INDIAN STATISTICAL INSTITUTE
Research and Training School
CENTRAL STATISTICAL ORGANISATION
Training Course in 'Official Statistics and related Methodology
B. Stat.(Hons.) Part IV: 1973-74
and
M. Stat. Part I: 1972-73
Final Examination

401 / 531

10.8.73

Max. marks: 100
Time: 3 hrs.

Attempt any five questions. All questions carry equal marks.

1. Describe the International statistical set up bringing out the role played by the Statistical Office of the United Nations.

OR

Describe the Statistical System in India bringing out the role played by the Central Statistical Organisation.

2. Describe the organisation and activities of either Directorate of Economics and Statistics, Ministry of Agriculture or Institute of Applied Manpower and Research. Mention the broad contents of one of the important publications of that organisation.

3. Describe the salient features of the 1971 Population Census. How it has been an improvement over 1961 census? Mention the various uses of the census data.

4. Describe the coverage, content and methods of collection of Industrial statistics. How are the data utilized for policy formulation and planning?

5. Describe briefly the method of collection and compilation of official statistics in any three of the following fields:-

    i) Foreign trade statistics
    ii) Health statistics
    iii) Road transport statistics
    iv) Educational statistics
    v) Family planning statistics

6. Describe the objectives and the procedures involved in conducting a Family Living Survey with particular reference to the working class family income and expenditure survey 1970-71.

7. Describe the various methods of estimation of National Income. Briefly indicate the method being adopted in India for one of the major sectors of economy.

OR

What is Capital formation? Explain briefly methods of estimation of Capital formation in India.

8. 'Statistics has an important role to play in the formulation and implementation of national plans for economic and social development'. Discuss.

9. Write notes on any two of the following:

    a) National occupational classification
    b) Employment/Unemployment surveys
    c) Index numbers of wholesale prices.

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PERIODICAL EXAMINATION

Statistics-II: Inference

Date: 12.11.73 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1.a) State and prove the Cramer-Rao inequality. (The regularity conditions need be stated explicitly).

b) A random sample $X_1, \ldots, X_n$ is available from a distribution with pdf.

$$p_\theta(x) = \begin{cases} \frac{x+1}{\theta(\theta+1)} \exp(-x/\theta), & x > 0, \theta > 0; \\ 0, & \text{otherwise} \end{cases}$$

Obtain an unbiased estimator of $(3+2\theta)(2+\theta)/(1+\theta)$ whose variance attains the Cramer-Rao bound. [15+10]=[25]

2.a) Let X be a Poisson (λ) variable, $\lambda > 0$. Find the unique unbiased estimator of $\exp.(-3\lambda)$. Comment on your result.

b) In Q.2(a), find an unbiased estimator of $3\lambda^3 + 7\lambda^2 + 9\lambda + 1$ whose variance attains some Bhattacharyya bound. [10+5]=[15]

3.a) Show that a necessary and sufficient condition for an estimator T to be a uniformly minimum variance unbiased estimator of an estimable parameter $h(\theta)$ is that for every g satisfying $E_\theta(g) = 0$ for all θ, $cov_\theta(T, g) = 0$ for all θ where $V_\theta(g) < \infty$ provided $V_\theta(T) < \infty$.

b) Show that a polynomial estimator, $\sum_{r=0}^{k} a_r T^r$, where a_i's are known constants is a uniformly minimum variance unbiased estimator of its expectation.

c) Let T be a uniformly minimum variance unbiased estimator of θ. Let T_1, T_2 be other unbiased estimators of θ with $V(T)/V(T_i) = e_i$ $(i = 1,2)$. Show that the correlation coefficient between T_1 and T_2 lies in the range

$$\sqrt{e_1 e_2} \pm \sqrt{(1-e_1)(1-e_2)}.$$

[10+7+8]=[25]

4.a) Define 'sufficient statistics' and 'completeness'. Illustrate these concepts with examples.

b) State the Factorization Theorem of sufficiency and prove it in the discrete case.

c) Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n $(n > 3)$ from a $N(\mu, \sigma^2)$ distribution, where μ and σ^2 are both unknown. Define $\bar{X} = n^{-1} \sum_1^n X_i$, $S^2 = \sum_1^n (X_i - \bar{X})^2$. Show that the unique uniformly minimum variance unbiased estimator of μ^2/σ^2 is

$$\frac{\bar{X}^2}{(n-3)S^2} - \frac{1}{n}.$$

[6+9+10]=[25]

5. Homework. [10]

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PERIODICAL EXAMINATION

Economics-3

Date: 19.11.73 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer all the questions.
All questions carry equal marks.

1. Write a short essay on the concept of economic development, distinguishing between the different senses in which the term can be used.

2. EITHER

Briefly trace the logical prerequisites for the occurrence of significant innovations in the field of production in an economy in a sustained manner.

OR

Give a brief account of the development of markets as an economic institution.

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PERIODICAL EXAMINATION

Measure Theory

Date: 26.11.73 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 120 marks. Answer as many questions as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

1. Let $\underline{\underline{A}}$ be a σ-field of subsets of a set X. Let f and g be two measurable functions. Let h_n for $n = 1,2,\ldots$ be a sequence of measurable functions defined as follows.

$$h_{2k} = g \text{ for } k = 1,2,\ldots \quad h_{2k+1} = f \text{ for } k = 0,1,2\ldots$$

Prove that $\{x : h_n(x) \text{ does not converge as } n \to \infty\}$ belongs to $\underline{\underline{A}}$. [15]

2. Let μ_1 and μ_2 be two measures on $(X, \underline{\underline{A}})$ such that $\mu_1(A) \geq \mu_2(A)$ for all A in $\underline{\underline{A}}$ and $\mu_1(X) = \mu_2(X) = 5$. Show that $\mu_1(A) = \mu_2(A)$ for all A in $\underline{\underline{A}}$. [10]

3. Let $\underline{\underline{S}}$ be a semifield of sets generating a σ-field $\underline{\underline{A}}$ of subsets of X. Let μ_1 and μ_2 be two measures on $\underline{\underline{A}}$ such that $\mu_1(A) = \mu_2(A)$ for all A in $\underline{\underline{S}}$. Then show that $\mu_1(A) = \mu_2(A)$ for all A in $\underline{\underline{A}}$. [20]

4. Let $(X, \underline{\underline{A}}, \mu)$ be a measure space. Let $A_n \in \underline{\underline{A}}$ for $n = 1,2,\ldots$ be a sequence of sets such that $\sum_{n=1}^{\infty} \mu(A_n) < \infty$. Prove that

a) $\mu\left(\bigcap_{n=1}^{\infty} A_n\right) = 0$

b) $\mu\left(\bigcap_{n=1}^{\infty} \bigcup_{m=n}^{\infty} A_m\right) = 0$

[Hint for part b: $\bigcup_{m=n}^{\infty} A_m \downarrow \bigcap_{m=n}^{\infty} \bigcup_{m=n}^{\infty} A_m$ as $n \to \infty$] [20]

5. Let $(X, \underline{\underline{A}}, \mu)$ be a measure space. Define the expectation of a measurable function f when

a. it is an indicator function
b. it is a step function
c. it is a nonnegative function
d. it is an integrable function. [15]

p.t.o.

6. Let $X = 1, 2, 3, \ldots$

$\underline{\underline{A}}$ = σ-field generated by doubleton subsets of X.

a. What are the measurable functions of $(X, \underline{\underline{A}})$

b. If P is a probability on $(X, \underline{\underline{A}})$, define $p_n = P(\{n\})$. Show that $p_n \geq 0$ for all n and $\Sigma p_n = 1$. Conversely if p_n is a sequence of real numbers such that $p_n \geq 0$ for all n and $\Sigma p_n = 1$ then show that there is a probability P on $(X, \underline{\underline{A}})$ such that $P(\{n\}) = p_n$. Also show that such a measure is unique.

c. Let P be a probability on $(X, \underline{\underline{A}})$. Let $p_n = P(\{n\})$. For any nonnegative measurable function f on $(X, \underline{\underline{A}})$ prove that

$$E(f) = \sum_{n=1}^{\infty} f(n) p_n .$$

[40]

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PERIODICAL EXAMINATION ..

Statistical Methods (Theory and Practical)

Date: 3.12.73 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 110 marks. Answer as many questions as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

1. State and prove Lapunov version of Central Limit Theorem. [20]

2. Let $\{T_n\}$ be a sequence of statistics such that $\sqrt{n}\,(T_n - \theta)$ is distributed asymptotically as $N(0, \sigma^2(\theta))$. Obtain the asymptotic distribution of

$$\sqrt{n}\,[g(T_n) - g(\theta)]/\,g'(T_n)\sigma(T_n)$$

where g is function of a single variable, such that g' exists. [20]

3. A sample (x_j, y_j) $j = 1, \ldots, 30$ of size 30 yielded the following summary of data

$\bar{x} = 39.5$ $\quad \Sigma x_j^2 = 50252$ $\quad \Sigma x_j y_j = 60225$

$\bar{y} = 49.5$ $\quad \Sigma y_j^2 = 81151$

Another sample of size 40 yielded

$\bar{x} = 25.5$ $\quad \Sigma x_j^2 = 40125$ $\quad \Sigma x_j y_j = 37125$

$\bar{y} = 30.5$ $\quad \Sigma y_j^2 = 42615$

a) Can you say that the samples are coming from two populations having the same correlation coefficient?

b) Can you say that the first sample is coming from a population in which the correlation coefficient is 0.223?

[20+10]=[30]

4. The following table gives the distribution of 353 individuals from a tribe T_1 and of 364 individuals from a tribe T_2 in four blood group classes O, A, B, AB.

	O	A	B	AB
T_1	121	120	79	33
T_2	118	95	121	30

Do the data suggest that the hypothetical proportions of the blood group classes in T_1 and T_2 are same? [15]

p.t.o.

5. Let $T_1, \ldots, T_k$ be k independent and consistent estimators of parameters $\theta_1, \ldots, \theta_k$ based on samples of sizes $n_1, \ldots, n_k$. Obtain the asymptotic distribution of the statistic

$$H = \Sigma w_i T_i^2 - \frac{(\Sigma w_i T_i)^2}{\Sigma w_i}$$

under $\theta_1 = \ldots = \theta_k$, where $w_i = n_i / s_i^2(T_i)$, $s_i^2(T_i)/n_i$ being the estimated asymptotic variance of T_i. It is given that $\sqrt{n_i}\,(T_i - \theta_i)$ is asymptotically normal $N(0, \sigma_i^2(\theta_i))$. Mention the assumptions (if any) which you may need in deriving the distribution. [15]

6. Three independent samples (x_{ij}) $j = 1, \ldots, n_i$ $i = 1, 2, 3$ of sizes 30, 40 and 50 drawn from the populations whose mean and variance exist. These samples yield

$\bar{x}_1 = 9.5$	$s_1^2 = 25.5$
$\bar{x}_2 = 15.5$	$s_2^2 = 49.5$
$\bar{x}_3 = 4.5$	$s_3^2 = 30.2$

s_i^2 being the variance of i^{th} sample. Do the data suggest that samples come from the populations whose mean are same. [10]

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PERIODICAL EXAMINATION

Econometrics (Theory and Practical)

Date: 10.12.73 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1. Discuss the problem of identification in the context of demand analysis based on time series market statistics of price and quantity for a single item. Suggest suitable conditions under which demand function could be estimated in such a situation. [20]

2. What is a Lorenz curve? Derive the equation of the Lorenz curve for a lognormal variate and find the expression for the Lorenz ratio. Demonstrate the properties of the Lorenz curve in this case. [20]

3.a) State the index number problem and discuss its role in the aggregation of economic relationships. [6]

b) What are the atomistic and functional approaches to the construction of price index numbers? Derive the Laspeyres, Paasche and Fisher price index numbers for binary comparisons and interpret them in the light of the above two approaches. [14]

4. The table below gives per capita expenditure on clothing for different levels of per capita total consumer expenditure from a Family budget survey.

monthly per capita expenditure class (in Rs.)	estimated %. of population	per capita expenditure on all items	per capita expenditure on clothing
(1)	(2)	(3)	(4)
0 - 8	25.03	5.86	0.19
8 - 11	20.63	9.48	0.56
11 - 13	11.33	11.98	0.77
13 - 15	8.57	14.00	1.14
15 - 18	9.34	16.50	1.48
18 - 21	6.71	19.46	2.06
21 - 24	4.99	22.37	2.29
24 - 28	3.63	25.82	3.72
28 - 34	4.44	30.30	3.27
34 - 43	2.68	37.63	5.05
43 - 55	0.99	48.73	9.27
55 -	1.63	75.49	12.73

i) Draw the unsmoothed engel curves for these data.

ii) Fit a constant elasticity engel curve to this data through weighted least squares procedure, compute also the coefficient of determination (R^2) and the standard error of the estimated engel elasticity. [25]

5. Class Exercise Records. [15]

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PERIODICAL EXAMINATION

Operations Research

Date: 17.12.73 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ]. The maximum marks you can score is 50 taking Groups A and B together.

Group A: Linear Programming

1.a) In the simplex algorithm for solving a LP problem, we want to make a non-basic variable x_j basic. Derive the rule for finding the variable to be removed from the basis.

b) Find the net change in the objective function in (a) and hence state the condition for optimality. When will the problem have unbounded solution.

c) Explain briefly the M-method with artificial variables when an initial basic feasible solution is not readily available. [8+7+5]=[20]

2.a) Write the dual problems of the following:

i) Max $z = 2x_1 + 3x_2 + x_3$
subject to
$$4x_1 + 3x_2 + x_3 = 6$$
$$x_1 + 2x_2 + 5x_3 = 4$$

ii) Min $z = -4x_1 - 3x_2$
$$x_1 \le 6$$
$$x_2 \le 8$$
$$x_1 + x_2 \le 7$$
$$3x_1 + x_2 \le 15$$
$$-x_2 \le 1$$
$$x_1, x_2 \ge 0.$$

b) Consider the primal problem Max $z = cx$ subject to $Ax \le b$, $x \ge 0$ and its dual Min $g = wb$, subject to $wA \ge C$, $w \ge 0$. Prove the following results.

i) If x is any feasible solution to the primal and w is any feasible solution to the dual, then $cx \le wb$.

ii) If $\hat{x}$ is a feasible solution to the primal and $\hat{w}$ is a feasible solution to the dual such that $c\hat{x} = \hat{w}b$, then $\hat{x}$ and $\hat{w}$ are optimal solutions to the primal and dual respectively.

iii) If either the primal or dual has an optimal solution, then the other also has an optimal solution. [7+13]=[20]

Group B: Inventory Control

3. What are the main problems of Inventory Control. Derive the Wilson square root formula for EOQ? [20]

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MID-YEAR EXAMINATION

Statistics-11: Inference

Date: 7.1.74 Maximum Marks: 100 Time: 4 hours

Note: The paper carries 120 marks. Answer as much as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [].

1.a) Give examples of (i) a sequence of unbiased inconsistent estimators and (ii) a sequence of consistent biased estimators.

b) Let $X_1, X_2, \ldots, X_{2n+1}$ constitute a random sample of size $2n+1$ $(n \geq 1)$ from a normal $(\theta, 1)$ distribution,

$$p_\theta(x) = \text{[illegible]}, \quad -\infty < x < \infty, \quad -\infty < \theta < \infty.$$

Show that the sample median is unbiased and consistent for θ.

c) Let X_1 and X_2 have a trinomial distribution with probability function

$$P_{\theta_1,\theta_2}(X_1 = x_1, X_2 = x_2) = \frac{n!}{x_1!\, x_2!\, (n-x_1-x_2)!} \times$$

$$\times\ \theta_1^{x_1}\ \theta_2^{x_2}\ (1 - \theta_1 - \theta_2)^{n-x_1-x_2},$$

where $x_1 = 0, 1, \ldots, n$; $x_2 = 0, 1, \ldots, n$; $x_1 + x_2 \leq n$;

$$0 < \theta_1, \theta_2 < 1.$$

Find the Cramer-Rao lower bounds for variances of unbiased estimators of θ_1 and θ_2, and show that there exist unbiased estimators of θ_1 and θ_2 attaining these bounds.

[5+8+12]=[25

2.a) Define 'minimal sufficient statistics'.

b) Let $X_1, X_2, \ldots, X_n$ be iid with pdf

$$p_\alpha(x) = \frac{1}{B(\alpha, \alpha)} x^{\alpha-1} (1-x)^{\alpha-1}, \quad 0 < x < 1, \quad \alpha > 0.$$

Show that $\prod_{i=1}^{n} (X_i(1-X_i))$ is minimal sufficient for α.

c) Show that if a sufficient statistic is boundedly complete, then it is minimally sufficient.

d) Let X be a rectangular $(\theta, \theta+1)$ variable, θ real. Show that X is minimally sufficient for θ, but that X is not complete.

[2+6+13+9]=[30

3.a) State and prove the Rao-Blackwell theorem.

b) Let X and Y be iid $N(0, \sigma^2)$ variables. Given n pairs of independent observed values (X_j, Y_j), $j = 1, \ldots, n$, $n \geq 2$ with this distribution, find the minimum variance unbiased estimator of the probability that the point (X_j, Y_j) falls inside the circle with centre $(0, 0)$ and radius R (R known).

p.t.o.

3.c) Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from a normal (μ, σ^2) population, where μ and σ^2 are both unknown. Let $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ denote the ordered X_i's. Define $\bar{X} = n^{-1} \sum_1^n X_i$, $S^2 = \sum_1^n (X_i - \bar{X})^2$. Show that $\sum_1^n X_i^2$ and $(X_{(n)} - X_{(1)})/S$ are independently distributed. [10+15+5]=[3

4.a) Let $X_1, X_2, \ldots, X_n$ be iid with probability function

$$P_\theta(X_1 = r) = a_r \theta^r / f(\theta), \quad r = 0, 1, 2, \ldots,$$

where $a_r > 0$ for all $r = 0, 1, 2, \ldots$; $\theta > 0$,

$$f(\theta) = \sum_{r=0}^{\infty} a_r \theta^r.$$

Show that in this case the maximum likelihood estimator of θ agrees with the estimator obtained by the method of moments.

b) Let $X_1, X_2, \ldots, X_n$ be iid with pdf

$$p_\theta(x) = \exp(\theta - x), \quad x \geq \theta,$$

where $\theta \in \Omega = \{0, \pm 1, \pm 2, \ldots\}$. Show that the maximum likelihood estimator of θ is given by the largest integer contained in T, where $T = \min(X_1, \ldots, X_n)$. Find the distribution of this estimator.

c) Give an example of a maximum likelihood estimator which is not consistent.

d) Starting with the consistency of a solution of the likelihood equation as an estimator of the true value of the parameter, prove its asymptotic normality. The regularity conditions need be stated carefully. [6+8+6+15]=[3

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MID-YEAR EXAMINATION.

Statistics-12: Statistical Methods (Theory and Practical

Date: 9.1.74 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 110 marks. Answer as many questions as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

1.a) Define the terms: order statistics, empirical distribution function, quantiles.

b) Obtain the asymptotic distribution of sample median mentioning the assumptions there in.

c) Let $\bar{x}$ be the mean of a random sample of size 50 taken from a normal population with its mean μ unknown. What size of sample would you need so that the median based on it may be equally efficient, for estimating μ, as the sample mean $\bar{x}$. [5+10+5]=[20]

2.a) Define Kolmogorov-Smirnov statistic d_n for testing the goodness of fit based on a sample of size n.

b) Thirty random observations are given below extracted from a table. Examine whether these can be regarded as a sample from a normal population with mean zero and variance unity. Upper 5 % point for limiting distribution of d_n is ... 1.36.

- 0.54	0.42	0.26	2.04	0.83	0.28
- 1.02	-1.08	0.58	0.09	-1.13	-0.62
- 0.37	-1.23	0.30	0.74	-1.59	0.06
- 0.19	1.16	-0.60	1.37	-1.08	1.30
0.22	-0.56	-0.00	-0.97	-0.55	0.30

[5+25]=[30]

3.a) Let T_n be a statistic such that it is distributed asymptotically normally around θ with variance depending on θ. Obtain a function g such that the asymptotic variance of transformed statistic $g(T_n)$ is independent of θ.

b) Describe square root transformation of Poisson variate and hyperbolic tan inverse transformation of correlation coefficient. [5+5]=[10]

4.a) Define Wilcoxan statistic T and Mann-Whitney statistic U for testing that two samples came from the same population. Obtain the relation between U and T.

b) Describe sign test for testing the hypothesis that population median is a given value MO, on the basis of a random sample of size n. [10+5]=[15]

p.t.o.

5. a) Let $T_1, \ldots, T_k$ be k-statistics with $ET_i = \theta_i$ $(i = 1, \ldots, k)$. Find the expressions for the asymptotic variance of a transformed statistic $g(T_1, \ldots, T_k)$ and co-variance between $g(T_1, \ldots, T_k)$ and $h(T_1, \ldots, T_k)$. Mention the assumptions needed in derivation.

b) Find the asymptotic variance of sample coefficient of variation. [10+10]=[20]

6. A sample of size 10 from a continuous distribution yields the observations 10, 11, 13, 20, 25, 11, 14, 15, 22, 18. Another sample of 15 from a continuous distribution yields the observations 11, 12, 13, 19, 20, 19, 27, 22, 13, 17, 19, 21, 23, 25, 28. Can you say that these samples are coming from the same population. [15]

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MID-YEAR EXAMINATION

Statistics-13: O. R. and S. Q. C.

Date: 11.1.74 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ]. Answer any three questions from Group A and all questions from Group B.

Group A

1.a) Consider the following LP problem: Max. $z = cx$, subject to $Ax = b$, $x \geq 0$. Let x_B be a basic feasible solution such that $z_j - c_j \geq 0$ for all non-basic variables. Prove that this basic feasible solution is optimal.

b) Suppose it is known that the demand for an item is as follows

$$(q_1, T_1), (q_2, T_2) \ldots (q_n, T_n),$$

where q_i is the quantity required at time T_i. It is assumed that the present stock is zero and it is required to work out a production schedule to meet this known pattern of demand up to time point T_n. It is given that the set up cost for the production run is 'C' and the cost of holding one unit of item for unit time is 'd'. Using Dynamic programming, derive the necessary recursion equations to obtain the optimum schedule. [12+13]=[25]

2. Suppose N Jobs are to be processed on two machines A and B in the same order. Let a_j and b_j be the processing times for the j^{th} job on machines A and B respectively.

a) Derive an expression for idle time on machine B for any sequence.

b) Show that it is better to have job j precede job j+1 when $\min(a_j, b_{j+1}) < \min(a_{j+1}, b_j)$.

c) State Johnson's rule for finding an optimal sequence or for minimising the total elapsed time. Find an optimal sequence for the problem

j	1	2	3	4	5
a_j	3	7	4	5	7
b_j	6	2	7	3	4

[25]

[8+7+10]=[25

3.a) Explain briefly the different types of replacement practices commonly used in industry.

b) An equipment costs A units of money and C_i is the cost of maintenance for the i^{th} period ($j > i \Rightarrow C_j > C_i$). Let V be the discount rate. The following rule gives the optimal replacement policy.

p.t.o.

'Do not replace if the next period's cost is less than the weighted average of previous costs. Replace if the next period's cost is greater than the weighted average of previous costs'.

Give a theoretical justification for this. [7+18]=[25]

4.a) Explain the meaning of different types of float, their calculation and use.

b) The following is a network for a project of six activities and five events

(i j)	Normal time (days)	Rapid (or crash time)(days)	Cost rate of compression per day
(1, 2)	15	12	10
(1, 3)	12	10	25
(2, 3)	5	2	30
(2, 4)	10	10	--
(3, 4)	8	6	50
(4, 5)	6	4	15

What is the minimum time in which you can complete the project and what is the minimum cost. [8+17]=[25]

Group B

5.a) Discuss and derive the economic order quantity formula for an inventory system when shortage is allowed to occur. [15]

b) An industrial concern needs a material at the rate of 250 tonnes a month. Procurement by the industry can be made instantaneously. The order cost per purchase is Rs.55. The storage cost is Rs.10 per month per tonne stored. It has been calculated that the shortage of the material costs Rs.100 per tonne short per month. Discuss the purchase policy of the industry. [10]

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MID-YEAR EXAMINATION

Statistics-14: Econometrics (Theory and Practical)

Date: 14.1.74 Maximum Marks: 100 Time: 4 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

1. How would you analyse cross-section data on consumer behaviour of households? How can this investigation be used in demand analysis. [25]

2. The following table gives the per capita demand for wheat, per capita income at constant prices, and retail price index for wheat and for all commodities during the period 1924-1936.

year	per capita demand for wheat (in seers)	per capita annual income (Rs.) at 1931-32 prices	retail price index (1931 = 100) all comments	wheat
(1)	(2)	(3)	(4)	(5)
1924	33.21	61.38	176	196
1925	35.99	62.15	175	208
1926	36.96	62.46	165	198
1927	31.09	60.64	159	192
1928	34.95	63.75	159	190
1929	42.71	65.30	147	159
1930	37.81	63.87	117	110
1931	36.56	62.21	100	100
1932	37.71	62.72	98	109
1933	36.92	63.24	94	104
1934	37.98	64.78	97	100
1935	36.54	65.61	99	106
1936	36.55	70.20	102	123

Estimate the constant elasticity demand function for wheat taking its price and per capita income as the explanatory variables. Calculate the standard errors of the estimated price and income elasticity, and also the coefficient of determination. [25]

3.a) Define the Geary-Khamis system of index numbers for multilateral comparisons. State the necessary and sufficient condition for the existence of these index numbers and interpret them.

b) Obtain the generalized Laspeyres', Paasche and Fisher's index numbers for multilateral comparisons using the concepts of 'exchange rate' and 'average price'. [15+ 10]=[25]

4.a) State a few tests of consistency of index numbers and comment on the usefulness these tests in the problem of choice an appropriate index number formula.

b) State and prove Leontief's theorem on the necessary and sufficient conditions for the functional separability of utility functions. [10+15]=[25]

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MID-YEAR EXAMINATION

Mathematics-7: Measure Theory

Date: 16.1.74 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 120 marks. Answer as many as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

Throughout $\underline{\underline{A}}$ stands for a σ-field of subsets of a set X.

1. Let $(X, \underline{\underline{A}}, \mu)$ be a measure space. For measurable functions f and g we write $f \backsim g$ if $\mu(\{x \mid f(x) \neq g(x)\}) = 0$.

   a) For measurable functions h_1, h_2 and h_3 if $h_1 \backsim h_2$ and $h_2 \backsim h_3$ prove that $h_1 \backsim h_3$.

   b) For a measurable function h if $h \backsim h^2$ prove that there is a set $A \in \underline{\underline{A}}$ such that $h \backsim \chi_A$. [15]

2. Let $(X, \underline{\underline{A}}, \mu)$ be a measure space.

   a) For a nonnegative measurable function f on $(X, \underline{\underline{A}})$ and a non-negative integer n prove that $\int_A f d\mu \geq n\mu(A)$ where $A = \{x \mid f(x) > n\}$.

   b) For an integrable function f on $(X, \underline{\underline{A}})$ prove that $\mu(\{x \mid f(x) > n\}) \longrightarrow 0$ as $n \rightarrow \infty$. [15]

3. Let $(Y, \underline{\underline{B}}, \lambda)$ be the measure space given by $Y = [0, 2)$, $\underline{\underline{B}}$ = Borel σ-field on Y and λ = Lebergue measure on $\underline{\underline{B}}$.
   Let f be the measurable function defined on Y by $f(y) = y$. Prove that $E(f) = 2$. [20]

4. a) State Jordan-Hahn Decomposition theorem for a signed measure.

   b) For a signed measure μ on $(X, \underline{\underline{A}})$ define its total variation measure $|\mu|$ on $\underline{\underline{A}}$ by
   $$|\mu|(A) = \mu^+(A) + \mu^-(A) \quad \text{for } A \in \underline{\underline{A}}$$
   Prove that (i) $|\mu(A)| \leq |\mu|(A)$.
   (ii) $|\mu|(A) = \sup \sum_{i=1}^{\infty} |\mu(A_i)|$
   where sup is taken over all countable partitions $A_1, A_2, \ldots$ of X where $A_i \in \underline{\underline{A}}$ for $i = 1, 2, \ldots$

   [Hint: You may use Jordan-Hahn Decomposition Theorem]. [25]

5. a) State Fubini's theorem for three measure spaces.

   b) Let $(Y, \underline{\underline{B}}, \lambda)$ be as in question 3. Let $\lambda \times \lambda \times \lambda$ be the product measure on $\underline{\underline{B}} \otimes \underline{\underline{B}} \otimes \underline{\underline{B}}$.
   Find $\lambda \times \lambda \times \lambda\,(B)$ where $B = \{(y_1, y_2, y_3) \in Y \times Y \times Y \mid y_2 \leq y_3\}$
   [Hint: Use the result of question 3]. [20]

6. Let $(X, \underline{\underline{A}}, \mu)$ be a measure space and A_n, $n = 1, 2, \ldots$ be a sequence of sets from $\underline{\underline{A}}$. If $\mu(\limsup A_n) = 0$, prove that $\liminf \mu(A_n) = 0$. [10]

7. Assignment. [15]

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MID-YEAR EXAMINATION

Economics-3

Date: 18.1.74. Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer any two questions out of question Nos. 1-3 and question No.4. Maximum allotted for each question are given in brackets [ ].

1. Give a clear exposition of the Ricardian theory of rent. Prove that the amount of rent increases as the size of output increases. Can you say anything about the behaviour of the relative share of rent as output grows? [12+4+4]=[20]

2. What are the basic assumptions of the classical theory of growth? Show how these assumptions lead to the conclusion that there is a long-run tendency of the economy towards a 'Stationary State'. [20]

3. Construct a simple model to explain the division of total labour force between agriculture and industry on the basis of Ricardian assumptions. In what sense does the rate of profit in agriculture govern the rate of profit in industry in this model? [15+5]=[20]

4. Write a short note on any one of the following:

   a) Tableau Economique

   b) The Reserve Army of Labour

   c) The relation between the distribution of product among different income-classes and the growth of the product. [10]

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PERIODICAL EXAMINATION

Economics

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Date: 25.3.74 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer any three questions. All questions carry equal marks.

1. What is an inter-industry transactions table? Show how it can be used for the purpose of national income accounting. Comment briefly on the valuation of product flows in the table.

2. Give a broad outline of the historical trends in the demographic patterns of developed as well as underdeveloped countries. What, if any, are the significant differences between the experience of underdeveloped countries in the last few decades and the experience of the present-day advanced countries during the early phase of their economic development in this regard?

3. Describe the major facts regarding the share of agriculture in the total labour force and the total product, as observed from both time-series data of individual countries and international cross-section data, pointing out the areas of both similarity and dissimilarity, if any.

4. Briefly review the major explanations cited for a secular decline in the share of agriculture in the total product of a country over a long period of growth in per-capita product.

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PERIODICAL EXAMINATION
Modern Algebra

Date: 8.4.74 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1. Define a permutation, a cyclic permutation and a transposition. Prove that every permutation can be expressed as a product of disjoint transpositions. Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 2 & 3 & 4 & 6 & 5 & 1 & 8 & 9 & 7 & 10 \end{pmatrix}$ as a product of cyclic permutations. [20]

2.a) Give two examples of a set with a non-associative composition. [5]

b) Show that the set of all transformations φ defined on complex numbers,
$$\varphi(z) = \frac{az+b}{cz+d}$$
where a, b, c, d are arbitrary complex numbers such that $ad - bc \neq 0$ forms a group with function composition. [15]

3. For any group G, show that

a) Right and left cancellation laws are valid. [4]

b) For any given pair of elements $a, b, \in G$ $\exists$ unique elements $x, y \in G$ such that $ax = b$, $ya = b$. [4]

c) $a, b \in G$, $(ab)^{-1} = b^{-1} a^{-1}$. [4]

d) The order of an element $a \in G$ is same as that of a^{-1}. [4]

e) The orders of elements a and $x^{-1}ax$ are same where $a \in G$, $x \in G$. [4]

4.a) Show that a set G with a composition is a group if for every pair $a, b \in G$, $\exists$ elements $x, y \in G$ such that $ax = b$, $ya = b$. [10]

b) A group has the property that for every $a \in G$, $a^2 = e$, e being the identity. Prove that G is abelian. [10]

5.a) Prove that every finite group G is isomorphic to a permutation group. Find the permutation group isomorphic to the multiplicative group of fourth roots 1, -1, i, -i of unity. [10]

b) Prove that the order of each subgroup of a finite group is a divisor of the order of the group. [10]

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PERIODICAL EXAMINATION

Inference

Date: 15.4.74 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1.a) What are the two types of error in testing statistical hypotheses? How are they controlled under the Neyman-Pearson formulation?

b) Show that a most powerful randomised level α test always exists for testing a simple hypothesis against a simple alternative.

c) Let $X_1, \ldots, X_n$ constitute a random sample of size n from a population with unknown continuous distribution function F and pdf f. Show that a UMP level α test for testing $H_0 : F = G$ against the alternatives

$$H : F = G^{1+\Delta} \; (\Delta > 0), \; G$$

specified, based on $X_1, \ldots, X_n$ is of the form $\prod_1^n G(X_i) > c$, where c is a suitable constant. Show also that the constant c does not depend on G.

d) Show that there does not exist a UMP level α test for testing $H_0 : \sigma = \sigma_0$ against the alternatives $H : \sigma \neq \sigma_0$ on the basis of random samples from a $N(0, \sigma^2)$ population.
[5+12+8+8]=[33]

2.a) When is a real parameter family of densities said to have monotone likelihood ratio?

b) Let $p_\theta(x)$ be a family of densities on the real line with monotone likelihood ratio in x. If ψ is a non-decreasing function of x, show that $E_\theta \psi(X)$ is a non-decreasing function of θ.

c) Let θ be a real parameter, and let the rv X have pdf $p_\theta(x)$ with monotone likelihood ratio in $T(x)$. Show that for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$, there exists a UMP test ϕ given by

$$\phi(x) = \begin{cases} 1 & \text{if} \quad T(x) > C \\ \gamma & \text{if} \quad T(x) = C \\ 0 & \text{if} \quad T(x) < C \end{cases}$$

where C and γ are determined by $E_{\theta_0} \phi(X) = \alpha$. Show also that the power function $\beta(\theta)$ of this test is strictly increasing for all θ for which $\beta(\theta) < 1$.

d) Show that a necessary and sufficient condition for densities $p_\theta(x)$ to have monotone likelihood ratio in x, if the mixed second derivative

$$\frac{\partial^2 \log p_\theta(x)}{\partial\theta \, \partial x}$$

exists, is that this derivative ≥ 0 for all θ and x.
[3+6+16+10]=[35]

p.t.o.

3.a) Describe the method of 'least favourable distribution' for testing a compositive hypothesis against a simple alternative. Prove a theorem which states that, under some conditions (to be stated by you) a distribution on the set of parameter values specified by the hypothesis is a 'least favourable' distribution and provides a UMP level α test of the given hypothesis.

b) Let $X_1, \ldots, X_m, Y_1, \ldots, Y_n$ be independent rv's, $X_1, \ldots, X_m$ iid $N(\xi, 1)$, and $Y_1, \ldots, Y_n$ iid $N(\eta, 1)$. Find a UMP level α test of $H_o : \xi = \eta$ against the alternative $H_1 : \xi = \xi_1, \eta = \eta_1$ $(\xi_1 < \eta_1)$.

[Hint: Try $\lambda : P(\xi = \eta = \frac{m\xi_1 + m\eta_1}{m+n}) = 1$].

Is this a UMP level α test of $H_o : \xi = \eta$ against $H : \xi < \eta$?

c) Let $f_1, \ldots, f_{m+1}$ be real-valued Lebesgue integrable functions. Let $\underline{\underline{C}}$ be the class of critical functions ϕ^* for which $\int \phi^* f_i \, dx = c_i$ $(i = 1, \ldots, m)$, where $c_1, \ldots, c_m$ are given constants. Assume $\underline{\underline{C}}$ to be non-empty. Show that a sufficient condition for a member ϕ of $\underline{\underline{C}}$ to maximize $\int \phi^* f_{m+1} dx$ is the existence of constants $k_1, \ldots, k_m$ such that

$$\phi(x) = \begin{cases} 1 & \text{if} \quad f_{m+1}(x) > \sum_1^m k_i f_i(x) \\ 0 & \text{if} \quad f_{m+1}(x) < \sum_1^m k_i f_i(x) \end{cases}$$

[12+14+6]=[32]

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PERIODICAL EXAMINATION

Statistical Quality Control

Date: 22.4.74 Maximum Marks : 50 Time: $1\frac{1}{2}$ hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1. a) Explain the meaning of statistical control.

   b) How do you justify the use of control charts

   c) Describe how to construct control charts for
      i) number defectives
      ii) mean

   when the subgroup size varies.

   [10 × 3] = [30]

2. For twenty days, 700 items of an electrical equipment are checked every day and the data below gives the number of defectives. Construct a p-chart. Is the manufacturing process under control with respect to its fraction defective.

Date	Defective	Date	Defective
1	161	11	54
2	118	12	64
3	220	13	53
4	67	14	52
5	85	15	51
6	82	16	56
7	78	17	46
8	74	18	42
9	60	19	66
10	45	20	46

[20]

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PERIODICAL EXAMINATION

Statistical Methods

Date: 29.4.74 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 110 marks. Answer as many questions as you can. The maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

1. Prove or disprove the following statements. Mention the needed assumptions in full while proving or disproving the statements.

i) S has a Wishart distribution $W_p(m, \Sigma)$ if and only if $L'SL/L'\Sigma L$ has χ^2- distribution with m degrees of freedom. [12]

ii) Let $S_1, S_2, \ldots, S_k$ be the dispersion matrices of K independent samples of sizes $n_1, \ldots, n_k$ respectively from a multivariate normal population, then the sum of these matrices would follow Wishart distribution with $\sum_1^k n_i - K$ degrees of freedom. [10]

iii) If R_o is the sample multiple correlation coefficient of X_1 and $(X_2, \ldots, X_p)$ based on a random sample of size n from a p-dimensional normal population then $R_o^2/(1 - R_o^2)$ would be distributed as central F with p-1 and n-p degrees of freedom. [12]

iv) Let $\bar{X}$ be the mean of a random sample of size n from $N_p(\mu, \Sigma)$ then $(\bar{X}-\mu)'\Sigma^{-1}(\bar{X}-\mu)$ would have chi-square distribution with p d.f. [8]

v) Multiple correlation coefficient is always non-negative. [8]

vi) The sample mean and sample variance covariance matrix are unbiased estimates of population mean vector and dispersion matrix respectively. [8]

2. Two random samples are drawn from trivariate normal populations for which it is known that

$\sigma_1 = 8.5$ $\rho_{12} = 0.75$
$\sigma_2 = 15.2$ $\rho_{13} = 0.24$
$\sigma_3 = 40.1$ $\rho_{23} = -0.55$

The samples yield the means of the variates as

first sample	$\bar{x}_1 = 14.5$	$\bar{x}_2 = 10.2$	$\bar{x}_3 = 25.1$
second sample	$\bar{x}_1 = 20.4$	$\bar{x}_2 = 7.5$	$\bar{x}_3 = 30.2$

i) Can you say that both the samples are coming from the same population?

ii) Can you say that both the samples are coming from a population with mean (15.5, 11.4, 30)? [10+10]=[20]

p.t.o.

3. A random sample of 30 Tehsils was selected and number of schools and colleges (x_1), number of educate people (x_2), number of educated unemployed (x_3) and population (x_4) were observed for these Tehsils. The sample yielded the means, s.d. and correlations as

$\bar{x}_1 = 60$	$s_1 = 40.5$	$r_{12} = 0.8$	$r_{24} = 0.3$
$\bar{x}_2 = 15$ thousand	$s_2 = 10.5$	$r_{13} = 0.7$	$r_{34} = 0.4$
$\bar{x}_3 = 5$ thousand	$s_3 = 30.2$	$r_{14} = 0.5$	
$\bar{x}_4 = 85$ thousand	$s_4 = 25.3$	$r_{23} = 0.8$	

Assume that (x_1, x_2, x_3, x_4) follow multivariate normal dist.

i) Find the partial correlation coefficient $r_{13.24}$

ii) Find the multiple correlation coefficient between x_1 and (X_3, X_4).

iii) Test the hypothesis that x_1 is independent of (X_3, X_4).

iv) Test the hypothesis that x_1 and x_4 are not correlated.

[8+8+8+8]=[32]

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PERIODICAL EXAMINATION

Econometrics

Date: 13.5.74 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1. Examine the methodological problems involved in estimating a production function from

   i) time series data

   ii) cross-section data. [12]

2. Give an account of Douglas' empirical studies of the production function and some of the major criticisms levelled against them. [9+15]=[24]

3. Derive the cost and supply functions. Comment on the difference in the methods of their derivations. [12]

4. Discuss the properties of the Cobb-Douglas production functions. [12]

5. In the following table Q indicates the volume of production, L the number of wage-earners and K the volume of capital in some suitable units in the manufacturing industries in U.S.A.

Year	Q	L	K
1900	102	105	107
1903	124	123	131
1906	152	133	163
1909	155	140	198
1912	177	152	226
1915	139	154	266
1918	223	200	366

Estimate the exponents of the production function $Q = AL^{\alpha} K^{\beta}$. Give your comments on the further interpretations of these estimates, assuming competitive market conditions. [40]

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PERIODICAL EXAMINATION

[420]

Applications of Statistics to Sciences

Dated: 3.6.74 Maximum Marks. 100 Time. 3 hours

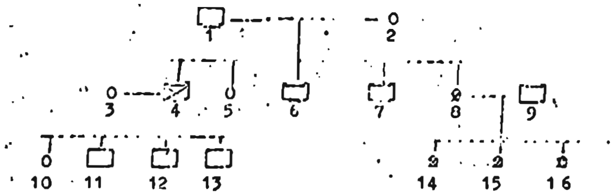
Note: Answer all questions. Question 1 should be answered in a separate answerscript. Marks allotted for each question are given in brackets [ ].

1.a) Describe one of the techniques commonly used for the measurement of grain sizes of sedimentary particles.

b) State the statistical problems involved in the analysis and interpretation of grain size data. [20]

2. A disease is determined by a recessive gene d of a pair of alleles D, d, that is, only an individual dd is affected by the disease.

The following is a pedigree where the X sign inside a circle or a rectangle denotes an affected female or male.



a) Write down the genotypes (or possible genotypes) of all the individuals in the pedigree. [12]

b) What is the probability that individual 5 is homozygous? [4]

c) If individual 13 marries an affected female, what is the probability for a child to be affected? [4]

d) Assuming that the a priori probability for individual 3 to be homozygous is 1/2, what is the a posteriori probability, given the observations in the pedigree of the four children 10, 11, 12, 13? [10]

3.a) State the Hardy-Weinberg law of genotypic equilibrium and prove this law for an initial population with equal male and female genotypic proportions

AA: u, Aa : v, aa : w ; $u + v + w = 1$ [7]

b) An initial population consists of the following genotypic proportions:

	AA	Aa	aa
male	u_1	v_1	w_1
female	u_2	v_2	w_2

i) What are the gene frequencies p_1, q_1 ; p_2, q_2 for males and females in this population?

Assuming random mating between males and females, [3]

p.t.o.

3.b) ii) Obtain the genotypic proportions in the offspring generation in terms of p_1, q_1, p_2, q_2. [5]

iii) Show that the gene frequencies for both males and females in the offspring generation are [3]

$$p = \frac{1}{2}(p_1 + p_2), \quad q = \frac{1}{2}(q_1 + q_2).$$

iv) Show that genotypic equilibrium is established from the F_2 generation onwards. [7]

c) Two populations P_1 and P_2 are as follows:

- the relative sizes are π and $1 - \pi$;
- P_1 consists of AA individuals only;
- P_2 is the Hardy-Weinberg equilibrium with respect to (A, a) with $p, q (= 1 - p)$ as the frequencies of the A and a genes.

If P_1 and P_2 are now mixed and random mating continues,

i) Obtain the gene frequencies in the mixed parental population. [4]

ii) Show that the proportion of heterozygotes in the offspring generation exceeds the proportion in the mixed parental population by $2\pi(1 - \pi)q^2$. [6]

4,a) If gene A has a mutation rate of μ per generation to a mutant form a, and if the relative fitness values of the genotypes AA, Aa and aa are 1, $1 - ht$ and $1-t$ respectively, show that under random mating

$$q_1 = \frac{q(1 - tq - pht) + \mu p(1 - qht)}{1 - 2htpq - tq^2},$$

where q_1 is the frequency of the mutant gene in the offspring generation and p, q the gene frequencies of A, a in the parental generation. [10]

b) In the above, if A is dominant and only the recessive genotype aa has a reduced fitness $1 - t$, show that q reaches an equilibrium value $\sqrt{\frac{\mu}{t}}$ [5]

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[illegible]

ANNUAL EXAMINATION

Inference

Date: 24.6.74. Maximum Marks: 100 Time: 4 hours

Note. Answer any five questions. All questions carry equal marks.

1.a) Let $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ denote the order statistics in a random sample of size n from a rectangular (θ_1, θ_2) distribution $(\theta_2 > \theta_1)$. Show that the statistics

$$T_1 = \frac{nX_{(1)} - X_{(n)}}{n-1} \quad \text{and} \quad T_2 = \frac{nX_{(n)} - X_{(1)}}{n-1}$$

are the unique uniformly minimum variance unbiased estimators of θ_1 and θ_2 respectively.

b) Show that the variance of every unbiased estimator of σ^2 in random samples of size n from a normal (μ, σ^2) distribution is at least $2\sigma^4/n$. Prove that this bound is not attainable.

c) Given a random sample of size n from a normal distribution with mean and variance both equal to $\theta(> 0)$, obtain the maximum likelihood estimator (MLE) of θ. [7+6+2+5]=[20]

2.a) State and prove Basu's independence theorem.

b) Let $X_1, \ldots, X_n$ constitute a random sample of size n from a population with pdf

$$p_{\mu,\sigma}(x) = \begin{cases} \sigma^{-1} \exp(-(x-\mu)/\sigma), & x \geq \mu, \ \sigma > 0 \\ 0 \quad \ldots, & \text{otherwise} \end{cases}$$

Define $T_n = \min(X_1, \ldots, X_n)$, $U_n = \max(X_1, \ldots, X_n)$ and $\bar{X}_n = n^{-1} \sum_1^n X_i$. Show that $(U_n - T_n)/(\bar{X}_n - T_n)$ is distributed independently of $\bar{X}_n^4$.

c) In Q.2(b), show that the differences of two consecutive order statistics are independently distributed. Hence, derive the distribution of $\bar{X}_n - T_n$. [2+5+5+[illegible]]=[20]

3.a) When is a test called biased? What is meant by a uniformly most powerful (UMP) level α test? On the basis of a random sample of size n from a Poisson (θ) distribution, find a UMP level α test for $H_0: \theta \leq \theta_0$ against $H: \theta > \theta_0$.

b) Let $X = (X_1, \ldots, X_n)$ constitute a random sample of size n from a uniform distribution on $(0, \theta)$. For testing $H_0: \theta = \theta_0$ against $H: \theta \neq \theta_0$, show that there exists a unique UMP level α test ϕ given by

$$\phi(x) = \begin{cases} 1, & \text{when } \max(x_1, \ldots, x_n) > \theta_0 \text{ or } \max(x_1, \ldots, x_n) \leq \theta_0 \alpha^{\frac{1}{n}} \\ 0, & \text{otherwise} \end{cases}$$

c) In Q.3(a) show that there does not exist a UMP level α test for testing $H_0: \theta = \theta_0$ against $H: \theta \neq \theta_0$.

[1+2+5+8+4]=[20]

p.t.o.

4.a) Define 'UMP unbiased tests' and 'tests similar on the boundary of the hypothesis and the alternative'.

b) Let Θ denote the parameter space, Θ_0 the hypothesis and Δ the common boundary of Θ_0 and $\Theta - \Theta_0$. Assume that the power function of every test is continuous for all points of Θ. Show that if a level α UMP similar test of Δ against $\Theta - \Theta_0$ is also level α for Θ_0 against $\Theta - \Theta_0$, then it is UMP unbiased for Θ_0 against $\Theta - \Theta_0$.

c) Given a random sample from a normal (μ, σ^2) distribution, where both parameters are unknown, show that the two-tailed t-test is a UMP similar test for $H_0 : \mu = 0$ against $H : \mu \neq 0$. [2+1+5+12]=[20]

5.a) Let $X_1, \ldots, X_m, Y_1, \ldots, Y_n$ be independent normal variables with known means ξ and η and unknown variances σ^2 and τ^2 respectively. Show that for testing $H_0 : \tau \leq \sigma$ against $H : \tau > \sigma$, there exists a UMP test given by the rejection region

$$\sum_1^n (Y_j - \eta)^2 / \sum_1^m (X_i - \xi)^2 \geq C.$$

b) Let X be a random variable with probability density (wrt some measure μ)

$$p_\theta(x) = K(\theta) \exp(\theta\, T(x)) h(x), \ \theta \text{ real}.$$

Develop a UMP unbiased test for testing $H_0 : \theta = \theta_0$ against $H : \theta \neq \theta_0$. [9+11]=[20]

6.a) Describe the likelihood ratio principle for testing composite hypotheses against composite alternatives.

b) Let X_{ij} ($j = 1, \ldots, n_i$; $i = 1, \ldots, c$) be independent random variables, X_{ij}'s ($j = 1, \ldots, n_i$) being iid $N(\mu_i, \sigma^2)$; $i = 1, 2, \ldots, c$. The parameters $\mu_1, \ldots, \mu_c, \sigma^2$ are all assumed to be unknown. Obtain the likelihood ratio test for $H_0 : \mu_1 = \ldots = \mu_c$ against all possible alternatives, and determine the distribution of the likelihood ratio test criterion under the null hypotheses.

c) Let X_{ij} ($j = 1, \ldots, n_i$; $i = 1, \ldots, c$) be independent random variables, X_{ij}'s ($j = 1, \ldots, n_i$) being iid with pdf

$$p_{\theta_i}(x) = \begin{cases} \exp(-(x - \theta_i)), & x \geq \theta_i \\ 0, & \text{otherwise} \end{cases}$$

$i = 1, \ldots, c$; $\theta_1, \ldots, \theta_c$, are all assumed to be unknown. Obtain the likelihood ratio test for $H_0 : \theta_1 = \ldots = \theta_c$ against all possible alternatives, and show that $-2 \log \lambda$ (where λ is the likelihood ratio test criterion) has an exact chi-square distribution under H_0. [4+4+4+3+5]=[20]

7.a) Let $X = (X_1, X_2, \ldots, X_n)$ be a random sample of size n from a univariate normal distribution with mean θ, $-\infty < \theta < \infty$, and variance unity. Obtain a confidence interval of the form $(\underline{\theta}(X), \bar{\theta}(X))$ for θ with confidence coefficient 0.95 such that it minimises

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ANNUAL EXAMINATION

Mathematics-7: Modern Algebra

Date: 25.6.74 Maximum Marks: 100 Time: 3 hours

Note: Answer any five questions. All questions carry equal marks.

1.a) Prove that every finite group G is isomorphic to a permutation group. Find the permutation group isomorphic to the multiplicative group of Fourth roots of unity.

b) Split up the permutation of P_4 as products of disjoint cycles and describe each as even or odd.

2.a) Let G be a group and H a subgroup of G. Demonstrate that the set of all right cosets of H constitute a decomposition of G into mutually disjoint classes. Decompose the additive group J of integers into the cosets of the subgroup consisting of all multiples of a given integer $m > 1$.

b) State and prove Lagrange's theorem and hence show that the order of every element of a finite group is a divisor of the order of the group.

3.a) Let G be a group. Define

i) a conjugate element of G (ii) a self conjugate element of G (iii) the normaliser of an element of G (iv) a self conjugate subgroup of G.

Let G be a group of order p^m, p being some prime integer. Show that G has atleast p self conjugate elements.

b) Show that every self conjugate subgroup of a group G is the kernel of some homomorphic mapping of G.

4. Let H be any normal subgroup of a group G. Let Σ denote the set of all those subgroups of G which contain H as a subgroup and let Σ' denote the set of all the subgroups of the quotient group G/H which we denote by G'. Then show that $K \to K/H = K'$, $K \in \Sigma$, $K' \in \Sigma'$ is a one-one mapping of Σ onto Σ'.

Also if K is a normal subgroup of G, then K' is a normal subgroup of G' and G/K is isomorphic to G'/K'.

5.a) Define (i) a ring (ii) a commutative ring (iii) a ring with or without zero divisor (iv) an integral domain (v) a field (vi) a divison ring (vii) A two sided ideal. Give two distinct examples of rings with zero divisors.

b) Define the quotient field of an integral domain and show that every given integral domain admits of a quotient field and that the same is unique.

6.a) Show that every homomorphic image of a ring is isomorphic to some quitient ring thereof. Also conversely, every quotient ring of a ring is a homomorphic image of the ring.

b) Let R denote the field of real numbers. Show that the product set R×R is a field for the two compositions defined as follows

$$(a,b)+(c,d) = (a+c,\ b+d)$$
$$(a,b)(c,d) = (ac-bd,\ ad+bc)$$

Show further that this field is isomorphic to the field C of complex numbers.

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- ANNUAL EXAMINATION -

Application of Statistics to Sciences

Date: 27.6.74 Maximum Marks: 100 Time:3 hours

Note: 1. Answer all questions.
2. Answer question 3 in a separate answerscript.
3. Marks allotted for each question are given in brackets [].

1.a) At a two-allele locus, A is dominant over a. A large number of couples of both dominant parents were located, each couple being found through one recessive child, implying that none of these couples is childless and the and that each couple has at least one recessive child. If the probability that such a couple has n children is p_n, show that the expected number of recessive children in a family is

$$\sum_{n \geq 1} p_n \frac{\frac{1}{4} n}{1-(\frac{3}{4})^n}.$$ [7]

b) If both parents are heterozygous for two pairs of non-homologous gene pairs (A, a) and (B, b), where A and B are dominant, what is the probability of having two double recessive children (aabb) in a family of 4 children ? [6]

c) In the case (b), derive the 9 : 3 : 3 : 1 proportions for the phenotypes of the offspring. [7]

2.a) Assuming random mating, and that the gene A is dominant over a, find the proportion of recessive phenotypes among the offspring of matings which are

i) dominant x dominant ; [4]

ii) dominant x recessive; [4]

b) For a sex-linked locus with two alleles (A, a), and with unequal initial gene frequencies in males and females of a random mating population, show that

i) the male and female gene frequencies reach an equal limiting value with the passage of generations; [6]

ii) with this limiting gene frequency, the population is in genotypic equilibrium both for males and females. [5]

3. Of a pair of alleles (A, a), the gene a is lethal and the relative fitness of the genotypes AA, Aa and aa are 1, 1-s and 0, the frequency of this gene in the initial generation being q_0. Under random mating, show that

a) the gene frequency in the n^{th} generation is given by the relation

$$\frac{1}{q_n} = \frac{1-2s}{1-s}\left[1+\frac{1}{1-s}+\dots+\frac{1}{(1-s)^{n-1}}\right]+\frac{1}{(1-s)^n}\cdot\frac{1}{q_0}.$$

Obtain an expression for q_n and show that when $s > 0$ and q_0 small, approximately

$$q_n = (1-s)^n q_0;$$ [9]

p.t.o.

b) if $s = 0$, the proportion of heterozygotes in the n^{th} generation is

$$\frac{2q_0 [1 - (n-1)q_0]}{(1 + nq_0)^2} ;$$ [5]

c) if the mutation rate from A to a is μ per generation, q_n approximately reaches an equilibrium value μ/s [6]

4.a) Under complete positive phenotypic assortative mating for a pair of alleles (A, a), show that the proportion of heterozygotes

i) has the value $2pH_0/(2p + nH_0)$ in the n^{th} generation, p being the gene frequency of A and H_0 the proportion of heterozygotes in the initial generation; [6]

ii) decreases with every successive generation. [4]

b) An initially random-mating population practises sib-mating in all successive generations. Assuming the result that the proportion of heterozygotes, H_n, satisfied the difference equation

$$4H_{n+2} - 2H_{n+1} - H_n = 0$$

and stating the appropriate initial conditions, show that

$$\frac{H_n}{H_0} = \frac{5+3\sqrt{5}}{10}\left(\frac{1+\sqrt{5}}{4}\right)^n + \frac{5-3\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{4}\right)^n$$

and evaluate the ratio for n = 2, 3, 4. [10]

5. A large area of coconut plantation in Kerala is infested with an unknown disease suspected to have been caused by deficiencies of one or more micro-nutrient elements. As is known, the perennial crop of coconut is unique in several respects, and inconvenient for regular manurial trials.

An experiment is to be undertaken in a given area of 20 hectares, each hectare having about 200 palms, of which one-third are healthy and the rest diseased at different stages.

i) Suggest a suitable design of experiment to study the effect of 7 micro-nutrients (A- G), each at two levels, on the disease. Give the complete layout plan of the design. [7]

ii) Indicate the statistical analysis appropriate for the design you suggest. [8]

iii) Enumerate the problems peculiar to the perennial crop like cocoanut for conducting manurial experiments in the natural field conditions. [5]

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ANNUAL EXAMINATION

Statistics-13: S.Q.C.

Date: 29.6.73 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 110 marks. Answer any part of any question. Maximum marks one can score is 100. Marks allotted for each question are given in brackets [ ].

1.a) Explain the role of 'rational subgroups' and 'control limits' in operating a control chart.

b) The following table gives the averages and ranges in samples of size 4 of test records of copper content in commercial brass sheets

Sample No.	$\bar{X}$	R	Sample No.	$\bar{X}$	R
1	11.10	0.6	16	11.45	1.3
2	11.70	1.2	17	11.55	1.6
3	11.35	1.0	18	9.88	0.9
4	11.25	1.0	19	10.78	1.2
5	11.40	2.0	20	11.23	0.7
6	11.00	0.6	21	10.93	1.7
7	11.20	1.0	22	11.50	2.7
8	11.35	1.2	23	10.78	0.7
9	11.50	2.0	24	10.95	1.1
10	10.88	1.1	25	11.48	2.9
11	10.85	1.0	26	10.80	0.4
12	11.53	1.2	27	10.20	2.0
13	11.15	0.8	28	11.88	1.5
14	11.28	1.0	29	11.23	0.8
15	11.00	0.8	30	11.30	0.6

Test whether the process is under statistical control. A minimum of 9 % in any sheet is the market specification for this characteristic. Excess of 0.1 % on an average results in a loss of Rs.8000 per annum to the factory. Estimate how much saving can be affected by maintaining statistical control at a proper level so as to satisfy market specification. [5+20]=[25]

2.a) What is average run length of a control chart?

b) Derive general expression $(1-p^{\lambda})/(p^{\lambda}(1-p))$ for ARL of $\bar{X}$-chart, where λ successive points beyond a control limit indicate lack of control and p is the probability to obtain a point beyond any one control limit. [5+15]=[20]

3. A stable process producing 'i' units per unit of time, has μ_0 as its initial setting. Its average level drifting linearly at the rate of θ per unit time, has one sided (upper) specification limit at $\mu_0+m\sigma$ where m is a constant and σ is the standard deviation of the process.

Assuming that σ remains unchanged and the characteristic follows normal distribution, show that the optimum production run 'a' which minimises the expected total loss, is given by the equation:

p.t.o.

$$mF(m) + f(m) = \frac{C_r \theta}{\sigma u I} + mF(\tau_a) + f(\tau_a)$$

where C_r = resetting cost
u = loss incurred per defective item
$\tau_a = m - \frac{a\theta}{\sigma}$. [20]

4.a) Explain the following terms:
(i) AOQ (ii) ASN and (iii) AOI

b) Obtain expression for average sample number (ASN) for single sampling plan by attributes when the inspection is curtailed with respect to rejection i.e. inspection is stopped and the lot is rejected as soon as k defectives are found on sampling inspection and the lot is accepted if in a sample of n items, k-1 or less defectives are found. [10+15]=[25]

5.a) Explain the mathematical basis underlying the construction of Dodge and Romig's single sampling LTPD plan by attributes.

b) A lot contains large number of items. Devise a single ... sampling attribute classification plan for it to meet the stipulations

AQL = 0.01 LTPD = 0.09
α = 0.05 β = 0.10

[10+10]=[20]

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ANNUAL EXAMINATION

Statistics-14: Econometrics

Date: 1.7.74 Maximum Marks: 100 Time: 3 hours

Note: Answer Q.No.7 and any four of the rest.
Marks allotted for each question are given in brackets [ ].

1. Explain how the input-output model can be related to statistical production function analysis. How do isoquants for the input-output model and those for the Cobb-Douglas model differ? [15]

2. Suppose that the output of firms in an industry follows the Cobb-Douglas relation. How would you use this relation to estimate the aggregate capital in this industry, given the corresponding aggregates of employment and output? You may make reasonable assumptions regarding the size distribution of the firms. [15]

3. Examine the effect of using moving averages on the cyclical and random components of a time series. [15]

4. Discuss the method of periodogram analysis for investigating cyclical components of a time series. [15]

5. Discuss the properties of the Cobb-Douglas production function. Why is this function widely used ? [15]

6. Write short notes on any two .

a) Correlogram analysis

b) measuring seasonal fluctuations by moving average method

c) trend measurement in time series analysis. [15]

7. The matrix of input coefficients for a certain economy is given below:

producing sector	using sector S	A	B	C	final use (in some appropriate units)
services (S)	0.1	0.1	0.1	0.2	60
agriculture (A)	0.0	0.1	0.0	0.3	105
basic industries (B)	0.0	0.1	0.3	0.1	40
consumer goods industries (C)	0.0	0.0	0.0	0.2	32
value added	0.9	0.7	0.6	0.2	

Find out the levels of production in the four sectors listed above. [30]

Practical Records [10]

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ANNUAL EXAMINATION

Statistics-12: Statistical Methods

Date: 3.7.74 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 110 marks. Answer as many questions as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

1. Prove or Disprove the following statements, mentioning the needed assumptions (if any).

a) Let $S \sim W_p(n-1, \Sigma)$ be partitioned as

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$ where S_{11} is $(r-1) \times (r-1)$. And let

$$S = \begin{pmatrix} S^*_{11} & S^*_{12} \\ S^*_{21} & S^*_{22} \end{pmatrix}$$ where S^*_{11} is 1×1. Then

$S^*_{11} - S^*_{12} S^{*}_{22} S^*_{21}$ is distributed as chi-square with $(n-1-p+r)$ degrees of freedom. [12]

b) Let $X_{(1)}, \ldots, X_{(n)}$ be a random sample from $N_p(\mu, \Sigma)$. Then generalized variance of the sample is distributed as the product of p independent chi-square variates. [8]

c) Distribution of sample partial correlation coefficient $r_{ij;q+1,\ldots,p}$ based on n observations from a $N_p(\mu, \Sigma)$ is same as that of total correlation coefficient r_{ij} based on $n-(p-q)$ observations. [10]

d) Let $\bar{x}$ and S denote the sample mean and dispersion matrix based on n observations from $N_p(\mu, \Sigma)$. Let x be an additional observation from $N_p(\mu, \Sigma)$ made independently. Then $(x-\bar{x})' S^{-1} (x-\bar{x})$ has F distribution with p and $n-p$ degrees of freedom. [10]

e) Let $\{X_{(\alpha)}\}$, $\alpha = 1, \ldots, n$ be a random sample from $N_p(\mu, \Sigma)$. Then $S = \sum_{\alpha=1}^{n} (X_{(\alpha)}-\bar{X})(X_{(\alpha)}-\bar{X})'$ can be written as $\sum_{\alpha=1}^{n-1} Z_{(\alpha)} Z'_{(\alpha)}$ where $Z_{(\alpha)}$ are independently distributed as $N_p(0, \Sigma)$. [8]

f) Let $r(n)$ denote sample correlation coefficient based on a sample of size n from a normal distribution with correlation ρ then

$$[r(n) - \rho] \sqrt{n-1} / (1-\rho^2)$$ is asymptotically

distributed as $N(0, 1)$. [12]

p.t.o.

2. Let x_1, x_2 and x_3 be the amounts of cork in a boring from the three directions into a cork tree. The amounts of boring on a tree is considered as an observation from a three-variate normal distribution. Thirty random observations are made yielding the sample mean and dispersion matrix (without any divisor) as

$$\bar{x}' = (8.86 \quad 4.50 \quad 0.86)$$

$$S = \begin{pmatrix} 128.72 & 61.41 & -21.02 \\ \cdots & 56.93 & -28.30 \\ \cdots & \cdots & 63.53 \end{pmatrix}$$

a) Is it true that $\mu_1 - 2\mu_2 + \mu_3 = 0$ where μ_1 is the population mean of x_1?

b) Can we say that $\mu = (8 \quad 5 \quad 1)'$?

c) Another independent sample of 25 observations yielded

$$\bar{x}' = (10.2 \quad 6.4 \quad 1.4)$$

$$S = \begin{pmatrix} 130.1 & 60.4 & -18.1 \\ \cdots & 53.3 & -30.0 \\ \cdots & \cdots & 60.0 \end{pmatrix}$$

Can you say that both the samples are coming from the same population? [12+8+15]=[35]

3. In an industrial plant 5 machines are turning out similar type of items. Thirty, thirty-five, forty, forty-five and fifty items respectively, were selected randomly from these machines and number of defective items were found to be two, three, four, five and six respectively. Can you say that all machines are equally efficient in terms of turning non-defective items? [6]

4. A sample of 13 observations from a bivariate normal population yielded the sample mean and dispersion matrix (without divisor) as

$$\bar{x}' = (10.15 \quad 7.45)$$

$$S = \begin{pmatrix} 11.2 & 10.1 \\ \cdots & 15.5 \end{pmatrix}.$$

San you say that $\mu_1 = \mu_2$? [9]

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ANNUAL EXAMINATION

Economic Development, Planning and Growth

Date: 5.7.74 Maximum Marks: 50 Time: 2 hours

Note: Answer Q.No.4 and any two of the rest. Marks allotted for each question are given in brackets [ ].

1. Critically discuss the specific advantages/disadvantages for the economic development of underdeveloped countries of the world to-day that follow from the fact that these countries are 'late-comers' in the process of industrialization. [20]

2. Explain the basic logic of the argument that an under-development country operating under market mechanism will typecally suffer from lack of investment demand. Can you cite any empirical evidence in support of the argument? State briefly the policy-prescriptions that follow from the argument. [10+5+5]=[20]

3. Discuss the relation between the degree of capital-intensity of an investment project and the rate of growth of output from it in a planned labour-surplus economy, clearly specifying the assumptions you make. [20]

4. Write a note on any one of the following concepts:

a) Demonstration Effect.

b) External Economics.

c) Primitive Socialism. [10]
