7. Describe the nitrogen cycle in brief with suitable diagram.

[10]

8. Write short notes on (any five):

   (a) Gravitational water

   (b) Monsoon onset.

   (c) Water balance

   (d) Field capacity

   (e) Compound fertilizer

   (f) Micro nutrients.

   (g) Clay-loam soil.

(3 x 5) = [15]

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:bcc:

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III YEAR:1996-97
SEMESTRAL-II EXAMINATION
ECONOMICS-IV

Date:9.5.97 Maximum Marks:100 Time: 3 Hours

Note: Answer question no.1 and any four from the rest of the questions. Allotted marks are given at the end of each question.

1. Examine whether the following statements are true (T), false (F) or neither true nor false (U). Give a short explanation in each case.

(a) In the presence of heteroscedasticity the usual OLS method always overestimates the standard errors of estimators.

(b) If a nonstochastic regressor is omitted from a linear regression model, the residuals will be heteroscedastic.

(c) When the least - squares technique is applied to a regression model estimated on the basis of economic time series data, it yields biased estimates because many economic time series are autocorrelated.

(d) The method of instrumental variables gives unbiased estimates of the coefficients of a linear regression model.

(e) An estimation of the demand function for steel gave the price elasticity of demand for steel as +0.3. This finding may be interpreted to mean that the price elasticity of supply is at least +0.3.

(f) In a simultaneous equation system, the more the number of exogenous variables, is the better.

(g) In the 2SLS method, only the endogenous variables on the right hand side of the equation should be replaced by their estimated values from the reduced form. One should not replace the endogenous variable on the left hand side by its estimated value from the reduced form.

(h) If the disturbance term in a linear regression model is not independently distributed with a common variance σ^2, the OLS estimates of the regression coefficients are always less efficient than the GLS estimates.

[$8 \times 2\frac{1}{2} = 20$]

p.t.o.

2. Consider the following regression results based on 20 observatio

$$\hat{y}_t = \hat{\beta}_1 + \hat{\beta}_2 x_t$$

where $\hat{\beta}_1 = -261.09$, $\hat{\beta}_2 = 0.2453$ s.e.$(\hat{\beta}_1) = 31.327$.

The t-value for testing significance of β_2 has been found to be 16.616.

(a) Find s.e. $(\hat{\beta}_2)$, r^2 and t-value for testing significance of β_1

(b) How do you interpret $\hat{\beta}_1$ and $\hat{\beta}_2$?

(c) Test the hypothesis that $\beta_2 = 0$.

(d) Test significance of r^2.

(e) Are the answers you have obtained in (c) and (d) in conflict? If they are not, what explains the harmony between these answ

[10+4+2+2+2=20]

3. Assume that in the following regression model

$$y_t = \beta_1 x_{t1} + \beta_2 x_{t2} + e_t,$$

all the variables are measured from their respective means. If β_1 is estimated from the regression of y on x_1 with x_2 omitted, show that the resulting estimate is, in general, biased but has smaller variance than the estimate detained with x_2 included. Wh will the MSE, $E(\hat{\beta}_1 - \beta_1)^2$ be smaller for the regression with x_2 omitted? [2

4. Describe an iterative procedure for estimating the coefficients of a linear regression model when the disturbance term follows first order autoregressive process and the structure of autocorrelation is unknown. Indicate its properties.

5. Define heteroscedasticity. Prove that the best linear unbiased estimators of the paramiter v ctor of a linear regression model are provided by the method of weighted least squares, provided heteroscedastic error variances σ_t^2, are known. Describe any tw of the diagnostic tests of heteroscedasticity.

6. Tintner's model of the American meat market is

$$y_1 = b_{12} y_2 + a_{11} x_1 + u_1 \quad \text{(demand)}$$

$$y_1 = b_{22} y_2 + a_{22} x_2 + a_{23} x_3 + u_2 \quad \text{(supply)}$$

(a) Examine the identifiability of the two equations.

6. (b) Suppose that it is known a priori that $a_{22}/a_{23}=k$, where k is a known number. Will the identification properties of the model change? Give reason for your answer.

   (c) Give an outline of a suitable estimation procedure along with the estimators of standard errors of the coefficients of the demand equation in case of (a). [8+8+4=20]

7. What is the Least Variance Ratio method of estimation? Does it give consistent estimate? Show that it is a member of k-class estimator. [20]

8. Derive the rank and order conditions for identification of individual equations of a simultaneous linear equations model under exclusion restrictions. [20]

INDIAN STATISTICAL INSTITUTE
B. STAT. (HONS.) III YEAR: 1996-97
SEMESTRAL-II BACKPAPER EXAMINATION
INTRODUCTION STOCHASTIC PROCESSES

Date: 26.6.97 | Maximum Marks: 100 | Time: 3 Hours

Note: You may answer as many questions as you like. The total of marks assigned against the questions in this paper is 105; but the maximum you can score is 100. All Markov Chains considered in this paper are time-homogeneous.

1. A MC $\{X_n,\ n=0,1,2,\dots\}$ on states $\{0,1,2,3,4,5\}$ has transition probability matrix

$$P = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 3/4 & 0 & 0 \\ 0 & 0 & \frac{1}{5} & \frac{4}{5} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

(a) Find all the recurrent and transient classes.

(b) Find the probability of eventual absorption into the class containing the state 0, starting from each of the transient states.

(c) Find $\lim_{n\to\infty} p_{5i}^{(n)}$ for $i=0,2,4,5$. [5+8+12]

2. Consider the symmetric free random walk on the line i.e. the MC $\{X_n,\ n=0,1,2,\dots\}$ with state space $S = \{0, \pm 1, \pm 2, \dots\}$ and transition probabilities

$$p_{i\ i-1} = p_{i\ i+1} = \frac{1}{2} \text{ for all } i \text{ in } S.$$

(a) Show that this MC is recurrent.

(b) Consider the system of equations

(*) $\underset{\sim}{u} = \underset{\sim}{u} P, \quad \underset{\sim}{u} = (u_i)_{i \in S}$, a row vector.

Show that u_i = constant for all $i \in S$ are the only bounded solutions of (*). Hence conclude that this MC is null recurrent. [7+8]

contd. ....2.

3. Consider the MC $\{X_n, n=0,1,2,\ldots\}$ with state space $S=\{0,1,2,\ldots,d$ and transition probabilities given by

$$p_{00}=1 \quad , \quad p_{0j}=0 \quad \text{for } 1 \le j \le d$$

$$p_{jk} = \frac{1}{j} \quad \text{for} \quad 0 \le k \le j-1, \quad 1 \le j \le d$$

Find $E(T \mid X_0=i)$ for $i = 1,2,\ldots d$ where the random variable T is defined by $T = n$ if and only $X_n=0$ and $X_\nu \neq 0$ for $1 \le \nu \le n-1$

for $n = 1,2,\ldots,$. [10

4. (a) Show that an irreducible MC $\{X_n, n=0,1,2,\ldots\}$ is positive recurrent if and only if

$$\Sigma \, \pi_{ii} = 1$$

where S is the state space and

$$\pi_{ii} = \lim_{n\to\infty} \frac{1}{n+1} \sum_{=0}^{n} p_{ii}^{(\nu)}$$

(b) Show that if i, j are 2 states in the same recurrent class, then the probability of visiting j infinitely often starting from i is 1. [7+8]

5. Men and women enter a bank according to independent Poisson Processes having respective rates 2 and 4 per minute. Starting at an arbitrary time, compute the probability that at least 2 men enter before 3 women enter the bank. [10

6. A system is composed of N identical components; each component, independent of other components operates a random length of time until failure. Suppose the failure time distribution is exponentia with parameter λ ($\lambda>0$). When a component fails it undergoes repair immediately (enough repairmen). The repair time distribution is exponential with parameter μ ($\mu>0$). The system is said to be in state n at time t if there are exactly n components under repair at that time. Set this up as a birth and death process. Find the infinitesimal parameters. Also, find the stationary distribution. [5+10]

3. Write down the forward differential equations for a continuous time Markov Process with states $\{0,1,2\}$ and infinitesimal paramete given by

$$Q = \begin{pmatrix} -\lambda, & \lambda, & 0 \\ 0, & -\mu, & \mu \\ 0, & 0, & 0 \end{pmatrix}$$

where $\lambda,\mu > 0$.

Solve these equations to find explicitly $P_{ij}(t)$, $i=0,1,2$, $j=0,1,2$. [5+10

INDIAN STATISTICAL INSTITUTE
Back-paper Examination: Semester II, (1996-97)
B.Stat. (Hons.) III Year
Statistical Inference III

Date: 27.6.1997 Maximum Marks: 100 Time: 3 hours

1. Consider a sequential test whose stopping rule N is the minimum value of n for which $S_n = \Sigma_{i=1}^{n} Z_i$ is $\geq a$ or $\leq b$, where $\{Z_i, i \geq 1\}$ is an i.i.d. sequence of r.v.'s satisfying $P[\varepsilon_1 \leq Z_1 \leq \varepsilon_2] = 1$, for some $\varepsilon_1 < 0 < \varepsilon_2$. Show that

$$\frac{1 - e^{t_o b}}{e^{t_o(a+\varepsilon_2)} - e^{t_o b}} \leq P[S_N \geq a] \leq \frac{1 - e^{t_o(b+\varepsilon_1)}}{e^{at_o} - e^{t_o(b+\varepsilon_1)}}$$

where t_0 is the unique nonzero root of $M(t) = E(e^{tZ}) = 1$. [10]

2. Two judges rank 5 competitors in the following way:

Judge A	5	3	1	2	4
Judge B	5	2	3	1	4

Compute two nonparametric measures of dependence between the two judges' rankings. [10]

3. Let $X_1, X_2, \ldots, X_n$ be i.i.d. observations from a continuous distribution and let $X_{(1)}$ and $X_{(n)}$ be the smallest and largest order statistics.
Find the smallest n such that $(X_{(1)}, X_{(n)})$ contains the median with probability ≥ 0.99. [10]

4. Define the kernel density estimator. Describing the conditions of asymptotics, find the expressions of its mean and variance. For what choice of the bandwidth h, the asymptotic mean square error of the estimator is minimum ? Show that for such an h, the square of the asymptotic bias is a constant multiple of the asymptotic variance, and hence find an appropriate expression of the asymptotic mean square error. [15]

5. Describe the Wilcoxon-Mann-Whitney test in a two sample location problem. Explain how the p-value can be computed exactly for this test. Also suggest suitable approximation when the sample size are large. [10]

p.t.o.

6. Consider the sampling without recall problem with 6 candidates. Describe the optimal strategy to maximize the probability of selecting the best or the second best candidate. What is the probability of success in this strategy ?

[15]

7. Define sample quartile. Prove that under suitable conditions it is asymptotically normal. [15]

8. $\{X_i, i \geq 1\}$ is an i.i.d. sequence of observations from a normal distribution with mean θ and variance 4. Describe the SPRT to test $H_0 : \theta = 0$ vs. $H_1 : \theta = 1$ so that both type I and Type II errors are less than 0.05. Obtain its ASN and OC curve.

[15]

:bcc:

INDIAN STATISTICAL INSTITUTE
Backpaper Examination: Semester II (1996-97)
B.Stat.(Hons.) III Year
Differential Equations

Date: 27.6.97 Maximum Marks: ~~60~~ 100 Duration 3 hrs.

Note: *This paper carries 120 marks. You may answer all the questions. But the maximum you can score is 100.*

1. Let $f(x, y)$ be a continuous function that satisfies a Lipschitz condition

$$|f(x, y_1) - f(x, y_2)| \le K|y_1 - y_2|$$

on a strip defined by $a \le x \le b$ and $-\infty < y < \infty$. If (x_0, y_0) is any point on the strip, then show that the initial value problem

$$y' = f(x, y), \; y(x_0) = y_0$$

has one and only one solution $y = y(x)$ on the interval $a \le x \le b$.

[20]

2. *(a)* Find the orthogonal trajectories to the one-parameter family of curves given by

$$y^2 = 4c(x + c).$$

(b) A tank contains 100 gallons of pure water. Beginning at time $t = 0$ brine containing 1 pound salt per gallon runs into the tank at the rate of 1 gallon per minute, and the mixture, which is kept uniform by stirring, flows out at the same rate. When will there be 50 pounds of salt in the tank?

[10+10=20]

3. Find the general solution of the following differential equations:
   (i) $(y^2 e^{xy} + \cos x)dx + (e^{xy} + xye^{xy})dy = 0.$
   (ii) $yy'' + (y')^2 - 2yy' = 0.$
   (iii) $(x^2 - 1)y'' - 2xy' + 2y = (x^2 - 1)^2.$

[6+7+7=20]

4. *(a)* Let $u(x)$ be a non-trivial solution of

$$u'' + q(x)u = 0,$$

where $q(x) > 0$ for all $x > 0$ and is continuous on $(0, \infty)$. If

$$\int_1^{\infty} q(x)dx = \infty,$$

show that $u(x)$ has infinitely many zeros on the positive x-axis.

P.T.O

(b) Find the normal form of the Bessel's equation

$$x^2y'' + xy' + (x^2 - p^2)y = 0$$

and show that every non-trivial solution has infinitely many positive zeros.

[10+10=20]

5. (a) Find the general solution of

$$(1 + x^2)y'' + 2xy' - 2y = 0$$

in terms of power series in x.

(b) Verify that the origin is a regular singular point of the differential equation

$$x^2y'' + xy' + x^2y = 0.$$

Show also that it has only one Frobenius series solution and find that series solution.

[10+10=20]

6. (a) Find the extremals for the integral

$$I(y) = \int \frac{\sqrt{1 + y'^2}}{y}\, dx.$$

(b) Find the form of the simple closed curve Γ of length l which will enclose the maximum area.

[10+10=20]

—x—