

INDIAN STATISTICAL INSTITUTE
First Semester Examination (Back Paper): 2019-20

M.S. (Q.E.) I YEAR

Game Theory I

Date: 14.01.20 Maximum Marks: 100

Duration: 3 hours

Write everything mathematically. Answer all questions.

1. (a) A mixed Nash equilibrium (m_1, m_2) of a two-player symmetric game is called symmetric if $m_1 = m_2$. Justify the following statement with a proof or a counter example: Every symmetric game has a symmetric mixed Nash equilibrium.
- (b) Consider the symmetric game A where $A_{11} = 0, A_{12} = 5, A_{21} = 1, A_{22} = 3$. Does it have an evolutionary stable strategy? Justify your answer.

[25]

2. Consider a game $G = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$ such that $S_i = [0, 1]$ and u_i is a linear function for all $i \in N$. Define the best response function in pure strategies as follows: for all $i \in N$ and all $s_{-i} \in S_{-i}$, $b_i(s_{-i})$ is the set of pure strategies s_i of i such that $u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$ for all $s'_i \in S_i$. Justify your answer to the following questions with a proof or a counter example.
- (a) Is $b_i(s_{-i})$ non-empty for all i and all s_{-i} ?
- (b) Is $b_i(s_{-i})$ convex for all i and all s_{-i} ?
- (c) Is $b_i(s_{-i})$ closed for all i and all s_{-i} ?
- (d) Can you use fixed point theorem to prove that a pure strategy Nash equilibrium will always exist for G ?

3. A Nash equilibrium is called *strong* if no group of players can profitably deviate from it, that is, for a game $G = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$, a strategy-profile $s \in \prod_{i \in N} S_i$ is a strong Nash equilibrium if for all $M \subseteq N$, all $s'_M \in \prod_{i \in M} S_i$, we have

$$u_i(s_M, s_{N \setminus M}) \geq u_i(s'_M, s_{N \setminus M})$$

for all $i \in M$.

Similarly, define strong Nash equilibrium in mixed strategies.

Justify the following statements with a proof or a counterexample.

- (a) If a game has a Nash equilibrium in pure strategies, then it has a strong Nash equilibrium in pure strategies.
- (b) Strong Nash equilibrium in mixed strategies always exists for 2×2 games.

(25)

4. For a game G , G^T denotes the T -period repeated game where in each period the game G is played. Consider the penalty shootout game (two-person zero-sum game) G . Justify your answer:
- (i) Is there a pure strategy Nash equilibrium of the game G^∞ ?
 - (ii) Is there a pure strategy subgame perfect Nash equilibrium of the game G^∞ ?

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$$u_i(s_M, s_{N \setminus M}) > u_i(s'_M, s_{N \setminus M})$$

for all $i \in M$.

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[25]

INDIAN STATISTICAL INSTITUTE

End-Semestral Examination: 2019-20

Course: Master's in Quantitative Economics Year I

Subject: Microeconomics I

Date: 18 November, 2019 Afternoon **Maximum Marks:** 60 **Duration:** 3 hours

Answer all questions. Students may consult their notes or study material, but not each other.

1. Suppose a union faces an inverse demand function for labour: $w = A - L$, where w is the wage rate and L is the demand for labour; $A > 0$. The demand parameter A can take the values $\bar{A}\delta, \bar{A}(1 - \delta)$ with equal probability, $1 > \delta > 0$. The union's preferences are given by a VNM utility function $u = \ln(wL)$. The union chooses the wage rate, w , so as to maximize its expected utility. Show how the wage level chosen by the union, and the corresponding employment level, change as (i) \bar{A} changes, and (ii) δ changes. Interpret your results. (20 marks)

2. Consider a competitive firm producing according to a strictly concave production function:

$$A = F(L),$$

where A represents the amount of output produced and L represents the amount of input. The (non-storable) output and input can be readily bought and sold at prices p and w , respectively. Future p can be either \bar{p} or \underline{p} with equal probability, $\bar{p} > \underline{p} > 0$. However, w is known with certainty. The firm chooses L to maximize expected utility, on the basis of a VNM utility function $u = A + B\pi^\alpha$, where π is its realized profit, with $1 > \alpha > 0$; $A, B > 0$. How do expected profit, output

and employment change with an increase in α ? Interpret your findings. What do your results imply for the direction of capital flow? (20 marks)

3. (i) Consider the utilitarian social welfare function $W(u^1, \dots, u^H) = \sum_{h=1}^H u^h$.

(a) Show that W respects cardinal unit comparable information on utility profiles, i.e., utilitarianism is a coherent concept of social welfare even if utility information is only cardinally measurable and unit comparable.

(b) Show that utilitarianism does not respect ordinal non-comparable (ONC) information, i.e., utilitarianism is incoherent given only ONC information on utility profiles.

(c) Show that utilitarianism does not respect ordinal fully comparable information.

(15 marks)

(ii) Consider the maximin social welfare function $W(u^1, \dots, u^H) = \min_h u^h$.

Show that W does not respect cardinal unit comparability, but it respects ordinal full comparability.

(5 marks)

INDIAN STATISTICAL INSTITUTE

Back paper: 2019-20

Course: Master's in Quantitative Economics Year I

Subject: Microeconomics I

Date: 13/1/20 **Maximum Marks:** 100 **Duration:** 3 hours

Answer all questions.

1. Prove that a choice function satisfies Houthakker's WARP, if and only if it also satisfies Sen's α and β conditions. (20 marks)
2. Prove that Samuelson's Weak Axiom of Revealed Preference implies the Law of Demand, but that the converse is not true. (20 marks)
3. Prove that the Consistent Firm Choice and Non-reversibility conditions are, together, equivalent to the Weak Axiom of Profit Maximization. (20 marks)
4. Explain how the relative measure of (local) risk aversion may be interpreted in terms of an agent's willingness to accept gambles. (20 marks)
5. (i) Set up a simple model to show how a risk-averse agent's choice of optimal insurance depends on the insurance premium. (10 marks)

(ii) Explain whether the following statement is true, false or uncertain: if an individual exhibits a diminishing coefficient of absolute risk aversion, she must exhibit a diminishing coefficient of relative risk aversion as well. (10 marks)

INDIAN STATISTICAL INSTITUTE

Semester Examination: 2019-20

MS (QE) I YEAR

Statistics

Date: 25 November 2019

Maximum Marks: 100

Duration: 3 hours

[Note: Answer any five questions. Each question carries 20 marks]

1. (a) State and prove Bayes theorem.
(b) Suppose that the questions in a multiple choice examination have four alternatives each, of which a student picks one as the correct alternative. A student either knows the truly correct alternative with probability 0.8 or he randomly picks one of the four alternatives as his choice. Suppose a particular problem was answered correctly. Find the probability that the student really knew the correct answer.
(c) If there are 30 questions, then find the probability that the student will correctly answer at least 28 questions. [4+8+8=20]
2. (a) Write down the inclusion-exclusion formula.
(b) There are three colours – Red (R), Blue (B) and Green (G). If each box of a 3×2 matrix is colored taking one of the three colours randomly (with equal probability), then find the probability that none of the rows and columns have all the cells with the same colour.
(c) Let x_1, x_2, \dots, x_n be i.i.d. r.s. from a normal distribution $N(\mu, \sigma^2)$, σ^2 is known. Find a non-trivial sufficient statistic for μ . [2+14+4=20]
3. (a) Let x_1, x_2, \dots, x_n be i.i.d. r.s. from a Uniform distribution $U(0, \theta)$. Show that $T = (\prod x_i)^{\frac{1}{n}}$ is a consistent estimator of $E(T)$.
(b) Let x_1, x_2, \dots, x_n be a r.s. from a pdf $f(x, \theta) = e^{-(x-\theta)}$, $x \geq \theta$, $-\infty < \theta < \infty$. Show that $T_n = x_{(1)}$ is consistent for θ . [10+10=20]
4. (a) Show that a necessary and sufficient condition that an unbiased estimator T has minimum variance at the value $\theta = \theta_0$ is that $\text{Cov}(T, S \mid \theta_0) = 0$ for every S with $E(S \mid \theta_0) = 0$ such that $V(S \mid \theta_0) < \infty$ provided $V(T \mid \theta_0) < \infty$.
(b) Let x_1, x_2, \dots, x_n be i.i.d. r.s. from $N(\mu, \sigma^2)$. Prove that there exists estimator T of σ^2 such that $\text{MSE}(T)$ is less than the $V(s^2)$. [10+10=20]

5. Let the random variables X_1 and X_2 having joint pdf

$$f(x_1, x_2) = K, \quad 0 < x_1 < x_2 < 1 \\ = 0 \quad \text{elsewhere.}$$

Obtain (i) the value of K.

(ii) marginal pdf of X_1 and X_2 ,

(ii) the conditional pdfs of either when the other is fixed, and

(iii) $E[X_2 \mid X_1 = x_1]$

[4+4+6+6=20]

6. (a) In a random sample of 1000 persons from town A, 400 are found to be consumers of wheat. In a sample of 800 from town B, 400 are found to be consumers of wheat. Do these data reveal a significant difference between town A and town B, so far as the proportion of wheat consumers is concerned?

(b) A restaurant can accommodate 50 customers. Experience indicates that 10% of those who make a reservation will not show. Suppose that the restaurant accepts 55 reservations. Let X denote the number of customers who show.

i. Find an expression for $P(X=x)$ which give the distribution of X .

ii. What are the mean and s.d. of X ?

iii. Use the normal approximation to calculate the probability that the restaurant will be able to accommodate all the customers that show.

iv. If instead of 50 customers, the restaurant can accommodate 52 customers then find the probability that the restaurant will be able to accommodate all the customers that show.

[6+(2+2+5+5)=20]

7. (a) Let x_1, x_2, \dots, x_n be a r.s. from $N(\mu, \sigma^2)$. Find the ML Estimator for σ^2 and find the variance of the estimate.

(b) Find a 95% confidence interval for σ^2 , if the value of s^2 is 10. [(4+8)+8=20]

Note: Give proper justification to your answers. Any result that you use should be stated clearly. This paper carries 65 marks. The maximum you can score is 60.

(1) Check whether

$$V_1 = \{(a, b, 0) : a, b \text{ are real numbers}\} \quad \text{and} \quad V_2 = \{(0, 0, c) : c \text{ is an integer}\}$$

are subspaces of \mathbb{R}^3 . If so, find their dimensions.

[2+2+1]

(2) Determine the rank and signature of the quadratic expression

$$3x^2 + 8y^2 - 24z^2 - 14xy + 10yz + 10xz.$$

Hence, comment on the nature of the quadratic form.

[4+1]

(3) If possible, diagonalize the following matrix and find an orthogonal basis with respect to which the corresponding quadratic form has a diagonal representation:

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

[3+2]

(4) Construct a bounded set of real numbers with exactly two limit points.

[3]

(5) Show that the set $(0, \infty)$ is **not** compact by finding an open cover of $(0, \infty)$ which has no finite subcover.

[4]

(6) Let $a_n = \sin n$ for $n \geq 1$. Show that the sequence $\{a_n\}_{n=1}^{\infty}$ is not convergent.

[4]

(7) Let $a_1 = 1, a_2 = 2$ and $a_{n+1} = \sqrt{a_{n-1}} + \sqrt{a_n}$ for $n \geq 2$. Show that the sequence $\{a_n\}_{n=1}^{\infty}$ is convergent. Find its limit.

[3 + 2]

(8) Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be two sequences of real numbers such that $\lim_{n \rightarrow \infty} a_n = 0$ and $\{b_n\}_{n=1}^{\infty}$ is bounded. Show that $\lim_{n \rightarrow \infty} a_n b_n = 0$.

[4]

(9) Find the supremum and infimum of the set $S = \left\{ \frac{mn}{1+mn} : m, n \in \mathbb{N} \right\}$.

[3]

(10) Prove that the equation $(1-x)\cos x = \sin x$ has at least one solution in $(0, 1)$.

[4]

(11) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be continuous at $a \in \mathbb{R}$ and $f(a) \neq 0$. Show that there exists a $\delta > 0$ such that $|f(x)| > 0$ for all $x \in (a - \delta, a + \delta)$.

[5]

(12) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be continuous and $E = \{x \in \mathbb{R} : f(x) = 0\}$. Show that E is a closed set.

[3]

(13) Maximize $f(x, y) = xy$ subject to

$$x + y^2 \leq 2,$$

$$x \geq 0,$$

$$y \geq 0.$$

[12]

(14) Classify an identified dynamical system for control purposes.

[3]

INDIAN STATISTICAL INSTITUTE

Semestral Examination: 2019-20

Course Name: M. S. (QE) I Year

Subject Name: Computer Programming and Applications

Date: 21/11/2019

Maximum Marks: 100

Duration: 3 hours

Note: Answer all questions. Default is ANSI C standard. You may use calculator.

1. Justify your answer

(a) Will the following code compile? If yes, what will be the output of the following programs?

i.

```
#include <stdio.h>
int main()
{
    int a;
    a = 1, 2, 3;
    printf("%d", a);
    return 0;
}
```

ii.

```
#include <stdio.h>
int main()
{
    int a = 10,
        b = 20,
        c = 30;
    if (c > b > a)
        printf("TRUE");
    else
        printf("FALSE");
    return 0;
}
```

(b) How will the following expressions be evaluated?

i. $(1 > 2 + 3 \ \&\& \ 4)$

ii. $(1 == 2 \ != \ 3)$

2.5x4=10

2. Answer any two. Use function(s) and mention proper parameter passing strategy. Avoid Global variable(s).

i. Write a complete C program which accepts three integers from the command line and finds their HCF and LCM. The values should be printed in a predefined file (named HCF_LCM.out).

For any invalid input(s) it should be able to print error message and quit.

ii. Write a complete well documented C program that accepts an integer from the command line and prints the prime factorization on screen and a file with filename as the input integer and extension as .txt.

Eg. If input number is 123, the file name should be 123.txt

For any invalid input, it should be able to print error message and quit.

iii. Consider the following variable declarations:

```
int *ipMatIP1, *ipMatIP2, iRow1, iCol1, iRow2, iCol2;
```

Assume two matrices are stored in the first two variables with iRow1, iCol1, and iRow2, iCol2 as their respective dimensions. Design a matrix multiplication C function and write the code.

30x2=60

3. Solve (correct to nine decimal places) the following using any technique from Numerical Analysis. Mention the name of the method you have used to solve.

i. Solve the system:

$$0.0003120x_1 + 0.006032x_2 = 0.003328$$

$$0.5x_1 + 0.8942x_2 = 0.9471$$

ii. Evaluate

$$I = \int_0^1 \sqrt{1-x^2} dx$$

15x2=30

Final Examination 2019 for MSQE I

Course name: **Basic Economics**

Date: **22 November 2019**

Maximum marks: **100**

Duration: **3 hours**

Instruction: This is a closed-books, closed-notes examination. Answer all questions in both the groups. Please use separate answer booklets for Groups A and B.

Group A

1. Following Quah (1993), consider the “mobility matrix” of 128 countries between 1982 and 2009, as given below:

Observations	Category	1	2	3	4	5
32	1	84	13	3	0	0
21	2	43	43	14	0	0
26	3	0	27	50	23	0
20	4	0	0	20	70	10
29	5	0	0	0	3	97

Refer to the above matrix to answer the following questions:

- (a) How many countries in 1982, out of the 128 countries, had per capita GDP less than a quarter of the world average? **(1 point)**
- (b) What percentage of countries make it from category 3 to category 4 in the 27 years period? **(1 point)**
- (c) What do the high numbers along the diagonal signify? **(1 point)**
- (d) Which set of countries have the least “mobility”? **(1 point)**

2. The following questions pertain to ‘conflict and development’.

- (a) What is a common index to measure ‘fractionalization’ in the society? Explain. **(3 points)**

b. What are the essential features of a 'polarized' society? (3 points)

c. Suppose 'inequality' in an economy increases. Is it always true that 'polarization' in that economy also increases? Explain why or why not, through diagrammatic examples. (4 points)

3. The following questions pertain to issues in measurement of inequality.

(a) What are the ethical principles that a measure of inequality is expected to satisfy? (4 points)

(b) Does mean absolute deviation satisfy them? Elucidate. (4 points)

(c) Consider two societies, each consisting of only three persons. Let the distribution of income in the two societies be (3, 12, 12) and (4, 9, 14), respectively. Calculate the coefficients of variation (CV) in the two societies. Which society is more unequal? Now calculate the Gini coefficients of the two societies. Does the Gini coefficients corroborate your conclusion regarding which of the societies is more unequal based on CV? Why or why not? Explain. (8 points)

OR

The following questions pertain to issues in measurement of poverty.

(a) Evaluate the following statements by providing a brief explanation or analysis. (4 + 4 = 8 points)

(i) The income gap ratio and the head count, as measures of poverty, may lead to very different uses of antipoverty resources by policy makers.

(ii) Both the poverty gap ratio and income gap ratio are insensitive to the inequality among the poor.

(b) Show that the FGT (Foster-Greer-Thorbecke) indices are increasingly sensitive to the income distribution among the poor, the greater is the value of α . In particular, assume that the Lorenz curve of incomes among the poor in a society worsens, while both the Head-Count Ratio and the Income-Gap Ratio remain unchanged. What happens to FGT for $\alpha = 2$? (8 points)

4. This question pertains to the theory of trade. Imagine there are only two countries that make up the world economy, call them North (N) and South (S). Both countries can produce only two commodities: computers and rice. Suppose labor is the only factor of production.

The following table describes how many units of labor are required to make one computer and one sack of rice. Suppose N and S each have a total of 600 units of labor.

Labour required	One computer	One sack of rice
In N	10	15
In S	40	20

Which country is relatively inefficient in the production of computers? Which country is relatively inefficient in the production of rice? Will the countries prefer to trade with each other? Explain. **(12 points)**

5. The following questions pertain to trade policies.

(a) How does an imposition of tariff by the Government substitute imports and protect an infant industry? **(6 points)**

(b) Can the government thereafter, always remove tariff and get the infant industry to cut costs and grow up? Elucidate using a simple game-tree with meaningful payoffs. **(6 points)**

Group B

1. "Macroeconomics is not without content and context." Discuss the statement in the light of developments of different schools of thought in macroeconomics. **(10 points)**

2. Discuss how different markets are accommodated in the Simple Keynesian model, IS-LM model, aggregate demand and aggregate supply model and Mundell-Fleming model. **(3+3+3+3 =12 points)**

3. Discuss the effects of expansionary monetary policy in Mundell-Fleming model under flexible exchange rate when (a) free flow of capital is allowed, (b) when restricted flow of capital is allowed, and (c) when capital flows are prohibited. **(4+4+4 = 12 points)**

4. Discuss the derivation of the Phillip's curve from aggregate supply curve. Discuss the concepts of adaptive expectation, menu costs and shoe-string costs of inflation. **(6+4+3+3 = 16 points)**