INDIAN STATISTICAL INSTITUTE

Mid-semester Examination: (2018-2019)

MS(QE) II

Individual and collective choice

Date: 03.09.2017 Maximum Marks: 100 Duration: 3 hrs.

(1) Suppose the data for the problem are preferences of the n ($< \infty$) agents of a society over two alternatives x and y. Answer the following questions.

(a) Define the following properties of a social welfare function: symmetry among agents, neutrality between alternatives, positive responsiveness and non-triviality. **(4)**

(b) Define the majority voting social welfare function and the dictatorial social welfare function. **(2)**

(c) State and prove May's theorem by giving all the relevant definitions. **(25)**

(2) State Arrow's impossibility theorem. Prove Arrow's impossibility theorem using the notion of "extremely pivotal agent". **(30)**

(3) Let Q be a binary relation defined on the finite set of alternatives A (where $|A| \geq 3$) and $\hat{Q}$ be its asymmetric part and $\overline{Q}$ be its symmetric part.

(a) Show that if $\hat{Q}$ and $\overline{Q}$ are transitive, then Q is an ordering. **(5)**

(b) Show that if Q on A is an ordering, then $\hat{Q}$ and $\overline{Q}$ are both transitive. **(4)**

(4) Define scoring rules. Show that scoring rules always generate an ordering on the finite set of alternatives A. **(4+8=12)**

(5) Define single-peaked preferences on a finite set of alternatives A. State and prove the Median Voter Theorem. **(4+14)**

INDIAN STATISTICAL INSTITUTE
Mid Semester Examination: (2018- 2019)

MS (Q.E.) II Year

International Economics I

Date: 05/09/2018 | Maximum Marks 40 | Duration 3 hours

Use separate booklets for group A & B

Group A

Answer all

1. Consider a Ricardian model of trade with complete specialization. Derive the model equilibrium and comment on the welfare changes if the factor of production is allowed to move freely across the countries. [10]

2. Derive the equilibrium for a Ricardian model of trade for the case where the home country imposes an ad-valorem tariff on its import. Assume that the trading equilibrium is characterized by complete specialization. [10]

Group B

Answer all

1. Show that in a two agent setting, Walras stability guarantees that the recipient of a transfer necessarily gains. [10]

2. Consider a 2 country, 2 commodity trading world, with perfectly competitive markets and show that imposing an ad- valorem export tax is the same as imposing an ad-valorem import tariff when the government redistributes all tax and tariff revenues lump sum. Also derive the optimal export tax.

[Note: An export tax on good i means the following: $p_i(1+\tau) = p_i^*$, where p_i is the domestic price and p_i^* is the international price of good i and τ is the ad valorem export tax rate.] [10]

INDIAN STATISTICAL INSTITUTE

Mid-term Examination: 2018-19

MSQE II 2018-19

Incentives and Organizations

Date: 6^{th} September 2018 Maximum marks: 40 Duration: 2 hours

Answer all questions

1. Consider the basic hidden information model done in class. Suppose the principal's benefit function is $S(q) = 12q$, and the agent cost function is $C(q, \theta) = \theta q^2$. The agent knows his own type, i.e., the value of θ, while the principal knows that θ is either 1 or 2 with equal probability. Output is observable and the principal can give a transfer to the agent. If the output is q, and the principal gives a transfer t, the net payoff of the principal is $12q - t$, while the net payoff of the agent is $t - \theta q^2$.

Assuming the agent as well as the principal have outside option 0, and that the principal is the residual claimant, has full bargaining power and offers contracts, what is the principal's net expected payoff from the interaction? [25]

2. Suppose P has x units of capital. There is a unit measure of workers. If P appoints a worker i, she can produce $2\alpha_i$ units of a consumption good which P can consume, and obtain utility $2\alpha_i$ If P does not appoint a worker, his utility is 0. P can appoint at most one worker.

If worker i is appointed, she can produce $2\alpha_i$ units of the consumption at cost α_i^2. If a worker is not appointed, her payoff is 0. A worker's type, i.e., the value of α, is known only to herself, while P knows that α is distributed uniformly over [0, 1].

P can offer a wage contract w, such that any worker who accepts it and is appointed produces $2\alpha_i$ and gets wage w. If he gets multiple acceptances, he appoints one of the accepting workers randomly.

a) What wage does P offer to maximise net expected payoff? [10]

b) How does your answer change if the workers' productivity parameter is 4 instead of 2? [5]

Indian Statistical Institute
Mid-Semester Examination: 2018-2019
MS(QE) II: 2018-2019
Industrial Organization

Date: 07/09/2018 Maximum Marks: 40 Duration: 3 Hours

Answer any FOUR questions

1. Let the market demand function be $Q = Ap^{-\varepsilon}$, $\varepsilon > 1, A > 0$. The cost function of the monopolist is $cQ; c > 0$. A consumer at a distance x from the monopolist's trading centre requires to pay tx as transport cost per unit of output to be bought. Under price discrimination what price (including transport cost) will the monopolist charge to the consumer? If x goes up, find its effect on the monopoly price net of transport cost.

[4+6=10]

2. Suppose two firms, 1 and 2, produce one good each at marginal cost c_i $(i = 1, 2)$. Each firm has a monopoly power in the production of its goods. The demand curve is $q = D(p)$ where $p \equiv p_1 + p_2$ is the price of the composite good and p_i is the price of good i $(i = 1, 2)$. Let $c \equiv c_1 + c_2$ and $\varepsilon = -D'p/D$ be the price elasticity of demand. Answer the following.
   (a) Derive the optimal p for the horizontally integrated structure in terms of c and ε.
   (b) Suppose now that the two firms choose their prices simultaneously and non-cooperatively. Show that $p = c/(1 - \frac{2}{\varepsilon})$.
   (c) Consider the non-integrated structure and suppose that that firm 1 chooses its price first and takes into account the effect of its choice on firm 2's price. Show that the composite price in this case will be given by $p = c/(1 - \frac{1}{\varepsilon})^2$.

[3+3+4=10]

3. Consider a monopolist that produces for two periods. The demand curves in both periods are $x_t = 1 - p_t$ for $t = 1, 2$. The marginal costs are $c > 0$ in the first period and $c - \gamma x_1$ in the second period. Here γ is a small and positive number. There is a discount factor of δ between the periods; $0 < \delta < 1$.
   (a) Explain briefly how the monopolist's problem changes compared with a situation when the marginal cost is c in both periods.
   (b) Find the quantities x_1 and x_2 that the monopolist chooses in the two periods.
   (c) Show that $x_1 > x_2$.

[3+5+2 =10]

4. There is a competitive fringe consisting of 10 firms. Each firm has cost function $C(q) = q^2/2$. A dominant firm first sets the price of the product but it can serve only the residual demand. The fringe firms take that price as given, and decide their sales. The market demand for the product is $Q = 120 - 5P$ and the dominant firm's marginal cost of production is 4. Find the equilibrium price in the market and sales of the different firms.

[10]

5. Consider a duopoly producing a homogeneous product. Firm 1 produces one unit of output with one unit of labor and one unit of raw material. Firm 2 produces one unit of output with two units of labor and one unit of raw material. The unit costs of labor and raw material are w and r. The demand is $p = 1 - x_1 - x_2$, and the firms compete in quantities.
   (i) Compute the Cournot equilibrium.
   (ii) How does firm 1's profit be affected if the price of labor goes up? Give economic intuition of the result.

[5+5=10]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: (2018-2019)
MS (Q.E.) II Year
Macroeconomics II

Date: 11/09/2018 | Maximum Marks 40 | Duration 3 hours

Use separate booklets for group A & B

Group – A

Answer question 1 and any one of 2 and 3.

1. Consider an economy existing for two periods and producing a single good y_i in period i using a single factor l_i (called labour) through the production function
$$y_i = \varepsilon_i l_i, i = 1,2.$$
Labour is paid according to its marginal productivity. Consumers have a total endowment of time equal to unity which they allocate between labour and leisure and a utility function
$$U_i = \ln c_i + b\ln(1 - l_i)$$
in each period and a discount rate ρ. Each Consumer maximizes her lifetime utility by choosing consumption c_i and labour supply l_i. There is an international market where the consumers can freely borrow or lend at a given rate of interest r.
(a) Write down the utility maximization problem of the representative consumer and the first order conditions of maximization.
(b) What are the wages in the two periods?

Now suppose there is a rise in ϵ_1 with ϵ_2 remaining constant.
(c) Show that leisure in period 1 falls relative to that in period 2.
(d) Noting that leisure is a normal good, show that output goes down but consumption goes up in period 2.
(e) What happens to output, consumption and labour supply in period 1?
[3 + 1 + 1 + 2 +3 =10]

2. Setting up the Lucas imperfect information model show that departure of output from its normal level is an increasing function of the surprise in the price level (the Lucas Supply Function). What implication does this have for the effectiveness of monetary policy?
[7 + 3 =10]

3. Setting up a real business cycle model with 100% depreciation and no government, show that if utility is log-linear in consumption and leisure and production function is Cobb-Douglas in labour and capital, both labour supply and the saving ratio are constant over time. Interpret the result.
[8 + 2 = 10]

P.T.O

Group - B

Answer all

1. Work out the dynamics of per capita assets for a Ramsey type small open economy, facing a constant rate of interest in the world capital market.
In this context discuss the problems associated with either a very low or a very high rate of interest.

[10]

2. a) In a linearized version of the Ramsey model, solve for the unique initial value of per-capita consumption that puts the economy on the saddle path, given the initial date per-capita capital stock

   b) Assuming a Cobb-Douglas production function, work out the transitional dynamics of the savings rate in the Ramsey model.

[5+5] = [10]

Indian Statistical Institute

Mid Semestral Examination: (2018 – 2019)
M.S.(QE) – II year
Econometric Applications I

Date:10.9.18 | *Maximum Marks – 50* | *Duration: 2 hours*

(Answer any two questions)

1. a) Suppose the income variate x follows a Gamma distribution with density function

$$f(x)=\frac{1}{\Gamma\alpha}\, e^{-x}\, x^{\alpha-1}, \qquad x\geq 0.$$

Find the Lorenz curve and Lorenz Ratio.

b) Let the Lorenz curve $L(p)$ $(0\leq p\leq 1)$ of a continuous type income distribution be given by

$$L(p)=1-(1-p)^{1-\frac{1}{a}}.$$

Find the underlying distribution.

c) Find the median of the income distribution represented by the distribution function $F(x)=\dfrac{x^a}{x_0^a+x^a}$. Why is it called the *sech square* distribution?

d) How do you graphically test the suitability of Log normal form for a given income distribution data?

[8+7+5+5 = 25]

2. a) Consider two income profiles $x=(x_1,x_2,\ldots,x_n)$ and $y=(y_1,y_2,\ldots,y_n)$ with $x_1\leq x_2\leq\cdots\leq x_n$ and $y_1\leq y_2\leq\cdots\leq y_n$.

(i) Define Lorenz dominance of x with respect to y. State the assumptions clearly.

(ii) Why is the ordering of income profiles, generated by the LC comparison, a quasi-ordering? Illustrate with an example.

(iii) What is a generalized Lornez curve (GLC)? What is the advantage of GLC over LC?

b) What do you mean by a "Bistochastic" matrix?

Suppose the income distribution $y = (1,6)$ is obtained from a distribution $x = (2,5)$ through income transfer. What kind of transfer is it? Show that if we write $y = Ax$, then A can not be Bistochastic.

c) Define 'hazard rate' and 'proportional failure rate' (PFR) in the context of income distribution. Show that the Pareto distribution has constant PFR throughout the domain.

[(3+3+3) + 8+ 8= 25]

3. a) State and explain the desirable properties that an income inequality index should satisfy.

b) What are 'relative' and 'absolute' measures of income inequality? Give examples.

c) Given the individual utilities

$$u(x_i) = \begin{cases} C + D\dfrac{x_i^{1-\varepsilon}}{1-\varepsilon}, & \varepsilon \neq 1 \\ C + D\log x_i, & \varepsilon = 1 \end{cases}$$

write down the expression for Atkinson's measure of inequality.

Show that Atkinson's income inequality index is invariant under positive linear transformation of the individual utility functions.

[9 + 6+ (5+5) =25]

INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700108
M.S.(Q.E.) II Year (2018– 19)
Semester I Examination
Econometric Methods II

Date: 12.11.2018 | Maximum Marks: 100 | Time: 3 hours

*This question paper carries a total of **110** marks. You can answer any part of any question. But the maximum that you can score is **100**. Marks allotted to each question are given within parentheses.*

Answer PART I and PART II on separate Answer Scripts.

PART – I

1. (a) Describe the ECM test for cointegration, and argue why this test is better than the residual –based test.

   (b) Discuss what you mean by Granger causality and why this is important in case of cointegration.

   [10 + 10 = 20]

2. (a) In the context of a two-variable VAR model, discuss what are impulse function, and the Choleski factorization.

   (b) Consider the following system of equations involving two time series variables x_t and y_t.

   $$x_t = u_t, \qquad u_t = u_{t-1} + \varepsilon_{1t}$$

   and $\quad y_t + \alpha x_t = \vartheta_t,\ \vartheta_t = \rho \vartheta_{t-1} + \varepsilon_{2t}$

   where ε_{1t} and ε_{2t} are uncorrelated white noise terms.

   Then show that x_t and y_t are *I (1)* variables regardless of the value of ρ. Further, do you think that x_t and y_t are cointegrated? Give explanations in support of your answer.

   [10 + 10 = 20]

3. (a) Describe a test for testing the null hypothesis of 'no conditional heteroscedasticity' against the alternative that the underlying conditional heteroscedasticity is given by a GARCH (*p*, *q*) model.

   (b) Discuss the basic statistical idea and result behind the White's Information Matrix test. Also state the test statistic with all relevant assumptions.

   [10 + 10 = 20]

P.T.O

PART – II

1. Consider the multiple linear regression model: $Y = X\beta + \epsilon$, where X is stochastic. Assume that data are independent across observations. Suppose $E(\epsilon_i|X_i) \neq 0$ but there are available instruments Z with $E(\epsilon_i|Z_i) = 0$ and $V(\epsilon_i|Z_i) = \sigma_i^2$, where $dim(Z) > dim(X)$. We consider the GMM estimator $\hat{\beta}$ that minimizes
   $G_N(\beta) = [\frac{1}{N}\sum Z_i(Y_i - X_i^{/}\beta)]^{/} W_N [\frac{1}{N}\sum Z_i(Y_i - X_i^{/}\beta)]$, where W_N is an appropriate weight matrix.
   a. Suggest an Instrumental Variable estimator for β using the entire vector of instruments.
   b. Show that the estimator suggested in (a) can be viewed as a GMM estimator as defined above.
   c. Suggest an optimal choice of W_N.

[10+10+5=25]

2. Consider the dynamic panel data model: $y_{it} = \alpha_i + \rho\, y_{i\,t-1} + \beta x_{it} + \varepsilon_{it}\, {}_{,t=1,2,....T;\; i=1,2,.....N}$. ε_{it} is i.i.d. with all ideal conditions. x_{it} is purely exogenous. $\alpha_i^{\prime}s$ are iid random variables with mean α and variance σ_α^2.
   a. Show that the OLS estimator of ρ is inconsistent for finite number of time series observations.
   b. Propose a GMM estimator of ρ . Write the moment conditions appropriately. Show that the proposed GMM estimator is consistent even when T is fixed.

[9+(4+12)=25]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: (2018-2019)

MS(QE) II

Individual and Collective Choice

Date: 14.11.2018 Maximum Marks: 100 Duration: 3 hrs.

Note: Answer all questions.

(1) State and prove the field expansion lemma by giving all the relevant definitions. **(3+12=15)**

(2) Define the plurality rule, the anti-plurality rule and the Borda count. Show that each of these three scoring rules fails to satisfy independence of irrelevant alternatives. **(3+12=15)**

(3) State and prove the Gibbard-Satterthwaite theorem (using the notion of an option set) when there are two agents and the individual preferences are rational and strict. Give all the relevant definitions. **(5+25=30)**

(4) Define single-peaked preferences (after first defining all the technical terms you will use in the definition). Assuming a given linear order, consider the social choice function, on the domain of all single-peaked preferences, that selects the rightmost peak under all profiles. Show that this social choice function is strategyproof. **(5+10=15)**

(5) Consider the pure public goods problem. Show that an efficient mechanism is dominant strategy incentive compatible if and only if it is from the class of VCG mechanisms. Give all the relevant definitions. **(25)**

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: (2018-2019)

MS (Q.E.) II Year

International Economics I

Date: 16.11.2018 Maximum Marks 60 Duration 3 hours

Group-A

Answer all questions

1. In a Ricardian model of trade, derive the conditions which lead to complete specialization. In such a model, is it possible that both the trading countries are incompletely specialized? Explain. [10]

2. Suppose that in a standard two country, two commodity, two factor Heckscher-Ohlin model of trade, the countries differ with respect to their preferences in a way that individuals in a particular country are identical within that country, but across the countries individuals exhibit a relative bias for a particular commodity i.e. given the same income and relative prices, individuals of one country consume a good relatively more that the individuals of the other country. Discuss the effect of this change in preferences on the commodity and factor prices, exports and imports of the countries. [10]

3. State and prove the Stolper Samuelson theorem in the context of a Heckscher-Ohlin model. In the same context, derive the Rybczynski theorem. In a two commodity, three factors specific factors model, derive a similar relationship relating the factor prices and the production of the commodities given the endowments. [10]

Group-B

Answer all questions

1. Show that in a three-agent setting, a transfer paradox might occur even when the equilibrium is Walras stable. In this context discuss the role of substitution effects in ensuring normal results. (15)

2. Show how in Krugman (1981) distributional conflict is related both to difference in factor endowment ratios and economies of scale. (15)

Indian Statistical Institute
First Semestral Examination: 2018-2019
MS(QE) II: 2018-2019
Industrial Organization

Date: 19/11/2018 Maximum Marks: 40 Duration: 3 Hours

Answer any THREE questions. Your total score cannot exceed 40.

1. Consider Hotelling linear city model with linear transport cost (in distance) (but there is no production cost). Consumers are uniformly distributed over the unit length of the city. Answer the following.
   (a) Suppose the two firms are located at two extreme points of the city and now the firms are to decide their optimal prices simultaneously. Find the optimal prices of the products.
   (b) Suppose the prices of the products are same and given to the firms and the firms want to decide their optimal locations simultaneously. Find the optimal locations.
   (c) Suppose that the social planner chooses locations for the firms, then the firms compete in prices. Find the socially optimal locations for the firms.
   (d) Suppose firm 1 chooses its location first, then firm 2 decides its location, and finally they simultaneously decide the prices of the products. Find their optimal locations and prices of the products.

[3 + 3 + 4 + 5 = 15]

2. (a) Consider a k-firm cartel in an n-firm oligopoly industry; $1 < k \leq n$. What conditions are needed to check for the cartel to be stable?
(b) Consider a three-firm industry; firms produce horizontally differentiated goods and compete in quantities. The market demand function of firm i is: $p_i = 1 - q_i - \frac{1}{2}\sum_{j \neq i} q_j\,; i, j = 1, 2, 3$. There are no costs of production. Check whether there can be a stable cartel comprising two or more firms. (Cartel's objective is to maximize joint profits of the members.)

[3 + 12 = 15]

3. Consider an n-firm Cournot oligopoly. Firm i has marginal cost $c_i; i = 1, 2, \ldots. n$. Assume that $\sum_i^n c_i$ is constant. Answer the following.
   (a) Define Herfindahl index (H) of market concentration.
   (b) Show that as firm asymmetry goes up, H goes up.
   (c) Show that as H increases, industry profit also increases.
   (d) When firm asymmetry goes up, what will be its effect on overall welfare (i.e., the sum of consumer surplus and industry profit)?

[2 + 4 + 4 + 5 = 15]

4. Consider two firms, firm 1 and firm 2, and two periods, $t = 1, 2$. Firm 2 can, however, enter only in the second period if entry is profitable. Hence firm 1 has monopoly in the first period. Market demand in each period is same and given by $p^t = 20 - x^t$, where x^t is the industry output in period t. Also in each period the cost function of firm i is $C_i^t(x_i^t) = 9 + 4x_i^t$; $i = 1, 2$. Firms maximize profits by setting quantities. Answer the following.
   (a) Determine the monopoly output and profit.
   (b) Suppose that for technical reasons, firm 1 has to choose the same quantity in each period (i.e., $x_1^1 = x_1^2 = x_1$ (say)). Then observing x_1^1, firm 2 is to consider entry in period 2. Derive the expression of firm 2's profit as a function of x_1^1 if it decides to enter. Show that firm 2 will make a positive profit if and only if $x_1^1 < 10$.
   (c) Assuming that firm 2 enters in period 2 (with a positive profit), determine firm 1's output, x_1 and its total profit.
   (d) Suppose firm 1 wants to deter entry of firm 2 in period 2. Which output will firm 1 choose (in the first period)? Does entry deterrence occur in a sub-game perfect equilibrium of the two period game?

[2 + 5 + 4 + 4 = 15]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2018-19

MSQE II 2018-19

Incentives and Organisations

Date: 19.11.18 November 2018 Maximum marks: 60 Duration: 3 hours

Answer all questions. All questions have equal marks.

Q1. Consider the competitive screening model with unproductive tasks and unobservable worker types. There are two firms. Any firm can offer contracts, any contract prescribing a wage and a task level. Firms first offer contracts, and workers then decide whether to follow self-employment or work for a particular firm.

(a) Suppose there are two types of workers. Show that any subgame perfect Nash equilibrium must fully separate between the types. [15]

(b) Would this conclusion change if there were three types of workers? [5]

Q2. Consider the principal agent model with hidden action. Assume the agent is risk-averse and can take two possible levels of effort. The principal wishes to implement high effort.

(a) Suppose there are two possible output levels. Show that the optimal contract is monotone, with transfer increasing in output. [10]

(b) Suppose now there are more than two possible output levels.

(i) Does monotonicity of the optimal contract necessarily hold? [5]

(ii) Does your answer in (i) depend on whether the number of possible output levels is finite? [5]

Q3. Consider the principal agent model with hidden action. Suppose there are two possible levels of effort, and two possible levels of output. The principal wishes to implement high effort. Assume the agent is risk-neutral and there is limited liability. The agent must be paid at least $\underline{w}$ if output is low, and at least $\overline{w}$ if output is high.

(a) Derive the optimal contract assuming the special case of $\underline{w} = \overline{w} = 0$. [15]

(b) Now consider the general case. Are properties of the optimal contract affected in comparison to (a)? [5]

Indian Statistical Institute
Semestral Examination: (2018 – 2019)
M.S.(QE) – II year
Econometric Applications I

Date: 22.11.2018 *Maximum Marks* – **100** *Duration: 3 hours*

Answer any ***four*** *questions.*

1. (a) Suppose some income is transferred from a poor person to a non poor person, other things remaining the same. What is expected to happen to a poverty measure? [state the axiom you use]. Examine the 'head count ratio' and 'income gap ratio' measures in light of this axiom.

   (b) In the context of measurement of poverty, state the axioms (i) Replication invariance, (ii) Subgroup consistency and (iii) Decomposability.

   (c) Consider the income profile $x_1 \leq x_2 \ldots \leq x_n$ and the poverty measure $P = \frac{1}{n}\sum_{i=1}^{q}(1-\frac{x_i}{z})^{\alpha}$, where q: number of poor; z: poverty line; and $\alpha = 0,1,2$.

   Examine the index in light of the axioms in (c) for different values of α.

   [6 + 6 + 13 = 25]

2. (a) Define 'income elasticity of demand'. How are the commodities classified based on this elasticity? Sketch the Engel curves corresponding to these classifications. Justify your answer.

   (b) If tax is proportional to the value of a consumer item, then show that taxing a luxury item is progressive.

   (c) Discuss the problems of 'identification' and 'least squares bias', likely to arise in estimation of demand functions from time series data.

   [10+5+10=25]

3. (a) Define the Specific Concentration Curve (SCC) for a particular commodity. What does the point (0.4,0.3) on a SCC signify?

   (b) Discuss the properties of SCC and derive its relationship with the Lorenz Curve (LC). When does the SCC reduce to LC?

P.T.O

(c) Assuming that income follows a Lognormal distribution and the Engel curve of an item is of constant elasticity form, describe alternative methods of estimating the Engel elasticity using the SCC.

[5+14+6=25]

4. (a) Explain the difference between a 'censored distribution' and a 'truncated distribution.

(b) Write down the Tobit model to incorporate zero consumption with full specification of the distribution of the error term. Why are the assumptions underlying the standard linear regression model not tenable in such a case?

(c) Given the dynamic model for 'clothing'

$$q(t) = \alpha + \beta s(t) + \gamma x(t)$$

where $q(t)$: rate of demand at time t

$x(t)$: income during the same time

$s(t)$: inventory of 'clothing' at time t.

and assuming that the stock is used up at a constant depreciation rate δ, find the short and long term derivatives of consumption with respect to income.

[3 + 12 + 10 = 25]

5. (a) Explain the 'deterministic' and 'stochastic' production frontiers assuming a log-linear relationship between inputs and output. State clearly the assumptions you make.

(b) Define the input and output oriented measures of 'technical efficiency' due to Debreu-Farrell (D-F) and Koopmans.

(c) "D-F technical efficiency is necessary, but not sufficient for Koopmans' technical efficiency" – explain diagrammatically for both input and output oriented measures.

(d) Describe the Corrected Least Squares (COLS) method of estimating the parameters of a deterministic production frontier.

[5+6+8+6 = 25]

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: (2018-2019)

MS (Q.E.) II Year

Macroeconomics II

Date: 26.11.18 Maximum Marks 60 Duration 3 hours

Group-A

Answer Question 1 and either 2 or 3.

1. Consider a two-period competitive financial economy with two primary Arrow securities. There are two dates 0 and 1 and there are two possible states at date 1. There is a single consumption good and consumption takes place only at date 1. There are two types of consumers in the economy. They are of equal numbers. Let w_s^h be the endowment of type h individual in state s, $h = 1, 2$ and $s = 1, 2$. The endowments are given by

$$\begin{bmatrix} w_1^1 & w_2^1 \\ w_1^2 & w_2^2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$

State 1 takes place with probability ¼ and state 2 with probability ¾. The probabilities are common knowledge. Both types of consumers are expected utility maximizers with an identical utility function

$$U^h(x_s^h) = \ln(x_s^h), \qquad h = 1, 2.$$

Here x_s^h is the consumption of type h in state s. The timing of the transactions are as follows:

(a) At date 0, markets for the two securities open.
(b) At date 1, spot market for the consumption good opens.

Now answer the following questions:

(i) Specify each consumer's optimization problem at date 0.

(ii) Derive the equilibrium of this economy.

PTO

[Hint: First solve the Arrow-Debreu equilibrium and then use the equivalence result to solve the equilibrium in the financial economy]

[3+12 = 15]

2. Show that there is a welfare loss if the government is unable to pre-commit the monetary rule as compared with the situation where it is able to do so. [15]
3. Prove that an Arrow-Debreu economy with complete contingent markets is equivalent to an economy where there are no contingent markets but a primary Arrow security for each state of the economy. [15]

Group-B

Answer all questions

1. Show that with investments having convex adjustment cost, the capital stock exhibits smooth transitional dynamics even when the country is small in the international capital market facing a constant rate of interest.

   Show that if the production function and the investment adjustment cost function are both linear homogeneous then Tobin's marginal q would be equal to the average q.

[10+5=15]

2. In the Blanchard-Yaari model with cohort *dependent* wage, what would be the effect of a sharper decline in wage with respect to age on the steady state capital accumulation?

   Consider the same model, but now with zero population growth, cohort *independent* wages and open to international asset market with a constant rate of interest. Now show that the aggregate savings in this model is negatively related to the level of assets?

[11+4=15]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination : 2018-19

M. Tech. (QR & OR)-II

Industrial Experimentation

Date: 26/11/2018 Maximum Marks: 100 Duration 3 hours

NOTE: (i) This paper carries 114 marks. Answer as much as you can but the maximum you can score is 100. The marks are indicated in [ ] on the right margin.

(ii) The symbols and notations have the usual meaning as introduced in your class.

(iii) Assume factors are fixed and normality assumptions are satisfied, unless indicated otherwise

1. Write a short note on the basic principles of experimental design and their usefulness in not more than one hundred and fifty (150) words.

[12]

2. Define a BIBD? When does one use a BIBD to conduct an experiment? Show that in such a design, number of treatments can never be more than the number of blocks.

[3+3+7 = 13]

3. The yield of a chemical process is determined in an experiment involving five two-level factors A, B, C, D, and E. The design and the data are given in Table 1.

Table 1. Data on yield of the chemical process

Sl.	Treatment combinations					Yield
No.	A	B	C	D	E	(%)
1	-	-	-	-	+	55.6
2	+	-	-	+	-	48.8
3	-	+	-	+	-	60.6
4	+	+	-	-	+	94.5
5	-	-	+	+	+	54.4
6	+	-	+	-	-	50.4
7	-	+	+	-	-	61.8
8	+	+	+	+	+	92.2

Identify the design. Write the complete defining relation and the alias relationships. What is the resolution of this design? Estimate the effects assuming that the factors D and E do not interact with each other and with other factors in the experiment. What other assumptions you may make to draw conclusions about the factors? Put out an analysis of variance table in line with your assumptions and test the significance of the factors.

[2+(4+3)+1+5+2+5 = 22]

4. A textile company weaves a fabric on a large number of looms. It would like the looms to be homogeneous so that it obtains a fabric of uniform strength. The process engineer suspects that, in addition to the usual variation in strength within samples of fabric from the same loom, there may also be significant variations in strength between looms. To investigate this, the engineer selects four looms at random and makes four strength determinations on the fabric manufactured on each loom. This experiment is run in random order.

a) Identify the design and write the associated model.

b) Find $E(SS_{Treatments})$ under your proposed model. [Just assume and do not derive the expression for $SS_{Treatments}$.]

[(2+3)+7 = 12]

5. A test engineer has three correlation devices (A), those he tested using four testers (C) randomly selected from each of the two facilities (B). Each correlation device was tested twice by each tester for its leakage current. Based on the collected data, an analysis of variance is done. The analysis of variance (ANOVA), shown in table 2, is worked out assuming all the factors to be crossed under a fixed effects model.

Table 2. ANOVA on Leakage Current

Source	Df	Sum Sq	Mean Sq	F value	Remarks
A	2	3.3117	1.65583	17.7411	Significant @ 1%
B	1	0.1633	0.16333	1.7500	Not Significant
C	3	0.3800	0.12667	1.3571	Not Significant
AB	2	0.7617	0.38083	4.0804	Significant @ 5%
AC	6	1.4950	0.24917	2.6696	Significant @ 5%
BC	3	2.4967	0.83222	8.9167	Significant @ 1%
ABC	6	1.1383	0.18972	2.0327	Not Significant
Error	24	2.2400	0.09333		
Total	47	11.9867			

Is the analysis correct and why? State the model you deem appropriate, with all assumptions, associated with the given problem. If wrong then correct the given ANOVA table and test for the significance of effects. Estimate appropriate variance components of the model.

[3+3+13+6 = 25]

6. Class Assignments. [30]

Table 3: *F* distribution (5%) Table

$F_{0.05,\ v1,\ v2}$

/	df_1=1	2	3	4	5	6	7	8	9	10	12	24
df_2=2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.64
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.77
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.53
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.84
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.41
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.12
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	2.90
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.74
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.61
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.51
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.42
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.35
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.29
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.24
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.19
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.15
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.11
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.08
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.05
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.03
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.01
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	1.98
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	1.96
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	1.89
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.52

INDIAN STATISTICAL INSTITUTE

Back Paper Examination: 2018-19

MSQE II 2018-19

Incentives and Organisations

Date: 04/01/19 Maximum marks: 100 Duration: 3 hours

Answer all questions

Q1. There is a firm (agent) which pollutes while producing. Suppose $\bar{q}$ is the production level and $\bar{R}$ is the revenue of the firm, both exogenously given. If $x \in [0, \bar{x}]$ denotes the level of pollution, then $D(x)$ is the damage to society. Let $c(x, \theta)$ be the pollution cost of the firm, where $\theta \in \{\theta_1, \theta_2\}$ is private information of the firm. The regulator (principal) does not know the realization of θ, but knows that $Pr(\theta = \theta_1) = p \in (0, 1)$. The level of actual pollution is observable. The regulator can propose a regulatory scheme, but the firm may or may not accept it. Assume if the firm does not accept, it will produce pollution $\bar{x}$. Further assume that

$$
\begin{aligned}
A1 &: c(\bar{x}, \theta_1) = c(\bar{x}, \theta_2) = \underline{c} \\
A2 &: c(x, \theta_1) < c(x, \theta_2), \forall x < \bar{x} \\
A3 &: c'(x, \theta) < 0, c''(x, \theta) \geq 0, \forall \theta \in \{\theta_1, \theta_2\} \\
A4 &: c'(x, \theta_1) > c'(x, \theta_2) \\
A5 &: D'(x), D''(x) > 0
\end{aligned}
$$

a) What is the socially optimal level of pollution (with society including the firm) $x^*(\theta)$, under full information? [10]

b) Can $x^*(\theta)$ be implemented under asymmetric information through a regulation scheme which depends only upon x? [10]

P. T. O

Q2. Consider the principal agent model with hidden information. Suppose the principal's benefit function is $S(q) = 15q$, and the agent cost function is $C(q, \theta) = \frac{\theta}{2}q^2$. The agent knows his own type, i.e., the value of θ, while the principal knows that θ is either 1 or 3 with equal probability. Output is observable and the principal can give a transfer to the agent. If the output is q, and the principal gives a transfer t, the net payoff of the principal is $15q - t$, while the net payoff of the agent is $t - \frac{\theta}{2}q^2$.

Assume that the agent and the principal have outside option 0. Further assume that the principal is the residual claimant, has full bargaining power and offers contracts. What is the principal's net expected payoff from the interaction? [40]

Q3. Consider the principal agent model with hidden action where output can take any realisation in $[\underline{q}, \overline{q}]$, the agent is risk-averse and can take any effort in $[0, \bar{e}]$. Show that the first-order or local approach to deriving the optimal contract is valid if two conditions, the monotone likelihood ratio property (MLRP) and the convexity of the distribution function condition (CDFC) hold. [40]

Indian Statistical Institute

Semestral Examination: (2018 – 2019)

M.S.(QE) – II year

Econometric Applications I

BACK PAPER

Date: 7.01.2019 *Maximum Marks – **100*** *Duration: 3 hours*

Answer **ALL** *questions.*

1. (a) Define 'income elasticity of demand'. How are the commodities classified based on this elasticity? Sketch the Engel curves corresponding to these classifications. Justify your answer.

(b) If tax is proportional to the value of a consumer item, then show that taxing a luxury item is progressive.

(c) Discuss the problems of 'identification' and 'least squares bias', likely to arise in estimation of demand functions from time series data.

[10+5+10=25]

2. (a) Define the Specific Concentration Curve (SCC) for a particular commodity. What does the point (0.4,0.3) on a SCC signify?

(b) Discuss the properties of SCC and derive its relationship with the Lorenz Curve (LC). When does the SCC reduce to LC?

(c) Assuming that income follows a Lognormal distribution and the Engel curve of an item is of constant elasticity form, describe alternative methods of estimating the Engel elasticity using the SCC.

[5+14+6=25]

3. (a) Explain the 'deterministic' and 'stochastic' production frontiers assuming a log-linear relationship between inputs and output. State clearly the assumptions you make.

(b) Define the input and output oriented measures of 'technical efficiency' due to Debreu-Farrell (D-F) and Koopmans.

PTO

(c) "D-F technical efficiency is necessary, but not sufficient for Koopmans' technical efficiency" – explain diagrammatically for both input and output oriented measures.

(d) Describe the Modified Least Squares (MOLS) method of estimating technical efficiency from cross section data using a deterministic production frontier and a Half Normal distribution of efficiency (u).

$$[u \sim HalfNormal(0,\sigma_u^2) \Rightarrow f(u) = \frac{2}{\sigma_u\sqrt{2\pi}} exp(-u^2/2\sigma_u^2)]$$

[5+6+8+6 = 25]

4. Write short notes on:
   (a) Measurement of 'Concentration' in business and industry.
   (b) Measurement of poverty.
   (c) Zero consumption and Tobit model.

[9+8+8=25]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: (2018-2019) (Back paper)

MS (Q.E.) II Year

Macroeconomics II

Date 14·01·2019 Maximum Marks: 100 Duration: 3 hours

Group-A

Answer any two

1. Develop a baseline real business cycle model and discuss how business fluctuations are created in such models through external shocks. [25]
2. Show how in the Lucas model of incomplete information, unanticipated monetary shocks have expansionary effects on output and employment. Can the government systematically use monetary policy to stabilize the economy? [20+5]
3. Following Rotemberg and Saloner show how prices remain sticky over the business cycle due to implicit collusion by firms. What does this theory predict about prices during a boom? [20+5]
4. Show that an economy with primary Arrow securities for each state is Pareto optimal. [25]

Group-B

Answer all questions

1. In an infinite horizon model, work out the dynamics of per capita asset for a small open economy, facing a constant rate of interest in the world capital market.
   In this context discuss the problems associated with either a very low or a very high rate of interest. [25]

2. Derive the steady state in the Blanchard-Yaari model and examine the effect of an increase in the probability of death per unit time on steady state capital accumulation.
   Also demonstrate how Ricardian equivalence is rendered invalid in this model.

   [20+5=25]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: (2018-2019) (Back paper)

MS (Q.E.) II Year

International Economics I

Date 14.01.2019 Maximum Marks: 100 Duration: 3 hours

Group-A

Answer all questions

1. Set up a Ricardian model of trade and show that trade leads to welfare gains. [15]

2. Suppose that in a standard two country, two commodity, two factor Heckscher-Ohlin model of trade, derive the factor price equalization, Stolper-Samuelson and Rybczynski theorems. Also comment on the pattern of trade. [20]

3. Setup a two commodity, three factors specific factors model as a baseline, and discuss the changes that occur if labour is allowed to move freely across the countries. [15]

Group-B

Answer all questions

1. Explain the mechanics through which capital inflow in Dei's model necessarily leads to welfare improvement. How is this result strikingly different from the Brecher, Diaz-Alejandro result, where the import competing capital intensive sector is tariff protected and the country is small. [25]

2. Show how in Krugman (1981) pattern of trade (intra/inter- industry) is related both to difference in factor endowment ratios and economies of scale. [25]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination : 2018-19
M.S.(QE) II YEAR
Mathematical Programming

Date: 18 February 2019 | Maximum Marks: 60 | Duration: $2\frac{1}{2}$ hours

---

Notation have usual meaning.

This paper carries 70 marks. However, maximum you can score is 60.

---

1 Suppose that you are responsible for scheduling the monthly production levels of a certain product for a planning horizon of twelve months. You are given the following information (for $j = 1, 2, \ldots, 12$):

(*i*) Demand for the product in month j is d_j. These could either be targeted values or be based on forecasts.

(*ii*) Cost of producing each unit of the product in month j is c_j. There is no set-up/fixed cost for production.

(*iii*) Inventory holding cost per unit for the month j is h_j. These are incurred at the end of each month.

(*iv*) Production capacity for month j is m_j.

Obtain a mathematical formulation to generate a production schedule that minimizes the total production and inventory holding costs over this planning horizon after satisfying the demand.

[10]

2 Consider the following optimization problem:

$$\begin{aligned} \min \quad & 2x_1 - 3x_2 + x_3 \\ \text{subject to} \quad & \\ -x_1 + x_2 \quad &= 4 \\ x_1 - 2x_2 + x_3 &\leq 6 \\ x_1 &\geq 1 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

(a) Give a feasible solution of the problem. (b) Find all the extreme directions (if there is any) of the solution space. (c) Hence or otherwise, argue whether the problem has an optimal solution.

[1+8+3 = 12]

[P.T.O.]

3 Prove that, in convex programming problem, every local optimal solution is global optimal.

[10]

4 The following statements are made in regard to the problem: min $\mathbf{c}^T\mathbf{x}$ subject to $\mathbf{x} \in \mathbf{X}$, where $\mathbf{X} = \{\mathbf{x} : \mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0\}$. State, for each one, whether it is **True** or **False**. Justify your answer by sketching graph, numerical example or proof.

(a) A feasible solution can be uniquely represented by the extreme points and extreme directions of $\mathbf{X}$.

(b) Even if $\mathbf{X}$ is unbounded, the problem need not be unbounded.

(c) Two-Phase method has been used to solve the problem. The optimal objective value of Phase I problem is found to be non-zero. Hence, the original problem must be infeasible.

[5+5+5 = 15]

5 Solving the following problem, by some simplex procedure, is of interest:

$$\begin{aligned} \max \quad & -x_1 - 2x_2 \\ \text{subject to} \quad & \\ & 3x_1 + 4x_2 \leq 12 \\ & 2x_1 - x_2 \geq 2 \\ & x_1, x_2 \geq 0. \end{aligned}$$

Name the procedure you wish to adopt with a brief justification. Set-up the first iteration table for your chosen procedure, and then identify the leaving and entering variables.

[5+8 = 13]

6 Consider the two linear programs as follows:

(P_1): min $\mathbf{c}^T\mathbf{x}$ subject to $\mathbf{A}\mathbf{x} = \mathbf{b}$, $\mathbf{x} \geq 0$,

(P_2): max $\mathbf{b}^T\mathbf{y}$ subject to $\mathbf{A}^T\mathbf{y} \leq \mathbf{c}$, $\mathbf{y}$ unrestricted.

Show that P_1 is infeasible if and only if P_2 with $\mathbf{c} = 0$ is unbounded.

[10]

——*** xXx ***——

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2018-19

Course: Masters in Quantitative Economics Year II

Subject: Economics of Conflict

Date: 19/2/2019; Afternoon **Maximum Marks:** 40 **Duration:** 3 hours

Answer all questions. Students may consult their own notes or study material, but not one another. Calculators are not allowed.

1. Suppose a peasant society consists of a single producer. The peasant consumes two goods, leisure (l) and food (f), and has a time endowment of 1. The peasant's preferences are given by $u = (\theta^{-1} f^{\theta} + kl)$; where $\theta \in (0,1), k > 0$. Food is produced by labour according to the production function: $f = (1-l)^{\beta}$; $\beta \in (0,1]$. Some proportion, t, of the peasant's produce is taken away by warlords. Warlord i has the ability to impose a tax t_i on the producer; $i \in \{1,2,\ldots,n\}$, so that $t = \sum_i t_i$. The warlords choose their tax rates simultaneously and non-cooperatively, while taking the peasant's response into account.

(a) Find the Nash equilibrium level of food production, total tax rate, total tax revenue, and the welfare of the peasant. **(4 marks)**

(b) Find out how these four values change as the number of warlords increases. Interpret your results. **(2 marks)**

(c) Find out how the total tax rate and tax revenue change as θ and β increase. Interpret your results. **(4 marks)**

2. Suppose two ethnic groups, A and B, are contesting for control over some mineral resource valued at v; $v > 0$. The probability of success of group $i \in \{A, B\}$ in the contest over the mineral resource is given by: $p_i = \frac{L_i}{L_i + L_{-i}}$; where $L_i \in [0,1]$ is the amount of labour allocated by the group to the contest and L_{-i} is the amount of labour allocated to the contest by the other group. Ethnic group i can produce (non-contestable) food according to the production function $D\theta_i^{\alpha}(1 - L_i)$, where $D > 0$ is a parameter representing the fertility of available agricultural land, of which $\theta_i \in [0,1]$ is the proportion controlled by i, and $\alpha > 0$ is a scale variable. The price of food is

normalized to unity. Each ethnic group maximizes its total income from production and appropriation.

(a) Find the total labour allocated to conflict and the success probability of ethnic group i in the Cournot-Nash equilibrium. (**4 marks**)

(b) Show how greater equalization of land-ownership across ethnic groups affects the distribution of the mineral resource and the total labour allocated to conflict. (**4 marks**)

(c) Assuming equal distribution of land-ownership, show how total social loss due to conflict is affected by an increase in the value of the contestable resource, and an increase in the agricultural productivity parameter D. (**3 marks**)

(d) Explain what your results in parts (b) and (c) suggest regarding social policy for restraining conflicts in a mineral-exporting economy which faces a sudden boom in international prices for its exports. (**4 marks**)

3. Suppose a worker's expected utility is given by:

$$u = A(1 - p^d)w - \frac{L^2}{2};$$

where L is the effort actually provided, A, the marginal utility of income, is some positive constant, w is the wage rate, and p^d is the probability of being dismissed, with $p^d = (1 - L)p^0(s)$, where s is the amount of supervisory input; $p^0(0) = 0, p' > 0, p'' < 0, p'(0) = \infty$. In case of dismissal, the worker receives zero income. All prices are unity. A profit-maximizing capitalist hires n identical workers to produce an output, using a production technology $Q = ZnL$, where $L \in [0,1]$ is the per worker labor effort actually provided, and Z is some positive productivity parameter.

(a) Find the worker's labor extraction function. (**3 marks**)

(b) Using your result in (a), and given the level of output, find how an increase in labour productivity (Z), and workers' valuation of income (A), individually affect the capitalist's profit (output net of supervision and labor costs) and the wage bill. Explain your results. (**4 marks**)

(c) Now suppose the firm is owned by a workers' state, which pays the entire output net of supervision cost to workers as wage, and maximizes the wage income of workers subject to the requirement of producing some given level of output, taking the labour extraction function in (a) as given. Show how an increase in labour productivity (Z), and workers' valuation of income (A), individually affect the wage bill. (**4 marks**)

(d) Explain what your results in (b) and (c) imply about the claim that a socialist society should discourage conspicuous consumption as a bourgeois vice. (**4 marks**)

Academic Year 2018-19

Economic Development

Mid-Term Examination

MSQE II

Maximum Marks 40

Time: 2 hours

Date: 20.2.19

Answer question 1 and either 2 or 3.

1. Consider an economy with a large number of agents distributed according to their inheritances X. Each agent lives for two periods. In period 1, an agent has the option of investing in education which involves a fixed expenditure h. Moreover, if an agent decides to undertake education, she has to forego work in period 1. However, mere expenditure on education does not guarantee that the agent successfully completes her education. She has to put in an effort denoted by e where $e \in [0,1]$. The cost of effort is $C(e) = \frac{1}{2}e^2$. The probability that the agent successfully completes her education is p where $p = e$. If the agent is successful in completing her education, then in the second period she earns an income normalized to be unity. If she is unsuccessful, then in the second period she earns $\alpha < \frac{1}{2}$. Again, the agent has the option of not undertaking education at all, in which case, she earns α in both periods. Finally, borrowing and lending is possible at exogenous rates of interest with the borrowing rate $i > r$, the lending rate. If an agent invests in education with borrowed funds, she pays back the principal with interest only if she is successful. In case of a failure, she pays nothing to the lender. The objective of an agent is to maximize her expected life time income. Her decision involves a choice of either undertaking education or not undertaking education and if the former option is chosen, then the agent also chooses how much effort she will put in.
   (a) Given that an agent has decided to undertake education, show that her effort increases with her inheritance. Also show that for agents with inheritances $X \geq h$, it is optimal to put in the maximum amount of effort.
   (b) Show that if $\frac{1}{2} > \alpha > \frac{1-h(1+i)}{2-h(1+i)}$, there exists X^* such that for inheritances $X < X^*$ the agent decides not to undertake education, for inheritances X such that $X^* \leq X <$

PTO

h, the agent undertakes education by taking a loan and for $X \geq h$ education is self-financing.

[10+10=20]

2. In a model of increasing returns and hierarchical preferences show how the existence of a middle class is necessary for industrialization.

[20]

3. Show how imperfect information regarding the increasing returns sector of an economy can remove the inderminacy associated with Krugman's History vs. Expectations model.

[20]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : 2018-19

Course Name: M.S. (Q.E.) II YEAR
Subject Name: The Theory of Mechanism Design
Date: 20-2-19 Maximum Marks: 40 Duration: 2 hours

Problem 1 Justify your answer by a proof or a counterexample.

(a) Suppose $A = \{a, b, c, d\}$ and the domains $\bar{\mathcal{D}}$ and $\hat{\mathcal{D}}$ are as follows:

$$\bar{\mathcal{D}} = \{abcd, acbd, adbc, badc, bcda, bdac, cadb, cbda, cdba, dabc, dbac, dcba\},$$

$$\hat{\mathcal{D}} = \{abdc, acdb, adcb, bacd, bcad, bdca, cabd, cbab, cdab, dacb, dbca, dcab\}.$$

Then,
(i) every unanimous and strategy-proof SCF $f : \mathcal{D}_1 \times \mathcal{D}_2 \to A$, where $\mathcal{D}_1 = \bar{\mathcal{D}}$ and $\mathcal{D}_2 = \hat{\mathcal{D}}$, is dictatorial.
(ii) every unanimous and strategy-proof SCF $f : \mathcal{D}_1 \times \mathcal{D}_2 \times \mathcal{D}_3 \to A$, where $\mathcal{D}_1 = \mathcal{D}_2 = \bar{\mathcal{D}}$ and $\mathcal{D}_3 = \hat{\mathcal{D}}$, is dictatorial.

(15)

(b) The matching function based on the men proposing deferred acceptance algorithm, where $|M| = |W| \geq 3$, is strategy-proof.

(10)

Problem 2 Suppose that there are three groups of students: MSQE, M.Stat, and M.Math. All these groups have equal number of students n. You need to form n teams consisting of exactly one student from each group (and hence total three students in a team). Each student has a preference over his/her possible partners (i.e., pairs of students one from each other group). For instance, if you are an MSQE student, then you have a preference over all pairs of (M.Stat,M.Math) students in M.Stat $\times$ M.Math.

(a) Model this problem as a matching problem, that is, define a suitable mathematical matching (as you have done in class by using m for one-sided or two-sided matchings).

(b) Define stability in this setting.

(c) Does stable matching always exist in this setting? Define an algorithm that produces a stable matching whenever that exists.

(15)

INDIAN STATISTICAL INSTITUTE

Mid-term Examination: 2018-19

MSQE II 2018-19

Auction Theory

Date: 21^{st} February 2019 Maximum marks: 40 Duration: 2 hours

Answer all questions

Q1. There are N bidders participating in a first-price auction. Bidder private valuations are independently drawn from $[0, 1]$ according to an increasing, differentiable distribution function F.

(a) Derive symmetric, increasing, differentiable equilibrium bid functions, assuming one exists. [10]

(b) Argue there is a unique such function, if it exists. [5]

(c) Is the bid function you have identified in parts (a) and (b) above an equilibrium? Argue rigorously. [5]

Q2. There are 2 potential bidders. Bidder private valuations are independently drawn from $[0, 1]$ according to the uniform distribution. The seller uses the second price auction. She cannot set a reserve price, but can set an entry fee. A bidder can participate in the auction only if he pays the entry fee. Entry is voluntary, publicly observable and simultaneous.

I: Suppose bidder private valuations are realized after entry decisions.

(a) What is a bidder's net expected payoff conditional on entry, given entry fee e? Specify any assumption you make for the derivation. [5]

(b) Characterize bidders' entry decisions in symmetric equilibrium, given entry fee e. [5]

(c) What is the seller's optimal entry fee given symmetric equilibrium bidder behavior? [5]

II: Suppose bidder private valuations are realized before entry decisions.

(d) What is the seller's optimal entry fee given symmetric equilibrium bidder behavior? [5]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: 2018-2019
MS (Q.E.) II Year
Sample Survey: Theory and Practice

Date: 22nd February, 2019 Maximum Marks 25 Duration 1.30 hours

All notations are self-explanatory. You can answer any part of any question.

1. Suppose a population has four units $\{U_1, U_2, U_3, U_4\}$. Consider the following six possible samples each of size 2:

$S_1 = \{U_1, U_2\},\ S_2 = \{U_1, U_3\},\ S_3 = \{U_1, U_4\},\ S_4 = \{U_2, U_3\}, S_5 = \{U_2, U_4\}, S_6 = \{U_3, U_4\}$.

Suppose $P(S_1) = \frac{1}{3}, P(S_2) = \frac{1}{6},\ and\ P(S_6) = \frac{1}{2};\ and\ P(S_3) = P(S_4) = P(S_5) = 0$.

(a) Find the *inclusion probability* ($\Pi_i = P(\text{unit } i \text{ is in sample})$) of each sampling unit.

(b) Consider the following estimator for the population mean of an underlying variable Y.

$T = \frac{Y_1}{2} + \frac{3Y_2}{4}$ if sample S_1 is selected

$= \frac{Y_1}{2} + Y_3$ if sample S_2 is selected

$= \frac{Y_3}{6} + \frac{Y_4}{2}$ if sample S_6 is selected

Find $E(T)$ and $var(T)$. [7+ (8+10)=25]

2. Discuss Systematic sampling scheme with relative merits and demerits. Propose an unbiased estimator for population mean based on systematic sampling. Find its variance. [3+3+4=10]

INDIAN STATISTICAL INSTITUTE

Semestral Examination: 2018-19

MSQE II 2018-19

Selected Topics II: Auction Theory

Date: 22^{nd} April 2019 Maximum marks: 50 Duration: 3 hours

Answer all questions. Please answer parts A and B in separate scripts.

Part A

1. a) What are the differences between discriminatory and uniform price auction rules in context of multiple object auctions? Is there any situation when uniform price auction and Vickrey auction become identical? Explain. [3 + 2]

b) Suppose there are 4 bidders and 8 units up for sale. The submitted bid vectors are $b^1 = (50, 47, 40, 32, 27, 22, 15, 5)$, $b^2 = (45, 42, 38, 31, 23, 18, 10, 7)$, $b^3 = (48, 44, 41, 35, 26, 20, 14, 6)$, $b^4 = (55, 46, 37, 30, 24, 19, 16, 8)$. Identify the winning bids. What will be the payments of the winning bidders under discriminatory, unifrom price and Vickrey auctions? [2 + 1 + 1 + 1]

c) Consider a situation involving 2 bidders and 2 objects. Both bidders have demand for both objects. Each bidder's value vector $X^i = (X_1^i, X_2^i)$ is independently and identically distributed on the set $\chi = \{\mathbf{x} \in [0,1]^2 : x_1 \geq x_2\}$ according to a density function f such that the marginal distributions are $F_1(x_1) = (x_1)^2$, $F_2(x_2) = (2 - x_2)x_2$. Formulate the payoff function for an individual bidder and show that it is optimal to shade the bid for the second unit in symmetric equilibrium. [5]

P.T.O.

Part B

2. Consider the common values model, and assume a second-price auction.

a) Characterize symmetric, increasing, differentiable equilibrium bid functions, assuming one exists, and show there is a unique such function. [10 + 5]

b) Show that equilibrium exists. [5]

3. Assume, following Burguet and Sákovics (1999), that there are two sellers, each with a single unit of a homogeneous good for sale. They each have 0 value for the objects and wish to sell using a second price auction. There are 2 potential buyers, each capable of consuming at most 1 unit. Buyer valuations for any object are i.i.d. draws from $[0, 1]$ according to the differentiable and increasing distribution function F, and private information. All parties are risk neutral and a buyer can attend at most one auction. Suppose seller i has set reserve price r_i, with $0 \leq r_1 \leq r_2 < 1$. The buyers now have to simultaneously decide which auction to enter, if any, and what to bid in an auction conditional on entry. Rigorously establish a symmetric equilibirum in this buyers' subgame: what does a buyer do as a function of her valuation, and why? [15]

Indian Statistical Institute
Second Semestral Examination: 2018 –2019
M.S. (Q.E) – II Year
Econometric Applications II

Date: **24.04.2019** Maximum Marks: **100** Duration: 3 hrs.

(Answer any four questions)

1. (a) Define True Cost of Living Index (TCLI). What is the difference between a TCLI and a standard price index number?
   (b) How can one estimate the sampling error of Laspeyres' price index using a regression framework?
   (c) What is 'Purchasing Power Parity (PPP)'? Why is it more appropriate than official exchange rate while making international comparison of level of living?
   (d) State the desirable properties that a PPP should satisfy. Describe the Geary-Khamis method of estimating PPP.

[4+5+5+ (3+8) =25]

2 (a) State and explain the properties of the Marshallian demand functions.
   (b) Diagramatically show the interrelationship among the demand functions, cost and indirect utility functions.
   (b) Show that for the Linear Expenditure System (LES), all commodities are price inelastic, that is, $|\gamma_{ii}| < 1$, γ_{ii} being the non-compensated own price elasticity of item *i*. State the assumptions clearly.
   (c) Show that for LES, the non-compensated own price elasticities are approximately proportional to the corresponding expenditure elasticities.

[6+3+8+8 = 25]

3. (a) Consider the following logarithmic form of the cost function *C(u, p)*:

$$\log C(u, p) = a_0 + \sum_{j=1}^{n} a_j \log p_j + \frac{1}{2}\sum_j \sum_k c_{jk}^{*} \log p_j \log p_k + u b_0 \prod_j p_j^{b_j},$$

where notations have their usual meanings.

   (i) Derive the demand system in budget share form from the above cost function. What is this system called?
   (ii) Derive the conditions under which the demand system satisfies adding–up, homogeneity and symmetry properties.

   (b) What are 'exact' and 'consistent' aggregations in the context of consumer demand analysis? For each type of aggregation, give an example of a demand system conforming to the respective type.

[(6+12) +7 = 25]

4. (a) Define homothetic preferences. Show that under linear homogeneity of preferences, the cost function is of the form

$$C(u,p) = \alpha(p)u^*,$$

where $u^* = f(u)$ and $\alpha(p)$ is linearly homogeneous in p, the vector of prices.

(b) Show that any unitary demand system derived from maximization of a utility function subject to a budget constraint satisfies negativity of the Slutsky matrix.

(c) Describe the Ramsey-Samuelson-Diamond-Mirlees approach to the determination of optimal commodity tax rates.

[7+8+10 = 25]

5. (a) Distinguish between the 'Unitary approach' and the 'Collective approach' for specification of household preferences. What are the different types of collective approach? Describe.

(b) Starting with maximization of a Pareto weighted sum of household members' utility subject to an income constraint, demonstrate why Slutsky symmetry fails. Hence, specify a test for validity of the unitary framework. [Assume that the Pareto weight is given by $\mu(p, I), p$ being the prices and I, the income].

(c)

(i) What is Distribution Factor Independence (DFI)?

(ii) Consider a general collective model specified by the following household utility function:

$$W(C^1, C^2, C^3, .., C^J, G; z, d) = \sum_j \mu_j(p, I, d)u^j(C^1, C^2, C^3, .., C^J, G; z),$$

where C^j is the consumption vector of the j-th member, G denotes consumption of public goods, z is the vector of demographic characteristics,

Enumerate the different special cases by imposing restrictions on $\mu_j(.)$. For each case specify the type of model and state whether Slutsky symmetry/ DFI is satisfied or not.

[6+8+ (3+8) = 25]

6. Write short notes on **any two**:

(a) The "Jackknife" and "Bootstrap" methods of resampling.

(b) Application of complete demand models to demand projection.

(c) Propensity Score Matching (PSM) and its applicability.

[25]

Indian Statistical Institute

MSQE II

Economic Development

Final Examination, 2019

Date: 26.4.19

Maximum Marks: 60

Time: 3 hours

Answer question 1 and any two from the remaining.

1. A borrower has a project of producing handicrafts and selling them to the international market. The international market requires that the quality of the products meets a certain pre-specified minimum standard denoted by $\hat{s}$. The actual quality $s = e + \varepsilon$ where e is the level of effort chosen by the borrower and ε is a random shock with $\varepsilon \sim N(0, \sigma^2)$. If the actual quality falls short of the pre-specified minimum standard, then the consignment is rejected and the borrower gets nothing. If, on the other hand, the minimum standard is met, the borrower can sell his products and earn a fixed revenue R. The borrower has to incur a cost $C(e), C' > 0, C'' > 0$ for expending effort. The borrower needs to borrow rupee 1 for his project from a lender who is competitive and has an opportunity cost ρ for each rupee lent. If the borrower can sell his products he has to repay a gross amount r. He pays nothing otherwise. All agents are risk neutral.

   (i) Set up a model to show how the effort level e, the probability of success $p(e)$ and the repayment amount r are determined.
   (ii) What happens to $e, p(e)$ and r if there is an increase in $\hat{s}$?
   (iii) What happens to $e, p(e)$ and r if there is an increase in R?

   [Assume that $F'' > 0$ around equilibrium where F is the distribution function of ε.]

   (8+6+6=20)

2. Setting up a suitable model, show how reciprocity can act as the basis of an informal insurance arrangement between two players with uncertain income streams which are (ex ante) identical, independent and infinite. What happens if the horizon is finite, that is, there is a period T after which both players die?

   (15+5=20)

3. Consider an agricultural commodity whose output is seasonal and demand is continuous. A small number of oligopolistic traders control the market. Show that the degree of price rise varies inversely with the degree of oligopoly of the market. Find the competitive sales path and show that it is socially optimal.

(10+10=20)

4. If assets are illiquid and can be pledged in the loan market for less than their actual values, show that the first best effort on the part of the borrower may be unattainable. Why is this problem especially relevant for less developed countries?

(17+3=20)

INDIAN STATISTICAL INSTITUTE

Final-Semestral Examination : 2018-19

Course Name: M.S. (Q.E.) II YEAR
Subject Name: The Theory of Mechanism Design
Date: 26.4.19 Maximum Marks: 50 Duration: 3 hours

1. Present the matching model with couples, where couples have non-separable preferences for togetherness. What can you say about the existence of stable matchings in such models. What happens if couples' preferences are separable? Justify your answer.

(10+5)

2. Consider the situation where there are exactly three objects in A and two agents in N. Suppose that each agent $i \in N$ has a normalized utility function $u_i : A \to [0, 1]$, that is, the maximum utility is always 1 and the minimum utility is always 0. Let U_i be the set of all such utility functions for $i \in N$ and let $U_N = U_1 \times U_2$. A mechanism (f, p) is individually rational if the net utility at every utility profile is non-negative for each agent, that is, $u_i(f(u_N)) - p_i(u_N) \geq 0$ for all $u_N \in U_N$ and all $i \in N$. For a mechanism (f, p), the expected revenue with respect to the uniform prior is defined as the expected value of $p_1(u_N) + p_2(u_N)$ with respect to the uniform probability distribution over U_N.

   (a) Find the set of all incentive compatible and individually rational mechanisms.

   (b) Identify the mechanism from the set of all individually rational and incentive compatible mechanisms for which the expected revenue with respect to the uniform prior is maximized.

(10+10)

3. Consider the (maximal) single-peaked domain $\mathcal{D}$ over the alternatives $\{a_1, \dots, a_3\}$. Let $U(\mathcal{D})$ be the set of all utility representations of the preferences in $\mathcal{D}$. Suppose there are two agents. What affine maximizers

are incentive compatible on $U^2(\mathcal{D})$? Does there exist an incentive compatible mechanism $\mu = (f, p)$ on $U^2(\mathcal{D})$ such that μ is not an affine maximizer and f is not a min-max rule? Justify your answer.

(10+5)

Indian Statistical Institute

Mid Semestral Examination: (2018 – 2019)

M.S. (QE) – II year

Econometric Applications II

Date: 25.02.2019 | *Maximum Marks – 50* | *Duration: 2 hours*

Answer any two questions

1. (a) Suppose you are required to find the distribution of income from a given set of income data on n (large) individuals, given by $x = \{x_1, x_2, x_3 \ldots . x_n\}$. Let $p(a)$ denote the true density evaluated at point 'a' and $p_{h_n}(a)$ denote the estimate of $p(a)$ based on bandwidth h_n. State the underlying conditions for $p_{h_n}(a)$ to converge to $p(a)$.

   (b) Explain how the Kernel density estimation procedure can be viewed as an extension of the concept of Histogram.

   (c) Show that $lim_{h_n \to 0} p_{h_n}(a) = p(a)$.

   (d) When will you use a "nearest neighbour approach" to estimate density? Describe the steps involved in estimating density by k-nearest neighbour (k-nn) method.

[6 + 6 + 8 + 5 = 25]

2. (a) Explain the concept of kernel regression.

   (b) Derive the Nadaraya–Watson kernel regression estimator for the following budget share function of the *h-th* household.

$$w_h = m(\log x_h) + \varepsilon_h, \quad h = 1,2,\ldots,H$$

   (c) Describe the k-nearest neighbour (k-nn) regression estimator.

   (d) Given the following data find the k-nn estimate for $x=4.5$ and $k=3$.

x	2.5	8.5	4	3.5	6	1	9.5	5	7
y	5	12	1	7	5	6	7	2	8

[4 + 8 + 4 + 9 = 25]

3. (a) What is 'selection bias'? What are the sources of such bias?

(b) Give an example of a bivariate sample selection model with a participation equation and an outcome equation. How would you estimate the parameters using Heckman's procedure? State the underlying assumptions clearly.

(c) Given a partial linear model of the form $y = z\beta + f(x) + \epsilon$, where z and x are scalars, how would you estimate β and $f(.)$?

[6 + 12 + 7 =25]

INDIAN STATISTICAL INSTITUTE

End-Semestral Examination: 2018-19

Course: MSQE Year II

Subject: Selected Topics III: Economics of Conflict

Date: 29/4/2019; Afternoon **Maximum Marks:** 60 **Duration:** 3 hours

Answer all questions. Students may consult their own notes or study material, but not one another. Calculators are not allowed.

1. Consider a society with two income levels: $I_H > I_L > 0$. Average income in this society is $\bar{I}$, total population is n and income class $i \in \{H, L\}$ has n_i members, with $n_L \equiv \rho n$, $\rho \in (0,1)$. Individuals have identical preferences, represented by the utility function $\ln x + \ln y$, where x is the amount of a private good consumed by the individual and y is the total amount of a public good generated by voluntary contributions; x, y > 0. All prices are unity. Define $I_H \equiv \bar{I}(1+\theta), \theta > 0$.

(i) Find the range of values of θ for which individual consumption bundles in the Cournot-Nash equilibrium are independent of the income distribution. Show how this range changes with a change in: (a) the total population (n), and (b) the population distribution (ρ). Explain your results. **(15 marks)**

(ii) Now (a) find the real incomes of both income classes; (b) find the real impact of a balanced budget redistribution proposal to increase all P nominal incomes marginally, and (c) show how this magnitude changes with increases in θ and ρ. **(15 marks)**

2. Consider a linguistic minority (N) of size $n \in (0, \frac{1}{2})$. This community's cost of assimilation to majority (M) linguistic norms is uniformly distributed in the $[0, k]$ interval; $k \in (0,1)$. For any minority individual i, the return from adopting M's linguistic norms is $\theta_M(1 - c_i)$, where θ_M is the proportion of the population that adopts M norms, and c_i is that individual's cost of assimilation. For such an individual, the return from persisting with N norms is $(1 - \theta_M)$.

(a) Suppose each community comes to be led by a warlord who allocates a fixed proportion of the community's total income, b, to consumption of each community member, and the remaining proportion $(1 - bs_j)$ to war against the other community; where s_j is the size of community j. Let D be the share of military expenditure in total income of the society. What happens to D as the size of the minority population, n, increases? Assume $b > \frac{k}{2}$ and that the size of the majority population is $(1 - n)$. **(15 marks)**

(b) Now suppose the state can develop and impose a third set of cultural-linguistic norms, with inputs from both communities. Thus, for community N, the cost of assimilating to this set of norms is uniformly distributed in $[0, kt]$, while it is uniformly distributed in $[0, k(1-t)]$ for community M, where $k \in (0,1), t \in [0,1]$. Thus, a higher value of t implies that the state's official norms are closer to those of the majority. Find the value of t that maximizes total income in society. How does this value change with changes in the size of the minority? **(15 marks)**

INDIAN STATISTICAL INSTITUTE
Semestral Examination: 2018-2019
M.S. (Q.E.) II Year
Sample Survey: Theory and Practice

Date: 30/4/2019 Duration 3 hours

The question paper has two parts, viz., Part A and Part B. You can answer any part of any question from Part A. But the maximum that you can score from Part A is 50. Part A carries a weightage of 20 %; and Part B carries 80 % weightage. All notations are self-explanatory.

Marks allotted to each question are given within parentheses.

Part A

1. Consider a pond with N (unknown) number of fish. Propose an appropriate sampling scheme to estimate N and find the variance of your proposed estimator. [7+8=15]

2. What are the advantages of the sampling method?. What is a sampling frame? [5+5=10]

3. Write a short note on non-sampling errors. [10]

4. Suppose that the population of size N (assuming that $N = nk$, n and k being integers) into n strata where the stratum h contains units with labels

$$G_h = \{(h-1)k + j,\ j = 1,2,..,k\},\ h = 1,2,..,n$$

and one unit is selected from each stratum randomly to get a sample of size n. The population values are modelled by the relation,

$$Y_i = a + bi,\ i = 1,..,N$$

where a and b are constants. Find the variance of the population total. [25]

Part B

1. **Project evaluation through presentation.** [62.5]

INDIAN STATISTICAL INSTITUTE
Second Semester Examination : 2018-19
M.S.(QE) II YEAR
Mathematical Programming

Date: 03 May 2019 Maximum Marks: 70 Duration: 3 hours

This paper carries 80 marks. Answer as much as you can.
However, maximum you can score is 70. Notations have usual meaning.

---

1 We are given a set of potential depot-locations (sites) $M = \{1, 2, \ldots, m\}$ and a set $N = \{1, 2, \ldots, n\}$ of clients. Opening a depot at site i involves a fixed cost f_i. Serving client j by a depot at location i costs c_{ij} units of money. The goal is to decide at which locations to open the depots and how to serve all the clients so as to minimize the overall cost. Give a mathematical formulation of the problem.

[10]

2 State balanced transportation problem in the form of mathematical programming problem. Show that it has an optimal solution.

[3+10 = 13]

3 Consider a network with the five nodes $\{s, 1, 2, 3, t\}$, where s and t are source and sink respectively. The set of directed arcs, on the network, along with respective capacity and existing flow are given as follows.

Sr. No.	Arc	(Capacity, Flow)
1	$(s, 1)$	(4,4)
2	$(s, 2)$	(4,4)
3	$(s, 3)$	(2,2)
4	$(1, 2)$	(8,0)
5	$(1, t)$	(6,4)

Sr. No.	Arc	(Capacity, Flow)
6	$(2, s)$	(6,2)
7	$(2, t)$	(4,4)
8	$(3, 2)$	(2,2)
9	$(3, t)$	(4,0)
10	$(t, 2)$	(4,0)

Draw the network. Starting with the existing arc flows, solve the *maximal flow problem* on this network. Hence or otherwise, identify a minimal cut.

[5+8+2 = 15]

4 Consider the problem: $\min f(\mathbf{x}) = 100\,(x_2 - x_1^2)^2 + (1 - x_1)^2$, $\mathbf{x} \in R^2$.

(a) Compute the gradient vector $\nabla f(\mathbf{x})$ and Hessian matrix $H(\mathbf{x})$.

(b) Find an optimal solution of the problem.

[(2+5)+5 = 12]

[P.T.O.]

5 Consider the mathematical formulation below, where x_j is the amount of product P_j for $j = 1, \ldots, 4$ to be manufactured over a month. The objective is to maximize total profit (in \$) by way of selling P_1, P_2, P_3 and P_4. Each of these products requires processing in two workshops (WS1 and WS2). The constraints describe that 400 hours of processing time is available in each of WS1 and WS2 during the period.

$$\max \quad z = 4x_1 + 6x_2 + 10x_3 + 9x_4$$

subject to

$$\begin{aligned} 3x_1 + 4x_2 + 8x_3 + 6x_4 &\leq 400 \\ 6x_1 + 2x_2 + 5x_3 + 8x_4 &\leq 400 \\ x_1, x_2, x_3, x_4 &\geq 0. \end{aligned}$$

The final (optimal) simplex tableau, for the corresponding minimization problem, is presented below, where x_5 and x_6 are slack variables.

	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	$-\frac{1}{2}$	0	-2	0	$-\frac{3}{2}$	0	-600
x_2	$\frac{3}{4}$	1	2	$\frac{3}{2}$	$\frac{1}{4}$	0	100
x_6	$\frac{9}{2}$	0	1	5	$-\frac{1}{2}$	1	200

Answer the following with regard to the original (maximization) problem.

(a) Write down the optimal basis, and identify its inverse from the tableau.

(b) Read from above the optimal solution and objective value.

(c) What is the maximum amount of money (in \$) you would spend to augment per hour of processing time in each of the workshops? Justify.

(d) Find the range of profit margin of P_1 for which the current basis remains optimal?

[(1+2)+(1+1)+5+5 = 15]

6 Consider the problem:

$$\min \; e^{-(x_1+x_2)}$$

subject to

$$\begin{aligned} e^{x_1} + e^{x_2} &\leq 20 \\ x_1 &\geq 0. \end{aligned}$$

(a) Give a feasible solution of the problem. Is it a regular point? Justify.

(b) Write down the KKT conditions for the problem.

(c) Find KKT point, if any, for the case $A(\mathbf{x}) = \{1\}$, where $\mathbf{x}$ is a feasible solution.

[(1+3)+5+6 = 15]

——*** xXx ***——

Indian Statistical Institute
Second Semestral Examination: 2018 –2019
M.S. (Q.E) – II Year
Econometric Applications II

Back Paper Examination

Date: 08.07.19 Maximum Marks: **100** Duration: 3 hrs.

(Answer all questions)

1. (a) Discuss the idea of Propensity Score Matching (PSM) and its applicability. Describe the different matching procedures.
   (b) What is 'sample selection bias'? Give an example of a bivariate sample selection model with a participation and an outcome equation. How would you estimate the parameters using Heckman's procedure?
   (c) How is the model in (b) different from standard Tobit model?

[10+12+3 = 25]

2. (a) Consider the following logarithmic form of the cost function *C(u, p)*:

$$\log C(u,p) = a_0 + \sum_{j=1}^{n} a_j \log p_j + \frac{1}{2}\sum_j \sum_k c_{jk}^* \log p_j \log p_k + u b_0 \prod_j p_j^{b_j},$$

where notations have their usual meanings.

   (i) Derive the demand system in budget share form from the above cost function. What is this system called?
   (ii) Derive the conditions under which the demand system satisfies adding–up, homogeneity and symmetry properties.

   (b) What are 'exact' and 'consistent' aggregations in the context of consumer demand analysis? For each type of aggregation, give an example of a demand system conforming to the respective type.

[(6+12) +7 = 25]

3. (a) Explain the idea of resampling. What are the possible situations when this may be needed?
   (b) Describe the "Jackknife" and "Bootstrap" methods of resampling.
   (c) Show that under certain condition the bias in the Jackknife estimator is zero. State the condition clearly.

[5+10+10=25]

4. (a) Distinguish between the 'Unitary approach' and the 'Collective approach' for specification of household preferences. What are the different types of collective approach? Describe.

(b) Starting with maximization of a Pareto weighted sum of household members' utility subject to an income constraint, demonstrate why Slutsky symmetry fails. Hence, specify a test for validity of the unitary framework. [Assume that the Pareto weight is given by $\mu(p, I)$, p being the prices and I, the income].

(c)

(i) What is Distribution Factor Independence (DFI)?

(ii) Consider a general collective model specified by the following household utility function:

$$W(C^1, C^2, C^3, .., C^J, G; z, d) = \sum_j \mu_j (p, I, d) u^j (C^1, C^2, C^3, .., C^J, G; z),$$

where C^j is the consumption vector of the j-th member, G denotes consumption of public goods, z is the vector of demographic characteristics,

Enumerate the different special cases by imposing restrictions on $\mu_j(.)$. For each case specify the type of model and state whether Slutsky symmetry/ DFI is satisfied or not.

[6+8+ (3+8) = 25]

INDIAN STATISTICAL INSTITUTE

Back Paper: 2018-19

Course: MSQE Year II

Subject: Selected Topics III: Economics of Conflict

Date: 08-07-19 **Maximum Marks:** 100 **Duration:** 3 hours

ANSWER ALL QUESTIONS

1. Using the model of competitive wage determination with worker-side moral hazard and costly monitoring discussed in class, identify how, and the exact sense in which, technology choice may turn out to be socially inefficient under capitalist control of the workplace. **(30)**

2. Using the model due to Dasgupta and Kanbur discussed in class, explain why greater philanthropy by the rich may exacerbate class antagonism against them. **(30)**

3. Develop a model to explain why relative income equality across groups and the presence of meta-communal identities may *both* be necessary to hold communal tensions in check. **(40)**

INDIAN STATISTICAL INSTITUTE

Back-paper Examination: 2018-19

MSQE II 2018-19

Selected Topics II: Auction Theory

Date: 09.07.19 2019 Maximum marks: 100 Duration: 3 hours

Answer all questions. Please answer parts A and B in separate scripts.

Part A

1. a) Explain the relations between multiple object Dutch auction and discriminatory price auction, and multiple object English auction and uniform price auction. Is there any relation between the multiple object English auction and the Vickrey auction? If yes, under what conditions? [2 + 2 + 4 + 2]

b) Suppose there are 5 bidders and 8 units up for sale. The submitted bid vectors are $b^1 = (50, 47, 40, 32, 27, 22, 15, 5)$, $b^2 = (45, 42, 38, 31, 23, 18, 10, 7)$, $b^3 = (48, 44, 41, 35, 26, 20, 14, 6)$, $b^4 = (55, 46, 37, 30, 24, 19, 16, 8)$, $b^5 = (65, 56, 50, 43, 34, 25, 17, 9)$. Identify the winning bids. What will be the payments of the winning bidders under respectively the discriminatory, unifrom price and Vickrey auctions? [4 + 2 + 2 + 2]

c) Consider a situation involving 2 bidders and 3 objects. Both bidders have demand for all 3 objects. Each bidder's value vector $X^i = (X_1^i, X_2^i, X_3^i)$ is independently and identically distributed on the set $\chi = \{\mathbf{x} \in [0, 1]^3 : x_1 \geq x_2 \geq x_3\}$ according to a density function f. Suppose the marginal distributions are denoted by F_1, F_2 and F_3 respectively. Formulate the payoff function for an individual bidder and show that it is optimal to shade the bid for the second unit onward in symmetric equilibrium. [10]

Part B

2. Consider the common values model, and assume a first-price auction.

a) Characterize symmetric, increasing, differentiable equilibrium bid functions, assuming one exists, and show there is a unique such function. [30]

b) Show that equilibrium exists. [10]

3. Assume, following Burguet and Sákovics (1999), that there are two sellers, each with a single unit of a homogeneous good for sale. They each have 0 value for the objects and wish to sell using a second price auction. There are N potential buyers, each capable of consuming at most 1 unit. Buyer valuations for any object are i.i.d. draws from $[0, 1]$ according to the uniform distribution, and private information. All parties are risk neutral and a buyer can attend at most one auction. Suppose seller i has set reserve price r_i, with $0 \leq r_1 \leq r_2 < 1$. The buyers now have to simultaneously decide which auction to enter, if any, and what to bid in an auction conditional on entry. Rigorously establish a symmetric equilibirum in this buyers' subgame: what does a buyer do as a function of her valuation, and why? [30]

Indian Statistical Institute

MSQE II

Economic Development

Back Paper Examination, 2019

Date: 10/07/2019

Maximum Marks: 100

Time: 3 hours

Answer all questions

1. Consider an informal insurance arrangement between two individuals A and B. Each individual has an uncertain income stream over an infinite horizon. In particular, for each person, in each period, income can take a value $y > 0$ with probability p $(0 < p < 1)$ and a value 0 with probability $(1-p)$. Every period, each person has a strictly increasing and concave utility function $U(y)$ with $U(0) = 0$. Also the individuals have a common discount rate $r > 0$.

   (a) Define the first best insurance contract and show that it is implementable if the rate of discount is sufficiently small.

   (b) Show that no insurance contract is implementable if the agents are risk neutral having a common utility function $U(y) = y$. Find the intuition behind the result.

   [12 + 13=25]

2. Give Marshall's argument as to why share tenancy might be an inefficient arrangement. Justify the existence of share tenancy as an optimal contract when the work effort of the tenant cannot be observed by the landlord.

   [10+20=25]

PTO

3. Show, in terms of a suitable model, that imperfect property rights might impede pledged collateral to mitigate moral hazard problems in a credit market. What kind of government intervention would be necessary in this context?

[20+5=25]

4. Consider an economy with a large number of agents distributed according to their inheritances X. Each agent lives for two periods. In period 1, an agent has the option of investing in education which involves a fixed expenditure h. Moreover, if an agent decides to undertake education, she has to forego work in period 1. However, mere expenditure on education does not guarantee that the agent successfully completes her education. She has to put in an effort denoted by e where $e \in [0,1]$. The cost of effort is $C(e) = \frac{1}{2}e^2$. The probability that the agent successfully completes her education is p where $p = e$. If the agent is successful in completing her education, then in the second period she earns an income normalized to be unity. If she is unsuccessful, then in the second period she earns $\alpha < \frac{1}{2}$. Again, the agent has the option of not undertaking education at all, in which case, she earns α in both periods. Finally, borrowing and lending are possible at exogenous rates of interest with the borrowing rate $i > r$, the lending rate. If an agent invests in education with borrowed funds, she pays back the principal with interest only if she is successful. In case of a failure, she pays nothing to the lender. The objective of an agent is to maximize her expected life time income. Her decision involves a choice of either undertaking education or not undertaking education and if the former option is chosen, then the agent also chooses how much effort she will put in.

(i) Assuming that all agents choose to undertake education, show how the effort choice changes with the level inheritance, that is, find $e = f(X)$.

(ii) For different levels of inheritance, determine whether an agent chooses education or not.

(iii) Now suppose that while the lending rate r is given exogenously (say, by the international market), the lending agencies (banks) can observe the inheritance level of each borrower and accordingly charge different borrowing rates to different borrowers. The banks are risk neutral, make zero expected profits and their opportunity cost of lending is r. Answer questions (i) and (ii) in the new set up.

(iv) Finally modify the set up described in (iii) by assuming that while banks can choose the borrowing rate, they cannot observe the inheritance level X of any individual borrower. However, the banks know the distribution of X. Answer questions (i) and (ii) in this modified set up.

[7+6+6+6=25]

INDIAN STATISTICAL INSTITUTE
Second Semester Examination : 2018-19
M.S.(QE) II YEAR
Mathematical Programming
Back Paper

Date: 11 · 07 · 2019 Maximum Marks: 100 Duration: 3 hours

Notations have usual meaning.

---

1 Consider a company that manufactures three products (A, B and C), each of which requires two types of resources (Machinery X and Machinery Y). Technology and resource capacity restrictions are given in the following table. For example, each unit of product A requires 7 units (say, of time) of Machinery X and 2 units of Machinery Y, and yields a profit of p_A.

Resource type (Machinery)	Amount of resources required to produce per unit of product			Current Available Capacity
	A	B	C	
X	7	3	1	≤ 28
Y	2	4	6	≤ 19
Unit Profit	p_A	p_B	p_C	

The company is considering capacity expansion by addition of new equipment. If the management decides to expand capacity of Machinery X, it must choose between adding either 5 or 15 new units of capacity, with an associated investment expense of 50 and 80, respectively. Similarly, if the management expands capacity of Machinery Y, the choices are 12 and 32 additional units of capacity, with investment costs of 30 and 90, respectively. However, the total investment expense must not exceed 150. The company is interested in the determination of production plan that maximizes its profit through consideration of possible capacity expansion.

(a) Formulate the above as an optimization problem.

(b) In addition, how do you modify your formulation under the following two different situations:

(i) the company does not want to expand capacity of Machinery Y unless capacity of Machinery X is increased.

(ii) the company does not want to expand capacity of Machinery Y unless it decides to increase capacity of Machinery X by 15 units.

[15+(3+2) = 20]

[P.T.O.]

2 Solve the following linear programming problem:

$$\min \ -5x_1 + 6x_2 - 3x_3 + 5x_4 - 12x_5$$

subject to

$$x_1 + 3x_2 + 5x_3 + 6x_4 + 3x_5 \le 90$$
$$x_1, x_2, x_3, x_4, x_5 \ge 0.$$

[10]

3 Consider the following linear programming problems:

$$\max \quad z = c^{\mathrm{T}}x$$
$$\text{subject to } Ax \le b$$

and

$$\min \quad z = c^{\mathrm{T}}x$$
$$\text{subject to } Ax \ge b.$$

Suppose that both these problems are feasible.

(a) Show that if one of these problems has a finite optimal solution then so does the other.

(b) Show that the first problem is unbounded if and only if the second problem is unbounded.

[8+7 = 15]

4 Consider a network with seven nodes: $\{s, 1, 2, 3, 4, 5, t\}$, and the set of directed arcs as follows:

Sr. No.	Arc	Arc Capacity
1	$(s, 1)$	6
2	$(s, 2)$	7
3	$(1, 2)$	1
4	$(1, 3)$	3
5	$(1, 4)$	4
6	$(2, 3)$	2

Sr. No.	Arc	Arc Capacity
7	$(2, 5)$	5
8	$(3, 4)$	3
9	$(3, 5)$	2
10	$(4, t)$	7
11	$(5, 4)$	2
12	$(5, t)$	4

Draw the network. Solve the *maximal flow problem* on this network.

[5+15 = 20]

[P.T.O.]

5 Show that the set of optimal solutions of convex programming problem is convex.

[10

6 Consider the problem:

$$\min\ x_1^2 - x_1x_2 + x_2^2 + x_3^2 - 2x_1 + 4x_2$$

subject to

$$\begin{aligned} -x_1 - x_2 &\leq 0 \\ 1 - x_3 &\leq 0. \end{aligned}$$

(a) Write down the KKT conditions for the problem.

(b) Find the KKT points.

(c) Is there a global optimal solution to the problem?

[7+13+5 = 25

———*** xXx ***———