# INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : 2018-19

Course Name: M.S. (Q.E.) I YEAR

Subject Name: Game Theory I

Date: 03-09-2018 Maximum Marks: 40 Duration: 2 hours

**Partial marks will be given**
**(i) if the approach is conceptually correct (even if the answer is incomplete), and**
**(ii) only if the writing is logically complete (so that a person with sound knowledge in Mathematics and English can understand your answers).**

**Problem 1.** Justify your answer with a proof or a counterexample.

Consider a finite zero-sum game with two players in which each player has at least three strategies. Then, there exists a unique MNE in which all the strategies of each player gets positive probability.

(10)

**Problem 2.** Consider the following auction.

There are three buyers: 1,2 and 3. The auctioneer opens the auction and Buyer 1 bids first. Then, depending on the bid of Buyer 1, Buyer 2 either challenges it by proposing a higher bid or quits the auction. Similarly, Buyer 3 either challenges the last standing bid or quits the auction. Finally, Buyer 1 gets a chance to challenge the last standing bid (if that bid is different from his/her own one) or quits the auction. The standing bid becomes the winning bid, and the item is sold to the corresponding bidder at a price equal to his or her bid.

**2.1.** Suppose that the valuations of the buyers are denoted by $v_1, v_2, v_3$. Suppose further that these valuations are known to each other (complete information). Consider all possible cases with respect to $v_1, v_2$ and $v_3$ for the following questions (wherever relevant).

(a) Is it possible to formulate this auction as a perfect information extensive form game with no chance? If yes, formulate it.

(b) Is it possible to formulate this auction as a normal form game? If yes, formulate it.

(c) Does this (auction) game have an NE? If yes, then find at least one NE of this game. Is there an NE of this game that is (i) subgame perfect (ii) not subgame perfect?

**2.2.** Suppose that the valuations are uniformly and independently distributed over $[0, 1]$. Is it possible to formulate this auction as a Bayesian game? If yes, formulate it. Is there a BNE of this game? Find at least one, if yes.

(30)

**INDIAN STATISTICAL INSTITUTE**

MidSemestral Examination: (2018-2019)

M. S. (Q.E.) – I Yr.

Computer Programming and Applications

Date: 07.09.2018 Maximum marks: 50 Duration: 2 hrs.

1. Give brief answers. [14×2 = 28]

a) Write two important features of system software.
b) What is called an operating system? Name two examples of OS.
c) Name two protocols used by the WWW.
d) What do you understand by cloud computing?
e) What is the format of an IP address in IPv4?
f) Is 1.160.44.260 a valid IP address? Justify.
g) State the source of adjacent string. 1017- 8D 08 10 0140 STA TEMP
h) What is the difference between multi-user and multi-tasking computer systems?
i) Name various components of the control unit of a computer.
j) What is the full form of SSD?
k) What is the difference between cache memory and random access memory?
l) Convert (computation to be shown) the decimal number $(1111)_{10}$ to its equivalent binary number.
m) Convert (computation to be shown) the hexadecimal number $(ABC)_{16}$ to its equivalent decimal number.
n) Write an expression whose computational complexity is $O(\log n)$.

2. Count the number of basic operations of each instruction and also the total number of instructions of the following pseudocode. [10]

Algorithm *prepareList* (*A*, *n*, *m*, *B*)

```
count ← 0
for i ← 0 to n do
   if A[i] >= m then
      { B[count] ← A[i]
        increase count by 1 }
  endif
  {increase i by 1}
endfor
return count
```

3. State what is computed by the pseudocode of question 2. [2]

4. Draw a flowchart to compute the value of ${}^{n}P_{r}$ . [10]

5. Write an algorithm to compute GCD of two integers. [10]

INDIAN STATISTICAL INSTITUTE

MSQE I, First Semester 2018-2019

Mid-Semester Examination

STATISTICS

Date: 04.09.2018 Duration: 3 hours.

[Total points 46. Maximum you can score is 40]

1. Two dice are rolled.

   (a) What is the probability that at least one is a six?

   (b) If the two faces are different, what is the probability that at least one is a six?

[3+4=7]

2. Suppose that $A$ and $B$ are events such that $0 < P(A) < 1$ and $0 < P(B) < 1$. Let $X$ and $Y$ be the corresponding indicator functions, i.e.,

$$X = \begin{cases} 1 & \text{if } A \text{ occurs} \\ 0 & \text{otherwise} \end{cases} \qquad Y = \begin{cases} 1 & \text{if } B \text{ occurs} \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Show that $Cov(X, Y) = P(X = 1, Y = 1) - P(X = 1)P(Y = 1)$.

   (b) Show that $Cov(X, Y) > 0 \Rightarrow P(Y = 1|X = 1) > P(Y = 1)$. Why is this result intuitively expected?

[3+3+1=7]

3. Suppose that $F_X(x)$ represent the cumulative distribution function of a random variable $X$ which is supported on the set of non-negative integers. Thus the support of $X$ is $\{0, 1, 2, \ldots\}$. Show that

$$E(X) = \sum_{i=0}^{\infty}(1 - F_X(i)).$$

[8]

4. Suppose that $X$ and $Y$ have a joint continuous distribution with the pdf

$$f(x, y) = \begin{cases} 3/4 & x^2 \leq y \leq 1 \\ & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $P[X \geq 1/2, Y \geq 1/2]$.

(b) Find $P[X \geq Y]$.

[4+4=8]

5. A random variable $X$ is said to have a Pareto distribution with parameters $\theta$ and $\alpha$ ($\theta > 0$ and $\alpha > 0$) if $X$ has the probability density function

$$f(x) = \begin{cases} \dfrac{\alpha\theta^{\alpha}}{x^{\alpha+1}} & x \geq \theta \\ 0 & \text{otherwise.} \end{cases}$$

Show that $\log(X/\theta)$ has an exponential distribution with rate parameter $\alpha$ (or mean parameter $1/\alpha$). [8]

6. The life (in hours) of electronic tubes of a certain type is supposed to be normally distributed with parameters $\mu = 155$ hours and $\sigma = 19$ hours. What is the probability that the life of the tube will be

(a) between 136 and 174 hours?

(b) between 117 and 193 hours?

(c) less than 117 hours?

(d) more than 193 hours?

[2+2+2+2=8]

# INDIAN STATISTICAL INSTITUTE

## Mid-Semestral Examination: 2018-19

**Course:** Master's in Quantitative Economics Year I

**Subject:** Microeconomics I

**Date:** 5th September, 2018 **Maximum Marks**: 40 **Duration:** 3 hours

**Answer all questions**

1. Prove that, for a tight deterministic demand function, the Weak Axiom of Revealed Preference is equivalent to Samuelson's Inequality. **(10 marks)**
2. Prove, for a deterministic supply function, that the Consistent Firm Choice and Non-reversibility conditions are, together, equivalent to the Weak Axiom of Profit Maximization. Prove also that Consistent Firm Choice and Non-reversibility are independent conditions. **(10 marks)**
3. Consider two competitive consumers facing identical price-wealth situations. Construct a deterministic demand function for each consumer such that each consumer individually violates the law of demand, but their aggregate stochastic demand function representation satisfies the law of demand in its stochastic version. **(10 marks)**
4. Suppose the universal set of alternatives is $X = \{a, b, c, d\}$, and consider a domain $Z = \{\{a, b, c, d\}, \{a, b, c\}, \{b, c, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, c\}\}$.
   (i) Construct a deterministic choice correspondence F defined over Z which satisfies Sen's $\alpha$ but violates Sen's $\beta$. **(5 marks)**
   (ii) Now construct another deterministic choice correspondence G over Z such that G satisfies Sen's $\beta$ but violates Sen's $\alpha$. **(5 marks)**

# Indian Statistical Institute

M.S.Q.E. 1st Year : 2018–2019
Mid-Semester Examination
Subject: Mathematical Methods
Date: 06/09/2018 Time: 2 hours Marks : 80

**Answer Group-A and Group-B on separate answer scripts.**

## Group-A

1. Suppose $A$ is a $3 \times 1$ matrix and $B$ is a $1 \times 3$ matrix. Prove that the product $AB$ is not invertible. [6]

2. For what values of $k$ does
$k.x + y = 1$
$x + k.y = 1$
have no solution, one solution or infinitely many solutions? [10]

3. Let $A, B, C$ are arbitrary matrices for which matrix multiplication is defined. Then show that $A \cdot (B \cdot C) = (A \cdot B) \cdot C$. [7]

4. Construct two $3 \times 3$ matrices $A$ and $B$, such that $AB \neq BA$. [5]

5. Let $A$ be an $n \times n$ invertible matrix with $n > 1$. Show that :
   (a) $\det(\text{adj } A) = (\det A)^{n-1}$.
   (b) $\text{adj}(\text{adj } A) = (\det A)^{n-2} A$. [4 + 8]

## Group-B

1. Show that the set $\{1, 2, 3, 4, 5, 7, 9, 11, 13, 15, ......\}$ is countable. [8]

2. Show that limit of $\frac{(n+1)}{(n^3 - 5n + 19)}$ as $n$ goes to infinity is 0. [10]

3. Consider the interval: $(a, b)$ where $a, b$ are real numbers and $b - a$ is extremely small. Show that:
   (a) The interval is uncountable.
   (b) There exists at least one rational number in the interval.
   (c) There exists at least one irrational number in the interval. [6 + 10 + 6]

**PGDBA 2018-20 Semester 1: Foundations of Database Systems**
**Mid Sem Exam**
**Time: 2 hours and 30 minutes**
All questions carry equal marks.

## Part A

Please refer to the SQL construct statements below for the questions in this part. These tables represent a simplified version of human resource data for an organization at a given point of time.

```
CREATE TABLE employees (
    emp_no      INT             NOT NULL,
    birth_date  DATE            NOT NULL,
    first_name  VARCHAR(32)     NOT NULL,
    last_name   VARCHAR(32)     NOT NULL,
    email       VARCHAR(64)     NOT NULL,
    profile     VARCHAR(2048),
    gender      ENUM ('M','F')  NOT NULL,
    PRIMARY KEY (emp_no)
);

CREATE TABLE managers (
    emp_no          INT             NOT NULL,
    manager_emp_no  INT             NOT NULL,
    PRIMARY KEY (emp_no),
);

CREATE TABLE salaries (
    emp_no      INT             NOT NULL,
    salary      INT             NOT NULL,
    FOREIGN KEY (emp_no) REFERENCES employees (emp_no) ON DELETE CASCADE,
    PRIMARY KEY (emp_no)
);
```

1. Suppose an employee's writes a profile description of 39 characters (1 character = 1 byte). How many bytes would be consumed to store his/her profile in the column `profile` of the table `employees`? Explain.
2. Write an SQL query to output the percentage of female employees in the organization.
3. Write an SQL query (with brief explanation) to output the names of all the employees who are not managers.
4. Write an SQL query (with brief explanation) to output the average salary of the managers.
5. Suppose every employee, except the general manager, must have exactly one manager, who is also another employee. The table `managers` stores this information, by storing the employee numbers of the employees and their manager's. Then, write an SQL query (with brief explanation) to output the full name and the profile of the general manager of the organization, in the following format

   ```
   full_name        profile
   ```

6. Is there any employee whose salary is greater than the salary of his/her manager? Write an SQL query (with brief explanation) to find out the answer.

   ```
   full_name        manager_email
   ```

## Part B

A distributed Hadoop file system contains customer transaction data for a retail store. Every time a customer makes a purchase, the tuples (`customer_id, product_id, timestamp`) are stored in the filesystem for all products purchased by the customer. Consider this data as input to design MapReduce algorithms (map and reduce functions) for the following problems. Write appropriate explanation for each of the solutions.

1. Describe a MapReduce algorithm to determine which customer has bought the most number of distinct products from the store.

2. Describe a MapReduce algorithm to compute all pairs of customers (A,B) such that A and B have bought the same product on the same day during the same hour of day from this store at least once. In your solution, you can assume there are obvious ways (functions) to convert timestamp into day and hour of day.

INDIAN STATISTICAL INSTITUTE

MSQE I, First Semester 2018-2019

Semester Examination

STATISTICS

Date: 12.11.2018 Duration: 3 hours.

[Total points 70. Answer as many as you can. Maximum you can score is 60]

1. (a) Suppose that two teams $A$ and $B$ are playing a best of 7 series; in this format the teams play a maximum of seven games and the first team to win four games wins the series. If the probability that team $A$ will win any particular game of the series against team $B$ is $1/3$ (independent of any other game), what is the probability that team $A$ will win the series.

   (b) Bus tickets in a certain city contain four numbers $U$, $V$, $W$, and $X$. Each of these numbers is equally likely to be any of the ten digits $0, 1, 2, \ldots, 9$. The four numbers are chosen independently. A bus rider is said to be lucky if $U + V = W + X$. What proportion of riders is lucky? [5+6=11]

2. Let $X_1, X_2, \ldots, X_n$ represent a random sample from a distribution having joint probability density function $f_\theta(\boldsymbol{x})$, $\theta \in \Theta$, where $\boldsymbol{x}$ belongs to the support of $(X_1, X_2, \ldots, X_n)$. Consider the problem of testing of hypothesis where the null $H_0 : \theta = \theta_0$ and the alternative $H_0 : \theta = \theta_1$ are both simple. Let $R$ be the rejection region of the test. Let $\alpha$ and $\beta$ be the probabilities of Type I error and Type II error, respectively.

   Suppose that, unlike the usual policy of fixing $\alpha$ to a suitable low level, and maximizing $1-\beta$ (or minimizing $\beta$) subject to that, the experimenter wishes to minimize the linear combination $a\alpha + b\beta$, where $a$ and $b$ are known positive constants. Show that this is achieved when the rejection region consists of such samples $\boldsymbol{x}$ for which

$$\left\{ \boldsymbol{x} : \frac{f_{\theta_1}(\boldsymbol{x})}{f_{\theta_0}(\boldsymbol{x})} > \frac{a}{b} \right\}.$$

[10]

3. Suppose that $X_1, X_2, \ldots, X_n$ form a sample of size $n$ from a given distribution with finite variance $\sigma^2$.



P.T.O

(a) Show that $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is an unbiased estimator of $\sigma^2$.

(b) Now suppose that $X_1, X_2, \ldots, X_n$ are drawn from a $N(\mu, \sigma^2)$ distribution, both parameters unknown. Show that $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is the maximum likelihood estimator of $\sigma^2$.

(c) Under the normal set up of part (b), check, between $\hat{\sigma}^2$ and $s^2$, which one has the smaller mean square error as an estimator of $\sigma^2$. [4+3+4=11]

4. Suppose that $X_1$ and $X_2$ are independent and identically distributed random variables defined on $\mathbb{R}^+$ each with common probability density function

$$f_X(x) = \sqrt{\frac{1}{2\pi x}} \exp\left\{-\frac{x}{2}\right\}, \; x > 0.$$

(a) Compute the joint pdf of random variables $Y_1 = X_1$ and $Y_2 = X_1 + X_2$.

(b) Find the marginal probability density function of $Y_2$. (You may use the result $\int_0^1 \frac{1}{\sqrt{t(1-t)}} dt = \pi$). [6+6=12]

5. Suppose that the random variable $X$ has moment-generating function

$$M_X(t) = \frac{1}{6}\exp\{-2t\} + \frac{1}{3}\exp\{-t\} + \frac{1}{4}\exp\{t\} + \frac{1}{4}\exp\{2t\},$$

find $P(|X| \leq 1)$. [6]

6. In order to study the reflective strength of three different paints, samples were taken from different painted surfaces for these three paints and the results are presented in the table below in appropriate coded units.

| Type of paint | Observations | | | | |
|---|---|---|---|---|---|
| I | 14.5 | 13.6 | 16.3 | 23.2 | |
| II | 16.2 | 14.7 | 17.3 | 16.8 | 14.6 |
| III | 15.2 | 14.4 | 18.7 | | |

Write down the one way ANOVA model with all the relevant assumptions. Test, at level 0.05, whether the average reflective strength of all the three paints are the same. [10 ]

7. In the following table, ten bivariate observations are represented; here $x_i$ represents the observed value of the response to the standard drug by the $i$-th individual, and $y_i$ represents the observed value of the response to the new drug by the $i$-th individual. We want to come up with a regression equation to predict the response to the new drug based on the response to the standard drug.

| $i$ | $x_i$ | $y_i$ |
|---|---|---|
| 1 | 1.9 | 0.7 |
| 2 | 0.8 | −1.0 |
| 3 | 1.1 | −0.2 |
| 4 | 0.1 | −1.2 |
| 5 | −0.1 | −0.1 |
| 6 | 4.4 | 3.4 |
| 7 | 4.6 | 0.0 |
| 8 | 1.6 | 0.8 |
| 9 | 5.5 | 3.7 |
| 10 | 3.4 | 2.0 |

(a) Construct the least squares regression line. (The following numbers are given: $S_{yy} = 26.309$, $S_{xx} = 36.081$, $S_{xy} = 24.717$).

(b) Test whether there is any regression effect.

(c) Find the 95% prediction interval for a new observation with $x = 2.0$. [3+3+4=10]

# Indian Statistical Institute
# First Semester Examination : 2018–2019

M.S.Q.E. 1st Year
Subject: Mathematical Methods
Date: 14 /11/2018 Marks: 100 Time: 3 hours

**Notations used are as explained in the class.**
**Answer Group-A and any five questions from Group-B on separate answer scripts.**

## Group-A

1. Let $V \subseteq \mathbb{R}^3$, $V = \{(x_1, x_2, x_3) : \frac{x_1}{3} = \frac{x_2}{4} = \frac{x_3}{2}\}$. Does $V$ form a vector space over $\mathbb{R}$? If so what is its dimension? [6]

2. Let $T : V \to W$ be a linear transformation from a vector space $V$ to a vector space $W$. Then prove that the kernel $Ker(T)$ and the image $Im(T)$ are subspaces of $V$ and $W$, respectively. [6]

3. If $x$ and $y$ are vectors in an inner product space $V$, then prove that $\langle x, y\rangle^2 \leq \langle x, x\rangle\langle y, y\rangle$. [6]

4. Let $A$ be an $n \times n$ matrix. Then prove that
   (a) the determinant of $A$ is the product of the $n$ eigenvalues, and
   (b) the trace of $A$ is the sum of the $n$ eigenvalues.

   [10]

5. Find two matrices $A$ and $B$ such that $det(A) = det(B)$, $tr(A) = tr(B)$, but $A$ is not similar to $B$. [6]

6. Show that a real symmetric $n \times n$ matrix $A$ is positive definite if and only if all the eigenvalues of $A$ are positive. [8]

7. Show that the characteristic polynomial of the $n \times n$ matrix

$$\begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{pmatrix}$$

   is $f(\lambda) := a_0 + a_1\lambda + \cdots + a_{n-1}\lambda^{n-1} + \lambda^n$. [8]

## Group-B

**Answer any five questions, each question carries 10 marks.**

1. Show that $\sqrt{3}$ is irrational. [10]

2. Find the cardinality of $[2.5, 17.3] \cup \{0\}$. [10]



P. T. O

3. Find from first principle, $\lim_{n\to\infty}(\frac{1}{n}+\frac{\sin n}{n+1})$. [10]

4. Show that $\lim_{x\to 1}\frac{x^2-\sqrt{x}}{\sqrt{x}-1}=3$. [10]

5. let $f:\mathbb{R}\to\mathbb{R}$ be continuous on $\mathbb{R}$ and $f(m)=0$ for every rational number $m$. Show that $f(x)=0$ for all $x\in\mathbb{R}$. [10]

6. On the cartesian plane, find points that are closest and farthest to $(1,2)$ and lying on the circle $x^2+y^2=80$. [10]

# Indian Statistical Institute

## Semester I Examination 2018-2019

## M. S. (Q. E.) - I year

### Subject: Computer Programming and Applications

Full Marks: 100 Duration: 3 hrs.

Date: 16/11/2018 **(Answer all questions)**

1. Answer the following questions in one or two sentences. [Max. 5×2 = 10]
   i. Which one is faster – SRAM or DRAM? Explain the reason.
   ii. What are the responsibilities of the Control Unit of a computer?
   iii. What are the different types of System Bus?
   iv. What is a computer server?
   v. Can a computer understand a program written in C language? – Discuss.
   vi. How does a compiler differ from an interpreter?

2. Write brief answers to the following questions. [Max. 10×2 = 20]
   i. Name two different data types used in C, one for each of two different storage sizes.
   ii. What is the use of extern keyword in C?
   iii. What is the concept of void type used in C programming?
   iv. What is called a global variable in C? What is its scope?
   v. What is the difference between structure type and union type variables?
   vi. Write a C for loop whose block of statements never gets executed.
   vii. What is the difference between a "do…while" loop and a "while" loop?
   viii. What is the limitation of a switch statement of C programming language? What is the role of 'break' command of a switch statement block?
   ix. Write down 4 different assignment operators of C language.
   x. Write down two different logical operators used in C programming language. When do these operators return true values?
   xi. Write the names of two functions used for dynamic memory allocations. Name the header file which contains their definition.
   xii. How can the fopen() statement be used to append to an existing file? What are the actions which get initiated by calling the fclose() function?

3. What will happen when the following statements of some C program get executed? Explain. [Max. 4×2 = 8]

*a.*
```
void swap(int a, int b){
int t;
t = a; a = b; b = t;
}

int main() {
int n1 = 10, n2 = 20;
swap(n1, n2);
printf("n1: %d, n2: %d\n", n1, n2);
}
```

P.T.O.

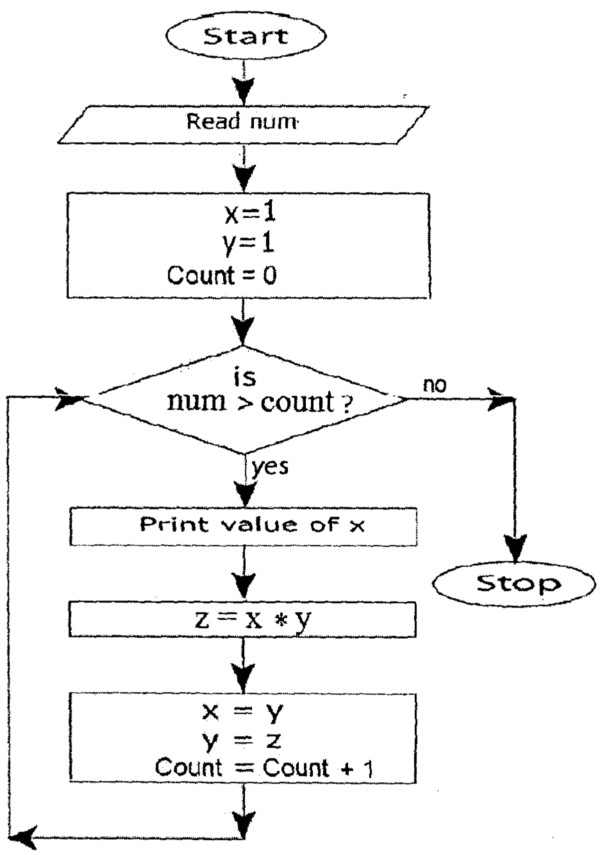
*b*. int *a;

```
a = (int *) malloc(4*sizeof(int));
a[0]=1; a[1]=2; a[2]=3; a[3]=4;
printf("Check the value: %d\n", *++a);
printf("Check the value: %d\n", *a++);
printf("Check the value: %d\n", *a);
```

*c*. int i=0;

```
printf("Values are: %d  \n", i, i++, ++i);
printf("Values are: %d  %d %d\n", i);
```

4. Write a C program implementing the algorithm shown by the following flowchart. If the input value is 4, what will be the output of the program? [12]




5. Write C programs to obtain the following: [15×2 = 30]
*a*. Display of an inverted pyramid with height = 5 and base = 9 formed by the symbol +.
*b*. Appending (at the end of the existing input file) the transpose of a matrix read from a given input file. [The matrix of the input file is preceded by a line containing its row and column numbers. Similarly, the output includes the transposed matrix preceded by a separate line containing its dimension.]

P. T. O.

6. Write down the output of the following C programs. [10×2 = 20]

*a*. Input to the following consists : Rohit, Madhuparna, Aastha, Soumyadeep, Jash

```
#include<stdio.h>
#include <string.h>
#define NO_OF_WORDS 5
#define LENGTH 50

int main()
{
   int i, j;
   char str[NO_OF_WORDS+1][LENGTH], temp[LENGTH];

   printf("Enter %d words:\n", NO_OF_WORDS);

  for(i=0; i<NO_OF_WORDS; ++i)
      scanf("%s",str[i]);


  for(i=0; i<NO_OF_WORDS-1; ++i)
     for(j=i+1; j<NO_OF_WORDS ; ++j)
      {
      if(strcmp(str[i], str[j])>0)
      {
         strcpy(temp, str[i]);
         strcpy(str[i], str[j]);
         strcpy(str[j], temp);
      }
    }

 printf("\nOutput: \n");
 for(i=0; i<NO_OF_WORDS; ++i)
 {
    puts(str[i]);
 }

 return 0;
}
```

*b*. Command line arguments to the following consists : 1. Rohit, 2. Madhuparna

```
#include <stdio.h>
#include <string.h>
#include <stdlib.h>
#define SYMBOL 26
```

P. T. O

```
int main(int argc, char *argv[])
{
  int i, j, cnt[SYMBOL];

  if (argc==1){
          printf("There is no input\n");
          exit(0);
  }

  for (i=0; i<SYMBOL; i++)
                 cnt[i] = 0;

  for (i=1; i<argc; i++)
          for (j=0; j<strlen(argv[i]); j++){
                     if((argv[i][j]<=90) && (argv[i][j]>=65))
                            cnt[argv[i][j]-65]++;
                           if((argv[i][j]<=122) && (argv[i][j]>=97))
                            cnt[argv[i][j]-97]++;
          }

  printf("Output values are:\n");
  for (i=0; i<SYMBOL; i++)
          printf("%d ", cnt[i]);
  printf("\n");

  return 0;
}
```

# INDIAN STATISTICAL INSTITUTE
## First Semester Examination: 2018-19

M.S. (Q.E.) I YEAR

Game Theory I

Date: 19.11.2018 Maximum Marks: 50 Duration: 3 hours

**Marks will be given for conceptually correct approach.**

1. A mixed Nash equilibrium $(m_1, m_2)$ of a two-player symmetric game is called symmetric if $m_1 = m_2$. Justify the following statement with a proof or a counter example: Every symmetric game has a symmetric mixed Nash equilibrium .
   Consider the symmetric game $A$ where $A_{11} = 0$, $A_{12} = 5, A_{21} = 1, A_{22} = 3$. Does it have any ESS? Justify your answer.

   [10]

2. Consider a game $G = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$ such that $S_i = [0, 1]$ and $u_i$ is a linear function for all $i \in N$.

   Define the best response function in pure strategies as follows: for all $i \in N$ and all $s_{-i} \in S_{-i}$, $b_i(s_{-i})$ is the set of pure strategies $s_i$ of $i$ such that $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ for all $s_i' \in S_i$.

   (a) Is $b_i(s_{-i})$ non-empty for all $i$ and all $s_{-i}$?

   (b) Is $b_i(s_{-i})$ convex for all $i$ and all $s_{-i}$?

   (c) Is $b_i(s_{-i})$ closed for all $i$ and all $s_{-i}$?

   (d) Can you use fixed point theorem to prove that a pure strategy Nash equilibrium will always exist for $G$? Justify your answer.

   [15]



PTO

3. A Nash equilibrium is called *strong* if no group of players can profitably deviate from it, that is, for a game $G = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$, a strategy-profile $s \in \prod_{i \in N} S_i$ is a strong Nash equilibrium if for all $M \subseteq N$, all $s'_M \in \prod_{i \in M} S_i$, we have

$$u_i(s_M, s_{N \setminus M}) \geq u_i(s'_M, s_{N \setminus M})$$

for all $i \in M$.

Similarly, define strong Nash equilibrium in mixed strategies.

Justify your answer with a proof or a counterexample.

(a) If a game has a Nash equilibrium in pure strategies, then it has a strong Nash equilibrium in pure strategies.

(b) Strong Nash equilibrium in mixed strategies always exists for $2 \times 2$ games.

(10)

4. For the following questions, $G^T$ denotes the $T$-period repeated game where in each period the game $G$ is played.

(a) If $G$ has $k$ Nash Equilibria where $k \geq 3$, then what can you say about the number of subgame perfect Nash equilibrium of $G^3$? Justify your answer.

(b) If a game $G$ does not have any pure strategy Nash equilibrium , then $G^3$ will not have any pure strategy Nash equilibrium. Justify your answer.

(c) In every Nash equilibrium of $G^3$, players play some Nash equilibrium of $G$ at every time-period of $G^3$.

(d) Consider the penalty shootout game (two-person zero-sum game) $G$. Justify your answer:

(i) Is there a pure strategy Nash equilibrium of the game $G^\infty$?

(ii) Is there a pure strategy subgame perfect Nash equilibrium of the game $G^\infty$?

[15]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2018-19

Course Name: **M. Tech. (QR&OR) - I Year**

Subject Name: **Quality Management & Systems**

**Date: 22 November 2018** **Maximum Marks: 100** **Duration: 2 hours**

**Note: Answer all questions.**

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1. Define quality function deployment. Describe the structure of House of Quality.

   Or, Define six sigma methodology and explain the various steps of implementation, in brief.

   [5 + 8 = 13]

2. Write short note on any four of the following terms.
   (a) Process Performance Measures
   (b) Different Views of Quality
   (c) Nonconformance Report Format
   (d) Disposition of Nonconforming Product
   (e) Internal Audit
   (f) Seven Principles of Quality Management defined by ISO

   [4 x 6 = 24]

3. Write short notes on any two of the following personalities, explaining their contribution to quality management.
   (a) Frederick Taylor
   (b) Max Weber
   (c) Henry Ford
   (d) Abraham Maslow

   [2 x 5 = 10]

4. Following are the objective evidences of an internal audit conducted according to ISO 9001: 2015 QMS. For any three objective evidences, provide your explanation and justification on being classified as a non-conformance or not.
   (a) Waste records show that waste was removed only once per week on 4 occasions during last three months instead of twice per week as stated in the procedure PR-6.8.
   (b) One batch of final product was dispatched before the completion of final inspection. However, it was found later, that the particular batch passed the final inspection.
   (c) Computerized stock shows 350 kg of material ZZZ are lying on the store but only 250 kg of material could be located during the audit.
   (d) Customer complaints are replied promptly and records are kept. Record shows similar complaints are recurring every month. When asked, the concerned Manager tells his duty only to reply the complaints promptly.

   [3 x 6 = 18]

5. Assignments.

   [35]

Indian Statistical Institute
Semester Examination 2018
Course Name: MSQE First Year, First Semester
Subject Name: Basic Economics

Date: 22/11/2018 | Full Marks: 60 | Duration: 2 Hours

Answer all the questions

1. (i) Consider a simple Keynesian model for an open economy with government, where aggregate planned investment, I = 200. Suppose at GDP (denoted $Y$) = 3000, aggregate saving ($S(Y = 3000)$) equals 500 and trade balance equals -100. Also, following exogenous increases in export (denoted $\bar{X}$) and autonomous import, $\bar{M}$, by 45 and 43 units respectively, $Y$ is found to increase by 4 units. Compute the equilibrium level of $Y$. Assume all relations to be linear. [20]

2. Suppose there takes place a labour saving technological change in the simple Keynesian model. How will it affect the level of GDP and employment, when consumption propensities differ across income classes? Explain your answer. [20]

3. Consider a hypothetical situation in a given period, where there are only two economies: the domestic economy and the foreign economy. We shall refer to the currencies of the former and the latter as domestic currency and foreign currency respectively. $F^d = D(e), D' = -20$ and $F^s = S(e), S' = 60$ are the demand and supply functions of foreign currency respectively and $e$ denotes the exchange rate. Foreign currency is traded for domestic currency in the foreign exchange market. Suppose the central bank of the domestic economy intervenes in the foreign exchange market and raises the equilibrium exchange rate by 2 units. It is given that the CRR (cash reserve ratio) and the currency-deposit ratio for the domestic economy are 0.5 and 0.5, respectively. Compute the resulting changes in the volumes of high-powered money and money supply indicating the associated changes in the balance sheets of the relevant banks. (Assume that all the relations are linear and initially the banks were fully loaned up.) [20]

INDIAN STATISTICAL INSTITUTE

**End-Semestral Examination: 2018-19**

**Course:** Master's in Quantitative Economics Year I

**Subject:** Microeconomics I

**Date:** 26.11.18 **Maximum Marks**: 60 **Duration:** 3 hours

**Answer all questions. Students may consult their notes or study material, but not each other.**

**1.** Consider a competitive firm producing according to a production function:

$$A = F(K, L),$$

where $A$ represents the amount of output produced, $K$ represents the amount of fixed input (capital), and $L$ represents the amount of variable input (labour). The quantity of capital cannot be changed in the short run, but the (non-storable) output and variable input can be readily bought and sold at prices $p$ and $w$, respectively. Future $p$ can be either $\overline{p}$ or $\underline{p}$ with equal probability, $\overline{p} > \underline{p} > 0$. However, $w$ is known with certainty. The firm chooses $L$ to maximize expected utility, on the basis of a VNM utility function $u = \pi^{\alpha}$, $1 > \alpha > 0$. How do expected profit, output and employment change with an increase in $\alpha$? Interpret your findings. What do your results imply for the direction of capital flow? **(20 marks)**

**2.** Consider a household endowed with $\bar{l}$ amount of labour, which it has to allocate between urban employment and rural employment, so as to maximize expected utility. The household's indirect utility function is given by: $Eu(m) = \overline{m} - \frac{kVar_m}{2}; k > 0$; where $\overline{m}$ is mean household income and $Var_m$ is the variance of household income. The rural wage rate is given by a function $w_r = \frac{z}{l_r}$ , $z > 0$, where $l_r$ is the amount of household labour allocated to rural

PTO

occupations. The urban wage rate can be either $W$ or 0 with equal probability. Show how the extent of urban migration changes with changes in (i) $\bar{l}$ , (ii) $W$ (iii) $z$ and (iv) $k$. Explain your results. **(20 marks)**

3. Suppose a union faces an inverse demand function for labour: $w = A - L_d$, where w is the wage rate and $L_d$ is the demand for labour; A > 0. Hence total income to labour is $wL = (Aw - w^2)$. The demand parameter $A$ can take the values $\overline{A} + \delta, \overline{A} - \delta$ with equal probability, $\bar{A} > \delta > 0$. The union's preferences are given by a VNM utility function $u = \ln(wL)$. The union chooses the wage rate, $w$, so as to maximize its expected utility. Show how the ratio $\frac{w}{A}$ chosen by the union changes as (i) $\overline{A}$ changes, and (ii) $\delta$ changes. Interpret your results. **(20 marks)**

Indian Statistical Institute
Back Paper Examination 2018
Course Name: MSQE First Year

Date of Examination: 21.01.2019 Full Marks: 100 Duration: 3 Hours

Answer all the following questions

1.(i) Is it possible for an economy to absorb more than what its purchasing power can command of the world NDP? Explain your answer. Also discuss the financial aspects.

(ii) Suppose in an economy in a given year the central bank had to sell foreign exchange worth Rs. 20,000 crore from its stock to hold the exchange rate at the target level. Some firms in the domestic economy borrowed from foreign financial institutions Rs. 30,000 crore, while some foreign firms borrowed Rs.12,000 crore from domestic financial institutions. In addition, foreigners purchased shares of domestic companies worth Rs.8000 crore. Domestic residents purchased land abroad worth Rs.200 crore. Domestic government's budget deficit was Rs.500 crore. Domestic households' expenditure on produced goods and services was Rs.90,000 crore of which Rs.10,000 was spent on buying houses from construction companies. Firms' investment in the domestic economy was Rs.60000 crore and depreciation of the capital stock of the economy was Rs.500 crore. From the data given above, compute the economy's private disposable income in the given period. [12+13=25]

2. Suppose the government cuts down subsidy and uses the resulting increase in saving to buy fighter planes from the USA. Examine the impact of this policy on the GDP of the domestic economy in the framework of the Simple Keynesian Model. [25]

3. Suppose the domestic economy is in recession. The gap between the current (equilibrium) GDP and the one that would have obtained in the absence of the recession is 1600 units. The RBI plans to tackle this recession by means of a monetary policy of reducing the repo rate. Sensitivity of investment with respect to the lending rate to the investors charged by the banks is – 40 units and marginal propensity to spend out of GDP on domestically produced goods is 0.5. By how much should the lending rate be changed from its existing level of 6 percent to eliminate the recession? In the light of this result, comment on the efficacy of the monetary policy in tackling the recession. [25]

4. Suppose the CRR and the currency-deposit ratio in an economy are 0.25 and 0.5 respectively. Start with an initial situation, where the banks are fully loaned up. The economy has a fixed exchange rate regime. Now suppose the central bank has to buy 100 units of foreign exchange from the market to keep the exchange rate at the target level. In the light of the information given above, answer the following questions:

(a) Show the resulting changes in the stocks of high-powered money and money supply, when the amount of excess demand for bank credit existing in the initial situation is adequate for the money multiplier process to work out fully.

(b) How will your answer to (a) change, if the excess demand for credit in the initial situation were 40 units? [12.5+12.5]

# INDIAN STATISTICAL INSTITUTE
## Mid-semester Examination (2018-2019)
## MS(QE) I
## Microeconomics II

Date: 18.02.2019 Maximum Marks: 100 Duration: 3 hours

(1) Show that if the preference relation $R$ on $X = \Re^L_+$ is complete and monotone, then it is locally non-satiated. **(5)**

(2) Define continuity of a preference relation $R$ on any consumption set $X$ in terms of limits. Also define continuity of a utility function $u : X \to \Re$ representing a preference relation $R$ on $X$. Show that if $u(.)$ is a continuous utility function representing $R$ on $X$, then $R$ must be continuous. **(3+2+10=15)**

(3) Let $R$ be a preference relation defined on $X$ and $u(.)$ be a utility function representing it. Show that $R$ on $X$ is strictly convex if and only if any utility function $u(.)$ representing it is strictly quasi-concave. **(10)**

(4) Suppose that $f(.)$ is the production function associated with a single-output technology, and let $Y$ be the production set of this technology. Show that $Y$ satisfies constant returns to scale if and only if $f(.)$ is homogeneous of degree one. **(15)**

(5) Suppose that $\pi(.)$ is the profit function of the production set $Y$ and that $y(.)$ is the associated supply correspondence. Assume that $Y$ is non-empty, closed and satisfies the free disposal property. Then we have the following:

(a) $\pi(.)$ is homogeneous of degree one.

(b) $\pi(.)$ is convex.

(c) $y(.)$ is homogeneous of degree zero. **(5+5+5=15)**

(6) Consider an economy consisting of $I$ consumers (indexed $i = 1, \ldots, I$), $J$ firms (indexed $j = 1, \ldots, J$) and $L$ commodities (indexed $l = 1, \ldots, L$). Each consumer $i$ is characterized by a consumption set $X_i \subset \Re^L$ and a *rational* preference relation $R_i$ defined on $X_i$. Each firm $j$ has the production possibilities summarized by the production set $Y_j \subset \Re^L$. We assume that $Y_j$ is non-empty and closed. The initial resources of commodities in the economy, that is, the economy's endowments are given by a vector $\hat{\omega} = (\hat{\omega}_1, \ldots, \hat{\omega}_L) \in \Re^L$. Thus, the basic data on preferences, technology, and resources for this economy are summarized by $(\{(X_i; R_i)\}_{i \in I}, \{Y_j\}_{j \in J}, \hat{\omega})$. Answer the following questions.



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(a) Define Pareto optimal and weak Pareto-optimal allocations. By imposing appropriate restrictions on the preferences, state and prove an equivalence result between Pareto-optimality and weak Pareto-optimality. **(4+16=20)**

(b) State and prove the first fundamental theorem of welfare economics by giving all the relevant definitions. **(20)**

# INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2018-19

MS (QE) I YEAR

Econometric Methods I

Date: 19 February 2019 Maximum Marks: 100 Duration: 3 hours

Note: Answer question 1 and any three from the rest of the questions]

1. Simplify the results of CLRM when K=2. I.e., for the following simple linear regression model

$$y = X\beta + e = x_1\beta_1 + x_2\beta_2 + e,$$

where $e \sim$ i.i.d. $(0, \sigma^2)$, $x_1$ is a $(T\times1)$ vector of '1's, $\hat{\beta}_1$ and $\hat{\beta}_2$ are LS estimators and $\bar{y}$ and $\bar{x}_2$ are the arithmetic means of y and $x_2$, show that

(a) $\hat{\beta}_1 = \bar{y} - \hat{\beta}_2\bar{x}_2$ and $\hat{\beta}_2 = \dfrac{\sum(x_{t2}-\bar{x}_2)(y_t-\bar{y})}{\sum(x_{t2}-\bar{x}_2)^2}$.

(b) $(y-X\hat{\beta})'(y-X\hat{\beta}) = \sum(y_t-\bar{y})^2 - \dfrac{(\sum(x_{t2}-\bar{x}_2)(y_t-\bar{y}))^2}{\sum(x_{t2}-\bar{x}_2)^2}$.

(c) $\text{Var}(\hat{\beta}_1) = \sigma^2[\dfrac{1}{T} + \dfrac{\bar{x}_2{}^2}{\sum(x_{t2}-\bar{x}_2)^2}]$,

$\text{Var}(\hat{\beta}_2) = \sigma^2/\sum(x_{t2}-\bar{x}_2)^2$,

and $\text{Cov}(\hat{\beta}_1,\hat{\beta}_2) = \sigma^2[-\bar{x}_2/\sum(x_{t2}-\bar{x}_2)^2]$. [8+8+12=28]

2. Write a brief account of different types of data that one comes across in econometric analysis. Describe the problems encountered with these data. Also describe some methods of refining the data. [10+8+6=24]

3. (i) Define coefficient of determination ($R^2$). (ii) Write down an alternative form of it and prove its equivalence. (iii) Interpret its value as a goodness of fit parameter keeping in views the alternative forms of R and/or $R^2$. (iv) Can you always use the value of $R^2$ to compare the goodness of fit of different forms of regression equations? Explain giving appropriate examples. [2+10+4+8=24]



PTO

4. (i) State the assumptions of completely independent linear stochastic regression model. (ii) Derive the ML estimator of the regression coefficient vector $\beta$. (iii) Show that it is unbiased and consistent. (iv) Find its variance. (v) Give an example of a partially independent linear stochastic regression model to show that LS estimate of the regression coefficient vector $\beta$ may be biased, but consistent. [2+4+6+2+10=24]

5. (i) Define "perfect" and "absence of" multicollinearity and discuss their consequences. (ii) What are the effects of multicollinearity? (iii) How will you detect the existence of multicollinearity? (iv) Do you agree with the statement that "if all the simple correlations are small then the problem of multicollinearity will not arise"? Give explanations for your answer. [6+8+8+2=24]

6. Write short notes on any three of the following:

   (a) Regression without intercept term.

   (b) Best Linear Unbiased Prediction under CNLRM.

   (c) Unbiased estimation of regression error variance in CLRM.

   (d) Partial correlation coefficient. [8×3=24]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: (2018-2019)
MS (Q.E.) I Year
Macroeconomics I

Date: 20/02/2019 | Maximum Marks 40 | Duration 3 hours

**Group A**

**Answer *all* the questions.**

1. Suppose Robinson Crusoe's preferences are given by

$$u(c,l) = c^{\gamma}(1-l)^{\gamma}$$

Where $0 < \gamma < 1$

His technology is $= f(l) = Al^{\alpha}$ , where $0 < \alpha < 1$.

Solve for Crusoe's optimal choice of consumption $c$ and labour $l$. (3)

2. Consider an economy with many identical households. Each household owns a business that employs both capital $k$ and labour $l$ to produce output $y$. The production function is $y = Ak^{3/10}l^{7/10}$. The stock of capital each household owns is fixed. It may employ labour at the prevailing wage rate $w$. Each household takes wage as given. The profit is $\pi = y - wl$.

   a) Determine the optimal amount of labour for each household to hire as function of its capital endowment $k$ and the prevailing wage $w$.
   b) Find the maximum profit of the household.
   c) Each household has utility $u(c,l) = c^{1/2}(1-l)^{1/2}$. Each household has an endowment of labour that can be rented out at the wage rate $w$. Set up the household's optimization problem.
   d) Derive the optimal supply of labour by household.
   e) Determine the equilibrium wage in the economy.
   f) What does this model imply about the wage differential between India and Bangladesh?

(6X2=12)

*PTO*

3. Suppose the following is a periodic flow budget constraint

$$(1+\tau_t^c)c_t + k_{t+1} = w_t + \left(1+\left(-\tau_t^k\right)r_t\right)k_t \ , t = 0,1,2,\ldots\ldots\ldots\infty$$

The government charges a consumption tax $(\tau_t^c)$ and taxes interest on capital $\left(\tau_t^k\right)$.

a) Find the aggregative lifetime budget constraint from the above periodic flow budget constrain.

b) Find the transversality condition (end-point restriction). (3+2=5)

## Group B

## Answer all questions

1. In an appropriate new Keynesian model, derive the long run multiplier of a balanced budget increase in government expenditure. Subsequently show that with variety effect absent, such multiplier is smaller than what would obtain in the short run. (10)

2. Show that in the flexible price equilibrium that obtains in the Blanchard and Kiyotaki model, money is completely neutral. (10)

[Note: You need not derive the Dixit –Stiglitz demand function or the formula for the price index. Just use them directly.]

INDIAN STATISTICAL INSTITUTE
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MID- SEMESTER II EXAMINATION 2018 - 19
M.S.(Q.E.) 1st Year
Time Series Analysis & Forecasting

Date: 21.02.2019 Maximum Marks: 50 Time: 2 Hours

*[This question paper carries a total of **60** marks. You can answer any part of any question. But the maximum that you can score is **50**. Marks allotted to each question are given within parentheses.]*

1. (a) Explain what you mean by seasonality and cyclical component of a time series.

   (b) Describe how you would obtain seasonal indices of a monthly time series which also contains trend.

   [6 + 9 = 15]

2. (a) Explain what you mean by 'ergodicity' property of a time series. Is it related to 'stationarity' property of a time series in any sense? Given justifications for your answer.

   (b) Derive the conditions for stationarity (in terms of its parameters) of an AR (2) process.

   [9+6 = 15]

3. (a) Let $\{X_t\}$ be a quarterly time series given by

   $X_t = (1+0.3B)\,(1-0.7B^4)\,a_t, \quad a_t \sim WN(0,\sigma^2)$.

   Determine the coefficients $\Pi_j^{'}s$ in the representation

   $$a_t = \sum_{j=0}^{\infty} \Pi_j X_{t-j}.$$

   (b) Discuss the behavior of ACF and PACF of an ARMA (1, 1) process.

(c) Suppose a time series process is given by

$$\Delta x_t = \beta_0 + \beta_1 t + a_t, \quad a_t \sim WN(0, \sigma^2)$$

Show that the process in its level values i.e., $x_t$'s contains a quadratic trend.

[6 + 5 + 4 = 15]

4. (a) Consider the difference equation

$$y_t - 1.6\, y_{t-1} + 0.89\, y_{t-2} = 0, t = 2, 3, \cdots$$

(i) Find the roots of the underlying characteristic equation.

(ii) Give the expression for $y_t$ if $y_0 = 0.8$ and $y_1 = 0.9$.

(iii) If $a_t \sim IN\left(0, \sigma^2\right), \sigma^2 > 0, t = \{0, \pm 1, \pm 2, \cdots\}$, is there a stationary time series satisfying

$$y_t - 1.6\, y_{t-1} + 0.89\, y_{t-2} = a_t\ ?$$

Explain your answers.

(b) Suppose that $\{x_t\}$ and $\{y_t\}$ are stationary processes satisfying $x_t - \alpha\, x_{t-1} = w_t$ and $y_t - \alpha\, y_{t-1} = x_t + z_t$

where $\{w_t\} \sim WN\left(0, \sigma^2\right) \{z_t\} \sim WN\left(0, \sigma^2\right), |\alpha| < 1$, and $w_t$ and $z_t$ are uncorrelated. Find the ACF of $\{y_t\}$.

[9 + 6 = 15]

Indian Statistical Institute

Mid-semester Examination 2019, MSQE I

Course name: **Selected Topics I: Groups, Identities and Economic Development**

Subject name: **Economics**

Date: **22 February 2019**

**Maximum marks: 50**

**Duration: 2 hours**

**Instructions:** This is a closed-book, closed-notes examination. No calculators are allowed. Please answer Group A and Group B in **separate** answer booklets.

## Group A

**1.** This question pertains to lobbying by a special interest group (SIG) under three possible states of the world, namely $\theta_L, \theta_M$ and $\theta_H$, $\theta_L < \theta_M < \theta_H$, as discussed by Grossman and Helpman. But instead of the states being equally likely, let the probability of the states $\theta_L$, $\theta_M$ and $\theta_H$ now be 1/2, 1/3 and 1/6 respectively. Otherwise it is the Grossman-Helpman setting with a single policy variable and a single SIG. Recall $\delta > 0$ to be the size of divergence between the ideal policies of the policy maker and the SIG. Consider a partially-revealing/semi-separating equilibrium where a report of "low" by the SIG indicates that the state is $\theta_L$ whereas a report of "not low" by the SIG indicates that the state is either $\theta_M$ or $\theta_H$. In this context, answer the following questions:

(i) What is the incentive-compatibility constraint of the SIG if the state is $\theta_M$? **(7 points)**

(ii) In general, if the probability of state $\theta_M$ is $p_M$ and the probability of state $\theta_H$ is $p_H$, what policy, say $z$, does the policymaker implement when the SIG reports "not low"? **(3 points)**

(iii) Write down the incentive-compatibility constraint of the SIG when the state is $\theta_M$ more generally in terms of $z$. **(3 points)**

**2.** This question deals with allocation of public spending by the government when one of the groups is organised and lobbies as discussed by Grossman and Helpman. In this context, they conclude that "SIGs contribute to parties and politicians in a more or less direct attempt to influence their policy decisions." Elucidate the statement and show how policy decision is influenced when the policymaker allocates a budget of size $g$ between two groups 1 and 2, where only group 1 is politically organised and offers campaign support to



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the policymaker. Assume both the groups enjoy benefits they receive using the function $V(x) = \sqrt{x}$ to make your point. **(12 points)**

**OR**

This question pertains to costly lobbying as discussed by Grossman and Helpman. Consider a single interest group, a single policy variable and two possible states of the world, $\theta_H = 10$ and $\theta_L = 5$. Let $\delta = 5$ be the size of divergence between the ideal policies of the policy maker and the SIG. Find the range of exogenous lobbying costs $l_f$ such that a separating equilibrium exists. Is costly lobbying beneficial for the SIG in this case? **(12 points)**

## Group B

Answer any two

(1) (a) In an imaginary world (which is otherwise similar to the one described in Greif's paper) some members of Maghribi traders started doing trade with the Arabs. How would that change the equilibrium outcomes described in Grief (1993)?

(b) Explain why would trade cease to exist under BPS if $\tau = 1$.

(c) Suppose there was a market for information about cheater's past behaviour in medieval Europe. Between Genoese and Maghribis, which group will be interested in pay money for information?

(d) What will be the equilibrium price for the information (at least indicate the equations required to determine the equilibrium price)?

**(3+4.5+1+4)**

(2) (a) What are the possible concern endogeneity in original Weber hypothesis?

(b) What was the instrumental variable used by Becker and Woessmann and why is it a valid instrument?

(c) How would you expect the results to change if Calvin, another early pioneer of protestantism, was born in Germany too?

(d) Why did the authors include distance of the county capital from the Prussian capital of Berlin as one of the controls?

(e) If Prussia had experienced high level of migration in 1870s, how would that affect the results? **(2+3.5+2+2+3)**

(3) (a) Information sharing has become really easy with the rise of internet and social media. Do you think that will help in withering away of state run institutions of contract enforcement? Explain your position.

(b) What will happen to the equilibrium in Greif (1993) if $\tau = 0$?

(c) How did Becker and Woessmann test the relative strength of Weber's thesis vis-a-vis Protestant human capital theory? **(6+4+2.5)**

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SEMESETER II EXAMINATION 2018 - 19
M.S.(Q.E.) 1st Year
Time Series Analysis & Forecasting

Date: 22.04.19 Maximum Marks: 100 Time: 3 hours

**Answer any FIVE questions.**

*[Marks allotted to each question are given within parentheses].*

1. (a) Consider an MA (1) process given by
$x_t = a_t - \theta a_{t-1}$, $|\theta| < 1$ and $\{a_t\} \sim WN\ (0, \sigma^2)$.
Show that another MA (1) process with the parameter $1/\theta$ will have the same ACF as that of the given MA (1) process. Show further that for the given MA (1) process, $-0.5 < \rho_1 < 0.5$, where $\rho_1$ stands for the autocorrelation coefficient at lag 1.

(b) Suppose that for a sample of size 100 from an MA (1) process given by $x_t = \mu + a_t + 0.5 a_{t-1}$, where $a_t$'s are WN (0, 1), the sample mean was obtained as $\bar{x}_{100} = 0.21$.
Construct a 95% confidence interval for $\mu$. Are the data compatible with the hypothesis $\mu = 0$? Justify your answer.

(c) Let $x_t = a + bt + y_t$, where $\{y_t\} = 0, \pm 1, \pm 2, \ldots\ldots\ldots\ldots$ is an independent and identically distributed sequence of random variables with mean 0 and variance $\sigma^2$, and *a* and *b* are constants. Define a new time series $\{W_t\}$ as

$$W_t = \sum_{j=-q}^{q} X_{t+j} \Big/ (2q+1).$$

Is $\{W_t\}$ stationary? Justify your answer.

[5+$\overline{7+3}$+5 = 20]

2. (a) Find the 2- step ahead minimum MSE forecast at origin *n* of the following time series.

$$x_t = 0.8 x_{t-1} - 0.4 x_{t-2} + a_t - 0.3 a_{t-1}, a_t \sim WN(0, \sigma^2)$$

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where $a_t$'s are white noise with zero mean and unit variance. Also find the variance of the forecast error.

(b) Discuss what you mean by out- of -sample forecasts. Discuss how out-of-sample forecasts are used to compare between two models in terms of forecast performance.

$[\overline{6+4}+\overline{4+6}=20]$

3. (a) Derive the test statistic of Chow's test for testing the null hypothesis of 'no structural break', in a given time series. Also state the main limitations of this test with justifications as to why these are considered as limitations.

   (b) Describe how the presence of unit roots can be tested in a segmented time series.

$[\overline{10+5}+5=20]$

4. (a) Describe the augmented Dickey-Fuller test for testing the presence of unit root in a given time series. Discuss also how this test may be used to distinguish between a DSP and a TSP.

   (b) Bring out the distinctive features between the ADF and KPSS tests.

$[\overline{6+6}+8=20]$

5. (a) Suppose the time series process $\{x_t\}$ is represented by

$$x_t = \sqrt{2}\sum_{j=1}^{q} \sigma_j \cos\left(\lambda_j t - \gamma_j\right)$$

where $\gamma_1, \gamma_2, \ldots\ldots\ldots, \gamma_q$ are independently uniformly distributed in the interval $(0, 2\pi)$ and $\sigma_j$'s and $\lambda_j$'s are constants. Show that $\{x_t\}$ is a covariance stationary process.

   (b) Given a finite realization $\{x_1, x_2, \cdots, x_n\}$ of a stationary time series, obtain the periodogram of $\{x_1, x_2, \cdots, x_n\}$, and then show that this can be regarded as a sample analogue of $2\pi f(\lambda)$, where $f(\lambda)$ is the spectral density function.

[10 + 10 = 20]

6. Write short notes on the following.
   (i) ML method of estimation of an ARMA $(p, q)$ process;
   (ii) Transfer function-noise model.

[10 + 10 = 20]

# INDIAN STATISTICAL INSTITUTE

## End-Semestral Examination: (2018-2019)

## MS(QE) I

## Microeconomics II

Date: 24. 04. 2019 Maximum Marks: 100 Duration: 3 hrs

**Note:** Answer all questions

(1) State and prove the second fundamental theorem of welfare economics by giving all the relevant definitions. (35)

(2) Consider the labour market model of adverse selection, where the marginal and average productivity of a worker is $\theta$ and $\theta \in [\underline{\theta}, \bar{\theta}]$. Let the opportunity cost of accepting employment for a worker of type $\theta$ be $r(\theta)$. Assume that $r(\theta)$ is continuous and increasing and that there exists $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $r(\theta) < \theta$ for all $\theta \in [\underline{\theta}, \hat{\theta})$ and $r(\theta) > \theta$ for all $\theta \in (\hat{\theta}, \bar{\theta}]$. Show that a competitive equilibrium with unknown types will necessarily lead to Pareto inefficient outcome. (10)

(3) Consider the labor market signaling model where the marginal productivity of a worker is $\theta \in \{2, 18\}$ and $Pr(\theta = 18) = 1/2$. The cost of education is $c(e, \theta) = \frac{e^2}{20}$ for all $e \geq 0$. Let $u(w, e; \theta) = w - c(e, \theta)$ be the utility of a worker of type $\theta$ who chooses an education level $e$ and receives a wage $w$. Assume that both worker types earn zero by staying home, that is $r(2) = r(18) = 0$.

(a) Consider the belief function

$$\mu^a(e) = \begin{cases} 1 & \text{if } e \geq e^*, \\ 0 & \text{if } 0 \leq e < e^*. \end{cases}$$

Find all possible values of $e^*$ for which we can have a separating equilibrium. Justify your answer.

(b) Consider the belief function $\mu^b(e) = \frac{e - \max\{0, e-8\}}{8}$ for all $e \geq 0$. Can you find a separating equilibrium for the belief function $\mu^b(e)$? Justify you answer.

**(8+12=20)**

(4) Show that in any sub-game perfect Nash equilibrium of the screening game with unknown worker types, the low ability worker accepts the contract $(\theta_L, 0)$ and the high ability worker accepts the contract $(\theta_H, t^{(1)})$, where $t^{(1)}$, the task level assigned to the high type, satisfies $\theta_H - c(t^{(1)}, \theta_L) = \theta_L - c(0, \theta_L)$. Here the marginal and average productivity of a worker is $\theta \in \{\theta_L, \theta_H\}$ with $0 < \theta_L < \theta_H < \infty$. The probability that a worker is of high type is $\gamma \in (0, 1)$ and the opportunity cost of accepting employment to each type of worker is zero. **(35)**

Indian Statistical Institute

End-Semestral Examination 2018-19

Course name: MSQE I

Subject name: **Selected Topics I: Groups, Identities and Economic Development**

Date: **April 26 2019**

Maximum marks: **100**

Duration: **3 hours**

**Instructions:** This is a closed-book, closed-notes examination. No calculators are allowed. Please answer Group A and Group B in **separate** answer booklets.

## Group A

**Remark:** This part has questions carrying **60 points**. Answer **all** questions. The maximum you can score is **50**.

**1. (14 points)** Consider the complete information model of 'Guns and Butter' as laid down by Baliga and Sjöström in their survey of bargaining and war. Recall that $N$ and $S$ are two risk-neutral players, representing two countries North and South respectively. Also recall that player $i \in \{N, S\}$ has resources $x_i$ which can be used to produce guns $g_i \geq 0$ and butter $b_i \geq 0$. If there is a war, then each player $i$ suffers a cost $c_i > 0$. Moreover, $x_N < c_S$ but $x_S > c_N$, so that South would never want to start a war, but North might. However South can propose to transfer an amount $t$ of butter to North as appeasement. North either accepts this proposal or declares war. Answer the following questions based on this model:

(i) What is the smallest $t$ that North accepts? **(4 points)**

(ii) Is it feasible for South to offer such a $t$, i.e. does $t$ satisfy $t \leq b_S$? Elucidate. **(4 points)**

(iii) Argue that in any equilibrium, North will set $g_N > 0$. **(3 points)**

(iv) Briefly explain how "helping North become productive can be *counter*productive". **(3 points)**

**2. (10 points)** Consider the paper 'Bargaining Versus Fighting' by Skaperdas. Consider the static game where, in the first period, two sides $A$ and $B$ choose enforcement efforts

$e_A$ and $e_B$, out of their exogenously given resources $R_A$ and $R_B$ respectively, in order to compete for a exogenously given prize $T$. Thereafter, in the second period, each side chooses whether to go to war or bargain with the aim of peacefully settling. Suppose further that the two adversaries are risk-averse. Specifically, suppose that both sides have their valuation functions given by von Neumann-Morgenstern utility functions $U_i(.)$ and the income of each is linear in the contested resource $T$ and in their own resource $R_i - e_i$, $i = A, B$. Further suppose that the players have already made the enforcement effort choices of the first period. The following questions pertain to such a setting:

(i) What is the expected utility of player $i$ if War happens? **(2 points)**

(ii) Suppose the two adversaries consider dividing the contested resource according to the winning probabilities. Would each player prefer receiving the share arising from this division or would each rather prefer to go to War? Elucidate. (You may graphically depict as well.) **(4 points)**

(iii) Suppose $U_i(x) = \sqrt{x}, T = 100, p_i = 1/2, R_i = 50, e_i = 10, \forall i$. Write down the equation that player $i$ considers when choosing between War and peaceful settlement. **(4 points)**

**3. (20 points)** Consider, once again, the paper 'Bargaining Versus Fighting' by Skaperdas as discussed in class, and now consider the first period choices of enforcement efforts of the static game.

(i) What are the equilibrium values of enforcement efforts and payoffs under 'Armed Peace'? (You may assume the 'rule of division' as assumed by Skaperdas) **(7 points)**

(ii) What are the equilibrium values of enforcement efforts and payoffs under War? **(7 points)**

(iii) Is it possible that enforcement efforts under War are lower than those under 'Armed Peace'? Briefly explain. **(3 points)**

(iv) Is it possible that equilibrium payoffs under 'Armed Peace' are lower than those under War? Briefly explain. **(3 points)**

**4. (16 points)** Consider, once again, the paper 'Bargaining Versus Fighting' by Skaperdas as discussed in class, and now consider the infinite horizon game where a static game is played every period. Specifically, for the static game now, the contested resource $T$ loses a portion $1 - \theta$ of its value if War were to occur. Moreover, assume $R_i = 0$, $i = A, B$. In each period, the two sides follow the same sequence of moves as before: first, they choose enforcement efforts and then make choices between War and Armed Peace. If, however, they were to engage in War, its outcome would be permanent, in the sense that the winner would capture the contested resource thereafter, although in each period he would only receive a fraction of its benefit ($\theta T$). Since War decides the winner once and for all, there would be no enforcement efforts in future periods. By contrast, Armed Peace involves a division of the contested resource in each period without any loss of the contested resource, but it can be expected to typically involve some arming so as to better each side's bargaining position. Let $\delta \in (0, 1)$ denote the identical discount factor for the two sides. The following questions pertain to such a setting:

(i) What is the equilibrium choice of efforts and the expected payoff of each side $i$ under War? **(8 points)**

(ii) We can calculate that in a Markov perfect equilibrium, the equilibrium choices of effort under Armed Peace is given by $e_A^P = e_B^P = e^P = \frac{\theta T}{4(1-\delta)}$. Moreover the equilibrium share under Armed Peace is $\beta^P = 1/2$. What is the equilibrium payoff under Armed Peace? **(4 points)**

(iii) When is the equilibrium payoff under Armed Peace negative? Intuitively explain the conditions. **(4 points)**

## Group B

Answer any **three**

**5.** In the country of Pitunia (which of course does not exist) lived a few immigrant communities and a native community (who also immigrated five hundred years back, but everybody forgot that. So we will call them native). One of the immigrant community was known as Zericans who formed a tightly knit community and specialized in transport business. There job was to accept consignment and take this to a destination. Given the nature of the job, the truck driver has opportunity to steal the consignment. However, if a Zerican driver caught stealing by a Zerican business owener, no other Zericans will hire him again.

**a)** Write the above mentioned information in the form of a model to formulate the driver's decision problem to steal. **(7)**

**b)** Will a Zerican business owner prefer to hire a driver from his own community? Why? *(You may explain intuitively based on the structure written in question 5(a))* **(5.6)**

**c)** In the last year, one Mr. Bump got elected the president of Pitunia with the promise that he will deport all Zericans because Zerican business owners only employ drivers from their own community. How will this election change the driver's wage? (4)

**6.** In a student hostel near Kolkata, students are voting on the type of food to be served in the students' hostel and their monthly contribution. Once the decision is reached they have to give the money even if they decide not to eat at the hostel. The students come from different parts of India, and have different test and preference, the most important cleavage being the veg-non veg divide in food preference. *(You may assume for simplicity that the choice between veg-non veg is not a dichotomous choice, but a continuous one.)*

**a)** Set up this problem using any model that you have done in the course (or you may come up with something of your own). **(6.5)**

**b)** Compare two hostels – in one students from many parts of India live and the other where mostly (fish eating) Bengali students live. Using the structure set up in the above section, explain in which hostel students are likely to contribute more in the common hostel fund. **(6.5)**

**c)** Suppose you are made the hostel super. Can you think of any new institutional arrangement to solve the problem? **(3.6)**

**7. a)** Why were medieval port towns less riot prone than colonial port towns? **(7.6)**

**b)** What were the endogeneity concerns in the regressions used in Jha (2012)? (4)

**c)** Both Jewish moneylenders in medieval Europe and Muslim traders in medieval India had complementarity in production with respect to their Christian and Hindu counterparts respectively. However, European Jews faced violence from the Christians, while Muslim traders did not face such violence even if the state ruler was a Hindu. Using the arguments made by Jha (2012), can you reconcile this difference? **(5)**

**8.a)** How did Becker and Woesmann (2009) estimate the effect of Protestantism after controlling for literacy? Specifically, explain why they used the marginal return of education from other studies rather than estimates from their own data. **(8.6)**

**b)** Becker and Sussman chose income tax and teacher's salary as the main economic outcome variables. What are the problems associated with these outcome variables? **(6)**

**c)** If Weber's original hypothesis were true, how would that change the results obtained by Becker and Woesmann? **(2)**

# INDIAN STATISTICAL INSTITUTE

Second Semester Examination: 2018-19

MS (QE) I YEAR

Econometric Methods I

Date: 30 April, 2019 | Maximum Marks: 100 | Duration: 3 hours

Note: Answer question 1 and any **three** from the rest of the questions

1. Data on three-variable linear regression problem $y = b_1+b_2x_2+b_3x_3+e$ yield the following results:

$$X'X = \begin{bmatrix} 33 & 0 & 0 \\ 0 & 40 & 20 \\ 0 & 20 & 60 \end{bmatrix}, X'y = \begin{bmatrix} 132 \\ 24 \\ 92 \end{bmatrix} \text{ and } \Sigma(y-\bar{y})^2 = 150.$$

(a) What is the sample size?
(b) Write down the normal equations and solve for the regression coefficients.
(c) Estimate the standard error of $b_2$ and test the hypothesis that $b_2$ is zero.
(d) Compute $R^2$ and interpret it. Also interpret the values of the regression coefficients.
(e) Predict the value of y given $x_2 = -4$ and $x_3 = 2$.
(f) Comment on the possibilities of any of the regressors being dummy variable.

[1+9+8+6+2+2=28]

2. What do you mean by the problem of multicollinearity in the data? How will you detect the existence of multicollinearity? Write some possible solutions to this problem. Do you agree with the statement that "if all the simple correlations are small then the problem of multicollinearity will not arise"? Give explanations for your answer. [4+6+12+2=24]

3. Consider the following regression-through-the-origin model:

$$y_i = \beta x_i + e_i, \text{ for I} = 1, 2.$$

You are told that $e_1 \sim N(0, \sigma^2)$ and $e_2 \sim N(0, 2\sigma^2)$ and that they are statistically independent. If $x_1 = +1$ and $x_2 = -1$, obtain the weighted least squares (WLS) estimate of β and its variance. If in this situation you had assumed incorrectly that the error variances are the same as $\sigma^2$, what would be the OLS estimator of β? Compare the estimates with the estimates obtained by the method of WLS. [24]

4. (a) Write down the dummy variable regression models where (i) only slope differs, (ii) only intercept differs, and (iii) both slope and intercept differ. Prove that the situation where both slope and intercept differ is same as using separate regressions so far as estimation of regression coefficients are concerned.

(b) Suppose in a simple linear regression model

$$y = \alpha + \beta x + e,$$

where the regressor x can take only two values '0' and '1'. It takes the value '0' for the first $n_1$ observations and takes the value '1' for the remaining $n_2$ observations ($n_1 + n_2 = n$). The corresponding means for y are $\bar{y}_1$ and $\bar{y}_2$ respectively. Find $\hat{\alpha}$, $\hat{\beta}$, $V(\hat{\alpha})$ and $V(\hat{\beta})$. [10+14=24]

5. Examine the validity of the assumptions of CLRM under the presence of errors-in-variables in the regression set up. Discuss the identification problem in this model. Describe the methods of estimation in errors-in-variables model. [6+6+12=24]

6. Outline the salient features that distinguish each of the following estimation methods in the context of SEM.

(a) OLS, (b) ILS (c) LIML (d) 2SLS [24]

7. (a) Explain with examples what is meant by under-, exact and over- identification of an equation in a SEM. State the rank and order conditions for identification of the equations in a SEM.

(b) Consider the following hypothetical system of simultaneous equations in which the Y variables are endogenous and X variables are predetermined.

$$
\begin{aligned}
&Y_{1t} - \beta_{10} - \beta_{12} Y_{2t} - \beta_{13} Y_{3t} - \gamma_{11} X_{1t} = u_{1t} \\
&Y_{2t} - \beta_{20} - \beta_{23} Y_{3t} - \gamma_{21} X_{1t} - \gamma_{22} X_{2t} = u_{2t} \\
&Y_{3t} - \beta_{30} - \beta_{31} Y_{1t} - \gamma_{31} X_{1t} - \gamma_{32} X_{2t} = u_{3t} \\
&Y_{4t} - \beta_{40} - \beta_{41} Y_{1t} - \beta_{42} Y_{2t} - \gamma_{43} X_{3t} = u_{4t}
\end{aligned}
$$

Examine the Rank and Order conditions of identifiability of each of the above equations.

[12+12=24]

8. Write short notes on any **three** of the following:

(a) The problem of normalization in SEM.
(b) Testing for Outlying Observations and Prediction by Dummy Variables.
(c) Durbin-Watson Test
(d) Dummy variable trap.
(e) Stochastic Regressors. [3×8=24]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: (2018-2019)
MS (Q.E.) I Year
Macroeconomics I

Date: 03.05.19 Maximum Marks 60 Duration 3 hours

## Group A

Answer *all* the questions:

1. Assume that the Solow model accurately describes the growth experience of a country, named NOLAND. As a result of a war, much of the capital in NOLAND was destroyed. Now answer the questions below with brief explanations.
   a. What will be the effect of this event on per capita income in NOLAND in the next five years (medium term)?
   b. What will be the effect of this event on per capita income in NOLAND in the long run?
   c. What will be effect on the annual growth rate of per capita income in NOLAND in the long run?
   d. Will recovery in NOLAND occur faster if investment by foreigners is permitted, or if it is prohibited?

(4X2=8)

2. Consider the following version of the optimal growth model. The social planner's problem is

$$\underset{\{c_t,k_{t+1}\}}{\text{Max}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

Subject to

$$c_t + k_{t+1} = f(k_t)$$

$t = 0,1,2,....$

i) Write the Bellman's equation for this problem.

ii) Derive the Euler's equation from the Bellman's equation.

PTO

iii) Use the specifications such as $u(c_t) = log(c_t)$ and $f(k_t) = k_t^{\alpha}$. Guess the value function $V(k_t) = A + B\ log(k_t)$. Show that the decision rule for capital would be $k_{t+1} = \alpha\beta k_t^{\alpha}$.

(2+4+10 =16)

3. Suppose that a consumer maximizes

$$log(c_1) + E[log(c_2)]$$

Under the constraint $c_1 + c_2 = w_1 + w_2$. So here the discount rate of period 2 utility and rate of return on saving are both zero. When $c_1$ is chosen, there is uncertainty about $w_2$: the consumer will earn $w_2 = x$ or $w_2 = y$ with equal probability. What is the optimal level of $c_1$?

(6)

**Group B**

**Answer all questions**

1. Show how equilibrium unemployment is sustained in the Shapiro- Stiglitz model. What do you think would happen to equilibrium unemployment if the firms were to experience a technological progress (by this I simply mean that firms could produce more, say twice, the output they produced earlier, for any given combination of inputs).

[12+3]

2. a) In the Blanchard-Kiyotaki model demonstrate that the monopolistically competitive output is smaller than the competitive output.

b) Consider an economy with the representative agent having the utility function:

$U = [C^{\alpha}(1-L)^{1-\alpha}]^{\gamma}\left[\frac{M}{P}\right]^{1-\gamma}$, $0 < \alpha, \gamma < 1$

Where $C = n\left[\frac{1}{n}\sum_{1=1}^{n} c_i{}^{\rho}\right]^{1/\rho}$, $0 < \rho < 1$ and $c_i$ is the consumption of the $i^{th}$ variety. $L$ is the labour supply, $P$ is the price index of the varieties. Each agent is endowed with one unit of labour, thereby $(1-L)$ is the leisure enjoyed. $M$ is the money balances (and suppose $M_0$ is the initial endowment of money). The household budget constraint is given by:

$PC + w(1-L) + M = M_0 + w + \pi - T$ where *w* is the money wage rate and $\pi$ is the economy wide profits and *T* is the taxes. Production of varieties is given by:

$$Y_i = 0 \; if \; L_i \leq F$$
$$= \frac{L_i - F}{k} \; if \; L_i > F \quad \text{where } k>0$$

$Y_i$ is the output of $i^{th}$ variety and $L_i$ is the labour employed in the production of the $i^{th}$ variety.

Assume that there are no costs in adjusting prices (i.e. prices are fully flexible) and that there is no entry/exit of firms (fixed *n*).

(i) Derive the multiplier of a balanced budget (*PG*=*T*) increase in government expenditure where *G* takes the form:

$G = n\left[\frac{1}{n}\sum_{1=1}^{n} g_i{}^{\rho}\right]^{1/\rho}$ and $g_i$ is the government consumption of the $i^{th}$ variety.

(ii) What would be the effect of such increase in government expenditure on *P* ?

[Hint: Try to write down the goods market equilibrium (*Y*=*C*+*G*) in a form which does not involve money balances. That would require a look into the money market equilibrium ( $M = M_0$ ).]

**[7+8]**

**INDIAN STATISTICAL INSTITUTE**

Second Semestral Examination: (2018-2019) (Back paper)

MS (Q.E.) I Year

Macroeconomics I

Date 12.07.19 Maximum Marks: 100 Duration: 3 hours

**Group A**

**Answer all questions**

1. Suppose the representative agent has the following problem:

$$Max\ E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t u(c_t)$$

Subject to the periodic budget constraint

$$A_{t+1} = (1+r)(A_t + w_t + c_t)$$

$$t = 0,1,2,\ldots$$

Where $A_t$ and $w_t$ represent assets and wage income, respectively.

i) Write the Bellman's equation for the problem.

ii) Write the end-point restriction (transversality condition).

iii) Derive the consumption Euler's equation from the Bellman's equation.

iv) Use the condition $r = \rho$ in the above Euler's equation. Also use the specification $u(c_t) = c_t - \frac{1}{2}ac_t^2$. What is the form of the Euler's equation now?

v) Explain the equation in (iv) in the light of Robert Hall's Random Walk hypothesis.

(2+2+4+2+4=14)

2. With the help of a suitable model explain in detail the concept of the Ricardian equivalence theorem.

(15)

PTO

3. Consider the following version of Ramsey – Cass - Coopman's optimal growth model in continuous time.

$$Max \int_0^\infty e^{-\rho t} u(c)\, dt$$

Subject to

$$\dot{k} = f(k) - \delta k - c$$

i) Write the current value Hamiltonian for this problem.

ii) Write the first order conditions for optimization for this problem.

iii) Obtain the consumption Euler's equation for this problem.

iv) What is the golden rule level of capital stock for the model.

v) Draw the sketch of phase diagrams for this problem.

vi) What are the significant differences of this model from Solow's growth model?

(2+4+3+3+2+3 =17)

4. Briefly explain the concept of convergence in Solow's growth model.

(4)

## Group B
## Answer all questions

1. Show that in an OLG model, a higher contribution to a 'Pay as you go' pension scheme will reduce the steady state per worker capital stock.
What would be the effect of such higher contribution on the steady state welfare?

[25]

2. Assume that there are costs to adjusting prices (small menu cost) which makes non adjustment of prices/ wages optimal for firms/ labourers in the face of an expansionary policy brought about through an increased money supply. Under such circumstances show that a monetary expansion will expand output and will also raise welfare (evaluated in vicinity of the initial monopolistically competitive equilibrium).

[25]

# INDIAN STATISTICAL INSTITUTE
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## SEMESTER II EXAMINATION (2018 – 19) – *BACK PAPER*
M.S. (Q.E.) I Year
Time Series Analysis & Forecasting

Date: 13.07.19 Maximum Marks: 100 Time: 3 hours

*Answer **all** questions. Marks allotted to each question are given within parentheses.*

1. (a) Discuss what is meant by trend in a time series data. Show that the method of moving averages is an appropriate method for obtaining trend if the trend is linear.

   (b) Let $\{X_t\}$ be a process given by

   $$X_t = \mu + Z_t + \beta Z_{t-1},$$

   where $\mu$ is a constant and $\{Z_t\} \sim WN(0,1)$. Show that the ACF of this process does not depend on $\mu$.

   (c) Let $\{Z_t\}$ be i.i.d. normal (0, 1) noise. Define $\{X_t\}$ as

   $$X_t = \begin{cases} Z_t \text{ if } t \text{ is even} \\ (Z_1^2 - 1)/\sqrt{2} \text{ if } t \text{ odd.} \end{cases}$$

   Show that $\{X_t\}$ is WN (0, 1), but not i.i.d. (0, 1) noise.

   [7+4+9 = 20]

2. (a) Show that the conditional expectation of $X_{n+h}$, given observations $x_n.x_{n-1}, x_{n-2}, \ldots,$ is the $h$-step ahead minimum MSE forecast of $x_{n+h}$ made at origin $n$.

   (b) Find the 3-step ahead minimum MSE forecast at origin $n$ of the following time series

   $$(1-0.6B)^2 \ x_t = (1+0.5B)a_t$$

   where $a_t$'s are white noise with zero mean and unit variance.

   (c) Discuss how out-of-sample forecasts are obtained in a given time series data.

   [6+7+7 = 20]

P.T.O

. (a) Distinguish between deterministic trend and stochastic trend. What kind of assumptio of trend should be made while doing trend analysis for a given time series? Explain.

(b) In testing the null hypothesis of unit root in a time series, discuss about the mai statistical problem that arises in applying the usual test statistic based on the least square estimator of the coefficient parameter of an AR (1) model.

[12+8 = 20

. (a) Discuss about the nature of unit roots in a quarterly time series.
(b) Describe the HEGY test for detecting the presence of seasonal and non-seasonal unit roots in a quarterly time series data.

[7+13 = 20]

. (a) Define the spectral density function $f(\lambda)$ of a stationary process, and then show that it is nonnegative for all $\lambda \in [-\pi, \pi]$. Also find $f(\lambda)$ for an MA (1) process.

(b) Given a finite realization $\{x_1, x_2, \ldots, x_n\}$ of a stationary time series, obtain its periodogram, and then show that this can be regarded as a sample analogue of $2\pi f(\lambda)$ where $f(\lambda)$ is the spectral density function.

[10+10 = 20