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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part IV : 1979-80
PERIODICAL EXAMINATIONS

Statistics-7: Complex Analysis

Date: 10.9.79 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions
All questions carry equal marks.

1. Answer any FOUR of the following questions:

i) Show that $|\sin z| \le 1$ iff z is real.

ii) Evaluate $\lim_{z \to 0} \left(\frac{\sin z}{z}\right)^{1/z^2}$.

iii) Test for uniform continuity of the function $f(z) = \frac{1}{z}$ in the domain $|z| < 1$.

iv) Find the angle through which the tangent to any curve through the point $1+i$ is rotated by the transformation $w = z^3$.

v) Find the radius of convergence of the power series $\Sigma \left(1 - \frac{1}{n}\right)^{n^2} z^n$.

2.a) Characterize the image on the Riemann sphere under stereographic projection of

i) a family of parallel lines

ii) a family of concentric circles

iii) a family of lines passing through a fixed point.

b) Show that the family of curves

$$\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$$

with $-a^2 < \lambda < -b^2$ is orthogonal to the family with $-a^2 < -b^2 < \lambda$.

3.a) Prove that a power series represents an analytic function in its circle of convergence, and possesses derivatives of all orders there.

b) Show that the series $\Sigma \frac{e^{inz}}{n^a}$ is uniformly convergent if $Im(z) \ge 0$, and $Re(a) > 1$.

[Note that if a, b are complex numbers, the generalized power a^b is defined by

$$a^b = e^{b \operatorname{Log} a}]$$

4.a) If $u(x, y)$ and $v(x,y)$ are harmonic in a domain D, show that the function

$$\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

is analytic in D.

p.t.o

4.b) Can the function $e^x(x \cos y - y \sin y)$ be the real part of an analytic function $f(z)$? If so, find $f(z)$.

5.a) Prove that if $f(z)$ is analytic and $f'(z) \neq 0$ in a domain D, then the mapping $w = f(z)$ is conformal at every point of D.

b) Map conformally the upper half of the z-plane onto the unit disk $|w| \leq 1$ of the w-plane so that the point $z = i$ goes to the point $w = 0$, and

$$\left(\frac{dw}{dz}\right)_{z=1-i} = \sqrt{2} + i\sqrt{2} .$$

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80

MID-YEAR EXAMINATIONS

Mathematics-7: Complex Analysis

Date: 11.12.79 | Maximum Marks: 100 | Time: 3 hours

Note: Attempt four questions in all of which two must be from Group A, and two from Group B. Questions carry equal marks, and maximum marks is 100.
Good presentation counts!
It should be clear, concise and complete.

Group A

1.a) Evaluate $\oint_c \frac{dz}{z-a}$ when c is any simple closed path and $z = a$ is (i) inside c, (ii) outside c.

b) Show that if $f(z)$ is analytic in a simply connected domain D, and a, z are any two points in D, then the function

$$F(z) = \int_a^z f(z)dz$$

is analytic in D, and $F'(z) = f(z)$.

2.a) Prove that if $f(z)$ is analytic in a region R then the maximum of $|f(z)|$ is allained on the boundary, unless $f(z)$ is constant. Further, if the maximum is attained at an interior point, then $f(z)$ is constant.

b) Integrate $\frac{e^{iz}}{z}$ around a suitable contour to show

$$\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$$

3.a) Let D be a simply connected domain. Let $f(z)$ be meromorphic in D, and $g(z)$ analytic in D. Let c be a simple closed path in D such that $f(z)$ has no pole on c, and $|g(z)| < |f(z)|$ on c. Then show that $f(z)$ and $f(z) + g(z)$ have the same number of zeros inside c.

b) Let $f(\theta)$ be a real valued continuous function defined on the unit circle c. Then show that the function $u(r, \theta)$ defined within and on c by

$$u(r, \theta) = \frac{1}{2\pi} \int_0^{2\pi} \frac{(1-r^2) f(\phi) d\phi}{1 - 2r\cos(\phi - \theta) + r^2}$$

takes on the prescribed value $f(\theta)$ on the boundary. (You may assume that $u(r, \theta)$ is a harmonic function.)

Group B

4.a) Establish the Laurent series development

$$\exp\left\{\frac{\lambda}{2}\left(z + \frac{1}{z}\right)\right\} = a_0 + \sum_{n=1}^{\infty} a_n\left(z^n + \frac{1}{z^n}\right), \quad 0 < |z| < \infty$$

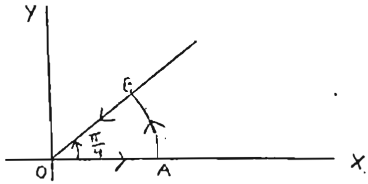
where

$$a_n = \frac{1}{\pi} \int_0^\pi e^{\lambda \cos t} \cos nt \, dt, \quad n \geq 0.$$

p.t.o.

4.b) Show that if f and g are analytic in a domain D, then f and g are identical iff the set $\{z \in D: f(z) = g(z)\}$ has a limit point in D.

5.a) Integrate e^{iz^2} around the contour



where AB is the arc given by $|z| = R$, $0 \leq \arg z \leq \frac{\pi}{4}$.
Hence show that

$$\int_0^\infty \sin x^2 \, dx = \int_0^\infty \cos x^2 \, dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}.$$

(You may assume that $\int_0^\infty e^{-x^2} dx = \frac{1}{2}\sqrt{\pi}$.)

b) Show by contour integration

i) $$\int_{-\infty}^{\infty} \frac{x^2}{1 + x^4} dx = \frac{\pi}{\sqrt{2}}$$

ii) $$\int_0^\pi \frac{d\theta}{a + \cos\theta} = \frac{\pi}{\sqrt{a^2 - 1}}, \quad a > 1.$$

6.a) Show that if $f(x)$ is periodic and with period 2π, and

$$f(x) = \begin{cases} -1 & \text{for } -\pi < x < 0 \\ 0 & \text{for } x = 0 \\ +1 & \text{for } 0 < x < \pi \end{cases}$$

then $f(x) = \frac{4}{\pi}\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots\right)$.

What is the value of the series for $x = \pm \pi$, and $x = 0$?

Deduce $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

b) Find the Fourier transform of $f(x) = e^{-|x|}$.
Use the Fourier inversion formula to show that $f(x)$ is the characteristic function of the probability density function

$$g(x) = \frac{1}{\pi(1 + x^2)}, \quad -\infty < x < \infty.$$

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV. 1979-80
and
M.Stat. Previous year : 1979-80
PERIODICAL EXAMINATIONS
Modern Algebra

Date. 10.3.80 Maximum Marks: 90 Time: 3 hours

Note: Answer as many questions as you can.
This paper carries 100 marks.
The maximum you can score is 90.

1.a) If G is a group, $x \in G$, $x \neq e$, define the term order of x.

b) If G is a finite group, $x \in G$, $x \neq e$, prove that the order of x is finite and it divides the order of G. (Justify your answer by stating precisely any theorems you use.)

c) G is an abelian group, $x, y \in G$, order of $x = m$, order of $y = n$. If m, n are relatively prime, prove that order of $xy = mn$. [5+8+7]=[20]

2. G is a finite abelian group. If the order of every non-identity element of G is p (where p is a fixed prime), prove that the order of G is p^n (for some positive integer n).

(Hint: Consider a suitable subgroup H, the quotient group G/H and use induction.) [10]

3.a) If G, H are groups, what does it mean to say $\phi: G \to H$ is a homomorphism of groups?

b) If N is a normal subgroup of H, prove that $N' = \{x \in G, \phi(x) \in N\}$ is a normal subgroup of G.

c) If ϕ is onto and H is abelian, is G abelian ? Justify your answer. [3+7+5]=[15]

4.a) Define the terms 'Field', 'characteristic of a Field'.

b) Let F be a field and $0 \neq f \in F[x]$. What does it mean to say that F_1 is a splitting field of the pair (F, f) ? [10]

5. Let Q denote the rationals and Z the integers. Prove that if $f \in Z[x]$ is irreducible over Z, then it is irreducible over Q. [15]

6. Let $\omega \in \mathbb{C}$ be one of the complex cube roots of unity. What is $Q(\omega)$? (i.e. write down the expression for a typical member of $Q(\omega)$). What is $[Q(\omega).Q]$? [15]

7. Let F be a field of characteristic 17. Suppose F contains all the roots of the equation $x^{35} - 1 = 0$. Prove that the set of roots of this equation forms a cyclic group of order 17. (Justify your answer by stating precisely any theorems you use.) [15]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80
and
M.Stat. Previous year : 1979-80
ANNUAL EXAMINATIONS
Modern Algebra

Date: 22.5.80 Maximum Marks: 100 Time: 3 hours

Note: The paper carries a total of 110 marks.
Answer as many questions as you can.
The maximum you can score is 100.

1. Let G be a finite group and H a subgroup of G. Prove that the order of H divides the order of G. [20]

2. If a group G has no non-trivial subgroups, show that G must be finite of prime order. (A subgroup H is said to be nontrivial if $H \neq e$ and $H \neq G$.). [10]

3. G is an abelian group of order 45. H is a subgroup of G and the order of H is 9. $x_1, x_2, \ldots, x_7$ are elements of G. Prove that there exists integers r, s, $1 \leq r \leq s \leq 7$ such that $(x_r \cdot x_{r+1} \cdots x_{s-1} \cdot x_s)$ is in H. [15]

4. Let G be the group of $n \times n$ real matrices (under the usual matrix multiplication). Let A be the set of $n \times n$ real matrices each of determinant 1.

a) Prove that A is a normal subgroup of G.

b) Prove that the quotient group G/A is isomorphic to the group of non-zero real numbers (with usual multiplication). [5+5]=[10]

5. Let Q be the field of rational numbers and R the field of reals. Let n be a positive odd integer. Prove that there exists an element a in R such that $[Q(a):Q]=n$. [15]

6. Let F be a finite field of order p^n. If F_1 is a subfield of F, prove that F_1 is of order p^m where m divides n. [15]

7.a) Let P_0 be a subset of $\mathbb{R}^2$ (i.e. the Cartesian plane). What does it mean to say that a point $Q \in \mathbb{R}^2$ is constructible from P_0 using ruler and compass constructions?

b) If $P_0 = \{(0,0), (1,0)\}$, prove that $(\sqrt[3]{2}, 0)$ is not constructible from P_0. [5+10]=[15]

8. Let F be a finite field and let F_0 be its prime subfield (i.e. F_0 is the smallest sub-field of F). Prove that $\exists\ a \in F$ such that $F = F_0(a)$. [10]

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B.Stat.(Hons.) Part IV: 1978-79
PERIODICAL EXAMINATIONS
Statistics-11: Measure Theory

Date: 3.9.79 Maximum Marks: 100 Time: 3 hours

Note: Solve all equations
Marks are given in brackets

1 (a) Define: Semi-ring, Ring, Field, σ-Ring

(b) Give examples of a semi-ring which is not a ring; ring which is not a field; a field which is not a σ-field. (One example for each case would do) [16]

2 S_1 is a semi-ring of subsets of a set X and S_2 is a semi-ring of subsets of a set Y. Let

$$S = \{A \times B : A \in S_1,\ B \in S_2\}$$

Show that S is a semi-ring of subsets of $X \times Y$ Show that S_1 and S_2 are rings need not imply that S is a ring. [16]

3 Let Q be the set of rational numbers in [0, 1] Show that if $I_1, I_2, \ldots, I_n$ are finitely many interval in $\mathbb{R}$ which cover Q, then $\sum_{k=1}^{n} L(I_k) \geq 1$ where $L(I_k)$ = length of I_k. Show that given $\epsilon > 0$ we can find countable number $(I_n)_{n=1}^{\infty}$ of intervals such that they cover Q and $\sum_{k=1}^{\infty} L(I_k) < \epsilon$. [16]

4 If m is a countably additive set function on a semi-ring S, show the m extends to a countably additive set function on the ring R generated by S. [16]

5 (a) Let m be a finite, countably additive measure, on a field $\mathcal{F}$ of subsets of a set X. Define m^* and sets measurable with respect to m^*. Show that collection of m^* measurable sets is closed under taking complements.

(b) Show that for $A, B \subseteq X$ $|m^*(A) - m^*(B)| \subseteq m^*(A \Delta B)$.

(c) A set $A \subseteq X$ is such that given $\epsilon > 0$ there exists $B \in \mathcal{F}$ such that $m^*(A \Delta B) < \epsilon$. Show that A is m^* measurable. [16]

6 Let $X = \{1,2,3,\ldots\}$ = set of all natural numbers. For any subset $A \subseteq X$ define U(A) and L(A) as follows

$$U(A) = \limsup_{n\to\infty} \frac{1}{n} C_n(A)$$

$$L(A) = \liminf_{n\to\infty} \frac{1}{n} C_n(A)$$

where $C_n(A)$ = number of integers in A less than or equal to n. U(A) is called the upper natural density of A and L(A) is called the lower natural density of A. Show that U is finitely subadditive on the power set of A but not countably subadditive. Show that for any infinite arithmatic progression $A = (a + nb)_{n=1}$ where $a, b \in X$ are fixed $U(A) = L(A) = \frac{1}{b}$. [1?]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80
MID-YEAR EXAMINATIONS
Statistics-11: Measure Theory

Date: 15.12.79 Maximum Marks: 100 Time: 3 hours

Note: Solve the questions.
Marks are given in brackets

1.a) Let μ be a real valued countably additive set function on a σ-field $\mathcal{F}$ of subsets of a set X. Let A be such that $\mu(A) \geq 0$. Show that there exists $B \subseteq A$ such that $\mu(B) \geq \mu(A)$ and for all $C \subseteq B$, $C \in \mathcal{F}$, $\mu(C) \geq 0$. Hence prove that X can be partitioned into two sets X_1 and X_2 in $\mathcal{F}$ such that $X_1 \cap X_2 = \emptyset$ and $\mu(A) \geq 0$ for all $A \subseteq X_1$, $A \in \mathcal{F}$ and $\mu(A) \leq 0$ for $A \subseteq X_2$, $A \in \mathcal{F}$. [16]

2.a)i) Define the notion of summable fnction for (i) simple function (ii) Borel measurable real valued function, where functions are defined on a finite measure space $(X, \mathcal{B}, m)$ (iii) Show that a bounded Borel measurable function on a finite measure space is always integrable. Is the same true if the measure space is not finite ? [16]

3. Let f be a function defined on $[0,1] \times [0,1]$ such that (i) for each y $f(\cdot, y)$ is summable in $[0,1]$.

ii) f is partially differentiable in y and for each y $|f(x,y+h) - f(x,y)| < K|h|$ where K is a constant independent of x. Show that

$$\frac{\partial}{\partial y} \int_0^1 f(x,y)\,dx = \int_0^1 \left(\frac{\partial}{\partial y} f(x,y)\right) dx$$ [16]

4. Define convergence almost everywhere and convergence in measure. Show either that convergence almost everywhere implies convergence in measure or that convergence in measure implies almost everywhere convergence for a subsequence. You may assume the measure space to be finite. [16]

5.a) Define covering in the sense of Vitali of a set E by a family M of closed intervals. State without proof the Vitali's covering theorem. [4]

b) Let f be a strictly increasing function on [a,b]. If at every point of a set E in [a,b] there exists at least one derived number $\leq$ p, where p is a fixed non-negative real number, show that

$$l^*(f(E)) \leq p\, l^*(E),$$

where l^* stands for outer lebesgue measure.

Or

State and Prove Radon-Nikodym theorem. [16]

6.a) Exhibit a bounded sequence $(f_n)_{n=1}^{\infty}$ of Riemann integrable functions on [0,1] converging everywhere to a non-Riemann integrable function f. Show that f is Lebesgue integrable on [0,1].

p.t.o.

6.b) Exhibit a sequence $(f_n)_{n=1}^{\infty}$ of Lebesgue summable function on [0, 1] converging pointwise to a summable function f such that

$$\lim_{n \to \infty} \int_0^1 f_n(x)\,dx \neq \int_0^1 f(x)\,dx$$

c) Exhibit a sequence $(f_n)_{n=1}^{\infty}$ of measurable function on [0, 1] which converges in measure (Lesborgue) but which does not converge almost everywhere. [16]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80
PERIODICAL EXAMINATIONS
Probability Theory and its Applications

Date: 18.2.80 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: The paper carries 115 marks. You may attempt as many questions or parts thereof as you like. The maximum you can score is 100. I want detailed proofs for Qs. 1 and 2. For the remaining questions, you may use without proof results proved in class, but in each case you must cite the result being used.

1. Let $(X, \mathcal{A}, \mu)$, $(Y, \mathcal{B}, \lambda)$ be finite measure spaces.
   a) Define the product σ-field $\mathcal{A} \textcircled{x} \mathcal{B}$ on $X \times Y$.
   b) Prove that, if $E \in \mathcal{A} \textcircled{x} \mathcal{B}$, then each vertical section of E is in $\mathcal{A}$.
   c) Prove that, if $E \in \mathcal{A} \textcircled{x} \mathcal{B}$, then the function $f(y) = \mu(E^y)$, $y \in Y$, is $\mathcal{B}$-measurable.
   d) Show that there is a unique measure m on $\mathcal{A} \textcircled{x} \mathcal{B}$ such that $m(A \times B) = \mu(A) \cdot \lambda(B)$ for each $A \in \mathcal{A}$, $B \in \mathcal{B}$.

   [2+5+6+7]=[20]

2. Let $(X, \mathcal{A}, \mu)$ be a measure space and let $(Y, \mathcal{B})$ be a measurable space. Let $T: X \longrightarrow Y$ be $(\mathcal{A}, \mathcal{B})$-measurable. Let $f : Y \longrightarrow [0, \infty)$ be $\mathcal{B}$-measurable and $E \in \mathcal{B}$. Prove that

$$\int_{T^{-1}(E)} f \circ T(x)\, d\mu(x) = \int_E f(y)\, d\mu T^{-1}(y).$$ [15]

3. Let $(X, \mathcal{A})$, $(Y, \mathcal{B})$ be measurable spaces.
   a) Prove that, if $\emptyset \neq E \subset X$, $\emptyset \neq F \subset Y$ and $E \times F \in \mathcal{A} \textcircled{x} \mathcal{B}$ then $E \in \mathcal{A}$ and $F \in \mathcal{B}$.
   b) Let $f : X \longrightarrow \mathbb{R}$, $g : Y \longrightarrow \mathbb{R}$ be, respectively, $\mathcal{A}$-measurable, $\mathcal{B}$-measurable. Prove that $\{(x,y) \in X \times Y : f(x) = g(y)\}$ belongs to $\mathcal{A} \textcircled{x} \mathcal{B}$.
   c) Suppose $f : X \longrightarrow \mathbb{R}$ is $\mathcal{A}$-measurable. Prove that $\{(x,y) \in X \times \mathbb{R} : f(x) = y\}$ belongs to $\mathcal{A} \textcircled{x} \mathcal{B}_{\mathbb{R}}$.

   [5+7+3]=[15]

4. Let X be a non-negative real-valued random variable on $(\Omega, \mathcal{A}, P)$. Let $\mathcal{B}$ be the Borel σ-field of $[0, \infty)$.
   a) Prove that $\{(w,x) \in \Omega \times [0, \infty) : x < X(w)\}$ belongs to $\mathcal{A} \textcircled{x} \mathcal{B}$.
   b) Show that $E(X) = \int_0^\infty (1 - F_X(x))\, dx$.

   [7+7]=[14]

5. Let F, G be continuous distribution functions and let $-\infty < a < b < \infty$.
   a) Prove that $\{(x,y) \in \mathbb{R} \times \mathbb{R} : a < x < b,\ y \leq x\}$ belongs to $\mathcal{B}_{\mathbb{R}} \textcircled{x} \mathcal{B}_{\mathbb{R}}$.
   b) Prove that

$$\int^b [illegible] + \int^b [illegible] = [illegible].$$

5.c) $\int_{-\infty}^{\infty} F \, dF = \frac{1}{2}$. [5+8+3] =[16]

6. Let X_n, $n \geq 1$, be a sequence of independent real-valued random variables on $(\Omega, \mathcal{A}, P)$.

a) Let $\mathcal{A}_1 = \mathcal{B}(X_n, n \geq 1)$ and $\mathcal{A}_2 = \bigcap_{n=1}^{\infty} \mathcal{B}(X_k, k \geq n)$.

Prove that $\mathcal{A}_1$ and $\mathcal{A}_2$ are independent.

b) Let $Z = \sum_{n=1}^{\infty} \frac{\cos X_{2n}}{n^2}$ and $W = \sum_{n=1}^{\infty} \frac{\sin X_{2n-1}}{n^2}$.

Show Z and W are independent random variables. [9+6]=[15]

7a) Let $(\Omega, \mathcal{A}, P)$ be a probability space. Let $A_t \in \mathcal{A}$, $t \in T$. Put $\mathcal{A}_t = \{\Omega, A_t, A_t^c, \emptyset\}$, $t \in T$. Say that the events $\{A_t\}_{t \in T}$ are <u>independent</u> if the σ-fields $\{\mathcal{A}_t\}_{t \in T}$ are independent. (This is a definition). Now prove that $\{A_t\}_{t \in T}$ are independent if and only if for every finite non-empty set $S \subset T$, $P(\bigcap_{s \in S} A_s) = \prod_{s \in S} P(A_s)$.

b) Let $(\Omega, \mathcal{A}, P)$ be a probability space. Let $A_n \in \mathcal{A}$, $n \geq 1$, and assume that $\{A_n\}_{n \geq 1}$ are independent. Show that

$$P\left(\bigcap_{n=1}^{\infty} A_n\right) = \prod_{n=1}^{\infty} P(A_n).$$

c) Let X, Y, Z be independent random variables, each having the uniform distribution over $[0, 1]$. Compute F_{X+Y+Z}.

[8+4+8]=[20]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80
PERIODICAL EXAMINATIONS
Statistics-11: Probability Theory and its Applications

Date: 14.4.80 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: Answer all questions. Give detailed proofs for Q.Nos. 1 - 5.

1. Suppose A_n, $n \geq 1$, are independent events. Show that, if $\sum_n P(A_n) = \infty$, then $P(\limsup A_n) = 1$. [15]

2. Let $X_1, X_2, \ldots, X_n$ be independent random variables such that $\sigma^2(X_i)$ is finite, $i = 1,2,\ldots, n$. Let $\epsilon > 0$. Prove that

$$P(\{\max_{1 \leq i \leq n} (S_i - E(S_i)) \geq \epsilon\}) \leq \frac{1}{\epsilon^2} \sum_{i=1}^{n} \sigma^2(X_i),$$

where $S_i = X_1 + X_2 + \ldots + X_i$, $i = 1,2,\ldots, n$. [15]

3. State and prove the Jessen-Wintner Trichotomy theorem. [15]

4. Let $X_1, X_2, \ldots$ be independent random variables. Assume $\sum_{n=1}^{\infty} \sigma^2(X_n) < \infty$. Prove that $\sum_{n=1}^{\infty} (X_n - E(X_n))$ converges a.s. [15]

5. State and prove Kolmogorov's Weak Law of Large Numbers. [15]

6. Let $X_1, X_2, \ldots$ be i.i.d. random variables such that $P(\{X_1 = 1\}) = P(\{X_1 = -1\}) = \frac{1}{2}$. Let $c_n \in \mathbb{R}$, $n \geq 1$. Prove that $\sum_{n=1}^{\infty} c_n X_n$ converges a.s. iff $\sum_{n=1}^{\infty} c_n^2 < \infty$. [12]

7. Let X_n, $n \geq 1$, be independent random variables such that $P(\{X_n = 1\}) = P(\{X_n = -1\}) = \frac{1-2^{-n}}{2}$,

$P(\{X_n = 2^n\}) = P(\{X_n = -2^n\}) = 2^{-n-1}$, $n \geq 1$.

Does the Strong Law of Large Numbers hold for this sequence? Justify your answer. [13]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80

Statistics-11: Probability Theory and its Applications

ANNUAL EXAMINATIONS

Date: 12.5.80 Maximum Marks: 100 Time: 4 hours

Note: The paper carries 125 marks. The most you can score is 100 marks. Attempt as many questions or parts thereof as you like.

1. Let $\mathcal{M}$ be the set of probability measures on $(\mathbb{R}, \mathcal{B}_{\mathbb{R}})$. For $\mu, \lambda \in \mathcal{M}$, define

$$\mu * \lambda (E) = \int \mu(E-y)d\lambda(y), \quad E \in \mathcal{B}_{\mathbb{R}}.$$

a) Prove that if $\mu, \lambda \in \mathcal{M}$, then $\mu * \lambda \in \mathcal{M}$.

b) Prove that $\mu * \lambda = \lambda * \mu$ for $\mu, \lambda \in \mathcal{M}$.

c) Prove that $(\mu * \lambda) * \nu = \mu * (\lambda * \nu)$ for $\mu, \lambda, \nu \in \mathcal{M}$.

d) Show that $\mu * \nu = \mu \Rightarrow \nu(\{0\}) = 1$.

e) Show that μ absolutely continuous $\Rightarrow \mu * \nu$ is absolutely continuous. [5+5+5+10+5]=[30]

2. Let $Y_1, Y_2, \ldots,$ be i.i.d. random variables such that $P(Y_1 = 1) = P(Y_1 = 0) = \frac{1}{2}$. Let $X = \sum_{n=1}^{\infty} \frac{Y_{2n}}{2^{2n}}$.

a) Show that the distribution of X is continuous singular.

b) Give an example to show that the converse of 1(e) is false. [5+5]=[10].

3.a) State and prove the Kolmogorov 0 - 1 law.

b) State and prove the Borel 0 - 1 law.

c) Suppose $X_1, X_2, \ldots,$ are i.i.d. random variables such that $P(X_1 = 1) = p$, $P(X_1 = -1) = 1-p = q$, where $0 < p < 1$ and $p \neq \frac{1}{2}$. Let $S_n = X_1 + X_2 + \ldots + X_n$, $n \geq 1$. Prove that $P(S_n = 0$ infinitely often$) = 0$.

d) Suppose $X_1, X_2, \ldots$ are independent, discrete random variables. Assume c_n are reals, $\sum_{n=1}^{\infty} c_n$ converges and $\sum_{n=1}^{\infty} P(X_n \neq c_n) < \infty$. Show that $\sum_{n=1}^{\infty} X_n$ converges a.s. and that $\sum_{n=1}^{\infty} X_n$ has a discrete distribution. [10+10 +5+10]=[35]

4.a) State and prove Kolmogorov's strong law of large numbers for i.i.d. random variables.

p.t.o.

4.b) Let $X_1, X_2, \ldots$ be i.i.d. random variables such that $E(X_1) = \infty$. Prove that

$$\frac{X_1 + X_2 + \cdots + X_n}{n} \rightarrow \infty \quad \text{a.s.}$$ [10+7]=[17

5.a) State and prove the Helly-Bray lemma.

b) If F, F_n are proper distribution functions and $F_n \xrightarrow{w} F$, show that $\int g \, dF_n \longrightarrow \int g \, dF$ for all bounded continuous $g : \mathbb{R} \longrightarrow \mathbb{R}$.

c) Let F, F_n, $n \geq 1$, be proper distribution functions such that F is continuous. If $F_n \xrightarrow{w} F$, show that $F_n \rightarrow F$ as $n \rightarrow \infty$ uniformly in x. [9+6+8]=[23

6.a) Suppose f is the characteristic function of a proper distribution function F. Suppose $f = 1$ on $[-\partial, \partial]$ for some $\partial > 0$. Show that

$$F(x) = 0 \quad \text{if} \quad x < 0$$
$$= 1 \quad \text{if} \quad x \geq 1 .$$

b) State the Central Limit Theorem for i.i.d. random variables. [5+5]=[10

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part IV : 1979-80
PERIODICAL EXAMINATIONS
Statistics - 12: Advanced Linear Estimation and Inference

Date: 12.11.79 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions.
All questions carry equal marks.

1. Let X and Y be random variables with joint density

$$F_{X,Y}(x, y|\lambda, \mu) = \begin{cases} \lambda\mu e^{-\lambda x - \mu y} & \text{for } x>0,\ y>0 \\ 0 & \text{otherwise.} \end{cases}$$

Find a UMP unbiased test of size α for testing.

a) $H_0 : \lambda = \mu$ vs $H_1 : \lambda \neq \mu$

b) $H_0 : \lambda \leq \mu + 1$ vs $H_1 : \lambda > \mu + 1$.

2. Let $X_1, \ldots, X_n$ be a sample of size n from $U(\theta_1\ \theta_2)$, the uniform distribution on the interval (θ_1, θ_2). For testing $H_0 : \theta_1 \leq 0$ vs $H_1 : \theta_1 > 0$ find UMP unbiased size $\alpha = .05$ test.

3. Let $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ be independent samples from populations with distribution $N(\mu, \sigma^2)$ and $N(\eta, \sigma^2)$, respectively. Find UMP invariant tests for testing

$$H_0 : \mu = \eta \quad \text{vs} \quad H_1 : \mu \neq \eta.$$

4. Let X and Y be random variables with joint density

$$f_{X,Y}(x, y|\mu, \lambda) = \begin{cases} \lambda\mu e^{-\lambda x - \mu y} & \text{for } x > 0,\ y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find UMP invariant test for

c) $H_0 : \lambda \leq \mu$ vs $H_1 : \lambda > \mu$.

b) $H_0 : \lambda = \mu$ vs $H_1 : \lambda \neq \mu$.

5. Let X_1 and X_2 be independent and identically distributed random variables. Let H_0 be the hypothesis that the underlying distribution is $N(\theta, 1)$, a normal distribution with variance 1 and let H_1 be the hypothesis that the underlying distribution is Cauchy with location parameter θ, that is

$$f(x, \theta) = \frac{1}{\pi} \cdot \frac{1}{1 + (x-\theta)^2} \qquad -\infty < x < +\infty$$

Find a UMP invariant test for H_0 vs H_1.

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV. 1979-80
MID-YEAR EXAMINATIONS
Statistics-12. Advanced Linear Estimation and Inference

Date: 13.12.79 Maximum Marks. 100 Time: 3 hours

Note: Answer any four questions.
All questions carry equal marks.

1. Let $Y_1, \ldots, Y_n$ be independent random variables having normal distribution with a common variance σ^2 and with means $EY_i = \mu_i$ for $i = 1, \ldots, k$ and $EY_i = 0$ for $i = k+1, \ldots, n$. We want to test the hypothesis.

$$H_0: \mu_1 = \cdots = \mu_r = 0 \quad \text{for} \quad r \leq k$$

a) Find the groups of transformations under which the problem is invariant.

b) Find the maximal invariant for problem.

2. If $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$ are samples from $N(\xi, \sigma^2)$ and $N(\eta, \tau^2)$ respectively and we want to test

$$H_0: \tau^2 = \sigma^2 \quad \text{vs} \quad H_1: \tau^2 \neq \sigma^2$$

Find a U.M.P invariant size α test.

3. Let X be a Poisson random variable with parameter θ i.e.

$$P(X = x) = \frac{\theta^x e^{-\theta}}{x!} \qquad x = 0, 1, 2, \ldots$$

If the loss function $W(\delta(X), \theta) = \frac{(\delta(X) - \theta)^2}{\theta}$, show that the estimate $\delta(X) = X$ is minimax by using Cramer-Rao inequality.

4. Let X be a binomial random variable

$$P_\theta(x) = \binom{n}{x}\theta^x(1-\theta)^{n-x}$$

we want to estimate θ when the loss function is

$$W(\delta(X), \theta) = \frac{(\delta(X) - \theta)^2}{\theta(1-\theta)}.$$

a) Find a constant risk estimate for θ

b) If θ has a Beta prior

$$\lambda(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1}, \quad 0 \leq \theta \leq 1$$

find the Bayes estimate for θ.

c) Show that the constant risk estimator in (a) is minimax.

5. Let $X_1, \ldots, X_n$ be independent, each being $N(\theta, 1)$, where it is known that $\theta > \theta_0$ and the loss function is squared error,

$$W(\delta(X), \theta) = (\delta(X) - \theta)^2$$

Show that $\bar{X} = \frac{1}{n}\sum_1^n X_i$ is minimax.

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80
PERIODICAL EXAMINATIONS

Statistics - 12: Advanced Linear Estimation and Inference

Date: 25.2.80 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions.
All questions carry equal marks.

1. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\theta, 1)$. We want to estimate θ under absolute error loss i.e.,

$$L(\theta, a) = |\theta - a|$$

a) Show that the sample mean $\bar{X}_n$ is an admissible estimate of θ.

b) Is $\bar{X}_n$ minimax ?

2. Let X have a Binomial distribution with parameter n and θ, where n is known and $\theta \in (\underline{H}) = [0, 1]$. Suppose the loss function is

$$L(\theta, a) = (\theta - a)^2$$

a) Find the group under which the problem is invariant.

b) Characterize the invariant decision rules.

c) Find a minimax invariant rule for $n = 1$.

d) Is the invariant minimax rule, minimax in the whole class of decision rules D^* ?

3. Let X be a r.v. with uniform distribution on $[\theta, \theta + 1]$. Find the best invariant estimate of θ under squared error loss function $L(\theta, a) = (\theta - a)^2$. Is the best invariant estimate is also best unbiased estimate ?

4. Let $X_1, \ldots, X_n$ be a sample from the uniform distribution on $[0, \theta]$. Find the best invariant estimate of θ if the loss function is

$$L(\theta, a) = \left(\frac{a}{\theta} - 1\right)^2 .$$

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80

ANNUAL EXAMINATIONS

Statistics-12: Advanced Linear Estimation and Inference

Date. 29.5.80 Maximum Marks: 100 Time: 4 hours

Note: Answer any five questions.

1. Let X and Y be independent with densities $f_X(x \mid \lambda) = f(x-\lambda)$ and $f_Y(y \mid \mu) = f(y-\mu)$. Let $Z = X - Y$ and $\theta = \lambda - \mu$.

   a) Suppose that for $\theta > 0$, $f_Z(z \mid \theta)/f_Z(z \mid 0)$ is non-decreasing in z, find a UMP invariant size α test of $H_0 : \theta \leq \theta_0$ vs $H_1 : \theta > \theta_0$.

   b) Suppose that, for $\theta \neq 0$, $f_{|Z|}(z \mid \theta)/f_{|Z|}(z \mid 0)$ is nondecreasing in $z > 0$, find a UMP invariant size α test of $H_0' : \theta = \theta_0$ vs $H_1' : \theta \neq \theta_0$

2. Let $X_1, \ldots, X_n$ be independent and identically distributed as $N(0, \theta)$. Find a U.M.V unbiased estimate of θ. Is this estimate admissible under squared error loss function? If not, find an estimate that is better.

3. Let X be binomial (n, p), That is

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} .$$

Find a minimax estimate of p when the loss function is squared error

$$|\partial(X) - p|^2$$

Compare this estimate with U.M.V. unbiased estimate of p.

4. Let $X_1, X_2, \ldots, X_n$ be i.i.d. $N(\theta, 1)$. Show that $\bar{X}_n = \frac{1}{n} \sum_1^n X_i$ is an admissible estimate of θ under the loss function

$$L(\partial(\bar{X}_n) - \theta) = |\partial(\bar{X}_n) - \theta|$$

5. Let θ be a location parameter for the distribution of a random variable X with finite variance and suppose that X is a complete sufficient statistic for θ. Suppose the loss function is squared error $L(\partial(X), \theta) = (\partial(X) - \theta)^2$. Show that the best invariant estimate of θ is the best unbiased estimate of θ.

Let $X_1, X_2, \ldots, X_n$ be i.i.d. with density

$$f(x, \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}} I_{(0,\infty)}(x).$$

Find the best invariant estimate of θ when the loss function is

$$L(\partial(X), \theta) = (\partial(X) - \theta)^2/\theta^2 .$$

p.t.o.

7. Let $X_1, \ldots, X_n$ be a sample from uniform distribution $U(\theta, 2\theta)$, where $\underset{\sim}{\bar{H}} = (0, \infty) =$ and $L(a, \theta) = (\theta - a)^2/\theta^2$. Find the best invariant estimate for θ.

8. Let $X_1, \ldots, X_n$ be i.i.d. uniform distribution $U(0, \theta)$. Find a uniformly most accurate family of confidence sets for θ at level $1 - \alpha$.

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part IV : 1979-80

PERIODICAL EXAMINATIONS

Statistics-13: Advanced Design of Experiments

Date: 17.9.79 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks are indicated in brackets. The class notes are allowed to use during the examination.

1. A 2^3 factorial experiment is planned in a single replication of completely randomised design. Factors are denoted by F_1, F_2, F_3; treatments are

$$f_1^{x_1} f_2^{x_2} f_3^{x_3}, \ x_i = 0,1, \ i = 1,2,3; \text{ and}$$

factorial effects are $F_1^{y_1} F_2^{y_2} F_3^{y_3}$, $y_i = 0, 1, \ i = 1,2,3.$ The responses are given below.

	f_1^0		f_1^1	
	f_2^0	f_2^1	f_2^0	f_2^1
f_3^0	31	26	46	61
f_3^1	29	36	51	76

Estimate all factorial effects and comment on your findings. [20]

2. Consider a 3^3 factorial experiment. Give a plan of dividing 3^3 treatments into 3^2 blocks of 3 plots by completely balancing two-factor and three-factor interactions. In ANOVA table, complete the columns corresponding to source and degrees of freedom. [30]

3. Give a $1/3^3$ fraction of a 3^5 factorial experiment. [Display all the treatments in the fraction.] [15]

4. Consider a 2^8 factorial experiment. Give a $1/2^2$ fraction so that no two main effects and two-factor interactions are aliases of each other. Supply the reason. [No need to display treatments in the fraction.] [15]

5. Consider a 2^4 factorial experiment in two replications. Factors are F_1, F_2, F_3 and F_4; treatments are $f_1^{x_1} f_2^{x_2} f_3^{x_3} f_4^{x_4}$, $x_i = 0,1, \ i = 1,2,3,4.$ Denote $f_i^1 = f_i$ and $f_i^0 = 1.$ The plans are given below.

p.t.o.

5. (contd.)

Rep	Block 1	2	3	4
Rep I	(1)	f_3f_4	f_3	f_4
	$f_1f_3f_4$	f_1	f_1f_4	f_1f_3
	$f_2f_3f_4$	f_2	f_2f_4	f_2f_3
	f_1f_2	$f_1f_2f_3f_4$	$f_1f_2f_3$	$f_1f_2f_4$
Rep. II	(1)	f_2f_3	f_2	f_3
	$f_1f_2f_3$	f_1	f_1f_3	f_1f_2
	$f_2f_3f_4$	f_4	f_3f_4	f_2f_4
	f_1f_4	$f_1f_2f_3f_4$	$f_1f_2f_4$	$f_1f_3f_4$

Find the factorial effects which are confounded in each replicate. [20]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80
MID-YEAR EXAMINATIONS
Statistics-13: Advanced Design of Experiments

Date: 21.12.79 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks are indicated within brackets.

1. Consider a 2^4 factorial experiment in a completely randomised design. The design is given below.

$$T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

The response vector is (35,30,24,20,31,28,25,22). Estimate the main effects assuming (i) the general mean and two factor interactions to be nonzero, and (ii) the three factor and higher order interactions to be zero. [30]

2. Divide 2^4 treatments in a FE(2^4) into 2^2 blocks of size 2^2 each in different replications so as to get a complete balance over the two and three factor interactions with a loss of 1/6 of the information on the former and half the information on the latter and furthermore, the main effects and the four factor interaction remain unconfounded. State the plan and indicate the arrangement of treatments within block in any one replication. [20]

3.a) When would you say a second order response surface design in k dimension to be

i) a rotatable arrangement,

and

ii) a rotatable design ? Discuss in detail. [10]

b) Find a necessary and sufficient condition for a second order rotatable arrangement in k-dimention to be a rotatable design.

OR

Prove that a set of n points at a distance ρ from the origin in k dimension together with $n_0 (> 1)$ points at the centre form a second order rotatable design.

[Note that 'or' is within the question 3(b)] [30]

c) Consider a second order response surface design in three dimension with the following points

$(\pm a, \pm a, \pm a)$
$(\pm ca, 0, \pm ca)$
$(0, \pm ca, 0)$
$(0, 0, \pm ca)$

Find a value of c so as to make the above design a rotatable one. [10]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV. 1979-80
PERIODICAL EXAMINATIONS
Statistics - 13. Advanced Sample Surveys

Date. 17.3.80 Maximum Marks. 100 Time: 3 hours

Note. The paper carries 110 marks. The maximum you can score is 100 marks. Do as many questions as you can do. The marks allotted are given at the end of each question in brackets.

1. Prove or disprove the following statements.

   i) Sample mean based on SRSWOR is the best unbiased estimator, for $\bar{Y}$, in the T_1, T_2 and T_5-classes of estimators.

   ii) The sampling strategy $S \equiv$ (ppxwr, $\hat{Y}$), where $\hat{Y} = \sum_{i \in s} y_i/p_i$, $p_i = 1-(1-x_i/X)^n$ is admissible in the unbiased subclass of T_5-class of estimators.

   iii) The sampling strategy $S_1 \equiv$ (ppxwr, $\hat{Y} = \frac{1}{n}\sum_{1}^{n} (y_i/x_i)X$) is better than $S_2 \equiv$ (SRSWR, $N\bar{y}$) for Y in case y and x are perfectly correlated.

   iv) Estimator $\lambda\bar{y}$ is better than $\bar{y}$, in estimating $\bar{Y}$, for some values of λ, where $\bar{y}$ is the simple random mean. [15+10+10+5]=[40]

2. Give suitable sampling strategies for $\bar{Y}$ in case population is quite hetrogeneous requiring stratification and (i) the number of population units N_h in various strata are known but sampling frames are not available for strata (ii) neither N_h are known nor frames are available. [15]

3. Give an estimate for the relative efficiency of the actual strategy T_1 = (ppxwr, $\frac{1}{Nn}\sum (y_i/x_i)X$ over the hypothetical strategy T_2 = (srswr, $\bar{y}$) for $\bar{Y}$. [10]

4. Let $\hat{Y}$ and $\hat{X}$ be unbiased estimators of Y and X respectively. Consider a class of estimators d / $d = \hat{Y} - t(\hat{X} - X)$ for Y, where t is a suitably chosen constant or statistic. Show that under moderate conditions on t (to be mentioned), the regression type estimator $d = \hat{Y} - \hat{\beta}(\hat{X} - X)$ has the smallest possible mean square error which could be achieved by any estimator in the class d, provided sample is large, and $\hat{\beta}$ is asymptotically unbiased estimate for the regression coefficient of $\hat{Y}$ on $\hat{X}$. [15]

p.t.o.

5. Let $U = \{1,2,3\}$ be a population of three units. Consider the problem of estimating the population mean of a character y by usual mean, regression, product and ratio estimators based on simple random samples (WOR) of two units. Let $y_1 = 1$ $y_2 = 2$ $y_3 = 3$ and $x_1 = 4$, $x_2 = 12$ $x_3 = 14$. Obtain the relative efficiency (in per cent) of the regression estimator (using sample regression coefficient) over the others. [20]

6.a) What do you mean by super population models? Give your view on their importance from practical point of view.

b) Give your views in support or in criticism of the unified approach (in sampling theory) developed by Godambe etc. [5+5]=[10

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part IV.1979-80
ANNUAL EXAMINATIONS

Statistics-13: Advanced Sample Surveys

Date: 19.5.80 Maximum Marks: 100 Time: 3 hours

Note: Do as many questions as you can. The paper carries 110 marks. The maximum you can score is 100 marks. The marks are given in brackets at the end of each question.

1. Prove or disprove the following statements.

i) There does not exist any UMVU estimator in the unbiased subclass of T_5-class of linear estimators,

ii) H-T estimator is admissible in T_7-class of linear estimators .

iii) Unbiased estimators in T_5-class are better than sample mean in case of SRSWOR.

iv) Let t be an unbiased estimator, for a parameter $\phi(y)$, based on any sampling design and W be a constant. The estimator Wt can never be better than t unless $W < 1$.

v) Let d_1 and d_2 be two estimators for population mean. Then $d = (1-a)d_1 + ad_2$ is better than d_1 and d_2, where a is a positive proper fraction. [5+5+5+5+5]=[25]

2. Let a population consist of three units (U_1, U_2, U_3). Define a sampling design as:

$P(U_1,U_2) = 0.4$, $P(U_2,U_3) = 0.3$, $P(U_1,U_3) = 0.2$,
$P(U_1) = 0.1$, $P(U_2) = P(U_3) = P(U_1,U_2,U_3) = 0$.

Let $(y,x) = (10,3), (20,5), (15,10)$ for the units in the population. Obtain expected values (mean) of sample mean estimator, H-T-estimator and ratio-estimator. Find the relative efficientics of the H-T estimator over the others. [20]

3.a) What do you mean by double sampling? When do you need to use it ?

b) It is proposed to use auxiliary information both at selection stage and estimation stage. Give an unbiased sampling strategy for the population total in case the total X of an auxiliary character is known but information on the selection-variate (on the basis of which probability set is to be defined) is lacking. Obtain the variance of the sampling strategy. [5+10]=[15]

p.t.o.

4. A SRSWOR of 50 units is drawn from a population of 200 units to estimate mean of a character y using the information on two other characters x_1 and x_2. The sample yielded.

$\Sigma y_j = .1220$ $\Sigma y_j x_{1j} = 46500$ $\Sigma y_j^2 = 93750$

$\Sigma x_{1j} = 1450$ $\Sigma y_j x_{2j} = 40500$ $\Sigma x_{1j}^2 = 95500$

$\Sigma x_{2j} = 1640$ $\Sigma x_{1j} x_{2j} = 72200$ $\Sigma x_{2j}^2 = 175500$

$\bar{X}_1 = 35$ $\bar{X}_2 = 50.$

i) Obtain the weighted ratio estimate, using optimum weights.

ii) Obtain estimated mean square error of the estimate.

iii) Estimate the relative efficiency of the weighted ratio estimate over (a) sample mean (b) ratio estimate using x_1 alone (c) ratio estimate using x_2 alobe, [15+10+10]=[35]

5.a) What do you mean by sampling and non-sampling errors?

b) Mention different types and sources of non-sampling errors.

c) Give an unbiased sampling strategy for population mean when non-response is present. [5+5+5]=[15]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part IV: 1979-80
PERIODICAL EXAMINATIONS
Statistics-14: Multivariate Analysis

Date: 15.10.79 Maximum Marks: 100 Time: 3 hours

Note: Answer any Five questions. Complete and precise answers carry more weight

1.a) Let $\underset{\sim}{X}$ be a random vector with n component $X_1,\ldots,X_n$ with mean vector $\underset{\sim}{\mu}$ and covariance matrix Λ which is positive definite. Obtain the equation of the best linear predictor of X_n in terms of $X_1,\ldots,X_{n-1}$.

b) Obtain the expression for the multiple correlation of X_1 and $(X_2,\ldots,X_{n-1})$.

2.a) Defining $\underset{\sim}{X} = A\underset{\sim}{Y} + \underset{\sim}{\lambda}$ where $\underset{\sim}{Y}$ is $N_k(0,I)$, $\underset{\sim}{\lambda}$ is a $p\times 1\ (p > k)$ vector of constants and A is a $(p\times k)$ matrix of rank k, show that $\underset{\sim}{Z} = D\underset{\sim}{X}$ where D is an $(m\times p)$ matrix, has Normal distribution and obtain its parameters.

b) Derive the characteristic function of $\underset{\sim}{X}$. What is the frequency function of $\underset{\sim}{X}$?

3.a) Let $\underset{\sim}{X}$ be $N_p(\underset{\sim}{\mu}, \Sigma)$ where Σ is positive definite. Obtain the conditional density functions of X_1 given $X_3 = x_3 \ldots X_p = x_p$, and of X_2 given $X_3 = x_3,\ldots,X_p = x_p$. Hence or otherwise derive an expression for the partial correlation of X_1 and X_3 given $X_3 = x_3,\ldots,X_p = x_p$.

b) Show that the best predictor of X_1 on $X_3,\ldots,X_p$ is linear and obtain the expression for that.

4. Let $\underset{\sim}{X}$ be $N_p(\underset{\sim}{\mu}, \Sigma)$ where Σ is positive definite. Based on a sample of $N(> p)$ observations $\underset{\sim}{X}_1,\ldots,\underset{\sim}{X}_N$ on $\underset{\sim}{X}$, derive the maximum likelihood estimators of μ and Σ. Show that they actually maximize the likelihood function.

5.a) Let $\underset{\sim}{X}$ be $N_p(\underset{\sim}{0},I)$. Show that a necessary and sufficient condition that the quadratic form $X'AX$ to have the χ^2 distribution is that A is idempotent. What will be the degrees of freedom?

b) Hence or otherwise obtain a necessary and sufficient condition that a quadratic form $\underset{\sim}{Y}'B\underset{\sim}{Y}$ is distributed as a χ^2-variable where $\underset{\sim}{Y}$ is $N_p(\underset{\sim}{0}, \Sigma)$. What will be the degrees of freedom in this case ?

6. Let $\underset{\sim}{X}$ be a multivariate Normal with mean vector $\underset{\sim}{0}$ and covariance matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & -4 \\ 0 & 0 & -4 & 6 \end{pmatrix}$$

p.t.o.

(1) Find the density function of the joint marginal distribution of X_3 and X_4.

(2) Find the conditional distribution of X_4 given X_1, X_2 and X_3.

(3) Obtain the partial correlation of X_1 and X_2 given X_3 and X_4.

(4) Obtain the multiple correlation of X_1 and (X_2, X_3, X_4).

(5) Find the mean and variance of

$$2X_1 + 3X_2 - 4X_3 + X_4$$

[5 x 4]=[20]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80

MID-YEAR EXAMINATIONS

Statistics-14: Multivariate Analysis

Date: 8.12.79 Maximum Marks: 100 Time: 3 hours

Note: 1. Answer any FIVE questions. Complete and precise answers carry more weight.

2. All vectors used are column vectors and row vectors are denoted with the transposes.

1.a) Obtain a characterization that a random vector $\underset{\sim}{X}' = (X_1, \ldots, X_k)$ will have a k-variate normal distribution with given mean vector $\underset{\sim}{\mu}$ and covariance matrix Σ.

b) Give an example to show that though the marginal distribution of each component of a random vector $\underset{\sim}{X}$ has a univariate normal distribution, $\underset{\sim}{X}$ need not have a multivariate normal distribution.

c) Using (a) or otherwise show that, if $\underset{\sim}{X}$ has $N_k(\underset{\sim}{\mu}, \Sigma)$ then $\underset{\sim}{Z} = B_{(m \times k)} \underset{\sim}{X}_{(k \times 1)}$ also has a normal distribution and obtain its parameters. [10+6+4]=[20]

2.a) Let $\underset{\sim}{X}_1, \ldots, \underset{\sim}{X}_N$ be a random sample from $N_k(\underset{\sim}{\mu}, \Sigma)$. Show that $\bar{\underset{\sim}{X}} = \frac{1}{N} \sum_1^N \underset{\sim}{X}_i$ and $A = \sum_1^N (\underset{\sim}{X}_i - \bar{\underset{\sim}{X}})(\underset{\sim}{X}_i - \bar{\underset{\sim}{X}})'$ are independent. (Prove the main result you use here).

b) Let $\underset{\sim}{Y}_\alpha$ be distributed as $N_k(\underset{\sim}{\partial} c_\alpha, \Sigma)$ for $\alpha = 1, \ldots, N$ and independent. Let $\sum_1^N c_\alpha^2 > 0$. Show that $\underset{\sim}{U} = \sum_{\alpha=1}^N c_\alpha \underset{\sim}{Y}_\alpha / (\Sigma c_\alpha^2)$ is distributed as $N_k(\underset{\sim}{\partial}, \Sigma/(\Sigma c_\alpha^2))$. Show also that $E = \sum_1^N (\underset{\sim}{Y}_\alpha - c_\alpha \underset{\sim}{U})(\underset{\sim}{Y}_\alpha - c_\alpha \underset{\sim}{U})'$ is independently distributed of $\underset{\sim}{U}$. [12+8]=[20]

3.a) Obtain the sampling distribution of the sample partial correlation coefficient $r_{12.3,\ldots,k}$ based on a sample of size N from $N_k(\underset{\sim}{\mu}, \Sigma)$ when $\rho_{12.3,\ldots,k} = 0$. (You can assume the sampling distribution of the sample correlation coefficient.)

b) A random sample of 24 observations from $N_3(\underset{\sim}{\mu}, \Sigma)$ gave the following sample covariance matrix. Test whether the partial correlation between X_1 and X_2 given X_3 is zero against the alternative that it is positive. [10+10]=[20]

$$S = \begin{bmatrix} 11.25 & 9.40 & 7.15 \\ & 13.53 & 7.38 \\ & & 11.57 \end{bmatrix}.$$

4.a) Show that the sample correlation coefficient r from a bivariate normal distribution with correlation coefficient ρ is asymptotically distributed as $N(\rho, (1-\rho^2)^2)$. Obtain a variance stabilizing transformation for the asymptotic distribution of r.

b) A sample of 67 observations from a bivariate normal distribution give the sample covariance matrix as

$$\begin{pmatrix} 5.99 & -3.90 \\ & 10.34 \end{pmatrix}$$

Test the hypothesis $H_0: \rho = 0.5$ against $H_1: \rho \neq 0.5$. [12+8]=[20]

5.a) A stochastic matrix A is distributed as $W_p(A|N|\Sigma)$. Obtain the characteristic function of A. Hence show that if $A_1, \ldots, A_k$ are independent stochastic matrices, A_i being distributed as $W_p(A_i|N_i|\Sigma)$ for $i = 1, \ldots, k$, then $\sum_1^k A_i$ is also distributed according to the Wishart distribution.

b) A stochastic matrix A is distributed as $W_p(A|N|\Sigma)$ and $\underset{\sim}{h}$ is a $p \times 1$ vector of constants. Derive the distribution of $\underset{\sim}{h}' A \underset{\sim}{h} / \underset{\sim}{h}' \Sigma \underset{\sim}{h}$. [12+8]=[20]

6.a) Let $\underset{\sim}{X}_1, \ldots, \underset{\sim}{X}_N$ ($N > k$) be a random sample from $N_k(\underset{\sim}{\mu}, \Sigma)$ (Σ positive definite) and let $T^2_{(\mu)} = N(\bar{\underset{\sim}{X}} - \underset{\sim}{\mu})' S^{-1} (\bar{\underset{\sim}{X}} - \underset{\sim}{\mu})$ where $\bar{\underset{\sim}{X}} = \frac{1}{N}\sum_1^N \underset{\sim}{X}_i$ and $(N-1)S = \Sigma(\underset{\sim}{X}_i - \bar{\underset{\sim}{X}})(\underset{\sim}{X}_i - \bar{\underset{\sim}{X}})'$. Show that $T^2(\mu_0)$ is a function of the likelihood ratio test criterion for testing the hypothesis $\underset{\sim}{\mu} = \underset{\sim}{\mu}_0$ vs $\underset{\sim}{\mu} \neq \underset{\sim}{\mu}_0$ when Σ is unknown. Obtain the sampling distribution of T^2 (or a suitable multiple of T^2).

b) How would you use this statistic to obtain a confidence region for $\underset{\sim}{\mu}$? [16+4]=[20]

7. Observations on X_1: Arithmetic power. X_2: Intellectual interest, X_3: Social interest and X_4: Activity interest are taken on 24 school going children and the sample covariance matrix is as below. Based on the data test the hypothesis that the multiple correlation of X_1 on (X_2, X_3, X_4) i.e. $\rho_{1(234)} = 0$.

$$S = \begin{pmatrix} 2.279 & -0.510 & 0.446 & 0.301 \\ & 11.790 & -0.235 & 0.202 \\ & & 0.184 & 0.149 \\ & & & 0.500 \end{pmatrix}$$

[20]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80

PERIODICAL EXAMINATIONS

Statistics-14: Sequential Analysis
and
Nonparametric Methods

Date: 24.3.80 Maximum Marks: 100 Time: 3 hours

Note: Answer as much as you can. The maximum you can score is 100.

1. Let $f(x,\theta)$ be the frequency function of a random variable X, where $\theta \in (H)$ is an unknown parameter.

a) Define precisely what is meant by a sequential procedure of testing a hypothesis H_o: $\theta \in \omega$ against the alternative $\theta \in (H) - \omega$.

b) Define the operating characteristic (O C) function $L(\theta)$ and the power function $P(\theta)$ of the above sequential procedure.

c) If $Z_n = \sum_{i=1}^{n} \log \frac{f(x_i,\theta'')}{f(x_i;\theta')}$, where the observations are taken till a terminal decision is taken and θ', $\theta''\in (H)$ are two fixed points, show that

$E(\exp Z_n \mid H_o \text{ accepted}: \theta') = L(\theta'')/L(\theta')$ and

$E(\exp Z_n \mid H_o \text{ rejected}: \theta') = P(\theta'')/P(\theta')$. [6+8+6]=[20]

2. Let X have the frequency function $f(x,\theta)$ where θ is an unknown parameter belonging to (H). Let $S(b,a)$ denote the SPRT of strength (α,β) for testing the simple hypothesis H_o: $\theta = \theta_o$ against the alternative H_1: $\theta = \theta_1$.
Obtain the Wald's approximations for a and b. If (α',β') is the strength of the modified test, show that $\alpha' + \beta' \leq \alpha + \beta$.

Show further that $\log \frac{\beta}{1-\alpha} \leq \min(0,b)$ and $\log \frac{1-\beta}{\alpha} \geq \max(0,a)$.
[4+6+10]=[20]

3.a) Let X be a random variable whose moment generating function $\phi(h) = E(\exp.hX)$ exists for all real h. Suppose further that there exists some $0 < \delta < 1$ such that $P(X < \log(1-\delta)) > 0$ and $P(X > \log(1+\delta)) > 0$. Show that the algebraic equation $\phi(h) = 1$ has unique non-zero solution if $E(X) \neq 0$.

b) If X is a Bernoulli random variable with $P(X=1) = p$, $0 < p < 1$, use the above result to obtain the OC function of the SPRT of strength (α,β) for H_o: $p = p_o$ against H_1: $p = p_1$. [12+8]=[20]

4.a Define the Average Sample Number (ASN) function of a sequential procedure. Obtain the Wald's approximate formula for the ASN function of an SPRT for testing a simple hypothesis against a simple alternative.

b) Obtain the ASN function of the SPRT of strength (α,β) for testing H_o: $\theta = \theta_o$ against H_1: $\theta = \theta_1$ in $N(\theta,1)$. [10+10]=[20]

p.t.o.

5. X is a Bernoulli random variable with $P(X=1)=p_1$, $0 < p_1 < 1$ and Y is a Bernoulli random variable independent of X with $P(Y=1) = p_2$, $0 < p_2 < 1$ where p_1 and p_2 are unknown. Derive an appropriate SPRT of strength (α, β) for testing the hypothesis $H_0: p_1 \geq p_2$ against $H_1: p_1 < p_2$. Obtain the Wald's approximations for the OC and ASN functions for the above SPRT. [12+8]=[20]

6. Let X be a random variable having the Poisson distribution with unknown parameter λ. Derive the SPRT of strength (α,β) for testing $H_0: 0 < \lambda \leq \lambda_0$ against $H_1: \lambda > \lambda_0$. Obtain the Wald's approximations for the OC and ASN functions of the above SPRT. [10+10]=[20]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part IV : 1979-80
ANNUAL EXAMINATIONS

Statistics - 14: Sequential Analysis and Nonparametric Methods

Date: 26.5.80 Maximum Marks. 100 Time. 3 hours

Note: Answer as much as you can. The maximum you can score is 100. Complete and precise answers are given more weight.

1.a) Define in the general form, a linear rank statistic for testing the hypothesis that a sample $X_1, \ldots, X_N$ of i.i.d. observations come from a continuous distribution symmetric about 0. Show that its null distribution is symmetric. Obtain its mean and variance and also derive a sufficient condition for the rank statistic to be asymptotically normal.

b) Illustrate your results by taking any specific values for the scores. [12+8]=[20]

2.a) $X_1, \ldots, X_N$ are i.i.d. observations from a continuous distribution. Show that the rank vector $R = (R_1, \ldots, R_N)$ of $(X_1, \ldots, X_N)$ has uniform distribution over the space $\mathcal{R}$ of all permutations of $(1, \ldots, N)$.

b) If $S = \sum_{i=1}^{N} c_i a(R_i)$ where $c_1, \ldots, c_N$ and $a_1 \ldots a_N$ areknown constants, derive a sufficient condition for the distribution of S to be symmetric about its mean. Obtain the variance of S.

c) If $N \geq 4$ show that $S = R_1 + R_2 - R_3 - R_4$ is symmetrically distributed about 0. [6+10+4]=[20]

3.a) The amount of iron present in the livers of two randomly chosen groups of white rats is measured after they had been fed on two different diets for a specified period and the data are as follows:

Diet A: 2.23, 1.14, 2.63, 1.00, 1.35, 2.01, 1.64
1.31, 1.13, 1.01, 1.70, 5.59, 0.96

Diet B: 4.50, 3.92, 2.07, 1.23, 6.96, 1.61, 2.94
1.96, 3.68, 1.54, 2.59, 1.35, 1.06

By doing an appropriate rank test examine whether different diets appear to affect the amount of iron present in the liver.

b) A random sample of 10 people who drove automobiles was selected to see if alcohol affected their reaction time. Each driver's reaction time was measured in a laboratory before and after drinking a specified amount of beverage containing alcohol. The results are as follows:

p.t.o.

3.b) (contd.)

Subject:	1	2	3	4	5	6	7
Reaction time before :	0.68,	0.64,	0.67,	0.82,	0.58,	0.80,	0.72
Reaction time after :	0.73,	0.62,	0.66,	0.92,	0.69,	0.87,	0.78

(contd.)	8	9	10
Reaction time before	0.65,	0.86,	0.73
Reaction time after	0.78,	0.89,	0.58

Is there any effect of alcohol on the reaction time ?

4.a) Four job training programs were tried on 20 new employees where 5 employees were randomly assigned to each. The results at the end of the program are as follows.

Program 1 : 50.30, 45.35, 42.38, 38.00, 33.00
Program 2 : 64.28, 20.86, 62.50, 48.75, 35.00
Program 3 : 93.10, 83.10, 77.47, 68.03, 40.00
Program 4 : 80.94, 77.97, 76.20, 67.66, 49.65

Examine whether there is any difference in the effectiveness of the four training programs.

b) At the beginning of the year a K.G. class was divided randomly into two groups and two different methods were used for teaching. At the end of the year each student was given a reading test with the following results.

Group 1: 227, 176, 252, 149, 16, 55, 234, 194, 247, 89, 99
104, 147, 88, 161, 171, 174, 194, 248, 206

Group 2: 209, 14, 165, 171, 292, 271, 151, 235, 147, 98, 63
184, 53, 228, 271, 19, 127, 151, 101, 179

Use Wald-wolfowitz runs test to examine whether there is any significant difference in the two methods. [14+6]=[20]

5.a) Show that Spearman's rank correlation test is a linear rank test for testing the hypothesis that two random variables with continuous distributions are independently distributed.

b) Cochran compared the reactions of several patients with each of two different drugs to see if there is any positive correlation between the two reactions of each patient. Do the data given below agree with the conclusion that there is positive correlation between the reactions of the two drugs?

Patient	1	2	3	4	5	6	7	8	9	10
Drug A:	0.7,	-1.6,	-0.2,	-1.2,	-0.1,	3.4,	3.7,	0.8,	0.[illegible]	2.0
Drug B:	1.9,	0.8,	1.1,	0.1,	-0.2	4.3,	5.5,	1.6,	4.6,	3.4

[10+10]=[20]

6. Describe the following test statistics briefly giving their form, limiting distribution and explaining when they are used and how.

a) Van Der Waerden test.

b) Kendall's τ.

c) Kolmogorov-Surnov one sample test. [7+7+7]=[21]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80
and
M.Stat. Previous year: 1979-80
PERIODICAL EXAMINATION
Q.C and OR

79-80 |461/5-7|

Date: 5.11.79 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions or parts thereof as you can. The maximum you can score is 100. Marks allotted to each part are shown in brackets.

1.a) Explain the role of rational subgroups in operating a control chart.

b) A company manufacture Z cycline capsules. Capsules are preparations in which potent drugs mixed with suitable inert diluent are enclosed in hard or saft gelatin shells. The net weight (weight of contents in the capsule) is specified as 310± 5 mg. The following table provides the weight of empty capsules for 20 samples each of size 4.

Sample No.	Weight (mg.) 1	2	3	4
1	79.37	92.65	86.83	83.05
2	85.56	82.25	87.00	93.29
3	92.05	82.90	85.38	87.45
4	81.34	8163	81.56	82.38
5	88.35	80.12	84.53	88.33
6	82.53	78.82	82.64	79.42
7	81.00	82.82	94.57	88.34
8	82.42	82.84	77.35	81.35
9	82.05	80.42	83.73	85.85
10	89.82	79.00	88.42	86.37
11	95.00	76.23	87.64	82.76
12	87.75	84.96	93.00	80.57
13	83.26	91.42	95.32	83.57
14	90.09	80.02	81.10	85.73
15	77.61	81.94	82.57	91.86
16	91.04	83.45	79.80	95.78
17	89.30	79.45	80.52	79.58
18	84.00	83.22	80.92	83.57
19	88.42	93.54	79.75	80.60
20	87.53	85.40	84.98	87.83

Analyse the data by means of drawing an $\bar{X}$-R chart. On the bases of your analysis recommend the average gross weight (weight of contents plus the empty capsule) and the corresponding control limits using the sample size as 4. (You may assume the weight of contents and the weight of empty capsule as independent random variables. [8+32]=[40]

2.a) In order to control the number of blue spots in an enamel plate 16 samples each containing 4 plates were taken and the number of blue spots in each plate was counted. The data are given below:

Sample No.	Number of blue spots in plates 1	2	3	4
1	3	2	4	5
2	1	1	0	2
3	4	1	1	2
4	5	5	1	6
5	2	2	0	1
6	4	1	1	0
7	2	3	1	0
8	1	2	1	1
9	4	0	2	1
10	5	1	1	1
11	1	0	1	1
12	2	2	1	3
13	4	2	1	3
14	2	2	4	3
15	0	1	1	1
16	1	2	2	1

Draw a suitable control chart and give your suggestion for future production.

2.b) What is process capability? Discuss its relationship with design tolerance indicating various alternative courses of action which producer might take. [14+6]=[20]

3.a) Why would you prefer sampling inspection plan to traditional hundred percent inspection?

b) Given lot size N = 1500, AQL = 0.05, LTPD = 0.08, Producer's Risk = 0.05 and consumer's risk = 0.10, obtain a single sampling plan by attributes using an approximate method. [8+12]=[20]

4.a) Draw AOQ curve for the plan n = 25, c = 2 when lot size is 1000. Read the value of AOQL approximately from the curve and compare the same with AOQL obtained from Dodge and Romig's formula.

b) Explain the meaning of following terms

(i) IQL (ii) AQL and (iii) Consumer's Risk. [14+6]=[20]

5.a) Show that if a set of $k \leq m$ vectors $a_1, a_2, \ldots, a_k$ can be found that are linearly independent and such that $\sum_j a_j x_j = b$ and all $x_j \geq 0$ then the point $X = (x_1, x_2, \ldots, x_k, 0, 0, \ldots, 0)$ is an extreme point of the convex set of feasible solutions to a linear programming problem where all the symbols have the usual meaning.

b) Solve the following linear programming problems graphically

i) Maximize $Z = x_1 + 1.5\,x_2$

subject to $2x_1 + 3x_2 \leq 6$

$x_1 + 4x_2 \leq 4$

$x_1, x_2 \geq 0$

ii) Minimise $Z = 6x_1 + 4x_2$

subject to $2x_1 + x_2 \geq 1$

$3x_1 + 4x_2 \geq 1.5$

$x_1, x_2 \geq 0.$ [6+7+7]=[20]

| 79 - 80 | 462/583 |

INDIAN STATISTICAL INSTITUTE
M.Stat. Previous year : 1979-80
and
B.Stat.(Hons.) Part IV : 1979-80
MID-YEAR EXAMINATIONS
Statistics-15: SQCOF/SQCOR

Date: 24.12.79 Maximum Marks: 100 Time: 3 hours

Note: Attempt any four questions. The marks allotted for each question are given in brackets.

1.a) Prove that every extreme point of the convex set of feasible to $Ax=b$ is a basic feasible solution.

b) Prove that if $x_1, x_2, \ldots, x_k$ are k different optimal basic feasible solutions to a l.p.p., then any convex combination of $x_1, x_2, \ldots, x_k$ is also an optimal solution.

c) Define a supporting hyper plane. Prove that a closed convex set which is bounded from below has an extreme point in every supporting hyper plane. [6+4+10]=[20]

2.a) Given a b.f.s. $x_B = B^{-1}b$ with $Z_o = C_B x_B$ to the l.p.p. $Ax=b$, $x \geq 0$, max $Z=Cx$, such that $z_j - c_j \geq 0$ for every column a_j in A. Then Z_o is the maximum value of Z subject to the constraints and the b.f.s. is an optimal b.f.s.

b) Solve the following problem:

$$\text{Minimise } Z = 4x_1 + x_2$$

$$\text{subject to } \quad 3x_1 + x_2 = 3,$$
$$4x_1 + 3x_2 \geq 6,$$
$$x_1 + 2x_2 \leq 3,$$
$$x_1,\ x_2 \geq 0$$

[8+12]=[20]

3.a) Solve by simplex method the following l.p.p.

$$\text{Maximise } Z = 10x_1 + x_2 + 2x_3$$

$$\text{subject to } \quad x_1 + x_2 - 2x_3 \leq 10$$
$$4x_1 + x_2 + x_3 \leq 20,$$
$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

b) What are artificial variables ? Solve by Charne's Method or otherwise,

$$\text{Minimise } \quad Z = 4x_1 + 2x_2,$$

$$\text{subject to } \quad 3x_1 + x_2 \geq 27,$$
$$x_1 + x_2 \geq 21,$$
$$x_1 + 2x_2 \geq 30,$$
$$\text{and } \quad x_1 \geq 0, \quad x_2 \geq 0.$$

[10+10]=[20]

p.t.o.

4.a) If a l.p.p. has an optimal solution, prove that its dual has also an optimal solution.

b) Solve the following problem by solving its dual problem :

$$\text{Minimise } Z = 3x_1 + 2x_2 + x_3 + 4x_4$$

$$\text{subject to } \quad 2x_1 + 4x_2 + 5x_3 + x_4 \geq 10$$
$$3x_1 - x_2 + 7x_3 - 2x_4 \geq 2$$
$$5x_1 + 2x_2 + 3x_3 + 6x_4 \geq 15$$
$$x_1,\ x_2,\ x_3 \text{ and } x_4 \geq 0.$$

[6+14]=[20]

5.a) For an item, the production is instantaneous.
The unit cost of storage = Re 1 per month
Setup cost (ordering cost) = Rs 25 per setup
Demand ... . = 200 units per month.

Find optimum quantity to be produced per setup, and hence determine the total cost of storage and setup per month.

b) A purchasing agent for spare parts is to select one among three sources. Source A will sell a particular component for Rs.10 each regardless of the size of the order. Source B requires a purchase of 1000 units at least, but will sell each item at the rate of Rs.9.50 . Source C requires that orders be placed for at least 1200 units and will sell the items at the rate of Rs.9.00. A fixed cost of Rs.200 is incurred each time an order is placed. Which source and what quantity of order should be select? What will be annual cost of carrying inventory, ordering and purchasing the items? The demand rate is 500 units per annum and cost of carrying inventory I = 0.25. [6+14]=[20]

Sessionals and Educational visit report. [20]

79-80 | 472/582

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80
and
M.Stat. Previous year: 1979-80

Statistics-16: Computer Programming

MID-YEAR EXAMINATIONS

Date: 26.12.79 Maximum Marks: 100 Time: 3 hours

Note: All questions are compulsory.
Marks are indicated against each question.

1. What are the major 'types' of statements available in FØRTRAN ? Give their names and two examples of each type. [15]

2.a) Examine whether the following items are valid variable names and if not, specify why:

(i) DØ (ii) DØIF (iii) DØS12 (iv) 4DELHI (v) GØTØIF
(vi) GØTØ4 (vii) PRINT1, (viii) N4LTC (ix) RØ-AD
(x) TØGØ [10]

b) Write the output of the following program

```
      N = 0
      DØ 50  I = 1, 10
      N = N+1,
      DØ 50  J = 1, 10, 2
      N = N+1
      DØ 50  K = 1, 10, 7
      N = N-1
   50 CØNTINUE
      PRINT 110, N
  110 FORMAT (5X, I7, 4HØVER, I4 )
      STØP
      END
```
[10]

3. Write a FORTRAN program to print the prime numbers upto a given number N(N included). The numbers should be printed five per line with a gap of four spaces in-between numbers. Give the flow chart also. [20]

4. Given a square matrix having 10 rows and 10 columns, write a FORTRAN /program to evaluate a given power of the matrix. That is, if A is the matrix and n a positive integer, the program should evaluate the matrix A^n (A multiplied by itself n times) and print out its elements. [20]

5.a) Explain the concept of a FUNCTIØN subprogram. How do you transmit parameters implicitly ? [10]

b) Write any FUNCTIØN subprogram and its associated calling main program to illustrate the technique of implicit transmission of parameters. [15]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80
and
M.Stat. Previous year : 1979-80

Statistics-16: Biostatistics/Biostatistics

PERIODICAL EXAMINATIONS

Date: 21.4.80 Maximum Marks: 100 Time: 4 hours

Note: The paper carries 125 marks. You may attempt any part of any question. Any score over 100 will be counted as 100. You may use notes.

1. Consider a diploid at a single locus with two alleles A and a with A dominant. Suppose gamete a is less viable than gamete A in the ratio $a : A = \alpha : 1$ where $0 < \alpha < 1$ (i.e., a proportion α of a live compared to 1 of A). Find the genotypic and phenotypic arrays in the offspring of Aa x Aa mating. [10]

2. Let A and a be the dominant and recessive alleles at the locus A and similarly B and b at locus B. We assume that these loci are linked with recombination fraction λ. Find the phenotypic distribution in the offspring of the following crosses:

   (a) AB | ab x Ab | aB (coupling x repulsion)

   (b) Ab | aB x Ab | aB (repulsion x repulsion). [10]

3. Consider the ABO blood group system with respect to a receipient - donor pair (R, D). Let

   X : no. of alleles (in both chromosomes) w.r.t. which the receipient and a donor differ at this locus (X can be 0, 1 or 2).

   We say that D is compatible with R if D possesses no alleles which are absent in R. Let

   $Y = 0$ if D is compatible with R
   $= 1$ if D is incompatible with R.

   If p, q, r are the gene frequencies of A, B, O respectively, find the joint distribution of (X,Y) if R, D are chosen independently and at random from the population. [10]

4. Making a suitable formulation of the problem and deriving suitable results show that hemophilia will be less frequent in females in a population in equilibrium under panmixia. What will be the ratio of female: male hemophiles under equilibrium? [20]

5. Compute the coefficient of parentage between double first cousins. Obtain the recurrence relation for the coefficient of inbreeding in the nth generation of a regular double first cousin inbreeding system. [20]

6. Consider a sex-linked character in which selection is practised only in the homogametic sex and suppose a proportion k of the recessives are discarded. Evaluate the progress of the population. [20]

p.t.o.

7. Let $p_o = 0.90$ and $q_o = 0.10$ be the gene frequencies of A and a in an initial population. Let $\mu = 5.10^{-5}$ and $\nu = 1.10^{-6}$ be the mutation rates for $A \rightarrow a$ and $a \rightarrow A$ respectively. Assuming no selection and infinite population, calculate:

   a) the gene frequencies at equilibrium;

   b) the gene frequencies in the fifth generation;

   c) the number of generations required to increase q by 0.005. [20]

8. Suppose we have a pair of alleles A, a with A completely dominant to a and a sample of N dominant individuals. We obtain one progeny each by selfing each individual and find r have no recessive progeny and N - r have recessive progeny. Find the maximum likelihood estimate of the proportion of homozygous dominants in the population. [15]

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79 – 80: 474/574

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979–80
M.Stat. Previous year: 1979–80

ANNUAL EXAMINATIONS

Statistics-16: Biostatistics

Date: 30.4.80 Maximum Marks: 100 Time: 3 hours

Note: Each question carries 20 marks. You may attempt any part of any question. Any score over 100 will be counted as 100. You may use notes.

1. Consider a panmictic population in equilibrium at a single locus with two alleles, A and a, with the gene frequency of A being p. Derive the joint genotypic array of a pair of siblings. If A dominates a, derive the phenotypic distribution of a pair of siblings. Giving values 0,1,2 respectively for aa, Aa and AA compute the correlation coefficient from the joint array.

2. Complete self-pollination in plants is not a common phenomenon. Actually most so-called self-pollinated plants practise random mating also. Consider a diploid with two alleles A, a at a single locus. Consider a population in equilibrium. Let ϕ be the proportion practising selfing, $1-\phi$ random mating. Compute an expression for the proportion h_n of heterozygotes in the nth generation and show that as $n \to \infty$

$$h_n \to 2pq\left(1 - \frac{\phi}{2-\phi}\right),$$

where p: initial gene frequency of A, $q = 1-p$. Compute F_∞, the coefficient of inbreeding in the equilibrium population.

3. Consider an autosomal locus with s alleles. Let there be selection against homozygotes acting in such a way that a proportion k of homozygotes $(0 < k < 1)$ is not viable. Show that equilibrium is attained at uniform frequency of the s alleles (i.e., the gene frequency of each allele is $1/s$). Is this equilibrium stable (i.e., if in a generation the gene frequencies are not of the form $1/s$ for each allele, does the population return to this equilibrium ?

4. In a certain study, sampling is done through the parents. Here the affected individuals are dominants. The phenotypic mating Dominant x Dominant only is considered and observations are restricted only to parents with at least one normal child. Derive the theoretical distribution of the number of normal children in the selected families with size s under segregation probability θ. Find an unbiased estimator of θ.

Let A_1, A_2 be codominant alleles at locus A, and B_1, B_2 be two codominant alleles at locus B. Consider the population of intercrosses

$$A_1A_2B_1B_2 \times A_1A_2B_1B_2 .$$

p.t.o.

5. (contd.)

a) Write down the genotype and the phenotype distributions under the hypothesis of independent segregation at the two loci;

b) Write down the contrasts for testing the hypothesis that the segregation ratios at locus A are 1:2:1 for $A_1A_1 : A_1A_2 : A_2A_2$. Do similarly for locus B.

c) Set up the contrasts for linkage.

d) Write down the statistics and their distributions for testing these hypotheses.

6. Derive expressions for the scoring method of maximum likelihood estimation in the case of a quantal assay problem with the logistic tolerance distribution; also explain the scoring method.

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80
and
M.Stat. Previous year: 1979-80

Econometrics-4: Econometrics | Econometrics

PERIODICAL EXAMINATIONS

Date: 31.3.80 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: Answer all questions.
Marks are given in brackets.

1. Describe the usual least squares method of fitting a Pareto distribution to an empirical frequency distribution of income.

   Can you modify this method to guarantee that the expected frequancies add up to the total of observed frequencies over the range where the Pareto law holds ? [10]

2. State clearly the law of proportionate effect and show that it leads to a lognormal distribution of income. [10]

3. Bring out the assumptions and approximations generally made in estimating Engel curves of the following form using household budget data:

$$y = f(x)$$

   where y = per capita household consumption of a specified item and x = per capita household income. [15]

4. How would you compare different algebraic forms of Engel curves in respect of goodness of fit when fitting them to empirical data based on budget enquiries ? Give a detailed account mentioning in particular the adjusted coefficient $\bar{R}^2$ and the DW statistic. [25]

5. EITHER

The following shows an empirical size distribution of monthly incomes for employees in an industrial establishment :

income (Rs.):	0-100	101-200	201-300	301-400	401-500	501-
No. of of earners	135	207	112	62	27	6

Assuming that the underlying distribution is lognormal, estimate the parameters of that distribution by any suitable method. Hence estimate the C.V. of the distribution and also the ratio

$$I = \frac{\text{average income of the top } 20\%}{\text{average income of the bottom } 20\%}$$ [30]

OR

The following table is based on a budget enquiry. At the time of analysis, the sample households were ranked in ascending order of per capita income and then grouped in such a manner that each group included 20 per cent of the estimated population.

p.t.o.

5. [illegible]

quintile group from bottom	average per person per month	
	income (Rs.)	expenditure on education (Rs.)
1st	137	5
2nd	216	16
3rd	372	37
4th	528	68
5th	923	152

Fit a semi-logarithmic Engel curve $y = \alpha + \beta \log x$ by least squares method, taking x = per capita income and y = per capita expenditure on education. (Compute $\hat{\alpha}$, $\hat{\beta}$; but you need not compute any $\hat{y}$.) Then compute Engel elasticity at x = Rs.500 using the fitted curve. [30

6. Practical Record. [10

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part IV: 1979-80
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ANNUAL EXAMINATIONS
Econometrics: Theory and Practical

Date: 16.5.80 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: Answer any three questions from Group A and all questions from Group B. Marks are given in brackets.

1.a) Define Lorenz ratio in terms of the Lorenz diagram and show how it is related to the Gini mean difference.

b) Derive the expression for the Lorenz ratio for a Paretean distribution of income. [20]

2. Describe fully how you can carry out Engel curve fitting to household budget data making allowances for effect of age-sex composition of the household on its consumption pattern. [20]

3. Examine the following problems arising in the estimation of demand functions from time series data:-

a) multicollinearity, (b) identification. [20]

4. State clearly the properties of the Cobb-Douglas production function and bring out the advantages of this functional form.

Given a sample of n observation-sets on the variables, how would you test whether returns to scale are constant or not ? [20]

5. Write short notes on any two of the following:-

a) ML estimation of lognormal parameters from grouped frequency data,
b) Economy of scale in household consumption.
c) Integration of cross-section and time series data in demand analysis. [20]

Group B.

6. EITHER

Fit a Pareto distribution to the following data on distribution of incomes liable to Surtax in UK during 1953-54:

Income (£):	2000-2499	2500-	3000-	4000-	6000-	10,000-
percentage of taxreturns:	28.6	19.9	21.7	16.8	8.8	4.2

(Note:- Your fit should cover the entire range of incomes included in the data.).

p.t.o.

6.(contd.)

<u>OR</u>

The following matrix gives the corrected sums of squares and products of three variables included in a study based on 15 annual observations:-

$$
\begin{array}{c}
X_1 \\ X_2 \\ X_3
\end{array}
\begin{pmatrix}
30.36 & 12.48 & 107.96 \\
 & 116.64 & 123.20 \\
 & & 532.00
\end{pmatrix}
$$

$$
\qquad X_1 \qquad\quad X_2 \qquad\quad X_3
$$

Here X_1 = log (cereals consumption per capita), X_2 = log (relative price of cereals) and X_3 = log (real disposable income per capita).

Assuming that the demand function is of the constant elasticity form, estimate the income and price elasticities of demand. Also set up 95 %. confidence limits for each elasticity. [30]

7. Practical Record. [10]
