Price vs Quantity: Essays On Strategic Choice in Differentiated Oligopoly

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May, 2019

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Thesis submitted to the Indian statistical Institute in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy

Acknowledgment

I shall forever be grateful to my supervisor, Professor Manipushpak Mitra for his constant guidance and support. I believe if there is anything interesting in this thesis, it is solely attributable to him. His motivation and encouragement, has helped me propel my inquisitiveness and understanding. He never failed to lend me a patient, listening ear whenever I approached him; specially during those moments of repeated doubt clarifications which were an interference to his busy schedule. He not only structured my logical thinking on the subject but also taught the nitty-gritties of writing a paper. It is an honour for me to get acquainted with his approach towards the subject.

I would like to express my deep appreciation and respect for Professor Arnab Chakraborty, who helped me to shape up my mathematical understanding. I am indebted to Professor Rupayan Pal and P. M. Sharada for agreeing to write the first paper (Chapter 2) along with me and my supervisor Professor Manipushpak Mitra. I am grateful to Professor Arghya Ghosh for repeated interaction on very diverse topics relating to industrial organization and for agreeing to write the third paper (Chapter 4) along with me and my supervisor Professor Manipushpak Mitra. I express my gratitude to Professor Satya Ranjan Chakravarty, Professor Indraneel Dasgupta, Professor Tarun Kabiraj, Professor Soumyanetra Munshi and Professor Souvik Roy for their helpful comments and suggestions.

I am very grateful to my seniors, classmates and juniors at Indian Statistical Institute. I thank Conan-da, Sattwik-da, Sandip-da, Srikanta-da, Kushal-da, Mridu Prabal-da, Chandril-da, Debojyoti-da, Parikshit-da, Gopakumar, Tanmoy-da, Mahamitra-da and many others for their help and encouragement. Special thanks to Sreoshi and Chayanika for carefully going through the thesis and suggesting many corrections. Special thanks to Pinaki for improving my understanding of many mathematical concepts. I am thankful to the office staff of Economic Research Unit, especially Satyajit-da and resourceful Swarup-da who were always there to render any help. Fellowship from the Indian Statistical Institute is gratefully acknowledged.

Contents

Chapter 1

Introduction

Industrial economists are often interested in comparing different market structures which are primarily based on their market outcomes and then try to determine the best market structure considering either the society's welfare or the firm's profit and sometimes considering both[1]. In this context, the "Cournot-Bertrand comparison" is one such important comparison that has often been analyzed in the literature of industrial economics. The main structural difference between Cournot competition and Bertrand competition arises due to the strategic variable through which firms interact with each other in the market. To be more specific, in case of Cournot competition, firms compete with quantities while under the Bertrand competition they compete with prices. The first study with differentiated products was made by Singh and Vives (1984). They conclude that under Cournot duopoly each firm in the industry produces less, charges more and earns higher profit than under Bertrand duopoly. Further, they argued that the latter is efficient than the former in terms of welfare ranking. We refer to these rankings as the standard rankings. Subsequent studies in this literature have mainly concentrated in determining the circumstances where these standard rankings are either partially reversed or fully reversed. One such contribution by Häckner (2000) shows that the standard rankings are dependent on the duopoly assumption and they get reversed under sufficient quality differences with increasing number of firms. How-

[1] In recent years a new notion of 'market quality' has emerged that generalizes the notion of social welfare where the key factors determining market quality are "quality of competition", "quality of information" and "quality of products" (see Yano (2009) and Dastidar (2017)).

ever they do not consider the welfare rankings between Cournot and Bertrand. Hsu and Wang (2005) conclude that the standard rankings hold in case of welfare with any number of firms. Amir and Jin (2001), have extended the "Cournot-Bertrand comparison" by including the following market indicators:- mark-up output ratio, average output, average price and Herfindahl index. Except for Singh and Vives (1984), the aforementioned studies deals with oligopoly market with linear demand. On the other hand, Vives (1985) and Okuguchi (1987) have worked with oligopoly markets assuming general non-linear demand functions. Subsequent studies by Mukherjee (2005) and Cellini et al. (2004) for free entry; Symeonidis (2003) and Lin and Saggi (2002) for endogenous Research & Development expenditure; López and Naylor (2004) for the wage bargaining provided evidence on partial reversal of the standard rankings. Arya et al. (2008b) and Aliprанti et al. (2014) have shown the complete reversal of the standard rankings with a vertically related producer along with Ghosh and Mitra (2009) who get the same with mixed market. One important contribution with homogeneous product is by Dastidar (1997) where it is established that the standard Bertrand-Cournot rankings are sensitive to the market sharing rules.[2]

So far the discussion has been primarily based on separate analysis of both Cournot and Bertrand competition. However, this separate analysis is rigid in the sense that the strategic variable through which firms compete in the market is exogenously given. But it may also be possible that firms can endogenously determine their strategic variables. Consequently, the equilibrium outcome may be one of the following: (i) only Cournot competition (ii) only Bertrand competition (iii) neither Cournot nor Bertrand competition. Hence, if the endogenous competition prevails, then the classic comparison between Cournot and Bertrand competition does not provide any meaningful results. This endogenous choice of strategic variables have been modeled by allowing firms to optimally determine the strategic variable through which they compete in the market before the market competition begins. Like "Cournot-Bertrand comparison" Singh and Vives (1984) have also made their first attempt towards the litera-

[2]It must also be mentioned in this context that there are papers that address the question on existence of Bertrand equilibrium with homogeneous commodites (see Dastidar (1995) and Dastidar (2011)).

ture on endogenous strategy choice. They conclude that with substitute (complement) goods, Cournot (Bertrand) competition is the unique equilibrium mode of competition. Cheng (1985) generalizes the result of Singh and Vives (1984) with a weaker set of assumptions using an elegant geometric approach. Boyer and Moreaux (1987) have extended the story of endogenous strategy choice from simultaneous interaction to sequential interaction between firms and conclude that the leader chooses quantity strategy but the follower is indifferent between choosing price strategy and quantity strategy in equilibrium. Tanaka (2001) extends the analysis of endogenous choice of strategic variables given vertical product differentiation and also identifies the conditions on the population distribution for which the result of Singh and Vives (1984) holds. In another study Tanaka (2001) extends the analysis from duopoly to oligopoly and has argued that the Cournot competition will always be an equilibrium mode of competition while the Bertrand competition will not. But the uniqueness of the equilibrium outcome remains unanswered. Further, Reisinger and Ressner (2009) have extended this analysis with profit maximizers under uncertain demand function and conclude that Cournot competition may or may not emerge as an equilibrium outcome.

Throughout the previous paragraph we have discussed different situation considering only profit maximizers. Other studies have considered the firms whose objectives are different from profit maximization and analyze the problem of endogenous strategy choice. One such study by Matsumura and Ogawa (2012) shows that the emergence of price competition has exogenously allowed co-existence of public and private firms. Chirco et al. (2014) extend this study with managerial delegation. Subsequent studies analysis of the endogenous strategy choice can be seen in the works by Chirco and Scrimitore (2013), Manasakis and Vlassis (2014).

This thesis is mainly focused on the above two topics and along with detail discussion of the framework. At the very outset the results of this study are as follows:

(A) Under a differentiated product duopoly where a regulated firm competes with a private firm and the instrument of regulation is the level of privatization the regulator first determines the level of privatization to maximize social welfare. Both firms then endogenously choose the mode of competition (that is, whether to compete in price or quantity) and finally, the two firms compete in the market. Under a very general demand specification, we show that when the products are imperfect substitutes (complements), there is a co-existence of private and public (strictly partially privatized) firms. Moreover, in the second stage, the firms compete in prices.

(B) In a vertically related differentiated product oligopolistic industry where a single vertically integrated firm supplies a key input not only to its own downstream division but also to all it's downstream rivals. The comparison between quantity competition and price competition in the downstream market with fixed numbers of firms reveals that the profit ranking of the downstream firms other than vertically integrated firm is sensitive to the size of competition that prevails as well as to the degree of product differentiation. Further, in both these cases the vertically integrated firm always compensates more for the loss in his profit due to competition increase than by selling input.

(C) Consider a differentiated product oligopoly market where both the price setting firms and quantity setting firms co-exist. Further each firm posses a conjectural variation (linear) about all the other firm's strategic variable. Suppose this conjecture is subject to evolutionary selection, then under evolutionary stable solution the equilibrium price, equilibrium quantity and resulting profits are all identical across firms. Further, this invariance result is true for all combinations of price-setting and quantity-setting firms.

1.1 Equilibrium co-existence of public and private firms and the plausibility of price competition

In Chapter 2 we incorporate the issue of privatization with the literature on endogenous strategy choice by considering a differentiated product duopoly where a regulated firm competes with a private firm and the instrument of regulation is the level of privatization. Firms interact through the following sequences. First, the regulator determines the level of privatization to maximize social welfare. Then, both firms endogenously choose the mode of competition (that is, whether to compete in price or quantity). Finally, the two firms compete in the market. The existing literature can be broadly classified into three groups:

(i) Papers where Stage 1 is absent and both firms are private, for example Singh and Vives (1984).

(ii) Papers where Stage 2 is absent, like Fujiwara (2007) where Cournot competition is assumed (and we have positive privatization level) and Ohnishi (2010) where Bertrand competition is assumed (and we have no privatization).

(iii) Papers where Stage 1 is absent but the market is mixed, like Matsumura and Ogawa (2012) who show that Bertrand competition will emerge regardless of the types of the goods (where one firm is assumed to be public).

The contribution of Chapter 2 is to develop a model that combines all the three stages, that is, to apply two stages of endogenization. The first stage endogenization is the objective function of the partially privatized firm and, like Singh and Vives (1984) and Cheng (1985), the second stage endogenization is price and quantity strategies. The first stage endogenization of adding positive weights on welfare in a firm's objective function seems natural in the context of partially privatized firms (see, for example, the papers in the mixed-oligopoly literature by Anderson et al. (1997), Ghosh and Mitra (2010), Ghosh and Mitra (2014), Matsumura (1998) and Matsumura and Ogawa (2012)). This literature focuses on mixed markets where both private and partially privatized (or public) firms coexist. In the early stages of industrialization of developing economies, there is often an upper bound on the extent of private ownership. When a foreign firm tries to enter a domestic market, the government can ask the foreign firm to pursue an objective different from profit maximization that includes Corporate Social Responsibility (for example, taking initiative to assess and take responsibility for the company's effects on the environment and impact on social welfare). If we assume that the government cares about social welfare and private firms' care about profit, then it seems plausible to assume that the partially privatized firm maximize a weighted combination of profit and welfare. Therefore, objectives different from profit are quite important and prevalent in the industrial organization literature. A paper with a very general objective function that allows for altruism and informational asymmetry is by Heifetz et al. (2007). However, Heifetz et al. (2007) do not allow for either privatization based enodogeneity (like Stage 1 of our three stage game) or price-quantity based endogeneity (like Stage 2 of our three stage game). Even when we have fully privatized firms, we know from the managerial-delegation literature that managers maximize a weighted combination of profit and quantity/revenue/welfare and it is compatible with profit maximization (see Fershtman and Judd (1987), Miller and Pazgal (2001), Sklivas (1987) and Vickers (1985)).

With quadratic utility function there is a growing literature that studies the coexistence of partially privatized firm and a private firm in a differentiated product market. With quadratic utility, only Stage 1 endogeneity like ours was addressed by Fujiwara (2007) and by Ohnishi (2010). In Fujiwara (2007), it is argued that under Cournot competition it is optimal to choose a positive weight ($\theta > 0$) for the partially privatized firm. In Ohnishi (2010), it was argued that under Bertrand competition it is optimal to choose zero weight ($\theta = 0$) for the partially privatized firm. Our analysis shows that, in general, if we also endogenize mode of competition along with privatization ratio, then Cournot competition (Fujiwara (2007)'s analysis) is never achieved as an equilibrium outcome. With quadratic utility, only Stage 2 endogeneity like ours was addressed by Matsumura and Ogawa (2012) with an added assumption that one firm is fully public (that is, no privatization is exogenously fixed at 0). Matsumura and Ogawa (2012) argued that Bertrand competition is the SPNE of the two stage game regardless of whether goods are substitutes or complements. We show that Bertrand competition is the SPNE of the three stage game, which allows for endogenous determination of the level of privatization. Moreover, our results hold for a very general demand specification.

De Fraja and Delbono (1989) show that, in homogeneous goods Cournot oligopoly with decreasing returns to scale technology, coexistence of a fully public firm with one or more private firms results in lower social welfare compared to that in oligopoly with only private firms. However, full privatization of the public firm is not socially desirable either; instead partial privatization of the public firm is socially optimal (see Matsumura (1998)). These results hold true in the case of differentiated products mixed oligopoly with constant returns to scale technology as well (see Fujiwara (2007)). That is, when firms compete in quantities, it is inefficient to have a fully public firm in the industry and this inefficiency in mixed oligopoly can be mitigated by partially privatizing the public firm. On the other hand, when firms compete in prices, coexistence of a fully public firm with one or more private firms is socially desirable and, thus, privatization (partial or full) of the public firm looses its appeal under price competition (see Anderson et al. (1997); Sanjo (2009); Ohnishi (2010)), unless goods are complements (see Ohnishi (2011)). This paper shows that, the level of privatization of the public firm has important consequences on the nature of product market competition and when firms can choose the mode of product market competition, coexistence of a fully public firm with one or more private firms is socially optimal, except in case of complementary goods. That is, optimality of partial privatization cannot be sustained when the nature of product market competition is endogenously determined when the goods are imperfect substitute. We further show (in Section 5) that this result can be valid even when the public firm is relatively inefficient (but not "too" inefficient) compared to its private counterparts.

1.2 Bertrand-Cournot comparison for oligopolistic industry with vertically integrated firm

In Chapter 3 we revisit the classic comparison of Cournot competition and Bertrand competition in a vertically related differentiated oligopolistic industry. The industry consist of a vertically integrated firm and n downstream firms. Production of final commodity requires a key input on a one-to-one basis. In case of production of key inputs upstream division of vertically integrated firm has monopoly. The upstream division of vertically integrated firm not only supplies the necessary key input to its own downstream division directly but also to all it's other downstream rivals through the upstream market. This upstream interaction is followed by all firm competing in the downstream market. In case of downstream competition, we separately analyze both Cournot and Bertrand competitions.

As we have already mentioned, in case of differentiated duopoly, Singh and Vives (1984) makes their first attempt to compare the equilibrium outcome of Cournot and Bertrand competition and we call the ranking between Cournot competition and Bertrand competition they provided as the standard rankings. Thereafter, with vertically related market structure Arya et al. (2008b) show that this standard ranking gets reversed. Our study is in the spirit of Arya et al. (2008b) but in an oligopolistic framework. With this vertically related structure we show that the ranking of vertically integrated firm's profit and welfare is robust with oligopolistic market but the ranking of the profit of all downstream firms (other than vertically integrated firm) is sensitive to the prevailing degree of competition. We also show that as the degree of competition increases, there is a loss in profit for the vertically integrated firm due to new entry and there is also a gain in profit for the vertically integrated firm due to increase in sales of input to these new entrants. However, the resulting aggregate profit of the vertically integrated firm goes up since the loss from the downstream market is less than the gain from the upstream market.

Though there are several papers on vertically related market, the literature specific to Cournot-Bertrand comparison in which one vertically integrated firm competes with other downstream firms is rare. Arya et al. (2008a) assume homogeneous products and quantity competition and analyze the firm's decision of either to produce the necessary input of production itself or to outsource it.[3] Allowing for product differentiation, but considering only the quantity competition Qing et al. (2017) analyze the capacity allocation problem of a monopolist supplier under bargaining. Arya and Mittendorf (2013b) analyze the role of discriminatory discloser; Mukherjee and Zanchettin (2007) study the vertical integration and product innovation as independent strategic choice of vertically related firms and Constantatos and Pinopoulos (2016) discuss the choice of capacity of input vs choice of input price. On the other hand Kabiraj and Sinha (2016) establish the fact that under price competition with differentiated products it may be possible that outsourcing is optimal when in house production is relatively cheap. Alipranti et al. (2014) assume vertical separation and allow bargaining between input seller and output producer. They compare between the Cournot and Bertrand duopoly and conclude that the ranking of all key market outcomes provided by Singh and Vives (1984) get reversed. In this context we see a range of papers that compare the outcome of Cournot and Bertrand competitions.[4] We also have papers that not only study the Bertrand and Cournot duopoly but also consider general demand function (for instances see Aguelakakis and Yankelevich (2017), Moresi and Schwartz (2017) and Moresi and Schwartz). These papers allow endogenous choice of strategic contract (price strategy and quantity strategy). Lee et al. (2016), Rozanova (2017), Lee et al. (2016), Lee and Choi (2014), Chang et al. (2018). Tremblay et al. (2013)

[3]There are also a series of studies contributed to the vertically related market structure assuming homogeneous product and quantity competition such as Chen et al. (2011),Chen (2011),Chen and Sen (2015)

[4]For example Yang et al. (2015) analysis how the product substitutability and brand equality affect the equilibrium channel structure of manufacturers selling competing products; Lee and Oh (2014) make a comparison of the Cournot model and the Bertrand model in a vertically related duopoly market with asymmetric costs between downstream firm. By allowing endogenous R & D, Li and Ji (2010) revisit the argument on welfare effect of price and quantity competition in the presence of technology licensing; Arya and Mittendorf (2013a) discuss the role of partial forward integration on the strategic investment and show how things changes from price to quantity competition; Chen (2010) study the strategic outsourcing between rival but separately discuss quantity competition with homogeneous good and price competition with differentiated product; Lee and Choi (2016) consider upstream R & D investments; Polemis and Eleftheriou (2018) consider regulating upstream monopoly.

examine the Cournot and Bertrand mixed competition when advertising rotates the market demand and make a case study for Honda and Scion. Fanti and Scrimitore (2017) explore the role of managerial delegation when it influences downstream firms' incentives. They determine the endogenous choice of delegation under both Cournot and Bertrand in a market where a vertically integrated producer also supplies an essential input to it's retail rival. Allowing for quality choice Xiao et al. (2014), Miyamoto (2014) and Bourreau et al. (2007) discusses the issue of outsourcing in case of vertical product differentiation.

We also have studies that focus on the differentiated oligopolistic market structure with linear demand function. Assuming vertical separation, Pinopoulos (2011) consider the long run with free entry and show that input price depends on the down stream market structure, where as, without free entry input price is independent of the downstream competition. Our analysis differs from the analysis of Pinopoulos (2011) in two aspect: firstly we consider existence of vertically integrated firm rather than vertical separation; secondly, we compare price and quantity competition not only for input price but also for all other market outcomes such as industrial profit, welfare etc. Rossini and Vergari (2011) also focus on the oligopolistic industry but for quantity competition only. They compare input production joint venture (IPJV) and vertical integration (VI). Moreover, Rossini and Vergari (2011) also deals with doupolistic price competition but for duopoly. Assuming quantity competition Bourreau and Dogan (2012) analyze the free entry for broad band service.

1.3 A strong equivalence result with evolutionary stable conjectural variations

In Chapter 4 we determine the optimal strategy choice of oligopolistic firms under evolutionary stable conjectural variation (at the aggregate level). Conjectural variation defines, how any firm in the industry believes about the reaction of all it's other rivals, in terms of changing the value of their strategic variable due to change in the value of their own strategic variable. Moreover, we consider a differentiated oligopoly industry with any number of firms and any combination of price choosers and quantity choosers. We develop a model in which each firm's conjectural variation is regarded as it's type. The types of all the firms thus determined generates an aggregate function resulting from these conjectures and this aggregate realization is subject to evolutionary selection. Therefore, our model predicts the nature of long run interaction amongst the firms. Specifically, we try to answer the following questions: Which society of population will survive in the long run? What is the market outcome of this society? Does the strategic choice have any role to play in this market outcome?

Given this evolutionary stable selection of the aggregate conjecture we show that in each mode of competition the market outcomes are identical. That is, for each mode of competition each firm produces same output, charge same price and earns same profit. Moreover, this outcome is different from the standard Cournot and Bertrand outcome. Further, there is no role of the mode of competition in determining the market outcomes. Therefore, under evolutionary stable solution each mode of competition will mimic the symmetric modes of competition that lie on both ends.

The literature on conjectural variations mainly focuses on consistent conjectures (Bresnahan (1981)), where each firm rightly anticipates rival firms' reaction. Kamien and Schwartz (1983) show that Bertrand and Cournot outcomes are identical in a linear duopoly under consistent conjectures. In a similar setting, Müller and Normann (2005) show that consistent conjectures are also evolutionarily stable, that is, loosely speaking, conjectural variations that are implied by consistent conjectures constitute best response. Our model is in the spirit of Müller and Normann (2005) as we also focus on best responses in conjectural variations space though we go beyond duopoly. Possajennikov (2015), Possajennikov (2016) establish the evolutionary stability of the consistent conjectural variation with n players without endogenizing the strategy space. However, our model allows for co-existence of both price-setters and quantity-setters and thus, in our context, each firm needs to conjecture responses from rival price-setting firms as well as rival quantity-setting firms. In this chapter we also analyze the stability of evolutionary selection of aggregate conjectural variation. This is different from Dastidar (2000) and Tremblay and Tremblay (2011) where the stability issue for market outcomes were addressed.

Chapter 2

Equilibrium co-existence of public and private firms and the plausibility of price competition

2.1 Introduction

What happens if, instead of two profit maximizing firms, we consider a regulated firm and a profit maximizing firm in the duoploy market with differentiated product? Singh and Vives (1984) and Cheng (1985) considered a two-stage game for a differentiated product duopoly market where both firms are profit maximizers. In the first stage, the firms credibly announce to play in either quantity or price strategies. If the goods are substitutes (complements), then it is shown that quantity or Cournot (price or Bertrand) competition is the SPNE outcome of this two stage game (see Singh and Vives (1984) and Cheng (1985)). In this chapter we model the co-existence of a regulated firm and a profit maximizing firm and, in particular, we model the objective of the regulator and then (like Singh and Vives (1984)) allow the firms to decide on the mode of competition before competing in the market. In a static scenario this calls for a three stage game which to the best of our knowledge has not been done in the differentiated product literature.[1] Moreover, there are many papers that provide important results by assuming quadratic utility function or CES utility function of the representative consumer. We want to come out of this limitation as well and allow for more general demand specifications to provide our results with the three stage game.

The primary reason for this three-stage game stems from the fact that when the goods are imperfect substitutes, it is not always the case that we find profit maximizing firms operating in a market and competing in quantities (like the results in Singh and Vives (1984) and Cheng (1985) suggest). Objective different from profit maximization for imperfect substitutes is a special feature of many markets in many countries. Examples include the telecom sector, banking industry, airlines, postal services, health sector, and education sector (see for example Backx et al. (2002), Badertscher et al. (2013), Doganis (2005) and La Porta et al. (2002)). Even in developed countries we often find the co-existence of welfare maximizing public firm and profit maximizing private firms.[2] Therefore, one cannot deny the role of regulation in the differentiated products markets.[3]

Assuming a market where a private firm competes with a public firm, it was shown by Matsumura and Ogawa (2012) that, with quadratic utility function of the representative consumer, price (Bertrand) competition is the SPNE of the two stage game regardless of whether goods are substitutes or complements. Therefore, one cannot unambiguously confirm that quantity competition will always follow in a differentiated product market when at least one firm is not a profit maximizer. However, what guarantees the co-existence of public firm and private firm in a differentiated product market? This requires a more careful modeling of the regulatory instrument and it is also for this reason that our contribution is important.

[1] All the models in the existing literature either endogenize the mode of competition or endogenize the objective of the non-profit maximizing firm but not both. Hence, we only have two stage (and not three-stage) models in the existing literature.

[2] In case of China after early 1980s we have seen the coexistence of both public and private firms. For example, in the health sector in urban China we find such a co-existence. In case of USA and England, we find such a co-existence in both health and education sectors.

[3] In case of the aviation sector in India, Air India is a government regulated enterprise competing with other private enterprise such as Jet Airways, IndiGo etc. In the Indian banking sector there are nationalized (regulated) banks such as State Bank of India that competes with other private banks such as Axis Bank.

We first add an earlier (first) stage to the two-stage game of Singh and Vives (1984) and Cheng (1985). In the first stage, a (regulator) government decides how much weight the partially privatized firm must attach to its own profit and social welfare assuming that the competing firm is a profit maximizer. We show that in such a set-up, when the goods are substitutes we uniquely end up in the co-existence of welfare maximizing public firm and profit maximizing private firms, that is, no privatization Bertrand equilibrium is the SPNE outcome of this game where the government sets zero (full) weight to profit (social welfare) of the partially privatized firm and both firms compete in prices (that is, Bertrand competition). When the goods are complements we uniquely end up in an SPNE outcome which we call strictly partial privatization Bertrand equilibrium where, in Stage 1, the government adds non-zero weights to both Firm 1's own profit and social welfare and, in Stage 2, firms play price strategies.

The first stage regulatory instrument of the government is the weight θ (lying in the closed interval $[0, 1]$) attached to the profit of the partially privatized firm and the residual weight $(1 - \theta)$ attached to the welfare of the society. According to Vives (1985), when both firms are profit maximizers, then, with Cournot competition, there is less of a profit loss with price under-cutting than with Bertrand competition. However, when we have one partially privatized firm, then there exists a critical value of weight ($\theta \in (0, 1)$) such that for each weight below this critical weight, there exists a critical price of Firm 2 below which Vives (1985)'s argument holds and, more importantly, above this critical price the reverse argument holds, that is, with Bertrand competition there is less of a profit loss with price under-cutting than with Cournot competition. It is precisely this feature that drives our main result when the goods are substitutes.

Our results hold under very general demand specifications. Moreover, our results are true even when the quantity reaction functions transformed in the price space are non-monotonic. In particular, for substitute goods, our result hold under the set of assumptions made by Cheng (1985) and with an additional assumption on welfare which is general enough and was used in Ghosh and Mitra (2014). To prove our results we have at times made use of Cheng (1985)'s geometric approach and, to prove one lemma, we have also used the line integral techniques similar to the one used in Ghosh and Mitra (2010), Ghosh and Mitra (2014). Specifically, to find the exact cut-off weight (θ) for the optimal choice of mode of competition for Firm 2 changes we use line integral techniques and then we apply Cheng (1985)'s geometric approach to sequentially eliminate possibilities other than the price competition.

The chapter 2 is organized as follows. In Section 2.2, we introduce the basic framework, our assumptions with imperfect substitute goods and we explain the three stage game. In Section 2.3, we present our main theorem with imperfect substitutes. In Section 2.4, we present the result with complement goods. In Section 2.5, we address the robustness of our game with quadratic utility and we also address the issue of cost asymmetry. In Section 2.6 we provide our conclusions followed by an appendix section (Section 2.7) where we provide the proofs of all the results.

2.2 Preliminaries

We consider an economy with a competitive sector producing the numéraire good (money) y and with a imperfectly competitive sector where two firms operate. Each firm produces a differentiated good. For any firm $i \in \{1,2\}$, let p_i and q_i denote Firm i's price and quantity respectively. For convenience we define the following notations. Let $\Re_+$ represent the non-negative orthant of the real line $\Re$. For any $x = (x_1, x_2) \in \Re^2_+$ and any $y = (y_1, y_2) \in \Re^2_+$, $x \neq y$ means either $x_1 \neq y_1$ or $x_2 \neq y_2$, $x \geq y$ means $x_1 \geq y_1$ and $x_2 \geq y_2$, and, $x >> y$ means $x_1 > y_1$ and $x_2 > y_2$. We assume a representative consumer who maximizes $\mathcal{U}(q,y) := U(q) + y$ subject to $p_1q_1 + p_2q_2 + y \leq M$ where $q = (q_1, q_2) \geq (0,0)$, $p = (p_1, p_2) >> (0,0)$ and M denotes income of the representative consumer. For any function $G : \Re^2_+ \rightarrow \Re$, define for any $i \in \{1,2\}$, $\partial_i G(x) := (\partial G(x)/\partial x_i)$, $\partial_{ii} G(x) := (\partial^2 G(x)/\partial x_i^2)$ and for any $i, j \in \{1,2\}$ such that $i \neq j$, $\partial_{ij} G(x) := (\partial^2 G(x)/\partial x_j \partial x_i)$ and $\partial_{ij} G(x) = \partial_{ji} G(x)$.

ASSUMPTION 2.1 For $i, j = 1,2$ ($i \neq j$) and any $q >> (0,0)$, (i) $\partial_i U(q) > 0$, (ii)

$\partial_{ii}U(q) < 0$, (iii) $\partial_{ij}U(q) < 0$ and (iv) $|\partial_{ii}U(q)| > |\partial_{ij}U(q)|$.

Given $\mathcal{U}(q, y)$ is quasi-linear, there is no income effect and hence the representative consumer's optimization is to select q to maximize $U(q) - p_1q_1 - p_2q_2$. Utility maximization yields the inverse demand function $p_i = \partial_i U(q) := F_i^{QQ}(q)$ for all $q \geq (0, 0)$ and for each $i \in \{1, 2\}$. Using Assumption 2.1 it follows that $\partial_i F_i^{QQ}(q) = \partial_{ii}U(q) < 0$ and $\partial_j F_i^{QQ}(q) = \partial_{ij}U(q) < 0$ for $i \neq j$. From Assumption 2.1(iv) we know that the demand system is invertible. Therefore, given any price vector $p = (p_1, p_2) >> (0, 0)$, we get the direct demand function $q_i = F_i^{PP}(p)$ for each $i \in \{1, 2\}$. Let $|D| := \partial_{11}U(q)\partial_{22}U(q) - (\partial_{12}U(q))^2 > 0$. Given Assumption 2.1, it also follows that $\partial_i F_i^{PP}(p) = \partial_{jj}U(q)/|D| < 0$ and $\partial_j F_i^{PP}(p) = -(\partial_{ij}U(q)/|D|) > 0$ for $i, j \in \{1, 2\}$ with $i \neq j$. For any $i \in \{1, 2\}$, any quantity $q_i \geq 0$, the level set $Q_i(q_i) = \{p \mid p >> (0, 0), F_i^{QQ}(p) = q_i\}$ generates iso-quantity curve for Firm i in the price space. Due to Assumption 2.1, the slope of the iso-quantity curve at $q_i = \bar{q}_i$ is $\frac{dp_j}{dp_i} |_{\bar{q}_i} = -(\partial_i F_i^{PP}(p)/\partial_j F_i^{PP}(p)) > 0$. By Assumption 2.1, own effect dominates cross effect implying that Q_1 is steeper than Q_2 in the price space (see Cheng (1985)). We assume identical total cost of both the firms and it is given by $C(y) = my$ where $m > 0$ and $y \geq 0$. When both firms choose quantity as a strategic variable, profit of Firm i is given as $\pi_i^{QQ}(q) = (F_i^{QQ}(q) - m)q_i$ for $i, j = 1, 2$ with $i \neq j$. The profit function of Firm i when both chooses price as a strategic variable is given by $\pi_i^{PP}(p) = (p_i - m)F_i^{PP}(p)$ for all $i, j = 1, 2$ with $i \neq j$. The constant $m > 0$ is the marginal cost of production.

Assumption 2.2 For $i, j = 1, 2$ $(i \neq j)$ and any $q >> (0, 0)$, (i) $\partial_{ij}\pi_i^{QQ}(q) < 0$ and (ii) $\partial_{ii}\pi_i^{QQ}(q) + |\partial_{ij}\pi_i^{QQ}(q)| < 0$.

Assumption 2.3 For $i, j = 1, 2$ $(i \neq j)$ and any $p >> (0, 0)$, (i) $\partial_{ij}\pi_i^{PP}(p) > 0$ and (ii) $\partial_{ii}\pi_i^{PP}(p) + |\partial_{ij}\pi_i^{PP}(p)| < 0$.

Assumption 2.1, Assumption 2.2 and Assumption 2.3 are very standard and these are satisfied by any standard demand function when products are imperfect substitutes (see Cheng (1985) and Vives (2001)). Let $CS = U - p_1q_1 - p_2q_2$ denote the consumer surplus and $\pi = \pi_1 + \pi_2 = (p_1 - m)q_1 + (p_2 - m)q_2$ denote the aggregate profit with π_1 (π_2) representing profit of Firm 1 (Firm 2). The (social) welfare is given by $W = CS + \pi = U - m(q_1 + q_2)$. The welfare function when both firms choose quantity as a strategic variable is given by $W^{QQ}(q) = U(q) - m(q_1 + q_2)$ with $\partial_i W^{QQ}(q) = F_i^{QQ}(q) - m$, $\partial_{ii} W^{QQ}(q) = \partial_i F_i^{QQ}(q) < 0$, and, $\partial_{ij} W^{QQ}(q) = \partial_j F_i^{QQ}(q) < 0$. The welfare function when both firms choose price as a strategic variable is given by $W^{PP}(p) = W^{QQ}(F_1^{PP}(p), F_2^{PP}(p)) = U(F_1^{PP}(p), F_2^{PP}(p)) - m(F_1^{PP}(p) + F_2^{PP}(p))$ with $\partial_i W^{PP}(p) = (p_i - m)\partial_i F_i^{PP}(p) + (p_j - m)\partial_i F_j^{PP}(p)$.

ASSUMPTION 2.4 For $i, j = 1, 2$ and $(p_1, p_2) \geq (m, m)$, (i) $\partial_{ii} W^{PP}(p) < 0$ and (ii) $\partial_{ii} W^{PP}(p) + \partial_{ij} W^{PP}(p) < 0$.

An assumption similar to Assumption 2.4 was used in Ghosh and Mitra (2014). Assumption 2.4 (i) is necessary to satisfy the second order condition of any welfare maximizing firm. We consider two very standard utility specifications. Suppose that the utility function of the representative consumer is given by

$$U(q) = a(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2), \tag{2.1}$$

where a ($> m$) is a preference parameter, γ ($-1 < \gamma < 1$) is the product differentiation parameter (see Dixit (1979) and Singh and Vives (1984)). A positive (negative) value of γ indicates substitute (complement) goods. We first restrict attention to substitute goods case. One can show that the quadratic utility function given in (2.1) satisfies all our assumptions (that is, Assumption 2.1 to Assumption 2.4) when the goods are substitutes. Suppose that the utility function of the representative consumer is given by

$$U(q) = [q_1^s + q_2^s]^\gamma,\ s\gamma, \gamma, s \in (-\infty, 1), \tag{2.2}$$

where $\sigma = \frac{1}{1-s}$ measure the elasticity of substitution (see Dixit and Stiglitz (1977) and Vives (2001)). Goods are substitute if $\gamma, s \in [0, 1]$ and complement if $\gamma, s \in [-\infty, 0]$. We first restrict attention to substitute goods case. One can show that the CES utility function satisfies the first three assumptions (that is, Assumption 2.1 to Assumption

2.3). If $1 - 2s + \gamma s^2 > 0$, then Assumption 2.4 is satisfied by the CES utility functions given in (2.2).

REMARK **2.1** It is important to note that we consider a weaker set assumptions than what is required for the stability of the equilibrium according to Dixit (1986).

2.2.1 The three stage game

We assume that Firm 1 is partially privatized (maximizing a weighted sum of welfare and its own profit) and Firm 2 is a private firm (maximizing its own profit). Therefore, the payoff function of Firm 1 is $V_1 := \theta\pi_1 + (1-\theta)W$ where θ is the privatization ratio (see Matsumura (1998)) and that of Firm 2 is π_2. Specifically, if Firm 1 is a public (private) firm, then $\theta = 0$ ($\theta = 1$) and Firm 1 maximizes social welfare (its own profit). For any given weight $\theta \in (0,1)$, Firm 1 maximizes the weighted sum of its own profit and social welfare. We consider a three stage game Γ and the stages of the game are as follows.

- **Stage1:** The government decides the level of privatization ($\theta \in [0,1]$) in order to maximize social welfare.
- **Stage 2:** Each firm decides (simultaneously and independently) whether to adopt a price strategy (call it P) or a quantity strategy (call it Q). See Table 1.
- **Stage 3:** Firm 1 and Firm 2 compete in the market.

We solve the game using backward induction. Given the first stage choice of θ, let the optimal price and quantity of Firm i be $p_i^{XY}(\theta)$ and $q_i^{XY}(\theta)$ assuming Firm 1 adopts strategy X and Firm 2 adopts strategy Y where $X, Y \in \{P, Q\}$. We denote the consequent profit of Firm i at the optimal choice and contingent on XY by $\overline{\pi}_i^{XY}(\theta) = \pi_i^{QQ}(q_1^{XY}(\theta), q_2^{XY}(\theta)) = \pi_i^{PP}(p_1^{XY}(\theta), p_2^{XY}(\theta))$. Similarly, the consequent welfare at this optimal choice and contingent on XY is $\overline{W}^{XY}(\theta) = W^{QQ}(q_1^{XY}(\theta), q_2^{XY}(\theta)) = W^{PP}(p_1^{XY}(\theta), p_2^{XY}(\theta))$. So the optimal pay-off of Firm 1 and Firm 2 contingent on XY are $\overline{V}_1^{XY}(\theta) = \theta\overline{\pi}_i^{XY}(\theta) + (1-\theta)\overline{W}^{XY}(\theta)$ and $\overline{\pi}_2^{XY}(\theta)$ respectively. With this specification, in the second stage firms play the following stage game.

Table 2.1: **Stage 2 of** Γ

Firm 1 \ Firm 2	Price	Quantity
Price	$\overline{V}_1^{PP}(\theta), \overline{\pi}^{PP}(\theta)$	$\overline{V}_1^{PQ}(\theta), \overline{\pi}_2^{PQ}(\theta)$
Quantity	$\overline{V}_1^{PQ}(\theta), \overline{\pi}_2^{PQ}(\theta)$	$\overline{V}_1^{QQ}(\theta), \overline{\pi}_2^{QQ}(\theta)$

Sub-game perfect equilibrium of Γ**:** For any $X, Y \in \{P, Q\}$, any $x_1 \in \{p_1, q_1\}$, any $y_2 \in \{p_2, q_2\}$, and, any $\theta^{XY} \in [0, 1]$, a profile of strategies $(\theta^{XY}, (X, x_1^{XY}(\theta^{XY})), (Y, y_2^{XY}(\theta^{XY})))$ is a sub-game perfect Nash equilibrium (SPNE) of Γ if it induces a Nash equilibrium in every sub-game of Γ. First, in Stage 3, given θ^{XY} and given XY, $(x_1^{XY}(\theta^{XY}), y_2^{XY}(\theta^{XY}))$ is a Nash equilibrium choice vector (that is, $x_1^{XY}(\theta^{XY})$ and $y_2^{XY}(\theta^{XY})$ are respectively the optimum choice of X by Firm 1 given $y_2^{XY}(\theta^{XY})$ and the optimum choice of Y by Firm 2 given $x_1^{XY}(\theta^{XY})$). Second, in Stage 2, given θ^{XY}, X is a best response of Firm 1 against Y of Firm 2 and Y is a best response of Firm 2 against X of Firm 1. Finally, θ^{XY} induces XY in Stage 2 and maximizes $\overline{W}^{XY}(\theta)$ in Stage 1. Moreover, there does not exist θ that induces a mode of competition Z_1Z_2 (with $Z_i \in \{P, Q\}$ for $i = 1, 2$) and yields a higher welfare than $\overline{W}^{XY}(\theta^{XY})$.

We define four possible types of equilibria of Γ.

(i) Let $(\theta^{PP}, (P, p_1^{PP}(\theta^{PP})), (P, p_2^{PP}(\theta^{PP})))$ be a Bertrand equilibrium with equilibrium weight θ^{PP}. If $\theta^{PP} = 0$, then we call it the *no privatization Bertrand equilibrium*. If $\theta^{PP} \in (0, 1)$, then we call it the *strictly partial privatization Bertrand equilibrium*.

(ii) Let $(\theta^{QQ}, (Q, q_1^{QQ}(\theta^{QQ})), (Q, q_2^{QQ}(\theta^{QQ})))$ be a Cournot equilibrium with equilibrium weight θ^{QQ}.

(iii) Let $(\theta^{PQ}, (P, p_1^{PQ}(\theta^{PQ})), (Q, q_2^{PQ}(\theta^{PQ})))$ be a Type 1 equilibrium with equilibrium weight θ^{PQ}.

(iv) Let $(\theta^{QP}, (Q, q_1^{QP}(\theta^{QP})), (P, p_2^{QP}(\theta^{QP})))$ be a Type 2 equilibrium with equilibrium weight θ^{QP}.

2.3 The main result

THEOREM **2.1** Suppose Assumption 2.1, Assumption 2.2, Assumption 2.3 and Assumption 2.4 hold. The strategy combination $(\theta^{PP} = 0, (P, p_1^{PP}(\theta^{PP})), (P, p_2^{PP}(\theta^{PP})))$, that is, no privatization Bertrand equilibrium, is the unique SPNE outcome of Γ.

Before going to the proof of Theorem 2.1 we illustrate the relevant reaction functions that will be helpful for our analysis. If both firms compete in prices, then for any $\theta \in [0,1]$, let $\mathcal{SV}_1^{PP}(\theta) = \{p \mid p >> (0,0), \partial_1 V_1^{PP}(p,\theta) = 0\}$ be the reaction function of Firm 1 in the price space. Given, Assumption 2.3 and Assumption 2.4, $\mathcal{SV}_1^{PP}(\theta)$ is invertible. Hence, we can represent it as $p_1 = SV_1^{PP}(p_2, \theta)$. In Figure 2.1, we represent $p_1 = SV_1^{PP}(p_2, 0)$ by the $S_1^{PP}S_1^{PP'}$ curve and we represent $p_1 = SV_1^{PP}(p_2, 1)$ by the $R_1^{PP}R_1^{PP'}$ curve and, for any $\theta \in (0,1)$, the curve $p_1 = SV_1^{PP}(p_2, \theta)$ must lie between the curves $p_1 = SV_1^{PP}(p_2, 0)$ and $p_1 = SV_1^{PP}(p_2, 1)$ (since by Assumption 2.3 and Assumption 2.4 one can show that $\partial_{11} V_1^{PP} < 0$). The reaction function of Firm 2 is the locus of all points in the set $\mathcal{R}_2^{PP} = \{p \mid p >> (0.0), \partial_2 \pi_2^{PP}(p) = 0\}$. By Assumption 2.3, we know that $\mathcal{R}_2^{PP}$ is a positively sloped curve with slope less than unity (see Cheng (1985)) hence it is invertible. Therefore, we can represent it as $p_2 = R_2^{PP}(p_1)$. In Figure 2.1, we represent $p_2 = R_2^{PP}(p_1)$ by the $R_2^{PP}R_2^{PP'}$ curve.

Suppose that both firms are competing in quantities. For any $\theta \in [0,1]$, the reaction function of Firm 1 is the locus of all points in the set $\mathcal{SV}_1^{QQ}(\theta) = \{q \mid q >> (0,0), \partial_1 V_1^{QQ}(q,\theta) = 0\}$. By Assumption 2.1 and Assumption 2.3, it is possible to show that $\partial_{11} V_1^{QQ}(q,\theta) < 0$, $\partial_{12} V_1^{QQ}(q,\theta) < 0$ and $|\partial_{11} V_1^{QQ}(q,\theta)| > |\partial_{12} V_1^{QQ}(q,\theta)|$. Hence, in the (q_1, q_2) plane, the $\mathcal{SV}_1^{QQ}$ curve is negatively sloped and its slope is more than unity in absolute sense. Therefore, we can represent it as $q_1 = SV_1^{QQ}(q_2, \theta)$. The reaction function of Firm 2 is locus of all points in the set $\mathcal{R}_2^{QQ} = \{q \mid q >> (0,0), \partial_2 \pi_2^{QQ}(q) = 0\}$. By Assumption 2.2 the reaction function $\mathcal{R}_2^{QQ}$ (in the (q_1, q_2) plane) is strictly decreasing with slope less than unity in absolute sense (see Cheng (1985)) and hence is invertible. Therefore, we can represent it as $q_2 = R_2^{QQ}(q_1)$. The graphs of $\mathcal{R}_2^{QQ}$ and $\mathcal{SV}_1^{QQ}(\theta)$ in price space are respectively $\mathcal{P}(\mathcal{R}_2^{QQ}) = \{p \mid \partial_2 \pi_2^{QQ}(q) = 0 \text{ and } q_i = F_i^{PP}(p)\ \forall\ i = 1,2\}$ and $\mathcal{P}(\mathcal{SV}^{QQ}(\theta)) = \{p \mid \partial_1 V_1^{QQ}(q,\theta) =$

0 and $q_i = F_i^{PP}(p)\ \forall\ i = 1,2\}$ and their respective equations in implicate form are $F_1^{PP}(p) - SV_1^{QQ}(F_2^{PP}(p),\theta) = 0$ and $F_2^{PP}(p) - R_2^{QQ}(F_1^{PP}(p)) = 0$. In Figure 2.1, the set of points in $\mathcal{P}(\mathcal{SV}_1^{QQ}(0))$ is represented by the line $p_1 = m$. Like Cheng (1985), one can show that the set of points $\mathcal{P}(\mathcal{R}_2^{QQ})$ must lie above the $R_2^{PP}R_2^{PP'}$. One such representation is the r_2r_2' curve in Figure 2.1.

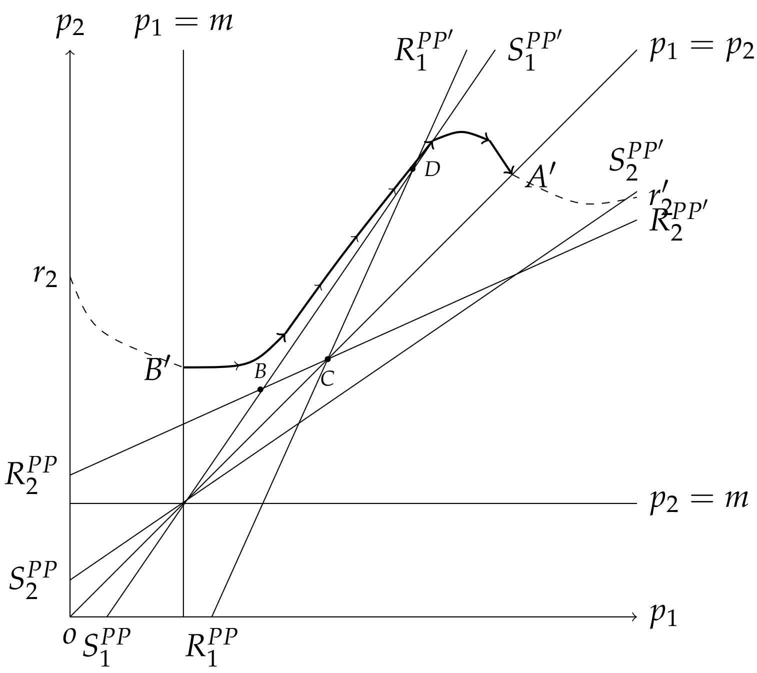


Figure 2.1: The case of imperfect substitutes

LEMMA **2.1** For any weight $\theta \in (0,1)$, $\partial_1 \pi_1^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta)) > 0$ and for any Firm i with $i \in \{1,2\}$, $\partial_i W^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta)) < 0$.

Lemma 2.1 states that with price competition and given any $\theta \in (0,1)$, at any equilibrium price vector $(p_1^{PP}(\theta), p_2^{PP}(\theta))$ it is always optimal for Firm 1 to increase (decrease) price given Firm 2's price remains at $p_2^{PP}(\theta)$ when Firm 1 is a profit (welfare) maximizer.

LEMMA **2.2** For any $\theta \in (0,1)$,

(i) $\partial_\theta q_1^{QQ}(\theta) < 0$ and $\partial_\theta q_2^{QQ}(\theta) > 0$.

(ii) $\partial_\theta p_1^{QQ}(\theta) > 0$, and, for any $i = 1,2$, $\partial_\theta p_i^{PP}(\theta) > 0$ and $\partial_\theta p_i^{PQ}(\theta) > 0$.

Lemma 2.2 provides the standard comparative static results.

LEMMA **2.3** (i) There exists a unique $\theta_1 \in (0,1)$ such that $\overline{\pi}_2^{PP}(\theta) \gtreqless \overline{\pi}_2^{PQ}(\theta)$ if and only if $\theta \lesseqgtr \theta_1$.

(ii) There exist $\underline{\theta}_3 \in (0,1)$ such that $\overline{V}_1^{PP}(\underline{\theta}_3) = \overline{V}_1^{QP}(\underline{\theta}_3)$ and, for any $\theta \in (0, \underline{\theta}_3)$, $\overline{V}_1^{PP}(\theta) > \overline{V}_1^{QP}(\theta)$.

(iii) There exist a unique $\theta_4 \in (0,1)$ such that $\overline{\pi}_2^{QP}(\theta) \gtreqless \overline{\pi}_2^{QQ}(\theta)$ if and only if $\theta \lesseqgtr \theta_4$.

Assume that Firm 1 chooses price strategy. Lemma 2.3 (i) states that there exist a unique $\theta_1 \in (0,1)$ for which Firm 2 is indifferent between choosing price strategy and quantity strategy. Moreover, if $\theta < \theta_1$, then price strategy is optimal for Firm 2, and, if $\theta > \theta_1$, then quantity strategy is optimal for Firm 2. Next, assume that Firm 2 chooses price strategy. Lemma 2.3 (ii) states that there exist $\underline{\theta}_3 \in (0,1)$ for which Firm 1 is indifferent between choosing price or quantity strategy. Moreover, if $\theta < \underline{\theta}_3$, then Firm 1 chooses price strategy. Lemma 2.3 (iii) states that when Firm 1 chooses quantity strategy, there exist an unique $\theta_4 \in (0,1)$ for which Firm 2 is indifferent between choosing price strategy and quantity strategy. For any $\theta < \theta_4$, price strategy is optimal and, for any $\theta > \theta_4$, quantity strategy is optimal. The cut-off point θ_1 (θ_4) is associated with the case where Firm 1 chooses price (quantity) strategy. These cut-off points in Lemma 2.3 (i) and (iii) reflects the reversal in the cost of adopting price strategy for Firm 2 compared to quantity strategy. For the privatization weights below these cut-off points the reverse intuition of Vives (1985) holds. To prove Lemma 2.3 (i) and Lemma 2.3 (iii) we use the line integral technique which is the two-variable asymmetric version of the one used in Ghosh and Mitra (2010), Ghosh and Mitra (2014).

LEMMA **2.4** Under price competition in Stage 2, the resulting welfare $\overline{W}^{PP}(\theta)$ is maximized at $\theta = 0$. Moreover, at $\theta = 0$, the government can uniquely induce price strategy for both firms.

Lemma 2.4 indicates that no privatization Bertrand equilibrium is a possible SPNE outcome of Γ. Specifically, if we can rule out the other modes of competition (that is, if we can rule out both firms choosing quantity strategy and if we can rule out one firm choosing price strategy and the other firm choosing quantity strategy), then from Lemma 2.4 it will follow that the no privatization Bertrand equilibrium is the unique SPNE outcome of Γ. The remaining lemmas together rule out other modes of competition and completes the proof of Theorem 2.1. Lemma 2.5 and Lemma 2.6 that follows rule out the possibilities of Type 1 and Type 2 equilibria.

LEMMA **2.5** There is no $\theta^{PQ} \in [0,1]$ such that $(\theta^{PQ}, (P, p_1^{PQ}(\theta^{PQ})), (Q, q_2^{PQ}(\theta^{PQ})))$ is an SPNE outcome of Γ.

LEMMA **2.6** There is no $\theta^{QP} \in [0,1]$ such that $(\theta^{QP}, (Q, q_1^{QP}(\theta^{QP})), (P, p_2^{QP}(\theta^{QP})))$ is an SPNE outcome of Γ.

Finally, to rule out the possibility of quantity competition, let us first generate the *Cournot equilibrium path* in the (p_1, p_2) space by varying θ from 0 to 1 and plotting the corresponding price vector. See the path $B'A'$ in Figure 2.1 where B' corresponds to $(p_1^{QQ}(0), p_2^{QQ}(0))$ and A' corresponds to $(p_1^{QQ}(1), p_2^{QQ}(1))$. The next lemma captures the exact behavior of the Cournot equilibrium path as we vary θ.

LEMMA **2.7** Let $(p_1^{QQ}(\theta), p_2^{QQ}(\theta))$ and $(p_1^{QQ}(\theta'), p_2^{QQ}(\theta'))$ be any two points on the Cournot equilibrium path. If $(p_1^{QQ}(\theta), p_2^{QQ}(\theta))$ is closer to $(p_1^{QQ}(1), p_2^{QQ}(1))$ than $(p_1^{QQ}(\theta'), p_2^{QQ}(\theta'))$ in terms of arch length of the path, then $\theta > \theta'$.

Lemma 2.7 can be explained in terms of the $B'A'$ segment of the r_2r_2' in Figure 2.1. For each point in the segment $B'A'$, we can associate a $(p_1^{QQ}(\theta), p_2^{QQ}(\theta))$ vector. Lemma 2.7 states that as we move along the $B'A'$ segment of the r_2r_2' curve (starting from B' and ending at A'), the underlying θ increases. Finally, to complete the proof of Theorem 2.1, we need to eliminate the possibility of quantity competition. Given Lemma 2.7 identifies the properties of the Cournot equilibrium path in terms of θ, we can use this path along with the cut-off point θ_4 (identified in Lemma 2.3 (iii)) to establish the impossibility of quantity competition. Hence, we have Lemma 2.8.

LEMMA **2.8** There is no $\theta^{QQ} \in [0,1]$ such that $(\theta^{QQ}, (Q, q_1^{QQ}(\theta^{QQ})), (Q, q_2^{QQ}(\theta^{QQ})))$ is an SPNE outcome of Γ.

2.4 Complements

To obtain the equilibrium outcome when the goods are complement we use the following assumptions.

ASSUMPTION 2.5 For $i, j = 1, 2$ ($i \neq j$) and any $q >> (0,0)$, (i) $\partial_i U(q) > 0$, (ii) $\partial_{ii} U(q) < 0$, (iii) $\partial_{ij} U(q) > 0$ and (iv) $|\partial_{ii} U(q)| > |\partial_{ij} U(q)|$.

ASSUMPTION 2.6 For $i, j = 1, 2$ ($i \neq j$) and any $q >> (0,0)$, (i) $\partial_{ij} \pi_i^{QQ}(q) > 0$ and (ii) $\partial_{ii} \pi_i^{QQ}(q) + |\partial_{ij} \pi_i^{QQ}(q)| < 0$.

ASSUMPTION 2.7 For $i, j = 1, 2$ ($i \neq j$) and any $p >> (0,0)$, (i) $\partial_{ij} \pi_i^{PP}(p) < 0$ and (ii) $\partial_{ii} \pi_i^{PP}(p) + |\partial_{ij} \pi_i^{PP}(p)| < 0$.

ASSUMPTION 2.8 For $i, j = 1, 2$, $i \neq j$ and any $p >> (0,0)$ such that $p_i \leq c \leq p_j$, (i) $\partial_{ii} W^{PP}(p) < 0$ and (ii) $\partial_{ii} W^{PP}(p) - \partial_{ij} W^{PP}(p) < 0$.

Assumption 2.5, Assumption 2.6 and Assumption 2.7 are very standard and these are satisfied by any standard demand function when the goods are complements (see Singh and Vives (1984) and Vives (2001)). Assumption 2.8 (i) is necessary to satisfy the second order condition of any welfare maximizing firm. With the quadratic (CES) utility function given by condition (2.1) (condition (2.2)), Assumption 2.5, Assumption 2.6, Assumption 2.7 and Assumption 2.8 are satisfied.

Before going to our result we explain the implications of Assumption 2.5, Assumption 2.6, Assumption 2.7 and Assumption 2.8 in terms of reactions functions in the price plane using Figure 2.2. In particular, we are interested in the function $p_2 = R_2^{PP}(p_1)$, the set $\mathcal{P}(\mathcal{R}_2^{QQ})$ for Firm 2 and, for $\theta \in \{0, 1\}$, we are interested in the function $p_1 = SV_1^{PP}(p_2, \theta)$ and the set $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$ for Firm 1. In Figure 2.2, the curve R_2R_2' represents the reaction function of Firm 2 when Firm 1 chooses price strategy, that is, $p_2 = R_2^{PP}(p_1)$. By Assumption 2.7, it is decreasing in p_1 with an absolute slope less than unity. Given Assumption 2.7 (ii), $\partial_{22}\pi_2^{PP}(p) < 0$ implying that in the region above the R_2R_2' curve $\partial_2\pi_2^{PP}(p) < 0$ and in the region below the R_2R_2' curve we have $\partial_2\pi_2^{PP}(p) < 0$. Therefore, given $\partial_1\pi_2^{PP}(p) = (p_2 - m)\partial_1 F_2^{PP}(p) < 0$, in the region above the R_2R_2' curve, the iso-profit curve of Firm 2 is decreasing and in the region below the R_2R_2' curve, the iso-profit curve of Firm 2 is increasing. In each point in the set $\mathcal{P}(\mathcal{R}_2^{QQ})$, Firm 2 maximizes profit $\pi_2^{PP}(p)$ subject to $q_1 = F_1^{PP}(p)$. Hence, each point in the set $\mathcal{P}(\mathcal{R}_2^{QQ})$ is a point of tangency between the iso-profit curve of Firm 2 and the iso-quantity curve of Firm 1. By Assumption 2.5, the iso-quantity curve of Firm 1 is negatively sloped implying that the tangency of the iso-quantity curve of Firm 1 with the iso-profit curve of Firm 2 must lie above the R_2R_2' curve. Therefore the set of points in $\mathcal{P}(\mathcal{R}_2^{QQ})$ lie above the R_2R_2' curve. Finally, as we move along the R_2R_2' curve towards the p_2 axis, the profit of the Firm 2 increases since $d\pi_2^{PP}(p_1, R_2^{PP}(p_1))/dp_1 = \partial_1\pi_2^{PP}(p_1, R_2^{PP}(p_1)) < 0$. In Figure 2.2, the R_1R_1' curve is the reaction function of Firm 1, that is, $p_1 = SV_1^{PP}(p_2, 1)$ for $\theta = 1$. By Assumption 2.7, it is decreasing and the slope is greater than unity. One can also show that each point in the set $\mathcal{P}(\mathcal{SV}_1^{QQ}(1))$ lies to the right of the R_1R_1' curve. By definition, $p_1 = c$ represents the set of points in the set $\mathcal{P}(\mathcal{SV}_1^{QQ}(0))$. In Figure 2.2, the S_1S_1' curve represents the function $p_1 = SV_1^{PP}(p_2, 0)$ and it satisfies the following condition.

$$(p_1 - m)\partial_1 F_1^{PP}(p) + (p_2 - m)\partial_1 F_2^{PP}(p) = 0. \tag{2.3}$$

By Assumption 2.5, $\partial_1 F_1^{PP}(p) < 0$, $\partial_1 F_2^{PP}(p) < 0$ and $|\partial_1 F_1^{PP}(p)| > |\partial_1 F_2^{PP}(p)|$ and hence using (2.3) it follows that the S_1S_1' curve must lie between the $p_1 = m$ line and the $p_1 + p_2 = 2m$ line (see line PP' in Figure 2.2). Similarly, the S_2S_2' curve represents the locus of points satisfying $\partial_2 W^{PP}(p) = 0$ and this curve lies between the $p_2 = m$ and the PP' lines. In Figure 2.2, point B is the intersection point between the R_1R_1' curve and the R_2R_2' curve representing the Bertrand equilibrium point for $\theta = 1$. Since both firm are facing symmetric demand and identical cost conditions, point B lies on the $p_1 = p_2$ line. Point A is the point of intersection between the S_1S_1' curve and the R_2R_2' curve representing the Bertrand equilibrium for $\theta = 0$. Given any $\theta \in [0, 1]$, the function $p_1 = SV_1^{PP}(p_2, \theta)$ lies between the S_1S_1' curve and the R_1R_1' curve. Therefore, for any θ, the equilibrium price vector $(p_1^{PP}(\theta), p_2^{PP}(\theta))$ must belong to the segment AB of the R_2R_2' curve.

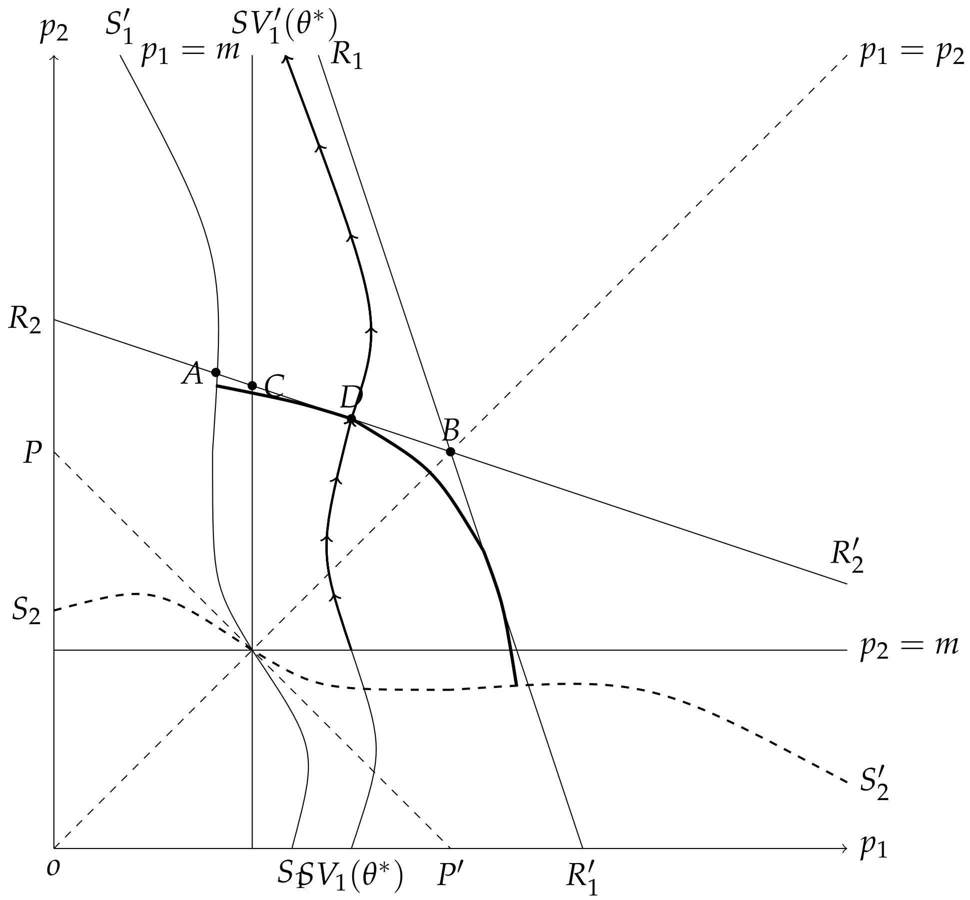


Figure 2.2: The case of complements

PROPOSITION **2.1** Suppose Assumption 2.5, Assumption 2.6, Assumption 2.7 and Assumption 2.8 hold. There exists $\theta^{PP} \in (0,1)$ such that the strictly partial privatization Bertrand equilibrium strategy combination $(\theta^{PP}, (P, p_1^{PP}(\theta^{PP})), (P, p_2^{PP}(\theta^{PP})))$ is the unique SPNE outcome of Γ.

2.5 Robustness

Following Kreps and Scheinkman (1983) argument on the importance of game form, we first check how important our three stage game Γ is in driving Theorem 2.1 and Proposition 2.1. We do this robustness check with quadratic utility function given by (2.1).

(a) Firstly, if we interchange Stage 1 and Stage 2 of Γ, then, in case of imperfect substitutes, we have no privatization Bertrand equilibrium as the unique SPNE outcome, and, in case of complements, we have strictly partial privatization Bertrand equilibrium with $\theta^{PP} = -\gamma(1+\gamma)/(4+3\gamma) \in (0,1)$ as the unique SPNE outcome.

(b) Keeping everything else unchanged, suppose in Stage 1 of Γ we replace the objective function of the government by $V_1 := \theta\pi_0 + (1-\theta)W$ where $\theta \in [0,1]$. For both imperfect substitutes and complements, no privatization Bertrand equilibrium is the unique SPNE outcome.

(c) We check the importance of our symmetric cost assumption. Suppose $C_i(q) = m_i q_i$ is the total cost function of Firm i for $i = 1,2$ and assume that $m_1 \neq m_2$. If the difference in the marginal costs of the two firms is 'large enough', then results can change for imperfect substitutes (see Zanchettin (2006)). Keeping the game Γ unchanged, if we assume cost asymmetry, then, with quadratic utility function given by (2.1), we have the following results. In case of imperfect substitutes, if $\gamma(3-\gamma^2)/2 < \alpha_1/\alpha_2 < 1/\gamma$, then we have no privatization Bertrand equilibrium as the unique SPNE outcome of Γ where $\alpha_i = a - m_i > 0$ for all $i = 1,2$. In case of complements, for any $\gamma \in (-1,0)$, we have the strictly partial privatization Bertrand equilibrium as the unique SPNE outcome of Γ with $\theta^{PP} = -\gamma(1-\gamma^2)(\alpha_2 - \gamma\alpha_1)/\left[(4-3\gamma^2)(\alpha_1 - \gamma\alpha_2) - \gamma(\alpha_2 - \gamma\alpha_1)\right] \in (0,1)$.

(d) If we have one regulated firm and more than one profit maximizing firms competing in a differentiated product market, then, by taking a general form of the quadratic utility function given by (2.1), we can show that in this three stage game it is optimal to select zero weight on profit of the regulated firm under price competition. However, in this scenario it was established by Haraguchi and Matsumura (2016) that one cannot always induce price competition. Specifically, Haraguchi and Matsumura (2016) show that for any given number of private firms greater than one, we always have a cut-off value of the substitution parameter γ below which one can induce price competition but above which one cannot.

Thus, even in an oligopoly framework, co-existence of a fully public firm and many profit maximizing firms is a possible equilibrium outcome under symmetric cost conditions and with sufficiently low values of the substitution parameter γ.

2.6 Conclusions

2.6.1 Government ownership as a policy instrument

Efficiency of a market crucially depends on the nature of strategic interaction among firms in the market. For example, unless firms are capacity constrained, price competition among firms results in higher social welfare than competition in terms of quantity. However, it is often difficult for a social planner to find appropriate policy instrument to influence the nature of firms' strategic interaction. Analysis of this paper reveals that, when firms are free to choose the strategic variable–price contract vis-á-vis quantity contract, the equilibrium modes of competition depends on the level of privatization of the public firm. It implies that the level of government ownership of one of the firms operating in a market is an effective policy instrument to influence the nature of strategic interaction among firms in that market in favor of the social planner.

2.6.2 Implementation aspect of the policy instrument

One can question the implementability aspect of regulating weight on profit of a partially privatized firm. The difficulty of implementability is a valid criticism if, as a policy, one has to sustain a weight on profit of the partially privatized firm which is neither zero nor one (like our SPNE outcome with complements). Specifically if, as a policy, the regulator has to maintain an exact weight $\theta \in (0, 1)$ on profit of the partially private firm, then it is difficult to implement it if the existing weight on profit of the partially private firm is $\theta' \neq \theta$ since the transition to θ calls for redistribution of private and public shares of the firm which may be difficult and costly. Moreover, there may be other legal difficulties in the form of upper bounds on private shares. However, by completely disallowing private stakeholders (that is, by retaining only government shares as a rule) in a partially privatized firm, the regulator can transform a partially privatized firm to a public firm. In that sense our result on imperfect substitute that prescribes the co-existence of a purely private firm and a purely public firm is easy to implement relative to our result on complement goods. However, the need for regulation to change the mode of competition is absent for complement goods.

2.6.3 Regulating both firms

If the government simultaneously regulates both the firms in an otherwise three stage game like ours, then we get marginal cost pricing implying that the equilibrium social welfare is higher than the social welfare associated with our SPNE outcome. Moreover, under such regulation, the mode of competition is also irrelevant. However, in reality we rarely see more than one regulated firm in a differentiated product duopoly (oligopoly) market. In that sense our approach to regulate only one firm is more realistic.

2.6.4 On the adverse effect of transforming the objective of a public firm towards more profit orientation

A public firm may choose to go private either for significant financial gain of the shareholders and CEOs' and/or to reduced regulatory requirements in order to focus on long-term goals. However, in developed countries (like the USA and the UK), the harmful effects of transition of a public firm towards private firm on the stakeholders was pointed out by Greenfield (see Greenfield (2008)).[4] It is also argued that a public firm going private may induce more overall efficiency in the long-run. Specifically, the

[4] According to Greenfield (see Greenfield (2008)), "There may be somewhat more freedom for private firms to operate with a view toward stakeholder interests, but the impact is likely to be marginal. And that freedom could cut the other way, giving private firms the ability to insulate themselves from stakeholder interests and public oversight, making them even more profit-oriented and less concerned about the public interest".

English government has radically restructured its school system under an assumption that school autonomy delivers benefits to schools and students. However, the paper by Eyles et al. (2017) shows that there is no evidence of improvement either in pupil performance or in teaching quality resulting from this conversion. The harmful short-run effects of more profit orientation in a differentiated product oligopoly market was pointed out by Anderson et al. (1997) (when only price competition is admissible and with CES utility function of the representative consumer). Our paper adds to this harmful effects argument of more profit orientation from the social welfare angle for the differentiated product market under symmetric cost conditions. From a policy perspective, our result suggests that if for some reason (other than welfare maximization) the regulator wants to change the orientation of the public firm (in a market with imperfect substitutes) towards more profit (by allocating non-zero weight on profit of the partially private firm), then we can have two types of welfare losses. Not only there is a certain welfare loss due to the increase in profit orientation of the partially private firm, there is a further chance of welfare loss due to a shift in the mode of competition from price to something else.[5] Since our results hold under very general demand specifications, when the goods are substitutes, the policy prescription is to try not to make a public firm more profit oriented.

2.6.5 Deficit financing

Often in many developing countries, government faces cash constraints due to budget deficit. In such a situation government can finance this deficit (totally or partially) from its public revenue. In our context, this means that given any privatization ratio $\theta \in [0,1]$, the government can retain $(1-\theta)\pi_1(\theta)$ of total profit for deficit financing. The government's optimization problem then is to $\max_{\theta \in [0,1]} W(\theta)$ subject to $(1-\theta)\pi_1(\theta) \geq D$. Observe that the government can finance at most $\bar{D} = (1/4)(2-\gamma)^3(\gamma+2)(2-\gamma^2)(a-m)^2/(\gamma^4-6\gamma^2+8)^2$ amount of deficit (since this amount is the unconstrained maximum of $(1-\theta)\pi_1$ in our three stage model).

[5]For complements, the first type of welfare loss is present but the second type of welfare loss is absent since price competition is a dominant strategy.

Therefore, we assume that $D \leq \bar{D}$. With quadratic utility of the representative consumer, one can show that if $\gamma \in (0, 3/4)$, then price competition will continue to emerge as the SPNE outcome with an equilibrium choice $\theta^D > 0$. If, however, $\gamma \in (0.9, 1)$ and D is close enough to $\bar{D}$, then it is possible to induce quantity competition in equilibrium with optimum privatization level $\theta^* (> \theta^D)$.

The government's motivation behind privatization may include the possibility to use receipts from privatization for debt financing. It may be interesting to extend the analysis to include such a possibility. Intuitively, we can say that if the government's objective function is a weighted average of social welfare and debt financing, then, for any given mode of competition, the optimal privatization will be higher than the social welfare maximizing level. However, it is not straight forward to infer the equilibrium outcomes in the case of endogenous mode of competition. We leave that analysis for future research.

Finally, a limitation of our work is that it rules out income effect. Specifically, demand for many products such as high-end cosmetics, apparel etc. sold by the firms in an oligopolistic market is likely to be sensitive to income. Since the analysis is likely to be significantly different and interesting, we plan to model such instances in the future.

2.7 Appendix

Proof of Lemma 2.1: We use two steps to prove the result.

Step 1: Given any weight $\theta \in [0, 1]$ in Stage 1 and given that firms compete in prices in Stage 2, the Stage 3 optimum choice $(p_1^{PP}(\theta), p_2^{PP}(\theta))$ is unique.

Proof of Step 1: In Stage 3, given p_2, Firm 1 chooses p_1 to maximizing $V_1^{PP}(p, \theta) = \theta\pi_1^{PP}(p) + (1-\theta)W^{PP}(p)$ and, given p_1, Firm 2 chooses p_2 to maximize $\pi_2^{PP}(p) = (p_2 - c)F_2^{PP}(p)$. The first order conditions are the following:

$$\partial_1 V_1^{PP}(p, \theta) = \theta F_1^{PP}(p) + (p_1 - m)\partial_1 F_1^{PP}(p) + (1-\theta)(p_2 - m)\partial_1 F_2^{PP}(p) = 0, \quad (2.4)$$

and

$$\partial_2 \pi_2^{PP}(p) = F_2^{PP}(p) + (p_2 - m)\partial_2 F_2^{PP}(p) = 0. \tag{2.5}$$

Using Assumption 2.3 (ii) and Assumption 2.4 (i) it follows that $\partial_{11} V_1^{PP} < 0$ and $\partial_{22}\pi_2^{PP} < 0$. Therefore, second order conditions for maximization are satisfied. Since $\partial_{12}\pi_2^{PP} > 0$, Firm 2's reaction function is increasing in (p_1, p_2). Moreover, $|\partial_{22}\pi_2^{PP}| > |\partial_{12}\pi_2^{PP}|$ implies that the slope of the reaction function of the Firm 2 is less than unity. The sign of $\partial_{12} V_1^{PP}$ can be anything. If for some $(p_1^{PP}(\theta), p_2^{PP}(\theta))$, $\partial_{12} V_1^{PP} > 0$, then by Assumption 2.4 (ii), the slope of the reaction function of the Firm 1 must be greater than unity implying that the intersection of this reaction function with Firm 2's reaction function is unique since, along the $\partial_1 V_1^{PP}(p) = 0$ curve, given any p_2 we have only one p_1, the locus of the function $\partial_1 V_1^{PP}(p) = 0$ will never intersect Firm 2's reaction function twice. If for some $(p_1^{PP}(\theta), p_2^{PP}(\theta))$, $\partial_{12} V_1^{PP} = 0$, then at that point Firm 1's reaction function has a slope of ∞. Given that the slope of the reaction function of Firm 2 is increasing (and is less than unity), we have a unique best response for Firm 1 given any p_2 implying uniqueness of the equilibrium point. Finally, if for some $(p_1^{PP}(\theta), p_2^{PP}(\theta))$, $\partial_{12} V_1^{PP} < 0$, then it is obvious that we will have a unique intersection.

Step 2: $\partial_1 \pi_1^{PP}(p_1^{PP}(0), p_2^{PP}(0)) > 0$.

Proof of Step 2: At $\theta = 0$, the equilibrium price vector $(p_1^{PP}(0), p_2^{PP}(0))$ satisfy following first order conditions

$$(p_1^{PP}(0) - m)\partial_1 F_1^{PP}(p_1^{PP}(0), p_2^{PP}(0)) + (p_2^{PP}(0) - m)\partial_1 F_2^{PP}(p_1^{PP}(0), p_2^{PP}(0)) = 0, \tag{2.6}$$

and

$$(p_2^{PP}(0) - m)\partial_2 F_2^{PP}(p_1^{PP}(0), p_2^{PP}(0)) + F_2^{PP}(p_1^{PP}(0), p_2^{PP}(0)) = 0. \tag{2.7}$$

By definition $q_2^{PP}(0) := F_2^{PP}(p_1^{PP}(0), p_2^{PP}(0)) > 0$ and, by Assumption 2.1, $\partial_2 F_2^{PP}(p_1^{PP}(0), p_2^{PP}(0)) < 0$. Therefore, from (2.7) we have $p_2^{PP}(0) > m$. Given $p_2^{PP}(0) > m$, and, $\partial_1 F_2^{PP}(p_1^{PP}(0), p_2^{PP}(0)) > 0$ and $\partial_1 F_1^{PP}(p_1^{PP}(0), p_2^{PP}(0)) < 0$ (by Assumption 2.1), from (2.6) we get $p_1^{PP}(0) > m$. By Assumption 2.1 we also have

$\partial_1 F_2^{PP}(p_1^{PP}(0), p_2^{PP}(0)) = \partial_2 F_1^{PP}(p_1^{PP}(0), p_2^{PP}(0)) < |\partial_1 F_1^{PP}(p_1^{PP}(0), p_2^{PP}(0))|$. Hence, from condition (2.6) we get $p_2^{PP}(0) > p_1^{PP}(0) > m$. Using $p_2^{PP}(0) > p_1^{PP}(0) > m$ and using the fact that the demands are symmetric with own effect dominant cross effect we have,

$$F_1^{PP}(p_1^{PP}(0), p_2^{PP}(0)) > F_1^{PP}(p_1^{PP}(0), p_1^{PP}(0)) = F_2^{PP}(p_1^{PP}(0), p_1^{PP}(0)) > F_2^{PP}(p_1^{PP}(0), p_2^{PP}(0)). \tag{2.8}$$

Finally,

$$\begin{aligned}
\partial_1 \pi_1^{PP}(p_1^{PP}(0), p_2^{PP}(0)) &= (p_1^{PP}(0) - m)\partial_1 F_1^{PP}(p_1^{PP}(0), p_2^{PP}(0)) + F_1^{PP}(p_1^{PP}(0), p_2^{PP}(0)) \\
&= -(p_2^{PP}(0) - m)\partial_1 F_2^{PP}(p_1^{PP}(0), p_2^{PP}(0)) + F_1^{PP}(p_1^{PP}(0), p_2^{PP}(0)) \\
&> (p_2^{PP}(0) - m)\partial_2 F_2^{PP}(p_1^{PP}(0), p_2^{PP}(0)) + F_1^{PP}(p_1^{PP}(0), p_2^{PP}(0)) \\
&> (p_2^{PP}(0) - m)\partial_2 F_2^{PP}(p_1^{PP}(0), p_2^{PP}(0)) + F_2^{PP}(p_1^{PP}(0), p_2^{PP}(0)) \\
&= 0.
\end{aligned}$$

Here the first equality is by definition, the second equality is due to (2.6), the first inequality follows from the fact $-\partial_1 F_2^{PP}(p_1^{PP}(0), p_2^{PP}(0)) > \partial_2 F_2^{PP}(p_1^{PP}(0), p_2^{PP}(0))$ and last inequality is due to (2.8). This proves Step 2.

To complete the proof we also use Figure 2.3. Given any θ, its (unique) corresponding equilibrium price vector $(p_1^{PP}(\theta), p_2^{PP}(\theta))$ is the intersection of the reaction function of Firm 1 $p_1 = SV_1^{PP}(p_2, \theta)$, and the reaction function of the Firm 2 $p_2 = R_2^{PP}(p_1)$. By condition (2.5), $R_2^{PP}(m) > m$ and $0 < dR_2^{PP}(p_1)/dp_1 < 1$ implying that $p_2 = R_2^{PP}(p_1)$ must intersect the $p_1 = p_2$ line from above (see Figure 2.3). Thus, to the left of the $p_1 = p_2$ line along Firm 2's reaction function $p_2 = R_2^{PP}(p_1)$ we have $p_2 > p_1$. Moreover, by symmetry of the firms, at $\theta = 1$ we have $p_1^{PP}(1) = p_2^{PP}(1)$. Hence, the intersection point of the curve $p_2 = R_2^{PP}(p_1)$ and the line $p_1 = p_2$ is also the intersection point of the curves $p_2 = R_2^{PP}(p_1)$ and $p_1 = SV_1^{PP}(p_2, 1)$. By Step 2, the intersection point of $p_2 = R_2^{PP}(p_1)$ and $p_1 = SV_1^{PP}(p_2, 0)$ must lie to the left of $p_1 = SV_1^{PP}(p_2, 1)$ and, for any $\theta \in (0, 1)$, $p_1 = SV_1^{PP}(p_2, \theta)$ is bounded between $p_1 = SV_1^{PP}(p_2, 0)$ and $p_1 = SV_1^{PP}(p_2, 1)$ (given Assumption 2.3 and Assumption

2.4(i)). As a result, every equilibrium price vector $(p_1^{PP}(\theta), p_2^{PP}(\theta))$ must belongs to the segment of $p_2 = R_2^{PP}(p_1)$ that lie between intersection of $p_1 = SV_1^{PP}(p_2, 0)$ and $p_1 = SV_1^{PP}(p_2, 1)$, that is, the over braced segment PP' in Figure 2.3.

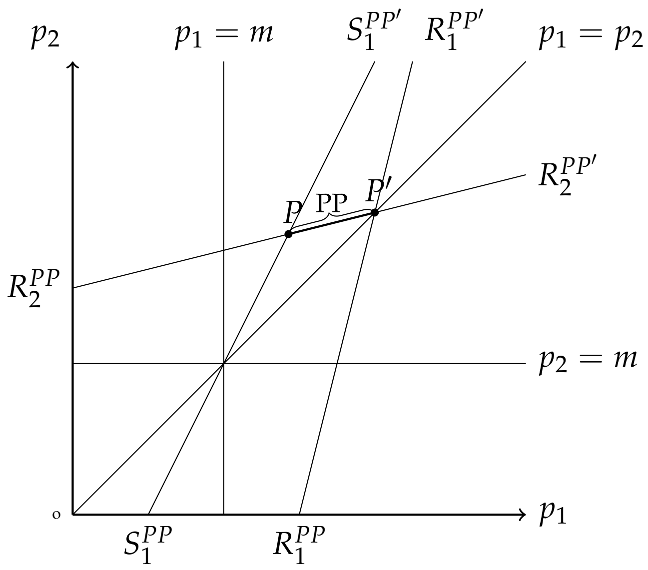


Figure 2.3: Region of potential Bertrand equilibria

The PP' segment in Figure 2.3 lies to the left of $p_1 = SV_1^{PP}(p_2, 1)$ implying $\pi_{1,1}^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta)) > 0$. Moreover, the PP' segment also lies to the right of $p_1 = SV_1^{PP}(p_2, 0)$ implying $W_1^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta)) < 0$. Finally, for $p_2 > p_1 > m$, the PP' segment in Figure 2.3 must lie completely above the $p_1 = p_2$ line implying $W_2(p_1^{PP}(\theta), p_2^{PP}(\theta)) < 0$.

Proof of Lemma 2.2: To prove $\partial_\theta q_1^{QQ}(\theta) < 0$ and $\partial_\theta q_2^{QQ}(\theta) > 0$, we differentiate the conditions $\partial_1 V_1^{QQ}(q_1^{QQ}(\theta), q_2^{QQ}(\theta), \theta) = 0$ and $\partial_2 \pi_2^{QQ}(q_1^{QQ}(\theta), q_2^{QQ}(\theta)) = 0$ with respect to θ and then solve for $\partial_\theta q_1^{QQ}(\theta)$ and $\partial_\theta q_2^{QQ}(\theta)$. This results in

$$\partial_\theta q_1^{QQ}(\theta) = -\frac{\partial_{22}\pi_2^{QQ}(q^{QQ}(\theta))\partial_{1\theta} V_1^{QQ}(q^{QQ}(\theta), \theta)}{|A^{QQ}|},$$

and

$$\partial_\theta q_2^{QQ}(\theta) = \frac{\partial_{12}\pi_2^{QQ}(q^{QQ}(\theta))\partial_{1\theta} V_1^{QQ}(q^{QQ}(\theta), \theta)}{|A^{QQ}|},$$

where for any $\theta \in [0, 1]$, $q^{QQ}(\theta) := (q_1^{QQ}(\theta), q_2^{QQ}(\theta))$, $\partial_{1\theta} V_{1,1}^{QQ}(q^{QQ}(\theta), \theta) =$

$\partial_1 \pi_1^{QQ}(q^{QQ}(\theta)) - \partial_1 W^{QQ}(q^{QQ}(\theta)) = q_1^{QQ}(\theta)\partial_1 F_1^{QQ}(q^{QQ}(\theta)) < 0$ and $|A^{QQ}| = \partial_{11} V_1^{QQ}(q^{QQ}(\theta), \theta)\partial_{22}\pi_2^{QQ}(q^{QQ}(\theta)) - \partial_{12} V_1^{QQ}(q^{QQ}(\theta), \theta)\partial_{12}\pi_2^{QQ}(q^{QQ}(\theta)) > 0$. Hence, we have $\partial_\theta q_1^{QQ}(\theta) < 0$ and $\partial_\theta q_2^{QQ}(\theta) > 0$.

Note that $\partial_\theta p_1^{QQ}(\theta) = \partial_1 F_1^{QQ}(q^{QQ}(\theta))\partial_\theta q_1^{QQ}(\theta) + \partial_2 F_1^{QQ}(q^{QQ}(\theta))\partial_\theta q_2^{QQ}(\theta)$ and that $\partial_\theta q_2^{QQ}(\theta)/\partial_\theta q_1^{QQ}(\theta) = -\partial_{12}\pi_2^{QQ}(q^{QQ}(\theta))/\partial_{22}\pi_2^{QQ}(q^{QQ}(\theta)) = dR_2^{QQ}(q_1^{QQ}(\theta))/dq_1$. From Assumption 2.1 and Assumption 2.2 we have $\partial_1 F_1^{QQ}(q^{QQ}(\theta)) + \partial_2 F_1^{QQ}(q^{QQ}(\theta))(dR_2^{QQ}(q_1^{QQ}(\theta))/dq_1) < 0$. Hence, using the earlier result $\partial_\theta q_1^{QQ}(\theta) < 0$, we get $\partial_\theta p_1^{QQ}(\theta) = \left(\partial_1 F_1^{QQ}(q^{QQ}(\theta)) + \partial_2 F_1^{QQ}(q^{QQ}(\theta))(dR_2^{QQ}(q_1^{QQ}(\theta))/dq_1)\right)\partial_\theta q_1^{QQ}(\theta) > 0$.

For any $\theta \in [0,1]$, define $p^{PP}(\theta) := (p_1^{PP}(\theta), p_2^{PP}(\theta))$. To show $\partial_\theta p_i^{PP}(\theta) > 0$ for $i = 1,2$, we first differentiate the functions $\partial_1 V_1^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta), \theta) = 0$ and $\partial_2 \pi_2^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta)) = 0$ with respect to θ and then solving for $\partial_\theta p_1^{PP}(\theta)$ and $\partial_\theta p_2^{PP}(\theta)$. This results in

$$\partial_\theta p_1^{PP}(\theta) = \frac{\partial_{22}\pi_2^{PP}(p^{PP}(\theta))(\partial_1 W^{PP}(p^{PP}(\theta)) - \partial_1 \pi_1^{PP}(p^{PP}(\theta)))}{|A^{PP}|},$$

and

$$\partial_\theta p_2^{PP}(\theta) = \frac{\partial_{12}\pi_2^{PP}(p^{PP}(\theta))(\partial_1 \pi_1^{PP}(p^{PP}(\theta)) - \partial_1 W^{PP}(p^{PP}(\theta)))}{|A^{PP}|}.$$

The term $|A^{PP}| = \partial_{11} V_1^{PP}(p^{PP}(\theta))\partial_{22}\pi_2^{PP}(p^{PP}(\theta)) - \partial_{12} V_1^{PP}(p^{PP}(\theta))\partial_{12}\pi_2^{PP}(p^{PP}(\theta))$ is positive due to Assumption 2.3 and Assumption 2.4. Given Lemma 2.1, for every $\theta \in (0,1)$, $\partial_1 \pi_1^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta)) - \partial_1 W^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta)) > 0$. Hence, for each $\theta \in (0,1)$, $\partial_\theta p_1^{PP}(\theta) > 0$ and $\partial_\theta p_2^{PP}(\theta) > 0$.

Next, we prove that $\partial_\theta p_i^{PQ}(\theta) > 0$ for $i = 1,2$. Suppose, given $q_2^{PQ}(\theta)$, Firm 1 chooses p_1 to maximize $V_1^{PQ}(p_1, q_2^{PQ}(\theta)) = \theta\pi_1^{PQ}(p_1, q_2^{PQ}(\theta)) + (1-\theta)W^{PQ}(p_1, q_2^{PQ}(\theta))$ and, given $p_1^{PQ}(\theta)$, Firm 2 chooses q_2 to maximize $\pi_2^{PQ}(p_1^{PQ}(\theta), q_2) = (F_2^{PQ}(p_1^{PQ}(\theta), q_2) - m)q_2$ where, for $i = 1,2$, $F_i^{PQ}(p_1, q_2)$ is the demand function of Firm i. The first order condition of Firm 1 is

$\partial_1 V_1^{PQ}(p_1^{PQ}(\theta), q_2^{PQ}(\theta)) = 0$ which results in

$$\theta F_1^{PQ}(p_1^{PQ}(\theta), q_2^{PQ}(\theta)) + (p_1^{PQ}(\theta) - m)\partial_1 F_1^{PQ}(p_1^{PQ}(\theta), q_2^{PQ}(\theta)) = 0.$$

Similarly the first order condition of Firm 2 is $\partial_2 \pi_2^{PQ}(p_1^{PQ}(\theta), q_2^{PQ}(\theta)) = 0$, which reduced to

$$(F_2^{PQ}(p_1^{PQ}(\theta), q_2^{PQ}(\theta)) - m) + q_2^{PQ}(\theta)\partial_2 F_2^{PQ}(p_1^{PQ}(\theta), q_2^{PQ}(\theta)) = 0.$$

Observe that the reaction function of Firm 1 can be written as $p_1 = F_1^{QQ}(SV_1^{QQ}(q_2, \theta), q_2)$ for all $\theta \in [0,1]$ and that of Firm 2 is $q_2 = F_2^{PP}(p_1, R_2^{PP}(p_1))$. For any $\theta \in [0,1]$, $p_1^{PQ}(\theta) - F_1^{QQ}(SV_1^{QQ}(q_2^{PQ}(\theta), \theta), q_2^{PQ}(\theta)) = 0$ and $q_2^{PQ}(\theta) - F_2^{PP}(p_1^{PQ}(\theta), R_2^{PP}(p_1^{PQ}(\theta))) = 0$. Differentiating the reaction function with respect to θ and then solving for $\partial_\theta p_1^{PQ}(\theta)$ and $\partial_\theta q_2^{PQ}(\theta)$ we get,

$$\partial p_1^{PQ}(\theta) = \frac{\partial_1 F_1^{QQ}(F_1^{PQ}(p_1^{PQ}(\theta), q_2^{PQ}(\theta)), q_2^{PQ}(\theta))\partial_\theta SV_1^{QQ}(q_2^{PQ}(\theta), \theta)}{|A^{PQ}|},$$

and

$$\partial_\theta q_2^{PQ}(\theta) = \frac{\begin{bmatrix} \partial_1 F_1^{QQ}(q_1^{PQ}(\theta), q_2^{PQ}(\theta))\partial_\theta SV_1^{QQ}(q_2^{PQ}(\theta), \theta) \\ \left(\partial_1 F_2^{PP}(p_1^{PQ}(\theta), p_2^{PQ}(\theta)) + \partial_2 F_2^{PP}(p_1^{PQ}(\theta), p_2^{PQ}(\theta))\frac{dR_2^{PP}(p_1^{PQ}(\theta))}{dp_1}\right) \end{bmatrix}}{|A^{PQ}|},$$

where $q_1^{PQ}(\theta) = F_1^{PQ}(p_1^{PQ}(\theta), q_2^{PQ}(\theta))$, $p_2^{PQ}(\theta) = F_2^{PQ}(p_1^{PQ}(\theta), q_2^{PQ}(\theta))$

$$|A^{PQ}| = 1 - \begin{bmatrix} \left(\partial_1 F_1^{QQ}(q_1^{PQ}(\theta), q_2^{PQ}(\theta))\frac{\partial SV_1^{QQ}(q_2^{PQ}(\theta), \theta)}{\partial q_2} + \partial_2 F_1^{QQ}(q_1^{PQ}(\theta), q_2^{PQ}(\theta))\right) \\ \left(\partial_1 F_2^{PP}(p_1^{PQ}(\theta), p_2^{PQ}(\theta)) + \partial_2 F_2^{PP}(p_1^{PQ}(\theta), p_2^{PQ}(\theta))\frac{dR_2^{PP}(p_1^{PQ}(\theta))}{dp_1}\right) \end{bmatrix} > 0$$

and $\partial_\theta SV_1^{QQ}(q_2^{PQ}(\theta), \theta) = -q_1^{PQ}(\theta)(\partial_1 F_1^{QQ}(q_1^{PQ}(\theta), q_2^{PQ}(\theta))/\partial_{11} V_1^{QQ}(q_1^{PQ}(\theta), q_2^{PQ}(\theta))) <$

0.[6] Hence, given $\partial_1 F_1^{QQ}(q_1^{PQ}(\theta), q_2^{PQ}(\theta)) < 0$, we get $\partial_\theta p_1^{PQ}(\theta) > 0$. Finally, $\partial_\theta q_2^{PQ}(\theta)/\partial_\theta p_1^{PQ}(\theta) = \left(\partial_2 F_1^{PP}(p_1^{PQ}(\theta), p_2^{PQ}(\theta)) + \partial_2 F_2^{PP}(p_1^{PQ}(\theta), p_2^{PQ}(\theta))\frac{dR_2^{PP}(p_1^{PQ}(\theta))}{dp_1}\right)$ implies that $\partial_\theta p_2^{PQ}(\theta) = (dR_2^{PP}(p_1^{PQ}(\theta))/dp_1)\partial_\theta p_1^{PQ}(\theta) > 0$.

Proof of Lemma 2.3: To prove part (i) and part (iii) of this result we use an application of the Fundamental (Gradient) Theorem of Line Integrals that states the following: Consider any function $f : \Re_+^2 \to \Re$ which is twice differentiable. For any $a = (a_1, a_2) >> (0,0), a' = (a_1', a_2') >> (0,0)$ and for any scalar $t \in [0,1]$ such that $a(t) = (a_1(t), a_2(t)) = (ta_1' + (1-t)a_1, ta_2' + (1-t)a_2) >> (0,0)$,

$$f(a') - f(a) = (a_1' - a_1)\int_0^1 \frac{\partial f(a(t))}{\partial a_1(t)}dt + (a_2' - a_2)\int_0^1 \frac{\partial f(a(t))}{\partial a_2(t)}dt. \tag{2.9}$$

Condition (2.9) specifies that given any smooth path $a(t)$ connecting points a and a' in the domain of a function f, the line integral through the gradient of the function f equals the difference in its scalar at the endpoints (that is, $f(a') - f(a)$) (see Apostol (1969) for a more detailed discussion on line integrals).

Proof of (i): In the price space, given any $\theta \in (0,1)$, if Firm 2 chooses price strategy, then Firm 1's reaction function is $p_1 = SV_1^{PP}(p_2, \theta)$, and, if Firm 2 chooses quantity strategy, then Firm 1's reaction function is the set of points $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$ and can be written in implicit form as $F_1^{PP}(p) - SV_1^{QQ}(F_2^{PP}(p), \theta) = 0$. Given Firm 1 chooses price strategy, Firm 2's reaction function is $p_2 = R_2^{PP}(p_1)$. Fix a $\theta \in [0,1]$. Consider $p_1(t) = tp_1^{PP}(\theta) + (1-t)p_1^{PQ}(\theta)$ defined for each $t \in [0,1]$. Applying the condition (2.9) on the function $\partial_2 \pi_2^{PP}(p)$ with endpoints $(p_1^{PP}(\theta), p_2^{PQ}(\theta))$ and $(p_1^{PQ}(\theta), p_2^{PQ}(\theta))$

[6] Specifically, $|A^{PQ}| = 1 - \left[\begin{array}{c} \left(\partial_1 F_1^{QQ}(q_1^{PQ}(\theta), q_2^{PQ}(\theta))\frac{dSV_1^{QQ}(q_2^{PQ}(\theta),\theta)}{dq_2} + \partial_2 F_1^{QQ}(q_1^{PQ}(\theta), q_2^{PQ}(\theta))\right) \\ \left(\partial_1 F_2^{PP}(p_1^{PQ}(\theta), p_2^{PQ}(\theta)) + \partial_2 F_2^{PP}(p_1^{PQ}(\theta), p_2^{PQ}(\theta))\frac{dR_2^{PP}(p_1^{PQ}(\theta))}{dp_1}\right) \end{array}\right]$

$$= \frac{\partial_{11}U(q_1^{PQ}(\theta), q_2^{PQ}(\theta))\left(\begin{array}{c} \partial_{22}U(q_1^{PQ}(\theta), q_2^{PQ}(\theta)) + \partial_{11}U(q_1^{PQ}(\theta), q_2^{PQ}(\theta))\left|\frac{dR_2^{PP}(p_2^{PQ}(\theta))}{dp_1}\right|\left|\frac{\partial SV_1^{QQ}(q_2^{PQ}(\theta),\theta)}{\partial q_2}\right| \\ -\partial_{12}U(q_1^{PQ}(\theta), q_2^{PQ}(\theta))\left|\frac{\partial SV_1^{QQ}(q_2^{PQ}(\theta),\theta)}{\partial q_2}\right| - \partial_{12}U(q_1^{PQ}(\theta), q_2^{PQ}(\theta))\frac{R_2^{PP}(p_1^{PQ}(\theta))}{dp_1} \end{array}\right)}{|D|}$$

$$> \frac{\partial_{11}U(q_1^{PQ}(\theta),q_2^{PQ}(\theta))\partial_{12}U(q_1^{PQ}(\theta),q_2^{PQ}(\theta))\left(1-\frac{dR_2^{PP}(p_1^{PQ}(\theta))}{dp_1}\right)\left(1-\left|\frac{dSV_1^{QQ}(q_2^{PQ}(\theta))}{dq_2}\right|\right)}{|D|} > 0.$$

we get

$$\partial_2\pi_2^{PP}(p_1^{PP}(\theta), p_2^{PQ}(\theta)) - \partial_2\pi_2^{PP}(p_1^{PQ}(\theta), p_2^{PQ}(\theta))$$
$$= (p_1^{PP}(\theta) - p_1^{PQ}(\theta)) \int_0^1 \partial_{12}\pi_2^{PP}(p_1(t), p_2^{PQ}(\theta))dt.$$

The point $(p_1^{PQ}(\theta), p_2^{PQ}(\theta))$ is on $p_2 = R_2^{PP}(p_1)$ implying $\partial_2\pi_2^{PP}(p_1^{PQ}(\theta), p_2^{PQ}(\theta)) = 0$. As a result we have

$$\partial_2\pi_2^{PP}(p_1^{PP}(\theta), p_2^{PQ}(\theta)) = (p_1^{PP}(\theta) - p_1^{PQ}(\theta)) \int_0^1 \partial_{12}\pi_2^{PP}(p_1(t), p_2^{PQ}(\theta))dt.$$

From Assumption 2.3 (i) it follows that $\int_0^1 \partial_{12}\pi_2^{PP}(p_1(t), p_2^{PQ}(\theta))dt > 0$. Therefore, $p_1^{PP}(\theta) \gtreqless p_1^{PQ}(\theta)$ if and only if $\partial_2\pi_2^{PP}(p_1^{PP}(\theta), p_2^{PQ}(\theta)) \gtreqless 0$. Observe first that

$$\lim_{\theta\to 0} \partial_2\pi_2^{PP}(p_1^{PP}(\theta), p_2^{PQ}(\theta)) = \partial_2\pi_2^{PP}(p_1^{PP}(0), p_2^{PQ}(0)) > 0. \tag{2.10}$$

Condition (2.10) holds since from the first order condition of profit maximization and welfare maximization we have $m = p_1^{PQ}(0) < p_1^{PP}(0) < p_2^{PP}(0)$ and since R_2^{PP} is increasing, that is, $p_2^{PQ}(0) < p_2^{PP}(0)$ therefore $(p_1^{PP}(0), p_2^{PQ}(0))$ lie below the R_2^{PP} hence implies (2.10). Also observe that

$$\lim_{\theta\to 1} \partial_2\pi_2^{PP}(p_1^{PP}(\theta), p_2^{PQ}(\theta)) = \partial_2\pi_2^{PP}(p_1^{PP}(1), p_2^{PQ}(1)) < 0. \tag{2.11}$$

Condition (2.11) holds since $p_1^{PP}(1) = p_2^{PP}(1) < p_2^{PQ}(1)$ implies that the point $(p_1^{PP}(1), p_2^{PQ}(1))$ lies above the R_2^{PP}. Conditions (2.10) and (2.11) implies that there exist θ_R,θ_S with $\theta_R \leq \theta_S$ such that for any $\theta \in (0, \theta_R)$ and any $\theta \in (\theta_S, 1)$ we have $p_1^{PP}(\theta) > p_1^{PQ}(\theta)$ and $p_1^{PP}(\theta) < p_1^{PQ}(\theta)$ respectively. Thus, $\partial_2\pi_2^{PP}(p_1^{PP}(\theta_S), p_2^{PP}(\theta_S)) - \partial_2\pi_2^{PP}(p_1^{PP}(\theta_R), p_2^{PP}(\theta_R)) = 0$ and applying condition (2.9) to this equality with end points $(p_1^{PP}(\theta_S), p_2^{PP}(\theta_S))$ and $(p_1^{PP}(\theta_R), p_2^{PP}(\theta_R))$ we get

$$\left[\begin{array}{c} (p_1^{PP}(\theta_S) - p_1^{PP}(\theta_R)) \int_0^1 \partial_{12}\pi_2^{PP}(p_1(t), p_2(t))dt \\ +(p_2^{PP}(\theta_S) - p_2^{PP}(\theta_R)) \int_0^1 \partial_{22}\pi_2^{PP}(p_1(t), p_2(t))dt \end{array} \right] = 0. \tag{2.12}$$

By Assumption 2.3 and Lemma 2.2 (ii) it follows that for condition (2.12) to hold we must have $p_1^{PP}(\theta_S) > p_2^{PP}(\theta_S) > p_2^{PP}(\theta_R) > p_1^{PP}(\theta_R)$ if $\theta_R < \theta_S$. But for each $\theta \in [0,1]$ we have $p_2^{PP}(\theta) \geq p_1^{PP}(\theta)$. Therefore, $p_1^{PP}(\theta_S) > p_2^{PP}(\theta_S)$ is a contradiction, hence we have $\theta_S = \theta_R = \theta_1$. Thus, there exists a unique $\theta_1 \in (0,1)$ such that $p_1^{PP}(\theta) \gtreqless p_1^{PQ}(\theta)$ if and only if $\theta \lesseqgtr \theta_1$.

Along Firm 2's reaction function $p_2 = R_2^{PP}(p_1)$, $\hat{\pi}_2^{PP}(p_1) := \pi_2^{PP}(p_1, R_2^{PP}(p_1))$. Given $R_2^{PP}(p_1) - m > 0$ and $\partial_1 F_2^{PP}(p_1, R_2^{PP}(p_1)) > 0$, $(d\hat{\pi}_2^{PP}(p_1)/dp_1) = \partial_1 \pi_2^{PP}(p_1, R_2^{PP}(p_1)) = (R_2^{PP}(p_1) - m)\partial_1 F_2^{PP}(p_1, R_2^{PP}(p_1)) > 0$. Therefore, along the reaction function $p_2 = R_2^{PP}(p_1)$, Firm 2's profit increases in p_1. For any $\theta \in [0, \theta_1)$, $p_1^{PP}(\theta) > p_1^{PQ}(\theta)$ holds. Hence, $\overline{\pi}_2^{PP}(\theta) = \pi_2^{PP}(p_1^{PP}(\theta), R_2^{PP}(p_1^{PP}(\theta))) > \pi_2^{PP}(p_1^{PQ}(\theta), R_2^{PP}(p_1^{PQ}(\theta))) = \overline{\pi}_2^{PQ}(\theta)$. Thus, if Firm 1 chooses price strategy, then Firm 2 optimally chooses price strategy. When $\theta = \theta_1$, if Firm 1 chooses price strategy, $p_1^{PP}(\theta_1) = p_1^{PQ}(\theta_1)$ implying $\overline{\pi}_2^{PP}(\theta_1) = \overline{\pi}_2^{PQ}(\theta_1)$ and Firm 2 is indifferent between price and quantity strategies. When $\theta \in (\theta_1, 1]$, if Firm 1 chooses price strategy, then $p_1^{PP}(\theta) < p_1^{PQ}(\theta)$ and by similar reasoning we can show that $\overline{\pi}_2^{PP}(\theta) < \overline{\pi}_2^{PQ}(\theta)$ so that it is always optimal for Firm 2 to choose quantity strategy.

Proof of (ii): Consider the difference $\overline{V}_1^{PP}(\theta) - \overline{V}_1^{QP}(\theta)$ evaluated at $\theta = 0$. It is quite easy to observe that $\overline{V}_1^{PP}(0) - \overline{V}_1^{QP}(0) = W^{PP}(p_1^{PP}(0), p_2^{PP}(0)) - W^{PP}(p_1^{QP}(0), p_2^{QP}(0)) > 0$. In particular, whatever be the shape of the locus of $\partial_1 W^{PP}(p) = 0$, starting from the point (m, m) as we move along that locus by increasing p_2, the welfare has to fall (see Figure 2.4) and, since the transformed reaction function ($\partial_1 \pi_1^{QQ}(q) = 0$) of Firm 1 in price space must lie above $p_1 = R_2^{PP}(p_2)$, we have $p_2^{PP}(0) > p_2^{QP}(0)$ and $(p_1^{PP}(0), p_2^{PP}(0))$ and $(p_1^{QP}(0), p_2^{QP}(0))$ lie on the locus of $\partial_1 W^{PP}(p) = 0$.

Consider $\overline{V}_1^{PP}(\theta) - \overline{V}_1^{QP}(\theta)$ at $\theta = 1$. We have, $\overline{V}_1^{PP}(\theta = 1) - \overline{V}_1^{QP}(\theta = 1) = \pi_1^{PP}(p_1^{PP}(\theta = 1), p_2^{PP}(\theta = 1)) - \pi_1^{QP}(p_1^{QP}(\theta = 1), p_2^{QP}(\theta = 1)) < 0$ since for a profit maximizing firm quantity strategy strictly dominates price strategy. Since $\overline{V}_1^{PP}(\theta) - \overline{V}_1^{QP}(\theta)$ is a continuous function of θ the result follows.

Proof of (iii): For this proof we restrict our attention to the quantity space (q_1, q_2). Given

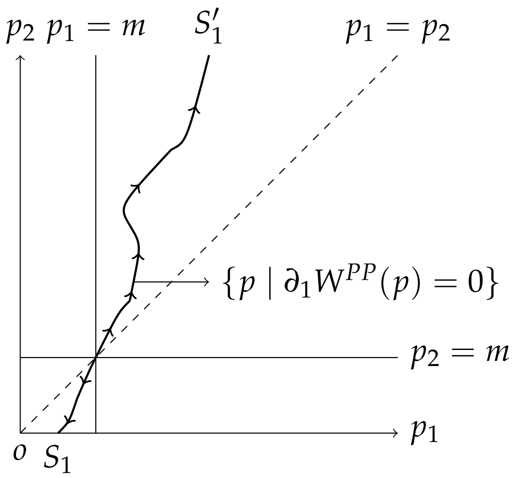


Figure 2.4: Welfare reaction function in price space

any $\theta \in (0,1)$, if Firm 2 chooses quantity strategy, then Firm 1's reaction function is $q_1 = SV_1^{QQ}(q_2, \theta)$. If Firm 2 chooses price strategy, then Firm 1's reaction function is $p_1 = SV_1^{PP}(p_2, \theta)$. If we transform $p_1 = SV_1^{PP}(p_2, \theta)$ to the quantity space, then we can be write it implicitly as $F_1^{QQ}(q) - SV_1^{QQ}(F_2^{QQ}(q), \theta) = 0$. Given Firm 1 chooses quantity strategy, Firm 2's reaction function is $q_2 = R_2^{QQ}(q_1)$.

Fix a $\theta \in [0,1]$. Consider $q(t) = tq_1^{QP}(\theta) + (1-t)q_1^{QQ}(\theta)$ defined for each $t \in [0,1]$. Applying the condition (2.9) on the function $\partial_2 \pi_2^{QQ}(q)$ with end points $(q_1^{QP}(\theta), q_2^{QQ}(\theta))$ and $(q_1^{QQ}(\theta), q_2^{QQ}(\theta))$ we have

$$\begin{aligned} &\partial_2 \pi_2^{QQ}(q_1^{QP}(\theta), q_2^{QQ}(\theta)) - \partial_2 \pi_2^{QQ}(q_1^{QQ}(\theta), q_2^{QQ}(\theta)) \\ &= (q_1^{QP}(\theta) - q_1^{QQ}(\theta)) \textstyle\int_0^1 \partial_{12} \pi_2^{QQ}(q_1(t), q_2^{QQ}(\theta)) dt. \end{aligned}$$

The point $(q_1^{QQ}(\theta), q_2^{QQ}(\theta))$ is on $q_2 = R_2^{QQ}(q_1)$ implying $\partial_2 \pi_2^{QQ}(q_1^{QQ}(\theta), q_2^{QQ}(\theta)) = 0$. Hence

$$\partial_2 \pi_2^{QQ}(q_1^{QP}(\theta), q_2^{QQ}(\theta)) = (q_1^{QP}(\theta) - q_1^{QQ}(\theta)) \int_0^1 \partial_{12} \pi_2^{QQ}(q_1(t), q_2^{QQ}(\theta)) dt. \tag{2.13}$$

Using Assumption 2.2 (i) it follows that $\int_0^1 \partial_{12} \pi_2^{QQ}(q_1(t), q_2^{QQ}(\theta)) dt < 0$ and hence we have $q_1^{QP}(\theta) \lesseqgtr q_1^{QQ}(\theta)$ if and only if $\partial_2 \pi_2^{QQ}(q_1^{QP}(\theta), q_2^{QQ}(\theta)) \gtreqless 0$. Let $r_1^{PP}(q_1)$ be the transformed price reaction of Firm 1. Given $q_2^{QP}(1) < q_2^{QQ}(1)$, we have $r_1^{PP}(q_2^{QP}(1)) >$

$R_1^{QQ}(q_2^{QP}(1)) > R_1^{QQ}(q_2^{QQ}(1))$ implying $q_1^{QP}(1) > q_1^{QQ}(1)$. Hence $(q_1^{QP}(1), q_2^{QQ}(1))$ must lie above the R_2^{QQ} curve. Thus, we have

$$\lim_{\theta \to 1} \partial_2 \pi_2^{QQ}(q_1^{QP}(\theta), q_2^{QQ}(\theta)) = \partial_2 \pi_2^{QQ}(q_1^{QP}(1), q_2^{QQ}(1)) < 0. \tag{2.14}$$

Also observe that

$$\lim_{\theta \to 0} \partial_2 \pi_2^{QQ}(q_1^{QP}(\theta), q_2^{QQ}(\theta)) = \partial_2 \pi_2^{QQ}(q_1^{QP}(0), q_2^{QQ}(0)) > 0 \tag{2.15}$$

Condition (2.15) holds since the price welfare reaction function of the Firm 1 in the quantity space must intersect the R_2^{QQ} curve to the left of $F_1^{QQ}(q) = m$. Therefore, $(q_1^{QP}(0), q_2^{QQ}(0))$ lies below the R_2^{QQ} curve. Condition (2.14) and (2.15) implies that there exist θ'_R, θ'_S with $\theta'_R \leq \theta'_S$ such that for all $\theta \in (0, \theta'_R)$ and $\theta \in (\theta'_S, 1)$ we have $q_1^{QQ}(\theta) > q_1^{QP}(\theta)$ and $q_1^{QQ}(\theta) < q_1^{QP}(\theta)$ respectively. Therefore, $\partial_2 \pi_2^{QQ}(q_1^{QQ}(\theta'_S), q_2^{QQ}(\theta'_S)) - \partial_2 \pi_2^{QQ}(q_1^{QQ}(\theta'_R), q_2^{QQ}(\theta'_R)) = 0$ and applying the condition (2.9) to this equality with end points $(q_1^{QQ}(\theta'_S), q_2^{QQ}(\theta'_S))$ and $(q_1^{QQ}(\theta'_R), q_2^{QQ}(\theta'_R))$ yields

$$\left[\begin{array}{c} (q_1^{QQ}(\theta'_S) - q_1^{QQ}(\theta'_R)) \int_0^1 \partial_{12} \pi_2^{QQ}(q_1(t), q_2(t)) dt \\ +(q_2^{QQ}(\theta'_S) - q_2^{QQ}(\theta'_R)) \int_0^1 \partial_{22} \pi_{2,22}^{QQ}(q_1(t), q_2(t)) dt \end{array} \right] = 0. \tag{2.16}$$

By Assumption 2.2 and Lemma 2.2 (i), it follows that for condition (2.16) to hold with $\theta'_R < \theta'_S$, we must have $q_1^{QQ}(\theta'_R) > q_2^{QQ}(\theta'_S) > q_2^{QQ}(\theta'_R) > q_1^{QQ}(\theta'_S)$. But we know that for all $\theta \in [0, 1]$, $q_2^{QQ}(\theta) < q_1^{QQ}(\theta)$ and we have a contradiction. As a result we must have $\theta'_S = \theta'_R = \theta_4$. Thus, there exists a unique $\theta_4 \in (0, 1)$ such that $q_1^{QP}(\theta) \lesseqgtr q_1^{QQ}(\theta)$ if and only if $\theta \lesseqgtr \theta_4$.

Along Firm 2's reaction function $q_2 = R_2^{QQ}(q_1)$ we have $\hat{\pi}_2^{QQ}(q_1) = \pi_2^{QQ}(q_1, R_2^{QQ}(q_1))$. Given that $R_2^{QQ}(q_1) > 0$ and $\partial_1 F_2^{QQ}(q_1, R_2^{QQ}(q_1)) < 0$ (by Assumption 2.1), $(d\hat{\pi}_2^{QQ}(q_1)/dq_1) = \partial_1 \pi_2^{QQ}(q_1, R_2^{QQ}(q_1)) = R_2^{QQ}(q_1) \partial_1 F_2^{QQ}(q_1, R_2^{QQ}(q_1)) < 0$. Therefore, along $q_2 = R_2^{QQ}(q_1)$, Firm 2's profit decreases in q_1. For any $\theta \in [0, \theta_4)$, $q_1^{QQ}(\theta) > q_1^{QP}(\theta)$ holds. Hence, we obtain $\overline{\pi}_2^{QQ}(\theta) = \pi_2^{QQ}(q_1^{QQ}(\theta), R_2^{QQ}(q_1^{QQ}(\theta))) < \pi_2^{QQ}(q_1^{QP}(\theta), R_2^{QQ}(p_1^{QP}(\theta))) = \overline{\pi}_2^{QP}(\theta)$. Thus, if Firm 1 chooses quantity strategy, then

Firm 2 optimally chooses price strategy. When $\theta = \theta_4$, if Firm 1 chooses quantity strategy, $q_1^{QQ}(\theta_4) = q_1^{QP}(\theta_4)$ implying $\overline{\pi}_2^{QQ}(\theta_4) = \overline{\pi}_2^{QP}(\theta_4)$ and Firm 2 is indifferent between price and quantity strategies. When $\theta \in (\theta_4, 1]$, if Firm 1 chooses price strategy, then $q_1^{QQ}(\theta) < q_1^{QP}(\theta)$ and by similar reasoning we can show that $\overline{\pi}_2^{QQ}(\theta) > \overline{\pi}_2^{QP}(\theta)$ so that it is always optimal for Firm 2 to choose quantity strategy.

Proof of Lemma 2.4: If we assume price competition in Stage 2, then, in Stage 1, the government chooses $\theta \in [0,1]$ to maximize welfare. Given $\overline{W}^{PP}(\theta) = W^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta))$, differentiating $\overline{W}^{PP}(\theta)$ with respect to θ we get,

$$\partial_\theta \overline{W}^{PP}(\theta) = \partial_1 W^{PP}(p^{PP}(\theta))\partial_\theta p_1^{PP}(\theta) + \partial_2 W^{PP}(p^{PP}(\theta))\partial_\theta p_2^{PP}(\theta). \tag{2.17}$$

By Lemma 2.2 (ii), $\partial_\theta p_i^{PP}(\theta) > 0$ and, by Lemma 2.1, $\partial_i W^{PP}(p^{PP}(\theta)) < 0$. Therefore, from equation (2.17), we get $\partial_\theta \overline{W}^{PP}(\theta) < 0$ for all $\theta \in (0,1)$. Since $\overline{W}^{PP}(\theta = 0) > \overline{W}^{PP}(\theta = 1)$, the optimal choice of θ in Stage 1 under price competition is $\theta = 0$.

If $\theta = 0$ is the optimal choice of Stage 1, then, given $\theta = 0 < \theta_1$, it is optimal for Firm 2 to choose price strategy when Firm 1 chooses price strategy (Lemma 2.3 (i)). Moreover, since $\theta = 0 < \theta_4$, it is optimal for Firm 2 to choose price strategy even when Firm 1 chooses quantity strategy (Lemma 2.3 (iii)). Therefore, with $\theta = 0$, choosing price is the dominant strategy for Firm 2 in Stage 2. Moreover, since $\theta = 0 < \underline{\theta}_3$ and since choosing price is the dominant strategy for Firm 2, it is optimal for Firm 1 to choose price strategy (Lemma 2.3 (ii)). Hence, given $\theta = 0$, in Stage 2 it is optimal for both firms to choose price strategy and it is the unique Nash equilibrium of the sub-game of Γ starting from Stage 2.

Proof of Lemma 2.5: We prove Lemma 2.5 using the following figure.

In Figure 2.5, the curve $R_1^{PP}R_1^{PP'}$ represents the function $p_1 = SV_1^{PP}(p_2, 1)$.[7] By Assumption 2.3 the curve $R_1^{PP}R_1^{PP'}$ is increasing in the price plane with slope greater than unity and hence must lie to the right of the $p_1 = m$ line. Since $S_1^{PP}S_1^{PP'}$ represents the function $p_1 = SV_1^{PP}(p_2, 0)$, it must lie between the $p_1 = m$ and $p_1 = p_2$ lines.

[7] In this Figure 2.5, we draw all curves as straight line just for simplicity of exposition.

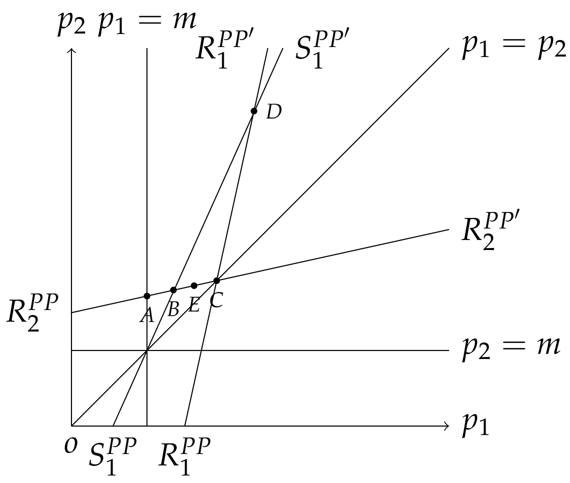


Figure 2.5: Impossibility of Type I equilibrium

Similarly, $R_2^{PP}R_2^{PP'}$ represents the function $p_2 = R_2^{PP}(p_1)$ and, by Assumption 2.3, it is always increasing in the price plane with slope less than unity and hence must lie above the $p_2 = m$ line. Therefore, the intersection point of $R_1^{PP}R_1^{PP'}$ and $R_2^{PP}R_2^{PP'}$ is the Bertrand equilibrium point C for $\theta = 1$ and by Assumption 2.3 this point is unique. Since firms have identical cost and symmetric demand conditions, point C must lie on the $p_1 = p_2$ line. By Assumption 2.3 and Assumption 2.4, the intersection of $R_2^{PP}R_2^{PP'}$ and $S_1^{PP}S_1^{PP'}$ is the Bertrand equilibrium point (B) for $\theta = 0$ and, by Step-2 of the Lemma 2.1, the point B must lie to the left of point C on R_2^{PP}. We do not impose any restriction on the locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(1))$ implying that it can take any shape and can intersect the curve $R_2^{PP}R_2^{PP'}$ more than ones. But the locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(1))$ must lie to the left of the $R_1^{PP}R_1^{PP'}$ curve (see Cheng (1985)). Hence, any intersection point between $R_2^{PP}R_2^{PP'}$ and the locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(1))$ must lie to the right of point C on the $R_2^{PP}R_2^{PP'}$ curve.

The line $p_1 = m$ is the locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(0))$. If Firm 1 select price strategy, then Firm 2's optimal reaction is to react along the $R_2^{PP}R_2^{PP'}$ curve (see Singh and Vives (1984)) in the price space. Again, given some $\theta \in [0, 1]$, if Firm 2 select quantity strategy, then Firm 1 optimally reacts (in terms of prices) according to the locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$ in the price space. Since $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$ must lie between the line $p_1 = m$ and the locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(1))$, for any given θ, when Firm 1 chooses price strategy and

Firm 2 chooses quantity strategy, the equilibrium point must lie on the $R_2^{PP}R_2^{PP'}$ curve and it must also lie on or to the right of point A.

By Lemma 2.3 (i), when Firm 1 chooses price strategy, there exist a $\theta_1 \in (0,1)$ at which Firm 2 is indifferent between choosing price strategy and quantity strategy, and, for $\theta < (>)\theta_1$, it chooses price (quantity) strategy. Hence, at θ_1 the Bertrand equilibrium price vector $(p_1^{PP}(\theta_1), p_2^{PP}(\theta_1))$ and Type-1 equilibrium price vector $(p_1^{PQ}(\theta_1), p_2^{PQ}(\theta_1)$ induces same profit for Firm 2. The point $(p_1^{PP}(\theta_1), p_2^{PP}(\theta_1))$ is the intersection point of $R_2^{PP}R_2^{PP'}$ and the locus of $p_1 = SV_1^{PP}(p_2, \theta_1)$ and the point $(p_1^{PQ}(\theta_1), p_2^{PQ}(\theta_1))$ is intersection point of $R_2^{PP}R_2^{PP'}$ and locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta_1))$ in the price space. Since along the $R_2^{PP}R_2^{PP'}$ curve, any two distinct points generate distinct profits, we must have $p_i^{PP}(\theta_1) = p_i^{PQ}(\theta_1)$. Hence, at $(p_1^{PP}(\theta_1), p_2^{PP}(\theta_1))$, the locus of $p_1 = SV_1^{PP}(p_2, \theta_1)$ and the locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta_1))$ intersect on the $R_2^{PP}R_2^{PP'}$ curve in the price space and the intersection point is unique by Lemma 2.3 (i). Since the locus of $p_1 = SV_1^{PP}(p_2, \theta_1)$ must lie between $S_1^{PP}S_1^{PP'}$ and $R_1^{PP}R_1^{PP'}$ and since $\theta_1 \in (0,1)$, the point $(p_1^{PP}(\theta_1), p_2^{PP}(\theta_1))$ must lie at the interior on the segment BC of the R_2^{PP} curve. Without loss of generality, let E be that point. By Lemma 2.2 (ii) $\partial_\theta p_i^{PQ}(\theta) > 0$, any point on the segment AE excepting point E corresponds to $\theta < \theta_1$. Hence, we cannot find any selection θ in Stage 1 for the government that can induce any (p_1, p_2) combination that lie in this segment of AE (except point E). Finally, the government won't induce any point on or to the right of E since each such point (on the $R_2^{PP}R_2^{PP'}$) generates less welfare than at point B. Since the point B can be induced by choosing $\theta = 0$ (by Lemma 2.4), the result follows.

Proof of Lemma 2.6: Consider Figure 2.6. In Figure 2.6 we introduce two new curves. The first one is the iso-welfare curve corresponding to welfare level of point B (that is, the welfare level $\overline{W}^{PP}(0)$). The second one is the $S_2^{PP}S_2^{PP'}$ curve which is the locus of $W_2^{PP}(p) = 0$. Point B is Bertrand equilibrium for $\theta = 0$ and, by Lemma 2.4, this point can be uniquely induced by choosing $\theta = 0$. If the resulting welfare from any strategy associated with Type II equilibrium yields a welfare less than the welfare level corresponding to point B, then the possibility of Type-II equilibrium is ruled out. By

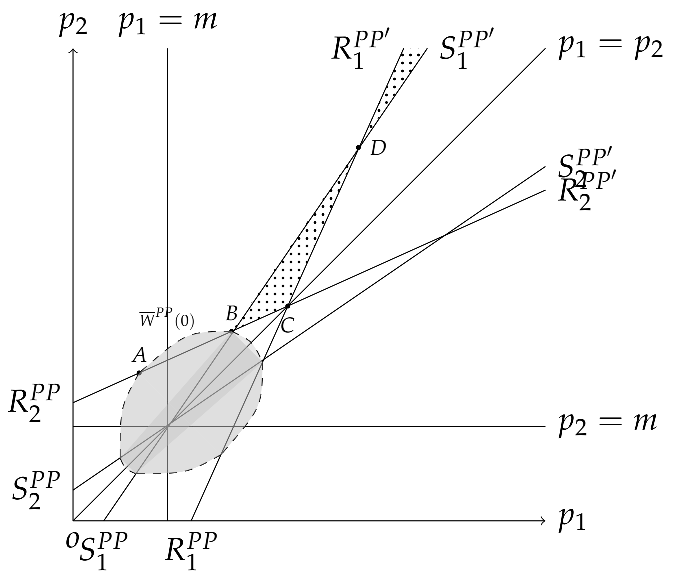


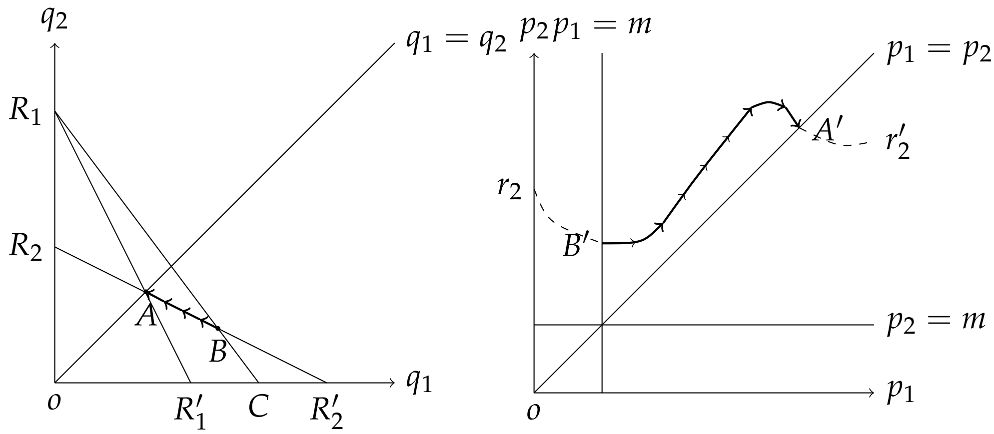
Figure 2.6: Impossibility of Type II equilibrium

Assumption 2.4 (i), in the regions above and below both $S_1^{PP}S_1^{PP'}$ and $S_2^{PP}S_2^{PP'}$ curves, the iso-welfare curve is upward sloping and in the region lying between these curves, the iso-welfare curve is downward sloping. The Bertrand equilibrium point at $\theta = 0$ (that is, point B) lies on the $S_1^{PP}S_1^{PP'}$ curve and is located above the $S_2^{PP}S_2^{PP'}$ curve. Therefore, to the left of point B the iso-welfare curve is increasing and to the right of point B it is decreasing. Since a consequence of welfare maximization in terms of quantity choice yields $(p_1 = m, p_2 = m)$ as the resulting price vector, it is the global maximum of $W^{PP}(p)$. Therefore, the upper contour set $\Omega_W^{PP} = \{p \mid W^{PP}(p) \geq \overline{W}^{PP}(0)\}$ of B is the region shaded in gray in Figure 2.6 that always includes point (m, m) as an interior point. When Firm 1 chooses quantity strategy and Firm 2 chooses price strategy, then the reaction function of Firm 1 is the locus of $p_1 = SV_1^{PP}(p_2, \theta)$ lying between the $R_1^{PP}R_1^{PP'}$ and the $S_1^{PP}S_1^{PP'}$ curves. The reaction function of Firm 2 is the locus of the set $\mathcal{P}(\mathcal{R}_2^{PP})$ that lies completely above the $R_2^{PP}R_2^{PP'}$. Therefore, any potential Type II equilibrium point must belong to the region lying between the $R_1^{PP}R_1^{PP'}$ curve and the $S_1^{PP}S_1^{PP'}$ curve and must also lie above the $R_2^{PP}R_2^{PP'}$ as shown in the Figure 2.6 by the dotted region (where the boundary is not included for the BC segment). Hence, the set in which the Type II equilibrium can occur is $E^{QP} = \{p \mid \partial_2\pi_2^{PP}(p) > 0, \partial_1 W^{PP}(p).\partial_1\pi_1^{PP}(p) \leq 0\}$. Since, due to Assumption 2.4, the $S_1^{PP}S_1^{PP'}$

curve can never bend back and since the only intersection of the closure of E^{QP} and the Ω_W^{PP} is point B and B is not in E^{QP}, the set Ω_W^{PP} and the set E^{QP} must be disjoint. Hence, for any price vector associated with Type II equilibrium, the resulting welfare is always less than the welfare corresponding to point B. Therefore, Type II equilibrium is ruled out.

Proof of Lemma 2.7: Consider Figure 2.7 where in Figure 3.4a we consider the quantity space and in Figure 2.7b we consider the price space. In Figure 3.4a, the curves R_1R_1', RC and R_2R_2' corresponds respectively to the function $q_1 = SV_1^{QQ}(q_2, 1)$, $q_1 = SV_1^{QQ}(q_2, 0)$ and $q_2 = R_2^{QQ}(q_1)$. Each curve is negatively sloped and both R_1R_1' and R_1C curves have an absolute slope of more than unity and the R_2R_2' curve has an absolute slope of less than unity. If $\theta = 1$, then firms are symmetric and hence we have $q_1^{QQ}(1) = q_2^{QQ}(1)$. Hence, the intersection point of R_1R_1' and R_2R_2' must lie on the $q_1 = q_2$ line (see point A in Figure 3.4a). For any point on the R_1C curve we have $p_1 = m$ and for any point on the R_1R_1' curve we have $p_1 > m$ excepting at point R_1 where we have $q_1 = 0$ and hence we also have $p_1 = m$. Since by Assumption 2.1 own effect on indirect demand is negative, the R_1C curve must lie to the right of the R_1R_1' curve . Consider point B (in Figure 3.4a) which is the point of intersection between the R_1C and the R_2R_2' curves. Point B must lie to the right of point A and both A and B are on R_2R_2'. Point B is the Cournot equilibrium vector $(q_1^{QQ}(0), q_2^{QQ}(0))$. Firstly, by Lemma 2.2, $\partial_\theta q_1^{QQ}(\theta) < 0$ and $\partial_\theta q_2^{QQ}(\theta) > 0$. Secondly, one can show that $\partial_\theta q_2^{QQ}(\theta) = \frac{dR_2^{QQ}(q_1^{QQ}(\theta))}{dq_1}\partial_\theta q_1^{QQ}(\theta)$ (see the proof of Lemma 2.2 (i)). Thirdly, for any $\theta \in [0, 1]$, the equilibrium point $(q_1^{QQ}(\theta), q_2^{QQ}(\theta))$ must lie on the R_2R_2' curve. Hence, for all $\theta \in [0, 1]$, $(q_1 = q_1^{QQ}(\theta), q_2 = q_2^{QQ}(\theta))$ is the parametric representation of the AB segment of R_2R_2' with A (B) representing the quantity vector corresponding to $\theta = 1$ ($\theta = 0$). As θ varies from 0 to 1 we move from point B to point A along R_2R_2' as shown by the arrows in Figure 3.4a.

Consider Figure 2.7b and let the curve r_2r_2' represent the set $\mathcal{P}(\mathcal{R}_2^{QQ})$. Point A' and B' in Figure 2.7b correspond to the points A and B respectively of Figure 3.4a. Since for any Cournot equilibrium the resulting price vector must satisfy $p_2 \geq p_1 \geq m$,



(a) Quantity Space (b) The Price Space

Figure 2.7: Quantity reaction function in the quantity and price space

the segment $B'A'$ must lie between the $p_1 = m$ line and the $p_1 = p_2$ line and above the $R_2^{PP}R_2^{PP'}$ curve (see Cheng (1985)). By Assumption 2.1 and Assumption 2.2, the Cournot equilibrium quantity vector $(q_1^{QQ}(\theta), q_2^{QQ}(\theta))$ is unique for each θ implying that $(p_1^{QQ}(\theta), p_2^{QQ}(\theta))$ is also unique. Therefore, for the segment $B'A'$, given any p_1 we must get a single p_2 and this segment can be represented as a function $p_2 = r_2^{QQ}(p_1)$ defined for $p_1 \in [m, p_1^{QQ}(1)]$. For each p_1, $r_2^{QQ}(p_1)$ is always well-defined and given continuity of AB segment, the $B'A'$ segment is also continuous. Starting from B' if we move towards A' along the segment $B'A'$, the underlying θ increases since the $B'A'$ segment has a functional representation it cannot be backward bending. Hence, given $\partial_\theta p_1^{QQ}(\theta) > 0$, $p_1^{QQ}(\theta)$ increases along the segment $B'A'$ when we start from B'.

Proof of Lemma 2.8: Consider Figure 2.8. Given any $\theta \in [0,1]$, if $q^{QQ}(\theta)$ is Cournot equilibrium quantity vector, then $q_2^{QQ}(\theta) = R_2^{QQ}(q_1^{QQ}(\theta))$ and $q_1^{QQ}(\theta) = SV_1^{QQ}(q_2^{QQ}(\theta), \theta)$ and the resulting price of Firm i is $p_i^{QQ}(\theta) = F_i^{QQ}(q^{QQ}(\theta))$ implying that the price vector $(p_1^{QQ}(\theta), p_2^{QQ}(\theta)) \in \mathcal{P}(\mathcal{R}_2^{QQ}) \cap \mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$. The graph $\mathcal{P}(\mathcal{R}_2^{QQ})$ must lie above R_2^{PP} in the price space and $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$ is bounded between $p_1 = m$ and the graph $\mathcal{P}(\mathcal{SV}_1^{QQ}(1))$. Again, since the firms face identical demand and cost conditions, the Cournot equilibrium price vector must lie in $E^{QQ} = \{p \mid \partial_1 \pi_1^{PP}(p) > 0,\ p_1 > m \text{ and } p_2 \geq p_1\}$. Therefore, the region $\mathcal{A}$ in Figure 2.8 represents the set $E^{QQ} \cap \Omega_W^{PP}$. This region $\mathcal{A}$ represents the set of points where

Cournot equilibrium can occur and resulting welfare is higher compared to point B. If $\mathcal{P}(\mathcal{R}_2^{QQ}) \cap \Omega_W^{PP} = \emptyset$, then, in Stage 1, the government's optimal choice of θ can never induce Cournot competition since, by choosing $\theta = 0$, the government can improve the level of welfare.

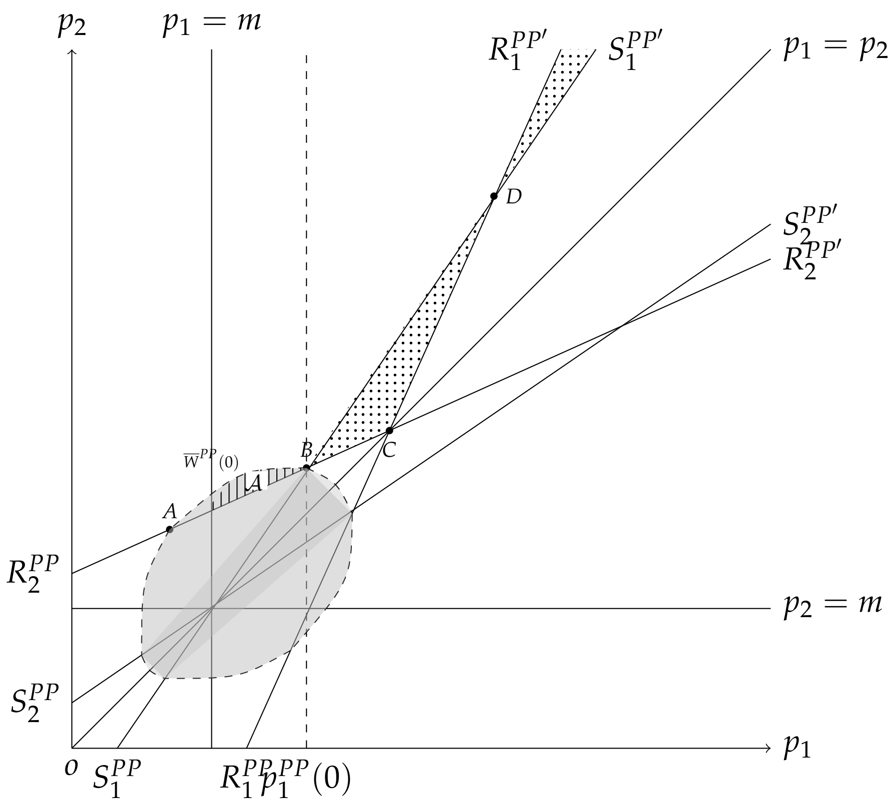


Figure 2.8: Case 1

If $\mathcal{P}(\mathcal{R}_2^{QQ}) \cap \Omega_W^{PP} \neq \emptyset$, then can the government induce quantity competition by choosing θ in such a way that the resulting price vector $(p_1^{QQ}(\theta), p_2^{QQ}(\theta)) \in E^{QQ} \cap \Omega_W^{PP}$? Consider the sets $E_{\geq}^{QQ} = \{p \mid p \in E^{QQ}$ and $p_1 \geq p_1^{PP}(0)\}$ and $E_{<}^{QQ} = \{p \mid p \in E^{QQ}$ and $p_1 < p_1^{PP}(0)\}$. Observe that $E_{\geq}^{QQ} \cap E_{<}^{QQ} = \emptyset$ and $E_{\geq}^{QQ} \cup E_{<}^{QQ} = E^{QQ}$. We consider two exhaustive cases.

Case 1: $E^{QP} \cap E_{<}^{QQ} = \emptyset$.

Case 2: $E^{QP} \cap E_{<}^{QQ} \neq \emptyset$

For Case 1, E^{QP} lies to the right of the vertical line $p_1 = p_1^{PP}(0)$ (see Figure 2.8). Since $E^{QP} \subset E^{QQ}$, we must have $E^{QP} \subset E_{\geq}^{QQ}$. Given $E^{QP} \cap \mathcal{P}(\mathcal{R}_2^{QQ}) \neq \emptyset$, $E^{QP} \cap \Omega_W^{PP} = \emptyset$ (by Lemma 2.6) and the continuity of the graph of $\mathcal{P}(\mathcal{R}_2^{QQ})$ in the price plane, there exists exactly one compact set $S_P(\subset \mathcal{P}(\mathcal{R}_2^{QQ}))$ such that (a) the interior of S_P is contained in the complement set of $E^{QP} \cap \Omega_W^{PP}$, (b) we can find (p_1, p_2) in the intersection of the boundaries of the sets S_P and Ω_W^{PP}, and, (c) we can find another (p_1, p_2) in the intersection of the boundaries of the sets S_P and E^{QP}. Using Lemma 2.7 we can now say that each θ for which $(p_1^{QQ}(\theta), p_2^{QQ}(\theta))$ in the interior of S_P is higher compared to every θ such that $(p_1^{QQ}(\theta), p_2^{QQ}(\theta)) \in \Omega_W^{PP} \cap \mathcal{P}(\mathcal{R}_2^{QQ})$ and is lower compared to every θ such that $(p_1^{QQ}(\theta), p_2^{QQ}(\theta)) \in E^{QP} \cap \mathcal{P}(\mathcal{R}_2^{QQ})$. By Lemma 2.3 (iii), $(p_1^{PP}(\theta_4), p_2^{PP}(\theta_4)) \in E^{QP}$. Hence, for every θ such that $(p_1^{QQ}(\theta), p_2^{QQ}(\theta)) \in \Omega_W^{PP} \cap \mathcal{P}(\mathcal{R}_2^{QQ})$, $\theta < \theta_4$. Thus, it is impossible for the government to induce Cournot competition by choosing θ such that resulting price vector belongs to $\Omega_W^{PP} \cap \mathcal{P}(\mathcal{R}_2^{QQ})$.

For Case 2, the entire E^{QP} does not lie to the right of the vertical line $p_1 = p_1^{PP}(0)$ (see Figure 2.9). Consider the set $E_<^{QP} = \{(p_1, p_2) \mid (p_1, p_2) \in E^{QP}, p_1 < p_1^{PP}(0)\}$. If $E_<^{QP} \cap \mathcal{P}(\mathcal{R}_2^{QQ}) = \emptyset$, then the analysis is similar to Case 1 and Cournot competition cannot be sustained. Finally, if $E_<^{QP} \cap \mathcal{P}(\mathcal{R}_2^{QQ}) \neq \emptyset$, then given $\mathcal{P}(R_2^{QQ}) \cap \Omega_W^{PP} \neq \emptyset$, $\Omega_W^{PP} \cap E^{QP} = \emptyset$ and continuity of the graph of $\mathcal{P}(\mathcal{R}_2^{QQ})$ in the price plane, we can find at least one $S^P \subset \mathcal{P}(\mathcal{R}_2^{QQ})$ for which we have three mutually exclusive sets S^{Pa}, S^{Pb} and S^{Pc} such that $S^{Pa} \cup S^{Pb} \cup S^{Pc} = S^P$, $S^{Pa} \subset E^{QP}$, $S^{Pb} \subset \Re^2_{++} \setminus \{\Omega_W^{PP} \cup E^{QP}\}$ and $S^{Pc} \subset \Omega_W^{PP}$. Assume that there are M such S^Ps'. Denote a representative S^P as S_m^P where $m \in \{1, 2, ..., M\}$. Therefore, for each S_m^P we have a set $S_m^{P'} \subset E^{QP}$. Can we find $(p_1^{QQ}(\theta_4), p_2^{QQ}(\theta_4)) \in S_m^{Pa}$? The following argument shows that the answer is no. By Lemma 2.7, along the graph of the set S_m^{Pa} in the price plane, p_1 is increasing (along the segment $B'A'$ in Figure 2.7b) and it must contain at least two points in the boundary of E^{QP} each of which corresponds to Type I equilibrium price vector for $\theta = 0$. Along the graph S_m^{Pa}, the behavior of $p_1^{QP}(\theta)$ is shown in Figure 2.10. Suppose $(p_1^{QQ}(\theta_4), p_2^{QQ}(\theta_4)) \in S_m^{Pa}$. By Lemma 2.3 (ii) θ_4 is unique and by Lemma 2.2, $p_1^{QQ}(\theta)$ is increasing in θ. Therefore $(p_1^{QQ}(\theta_4), p_2^{QQ}(\theta_4))$ is unique. Hence, if $(p_1^{QQ}(\theta_4), p_2^{QQ}(\theta_4)) \in S_m^{Pa}$, then there dose not exist any $k \in \{1, 2, ..., M\}$ with $k \neq m$ such that $(p_1^{QQ}(\theta_4), p_2^{QQ}(\theta_4)) \in S_k^{Pa}$.

Let $\overline{OT}$ denote the length of the OT segment in Figure 2.10. Given

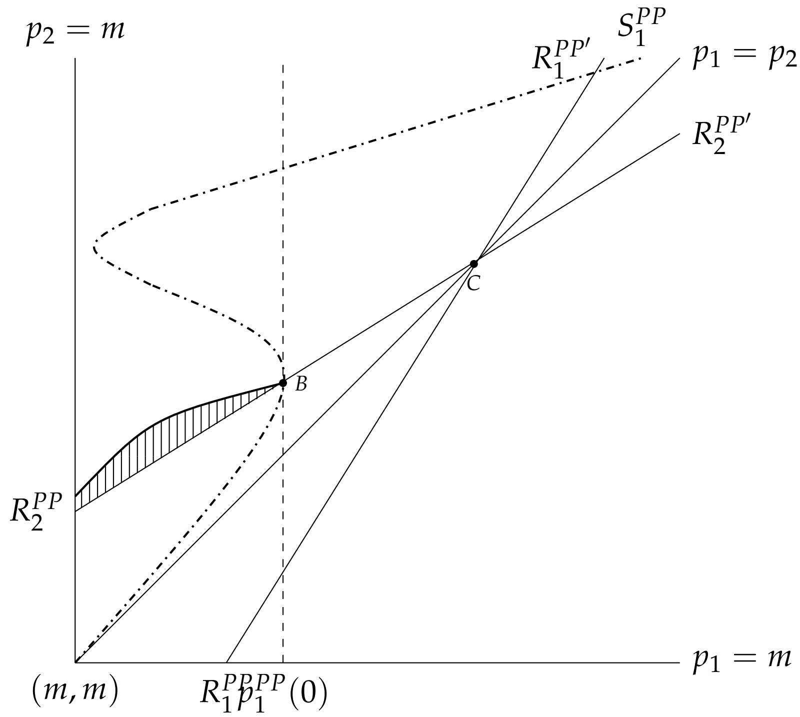


Figure 2.9: Case 2a

$(p_1^{QQ}(\theta_4), p_2^{QQ}(\theta_4)) \in S_m^{Pa}$, $\theta_4 > \overline{OT}$ is not possible. If $\theta_4 = \overline{OT}$, then for $p_1^{QQ}(\theta) = p_1^{QP}(\theta)$ at $\theta = \theta_4$ either $p_1^{QQ}(\theta)$ has slope of ∞ at θ_4 (which is impossible since $\partial_{11} V_1^{QQ} \partial_{22} \pi_2^{QQ} - \partial_{12} V_1^{QQ} \partial_{12} \pi_2^{QQ} \neq 0$) or $p_1^{QQ}(\theta)$ should intersect $p_1^{QP}(\theta)$ twice which is again a contradiction due to uniqueness of θ_4 (see Lemma 2.3 (iii)). If $\theta_4 < \overline{OT}$, then (given $p_1^{QQ}(\theta) = p_1^{QP}(\theta)$ holds for at most one θ) the only possibility is that $p_1^{QQ}(\theta)$ is tangent to the lower segment of $p_1^{QP}(\theta)$ at $\theta = \theta_4$ which is again a contradiction since, in that case, we can find at least one $\theta > \theta_4$ such that $p_1^{QP}(\theta) > p_1^{QQ}(\theta)$.

Proof of Proposition 2.1: We use four steps to prove the result.

Step (i): The value of θ that maximizes $\overline{W}^{PP}(\theta)$ must belongs to $(0, 1)$.

Proof of Step (i): The first order condition of Stage 1 under the assumption that firms select price strategy in Stage 2 is given by

$$\partial_\theta \overline{W}^{PP}(\theta) = \partial_1 W^{PP}(p^{PP}(\theta)) \partial_\theta p_1^{PP}(\theta) + \partial_2 W^{PP}(p^{PP}(\theta)) \partial_\theta p_2^{PP}(\theta). \quad (2.18)$$

Like Lemma 2.2, when goods are complement one can show that $\partial_\theta p_1^{PP}(\theta) > 0$,

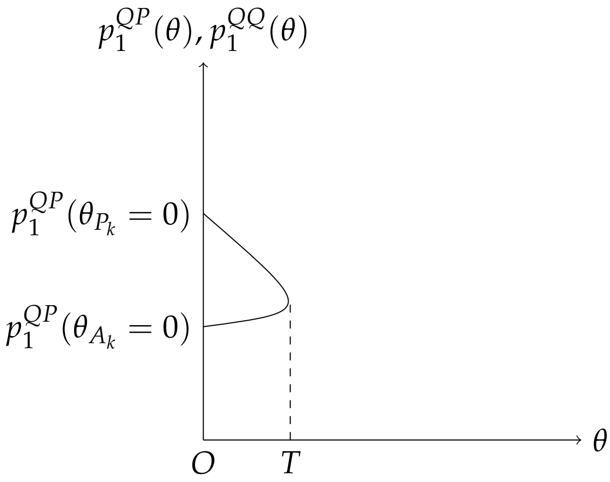


Figure 2.10: Case 2b

$\partial_\theta p_2^{PP}(\theta) < 0$, and, $\partial_\theta p_2^{PP}(\theta) = \frac{dR_2^{PP}(p_1^{PP}(\theta))}{dp_1}\partial_\theta p_1^{PP}(\theta)$. Therefore, from condition (2.18) we get,

$$\partial_\theta \overline{W}^{PP}(\theta) = \left(\partial_1 W^{PP}(p^{PP}(\theta)) + \frac{dR_2^{PP}(p_1^{PP}(\theta))}{dp_1}\partial_2 W^{PP}(p^{PP}(\theta))\right)\partial_\theta p_1^{PP}(\theta). \quad (2.19)$$

At $\theta = 0$, the price vector $(p_1^{PP}(0), p_2^{PP}(0))$ corresponds to point A in the Figure 2.2. At A $\partial_1 W^{PP}(p_1^{PP}(0), p_2^{PP}(0)) = 0$ (since, point A must lie on the S_1S_1' curve), $\partial_2 W^{PP}(p_1^{PP}(0), p_2^{PP}(0)) < 0$ (since, point A must lie above PP') and, by Assumption 2.7, we also have $(dR_2^{PP}(p_1^{PP}(\theta))/dp_1) < 0$. Hence, at $\theta = 0$, $\partial_\theta \overline{W}^{PP}(\theta) = \frac{d\overline{W}^{PP}(\theta)}{d\theta} > 0$. At $\theta = 1$, the price vector $(p_1^{PP}(1), p_2^{PP}(1))$ corresponds to point B in the Figure 2.2 where we have $\partial_1 W^{PP}(p_1^{PP}(1), p_2^{PP}(1)) < 0$ (since point B must lie to the right of S_1S_1'), $\partial_2 W^{PP}(p_1^{PP}(1), p_2^{PP}(1)) < 0$ (since point B must lie above S_2S_2'), $\partial_1 W^{PP}(p_1^{PP}(1), p_2^{PP}(1)) = \partial_2 W^{PP}(p_1^{PP}(1), p_2^{PP}(1))$ (since point B must lie on $p_1 = p_2$ line and the welfare function is symmetric) and (applying Assumption 2.7) we also have $-1 < (dR_2^{PP}(p_1)/dp_1) < 0$. Thus, at $\theta = 1$, $\partial_\theta \overline{W}^{PP}(\theta) = \frac{d\overline{W}^{PP}(\theta)}{d\theta} < 0$. Given $\frac{d\overline{W}^{PP}(\theta)}{d\theta} > 0$ at $\theta = 0$ and $\frac{d\overline{W}^{PP}(\theta)}{d\theta} < 0$ at $\theta = 1$, and, given the second order condition $\frac{d^2\overline{W}^{PP}(\theta)}{d\theta^2} < 0$, it follows that the optimal stage 1 choice of θ is some θ^* that lies in the open interval $(0,1)$. Hence, at $\theta = \theta^*$ the equilibrium price vector $(p_1^{PP}(\theta^*), p_2^{PP}(\theta^*))$ must belong to the interior of the segment AB (say some point like D in Figure 2.2) where the iso-welfare curve is tangent to the R_2R_2' curve.

Step (ii): On the (p_1, p_2) plane, for any given $\theta \in [0,1]$, all points in $\mathcal{SV}_1^{PP}(\theta)$ must lie to the left all points in $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$.

Proof of Step (ii): In Stage 2, if Firm 2 chooses price strategy, then Firm 1's reaction function is given by

$$\partial_1 V_1^{PP}(p,\theta) = (p_1 - m)\partial_1 F_1^{PP}(p) + \theta F_1^{PP}(p) + (1-\theta)\partial_1 F_2^{PP}(p) = 0. \tag{2.20}$$

In Stage 2, if Firm 2 chooses quantity strategy, then Firm 1's reaction function is

$$\partial_1 V_1^{QQ}(q,\theta) = F_1^{QQ}(q) - m + \theta q_1 \partial_1 F_1^{QQ}(q) = 0. \tag{2.21}$$

If $\hat{p} = (\hat{p}_1, \hat{p}_2)$ is a solution to (2.21), then $\hat{p}_1 - m + \theta F_1^{PP}(\hat{p}_1,\hat{p}_2)\partial_1 F_1^{QQ}(F_1^{PP}(\hat{p}_1,\hat{p}_2), F_2^{PP}(\hat{p}_1,\hat{p}_2)) = 0$ implying that $\hat{p}_1 - m = -\theta F_1^{PP}(\hat{p}_1,\hat{p}_2)\partial_1 F_1^{QQ}(F_1^{PP}(\hat{p}_1,\hat{p}_2), F_2^{PP}(\hat{p}_1,\hat{p}_2))$. At $(\hat{p}_1,\hat{p}_2)$, $\partial_1 V_1^{PP}(\hat{p}_1,\hat{p}_2,\theta) = (\hat{p}_1 - m)\partial_1 F_1^{PP} + \theta F_1^{PP} + (1-\theta)\partial_1 F_2^{PP} = \theta\left(1 - \partial_1 F_1^{PP}(\hat{p})\partial_1 F_1^{QQ}(\hat{q})\right) F_1^{PP} + (1-\theta)\partial_1 F_2^{PP}(\hat{p}) < 0$. Therefore, given any $\theta \in [0,1]$, we have $SV_1^{PP}(\hat{p}_2,\theta) < \hat{p}_1$, that is, all points satisfying $p_1 = SV_1^{PP}(p_2,\theta)$ must lie to the left of all points in $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$.

Step (iii): In Stage 2, choosing price is the dominant strategy for Firm 2.

Proof of Step (iii): When Firm 1 chooses price strategy, the curve R_2R_2' is the reaction function of Firm 2. Therefore, the singleton set $\mathcal{SV}_1^{PP} \cap \mathcal{R}_2^{PP}$ must lie to the left of all points in the set $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta)) \cap \mathcal{R}_2^{PP}$. Therefore, at any given θ, if Firm 1 chooses price strategy, then it is always optimal for Firm 2 to choose price strategy. When Firm 1 chooses quantity strategy, the set of points $\mathcal{P}(\mathcal{R}_2^{QQ})$ represent the reaction function of Firm 2 in terms of prices. Again, like Lemma 2.7, one can show that if we generate the Cournot equilibrium path in the price space by changing θ from 0 to 1 and plotting the corresponding price vector, then, along that Cournot equilibrium path, as we move from price vector $(p_1^{QQ}(0), p_2^{QQ}(0))$ to price vector $(p_1^{QQ}(1), p_2^{QQ}(1))$ the underlying θ increases. Like Lemma 2.2(ii), one can also show that $\partial_\theta p_1^{QQ}(\theta) > 0$. Hence, along that Cournot equilibrium path, p_1 also increases. By Assumption 2.5, $\partial_1 \pi_2^{QQ}(q) > 0$ and

$\partial_1 F_1^{PP} < 0$ implying $d\pi_2^{QQ}(F_1^{PP}(p), R_2^{QQ}(F_1^{PP}(p)))/dp_1 = \partial_1 \pi_2^{QQ}(q_1)\partial_1 F_1^{PP}(p) < 0$. Therefore, along the Cournot equilibrium path, the profit of the Firm 2 decreases as we move from $(p_1^{QQ}(0), p_2^{QQ}(0))$ to $(p_1^{QQ}(1), p_2^{QQ}(1))$. Since, by Step (ii) for any θ the set of point $\mathcal{SV}_1^{PP}(\theta)$ lie to the left of the set of points in $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$, the profit associated with the point in the singleton set $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta)) \cap \mathcal{P}(\mathcal{R}_2^{QQ})$ is less than profits from all point in the set $\mathcal{SV}_1^{PP}(\theta) \cap \mathcal{P}(\mathcal{R}_2^{QQ})$. Therefore, at any given θ, if Firm 1 chooses quantity strategy, then also it is optimal for Firm 2 to choose price strategy.

Step (iv): In Stage 2, if Firm 2 chooses price strategy, then Firm 1 also chooses price strategy.

Proof of Step (iv): On the region lying above the set $\mathcal{T} := \{p \mid p_2 \geq m, p_1 + p_2 \geq 2m\}$, $\partial_2 V_1^{PP}(p, \theta) = \theta \partial_2 \pi_1^{PP}(p) + (1 - \theta)\partial_2 W^{PP}(p) < 0$. Therefore, $V_1^{PP}(p, \theta)$ is decreasing in p_2 for all points in the set $\mathcal{SV}_1^{PP}(\theta) \cap \mathcal{T}$. Again, the reaction function of Firm 2 given Firm 1 chooses price (that is, the R_2R_2' curve in Figure 2.2) lies above the line $p_2 = m$ and each point in this reaction function lies below all points in the set $\mathcal{P}(\mathcal{R}_2^{QQ})$. Moreover, all points on the AB segment in Figure 2.2 is contained in $\mathcal{T}$ and for all points on the AB segment we have $dV_1^{PP}(SV_1^{PP}(p_2, \theta), p_2)/dp_2 < 0$. Therefore, at the intersection point of the R_2R_2' curve and the $p_1 = SV_1^{PP}(p_2, \theta)$ curve, we value of $V_1^{PP}(p, \theta)$ is higher compared to all points in the set $\mathcal{SV}_1^{PP} \cap \mathcal{P}(\mathcal{R}_2^{QQ})$. Hence, given Firm 2 chooses price strategy, it is always optimal for Firm 1 to choose price strategy. Hence, Step (iv) follows.

Step (iii) and Step (iv) shows that given any $\theta \in [0, 1]$, price competition is the only Nash equilibrium of the sub-game starting from Stage 2. By Step (i), at some $\theta^*(\in (0, 1))$, the government maximizes welfare under price competition. Therefore, the strategy combination $(\theta^{PP} = \theta^*, (P, p_1^{PP}(\theta^{PP})), (P, p_2^{PP}(\theta^{PP})))$ is the unique SPNE of Γ.

Chapter 3

Bertrand-Cournot comparison for oligopolistic industry with vertically integrated firm

3.1 Introduction

An important contribution in the Bertrand-Cournot comparison literature is by Arya et al. (2008b). Arya et al. (2008b) consider a vertically related duopoly market with imperfect substitutes that consists of one vertically integrated producer and a downstream rival firm. In the retail market, firms compete with each other while in the upstream market the vertically integrated firm is a monopolist. The vertically integrated firm supplies a necessary input to its downstream division and the vertically integrated firm also supplies the same input to its downstream rival. They show that the standard Bertrand-Cournot rankings (of Singh and Vives (1984)) get reversed.

There are many real life situations where firms compete through a vertically related market. In the gasoline market, gasoline refiners supply key input to its retail competitor. Under e-commerce, the manufacturers are in direct competition with its retailers (Arya et al. (2008b)). In telecommunication market, firms often purchase and rent their network to their competitor(s) (Weisman and Kang (2001)) and simplilar type of competition is also found in case of wireless telecommunication market (Banerjee and Dippon (2009)). In case of smart phone market, Samsung electronics not only acts as a subsidiary of Samsung smart phone manufacturer but also supplies microprocessors to Apple Inc to produce iPhone (Qing et al. (2017)).

In this chapter we extent the duopoly structure of Arya et al. (2008b) to oligopoly. Specifically, we consider a vertically related differentiated product industry where a vertically integrated firm acts as a monopolist supplying a necessary input not only to it's downstream division but also to all its downstream rivals. Given this market structure, we separately analyze both the short run and the long run outcomes of the industry under price and quantity competitions. Our first result shows that the ranking of profit of each downstream rival of the vertically integrated firm under price and quantity competitions (as provided in Arya et al. (2008b)) is sensitive to the degree of downstream competition. We also analyze how the results of Arya et al. (2008b) are affected with this change in the degree of competition. Our short run analysis not only provides some interesting facts about the equilibrium behavior of the vertically integrated firm but it also highlights some fundamental differences between price and quantity competition in this context. Our long run analysis shows that price competition always gives higher profit to the vertically integrated firm compared to quantity competition though the welfare ranking depends on the entry cost of the rival firms.

The chapter is organized as follows. In Section 3.2, we introduce the preliminaries of the chapter. In Section 3.3, we analyzes the short run of the model. In Section 3.4 we analyzes the long run of the model. Finally in Section 3.5 we have the appendix which contains all prove of this chapter.

3.2 Preliminaries

In this study we consider an economy consisting of a competitive sector and a vertically related sector. The former sector producing a numéraire good (money) y while the latter not only produces a imperfect substitute product (as a final commodity) but also the necessary input that is required to produce that final commodity. The vertically related sector, containing both the upstream market/ input market where the necessary input is traded and the downstream market/ retail market where the final commodity is traded. The vertically related sector consists of a set $N = \{0, D_1, \ldots, D_n\}$ of firms where firm 0 is a vertically integrated firm (VIP) whose upstream division U_0 has monopoly in the upstream market/ input market and suppling input not only to its own downstream division D_0 but also to all its n downstream rivals $D_1, \ldots, D_n$. The set $N_D = \{D_1, \ldots, D_n\}$ denotes the set of all downstream rivals of VIP. We assume that the input is traded through the upstream market and is used to produce final commodity on a one-to-one basis, that is, one unit of necessary input is required to produce one unit of the final output. Figure 3.1 below illustrates the market structure of vertically related sector where arrows depicts the flow of input from U_0 to all the rival firms of the vertically integrated firms. The vertical box refers to the fact that U_0 and D_0 are vertically integrated. Finally, the horizontal box refers all the rival firms of VIP operating in the downstream market. For each firm $i \in N$, the quantity

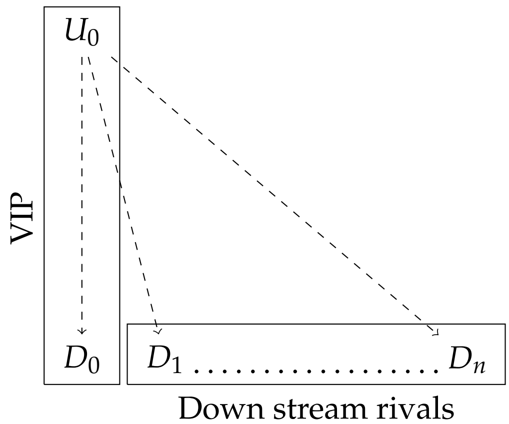


Figure 3.1: Market Structure

produced is denoted by q_i and the price charged is denoted by p_i.

3.2.1 Demand side

We assume that the utility function of the representative consumer is quasi-linear and of the form $\mathcal{U}(\boldsymbol{q}, y) = U(\boldsymbol{q}) + y$ where $\boldsymbol{q} = (q_0, q_1, \ldots, q_n)$ is the vector of outputs.

Specifically, we assume that $U(\boldsymbol{q})$ is quadratic and is of the following form:

$$U(\boldsymbol{q}) = a \sum_{k \in N} q_k - \frac{1}{2} \left(\sum_{k \in N} q_k^2 + s \sum_{k \in N} \sum_{k' < k} q_k q_{k'} \right), \tag{3.1}$$

where $a > 0$ is the taste parameter and $s \in (0,1)$ indicates the common degree of substitutability across the firm specific commodities. The representative consumer maximizes $\mathcal{U}(\boldsymbol{q}, y) = U(\boldsymbol{q}) + y$ by choosing $\boldsymbol{q}$ and y subject to the budget constraint $\sum_{k \in N} p_k q_k + y \leq I$ where I be the income of the representative consumer. Due to quasi-linear specification of the utility function, the consumer's problem reduces to maximize consumer surplus $CS = U(\tilde{\boldsymbol{q}}) - \sum_{k \in N} p_k q_k$. The first order condition of consumer's optimization gives us the inverse demand functions of each firm $i \in N$ and has the following form:

$$P_i(\boldsymbol{q}) = a - q_i - s \sum_{k \in N \setminus \{i\}} q_k. \tag{3.2}$$

Given the inverse demand system resulting from the use of condition (3.2) for each $i \in N$, one can obtain the direct demand function of each $i \in N$ and it is given by

$$D_i(\boldsymbol{p}) = \frac{a}{1+ns} + \frac{1+(n-1)s}{(1-s)(1+ns)} p_i + \frac{s}{(1-s)(1+ns)} \sum_{j \in N \setminus \{i\}} p_j. \tag{3.3}$$

The expression for the consumer surplus after incorporating conditions (3.1) and (3.2) yields

$$CS(\boldsymbol{q}) = \frac{\sum_{k \in N} q_k^2 + s \sum_{k \in N} \sum_{k' \in N \setminus \{k\}} q_k q_{k'}}{2}. \tag{3.4}$$

3.2.2 Supply side and welfare

Let m_0 denote the constant marginal cost of production of the final commodity for firm 0 and m denote the common constant marginal cost of all the rival firms. Further the production of the final commodity requires no fixed cost. Suppose z is the input price that VIP charges for each unit of input it sales in the upstream market to each of its downstream rivals. Therefore, the cost of firm 0 to produce q_0 unit of final commodity is $C_0(q_0) = m_0 q_0$ and that of any firm $i \in N_D$ to produce q_i unit of final commodity is $C_i(q_i) = (m+z)q_i$. We also assume that firm 0 has cost advantage over all its downstream rivals, that is, $m_0 < m < a$. Therefore, the profit of Firm 0 is

$$\pi_0 = (p_0 - m_0)q_0 + z \sum_{j \in N_D} q_j, \tag{3.5}$$

where $(p_0 - m_0)q_0$ is the profit earned from the downstream market by selling final good and $z \sum_{j \in N_D} q_j$ is the profit earned from the upstream market by selling necessary input of production. The profit of each rival firm $i \in N_D$ is

$$\pi_i = (p_i - z - m)q_i. \tag{3.6}$$

Define the joint profit from the industry as $\pi^J := \sum\limits_{k \in N} \pi_i$ and again using condition (3.2) we get

$$\pi^J = (a - m_0)q_0 + (a - m) \sum_{k \in N_D} q_k - \left[\sum_{k \in N} q_k^2 + s \sum_{k \in N} \sum_{k' \in N \setminus \{k\}} q_k q_{k'} \right]. \tag{3.7}$$

Finally, the welfare of the society is $W := CS + \pi^J$ and using conditions (3.4) and (3.7) we get

$$W(\boldsymbol{q}) = (a - m_0)q_0 + +(a - m) \sum_{k \in N_D} q_k - \frac{1}{2} \left[\sum_{k \in N} q_k^2 + s \sum_{k \in N} \sum_{k' \in N \setminus \{k\}} q_k q_{k'} \right]. \tag{3.8}$$

3.3 The short run

In this section, we analyze the short run interaction amongst the firms. By short run we mean that the number of firms in the market is fixed. We restrict our analysis to two very fundamental market games: First, the market where we have quantity competition in the downstream market which we call downstream Cournot competition and we denote it by Γ_n^C. Second, the market where we have price competition in the downstream market which we call downstream Bertrand competition and we denoted it by Γ_n^B. Though Γ_n^C and Γ_n^B differ in terms of the strategy space, they share the same stages which are defined below:

Stage-I: In this stage there is interaction only in the upstream market and the VIP decides at what common price it will sell the necessary input to all it's downstream rivals.

Stage-II: In this stage firms compete in the downstream market. In particular, in case of Γ_n^C (Γ_n^B) firms compete in quantity (price).

Before stating and elaborating our results, we briefly discuss all the stages of both downstream Cournot competition and downstream Bertrand competition.

3.3.1 The downstream Cournot competition (Γ_n^C)

Stage-II choice of Γ_n^C: In case of downstream Cournot competition (Γ_n^C), if in the Stage-I the upstream division of Firm 0 charges the input price z, then, in the Stage-II, the downstream division of Firm 0 along with all it's downstream rivals compete in the downstream market by producing optimal quantity of their respective retail output. Formally in Stage-II, Firm 0 chooses q_0 to maximize it's profit $\pi_0^C(\boldsymbol{q}) = (P_0(\boldsymbol{q}) - m_0)q_0 + z\sum_{j\in N_D} q_j$ given the output of all it's downstream rivals N_D and each rival firm $D_i \in N_D$ chooses q_i to maximize $\pi_i^C(\boldsymbol{q}) = (P_i(\boldsymbol{q}) - z - m)q_i$ given the retail output of the Firm 0 and the output of all other downstream firms. Let the Stage-II optimal output vector be $\boldsymbol{q}^C(z,n) = (q_0^C(z,n), q_1^C(z,n), \ldots, q_n^C(z,n))$. Then $\boldsymbol{q}^C(z,n)$ must satisfy the first order condition $\partial_0\pi_0^C(\boldsymbol{q}^C(z,n)) = 0$ for Firm 0 and $\partial_i\pi_i^C(\boldsymbol{q}^C(z,n)) = 0$ for each firm $D_i \in N_D$. Since all rival are symmetric, we get equal optimal choice of quantity across these firms, that is, $q_j^C(z,n) = q^C(z,n)$ for all $j \in N_D$. Optimization exercise gives

$$q_0^C(z,n) = \frac{(2-s)(a-m_0) + ns(m-m_0) + nsz}{(2-s)(2+ns)} \tag{3.9}$$

and

$$q^C(z,n) = \frac{(a-m) + \bar{G} - 2z}{(2-s)(2+ns)}, \tag{3.10}$$

where $\bar{G} := (a-m) - s(a-m_0)$. Hence the resulting two-stage profit of Firm 0 is

$$\pi_0^C(\boldsymbol{q}^C(z,n)) = \left(q_0^C(z,n)\right)^2 + nzq^C(z,n) \tag{3.11}$$

and that of any firm $D_i \in N_D$ is

$$\pi_i^C(\boldsymbol{q}^C(z,n)) = \left(q^C(z,n)\right)^2. \tag{3.12}$$

The Stage-I choice of Γ_n^C: In this stage upstream division of Firm 0 choose it's optimum input price $z^C(n)$ to maximizes it's stage-II profit given by (3.11). Therefore $z^C(n)$ we must satisfy the first order condition, $\partial_z \pi_0^C \left(\boldsymbol{q}^C(z^C(n), n)\right) = 0$. The solution is

$$z^C(n) = \frac{a-m}{2} - \frac{ns^2\bar{G}}{2\left[4\left(2+(n-1)s\right) - 3ns^2\right]}. \tag{3.13}$$

3.3.2 The downstream Bertrand competition (Γ_n^B)

Stage-II choice of Γ_n^B: Given the stages of the downstream Bertrand competition (Γ_n^B), if in the Stage-I, the upstream stream division of Firm 0 charges the input price z from it's downstream rivals, then, given this value of z, in Stage-II, the downstream division of Firm 0 along with all it's downstream rivals compete in the downstream market by optimally selecting price to be charged for their respective retail commodity. Specifically, in Stage II, Firm 0 chooses p_0 to maximize it's profit $\pi_0^B(\boldsymbol{p}) = (p_0 - m_0)D_0(\boldsymbol{p}) + z\sum_{j\in N_D} D_j(\boldsymbol{p})$ taking the price of all it's downstream rivals as given and each firm $D_i \in N_D$ chooses it's price p_i to maximize $\pi_i^B(\boldsymbol{p}) = (p_i - z - m)D_i(\boldsymbol{p})$ given the price of Firm 0 and the price of all others downstream firms. Let the Stage-II optimal price vector be $\boldsymbol{p}^B(z,n) = (p_0^B(z,n), p_1^B(z,n), \ldots, p_n^B(z,n))$. Then $\boldsymbol{p}^B(z,n)$ must satisfy the following set of first order conditions: $\partial\pi_0^B(\boldsymbol{p}^B(z,n))/\partial p_0 = 0$ for

Firm 0 and $\partial \pi_i^B(\boldsymbol{p}^B(z,n))/\partial p_i = 0$ for each $D_i \in N_D$. Optimal solution gives:

$$p_0^B(z,n) = m_0 + \frac{\begin{bmatrix} (2+(n-1)s)(1-s)(1+ns)(a-m_0) - ns(1+(n-1)s)\bar{G} \\ +ns(3+2(n-1)s)z \end{bmatrix}}{2(1+(n-1)s)(2+(n-1)s) - ns^2} \tag{3.14}$$

and

$$p^B(z,n) = z + m + \frac{(1-s)(1+ns)(a-m) + (1+(n-1)s)\bar{G} - 2(1-s)(1+ns)z}{2(1+(n-1)s)(2+(n-1)s) - ns^2}. \tag{3.15}$$

The resulting quantity of Firm 0 is

$$q_0^B(z,n) := D_0(\boldsymbol{p}^B(z,n)) = \frac{(1+(n-1)s)(p_0^B(z,n) - m_0)}{(1-s)(1+ns)} \tag{3.16}$$

and the resulting quantity of each downstream rival is

$$q^B(z,n) := D_i(\boldsymbol{p}^B(z,n)) = \frac{(1+(n-1)s)(p^B(z,n) - m - z)}{(1-s)(1+ns)}. \tag{3.17}$$

Thus, the resulting aggregate profit of Firm 0 (given z) is

$$\pi_0^B(\boldsymbol{p}^B(z,n)) = \frac{(1+(n-1)s)(p_0^B(z,n) - m_0)^2 - nsz(p_0^B(z,n) - m_0)}{(1-s)(1+ns)} + nzq^B(z,n) \tag{3.18}$$

and that of any firm $D_i \in N_D$ is

$$\pi_i^B(\boldsymbol{p}^B(z,n)) = \frac{(1+(n-1)s)\left(p^B(z,n) - m - z\right)^2}{(1-s)(1+ns)}. \tag{3.19}$$

Stage-I choice of Γ_n^B In Stage-I, Firm 0 selects the input price $z^B(n)$ to maximizes it's Stage-II profit given by (3.18). Therefore, the optimal $z^B(n)$ must satisfy following first order condition $\partial_z \pi_0^B(\boldsymbol{p}^B(z^B(n),n)) = 0$ and solving for $z^B(n)$ we get,

$$z^B(n) = \frac{a-m}{2} - \frac{ns^2\bar{G}}{2\left[4\left(1+(n-1)s\right)^2\left(2+(n-1)s\right) + ns^2\right]}. \tag{3.20}$$

3.3.3 The short run results

In this subsection we present all our short run results. The first one is relating to the foreclosure condition. Since Firm 0 is sole supplier of the input it has the ability to foreclose all other downstream rival and become an monopolist in the downstream market. However, foreclosure also terminates Firm 0's sales of input to its downstream rivals. Therefore, when wholesale profit by selling input is 'high' enough, Firm 0 will not foreclose. The non-foreclosure condition is stated in the next Lemma 3.1

LEMMA **3.1** Under both Γ_n^B and Γ_n^B, the vertically integrated firm forecloses all its downstream rivals if and only if $\bar{G} = (a-m) - s(a-m_0) \leq 0$ or $(a-m)/(a-m_0) \leq s$.

The foreclosure condition states that if products are sufficiently heterogeneous, then Firm 0 will never forecloses it's downstream rivals. Note that the foreclosure condition in 3.1 is identical to the foreclosure condition stated in the Lemma 1 of Arya et al. (2008b). Therefore, the foreclosure condition stated by Arya et al. (2008b) is robust for oligopolistic framework as well. Like Arya et al. (2008b), we also assume that the non-foreclosure condition holds, that is, we assume

$$\bar{G} = (a-m) - s(a-m_0) > 0. \tag{NF}$$

Given (NF) one can show that, the ranking of the equilibrium profit of Firm 0 and the welfare of the society, between Γ_n^C and Γ_n^C, as provided by Arya et al. (2008b) are also true for any number of firms. Like Arya et al. (2008b) (see Lemma 2 in Arya et al. (2008b)), in our context Firm 0 always sets higher input price under Γ_n^B than under Γ_n^C, that is, $z^B(n) > z^C(n)$ for all $n \geq 1$. But there are some fundamental differences between Γ_n^B and Γ_n^C which to the best of our knowledge is not available in the earlier literature. The Lemma 3.2 below show one of such difference.

LEMMA **3.2** For any positive integer $n \geq 1$, the following holds:

(i) $z^C(n)$ is monotonically decreasing in n and $z_\infty^C := \lim_{n\to\infty} z^C(n) = (a-m)/2 - (s\bar{G}/(8-6s))$.

(ii) If $\max\{1, \hat{n}(s)\} = 1$, then for all $n \geq 1$, $z^B(n)$ is increasing in n. If $\max\{1, \hat{n}(s)\} > 1$, then, for all $n > \hat{n}(s)$, $z^B(n)$ is increasing in n, and, for all $n < \hat{n}(s)$, $z^B(n)$ is decreasing in n where $\hat{n}(s) := [(-(2-s) + \sqrt{(2-s)(10-9s)})/4s]$. Further $z^B_\infty := \lim_{n\to\infty} z^B(n) = (a-m)/2$.

Lemma 3.2 states how the equilibrium input price behaves with n and establishes a fundamental difference between Γ^C_n and Γ^B_n. Specifically, Lemma 3.2 states that Firm 0 always induces its rivals to produce more under Γ^C_n as n increases by reducing the per-unit cost for the rivals and induces to produce less under Γ^C_n as n increases by increasing the per-unit cost of the rivals. Therefore, under Γ^B_n, Firm 0 becomes more protective to its downstream division as competition in the downstream market increases but, under Γ^C_n, Firm 0 becomes less protective to its downstream division as competition in the downstream market increases. In Figure 3.2, we illustrate the exact

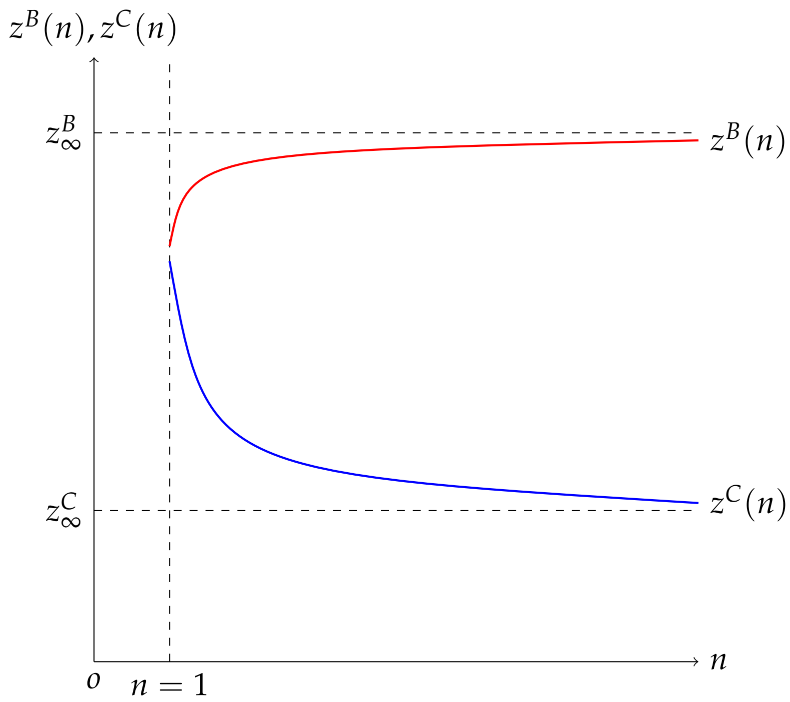


Figure 3.2: Pattern of $z^B(n)$ and $z^C(n)$.

pattern of $z^C(n)$ and $z^B(n)$ with respect n as explain by Lemma 3.1. Although given any number of firm, the ranking of equilibrium profit of Firm 0 and the equilibrium welfare between Γ^C_n and Γ^C_n are same as the ranking provided by Arya et al. (2008b), the ranking of the profit of each downstream rival depends on both size of the downstream competition and degree of product differentiation.

PROPOSITION **3.1** Under both Γ_n^B and Γ_n^C, the equilibrium profit of any downstream rival is decreasing in size of the number of firms competing in the downstream market. Further, the relative ranking of the equilibrium profit of any downstream rival under Γ_n^B and Γ_n^C is sensitive to the degree of substitution s as well as n.

Specifically, if $((a-m)/(a-m_0)) \geq 8/9$ holds for all $s \in (0, 8/9)$, then there exist a $\tilde{n}(s)$ such that (a) for all $n > \tilde{n}(s)$, Γ_n^C yields higher profit of any rival firm compared to Γ_n^B and (b) for all $n < \tilde{n}(s)$, the reverse holds. If $s \in [8/9, ((a-m)/(a-m_0)))$, then Γ_n^B always yields higher profit of any rival firm compared to Γ_n^C. Finally, if $((a-m)/(a-m_0)) < 8/9$, then for all $s \in (0, ((a-m)/(a-m_0)))$ there always exist a $\tilde{n}(s)$ such that for all $n > \tilde{n}(s)$, Γ_n^C yields higher profit of any rival firm compared to Γ_n^B.
Proposition 3.1 suggests that for large n, the ranking of the equilibrium profit of any rival, as provided by Arya et al. (2008b) may change. In Figure 3.5, the shaded region shows all possible combinations of (s, n) for which there is a ranking reversal relative to Arya et al. (2008b) (only in terms of profit of the rivals). In Figure 3.5, the thick curve is the implicit locus of the (s, n) for which the profit of any downstream firm under Γ_n^C is equal to that under Γ_n^B.

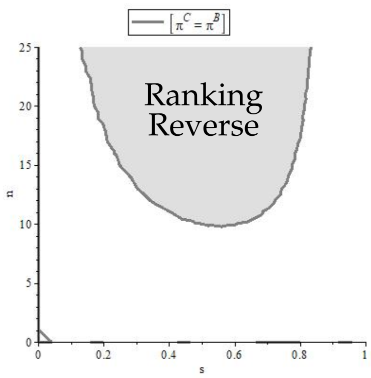
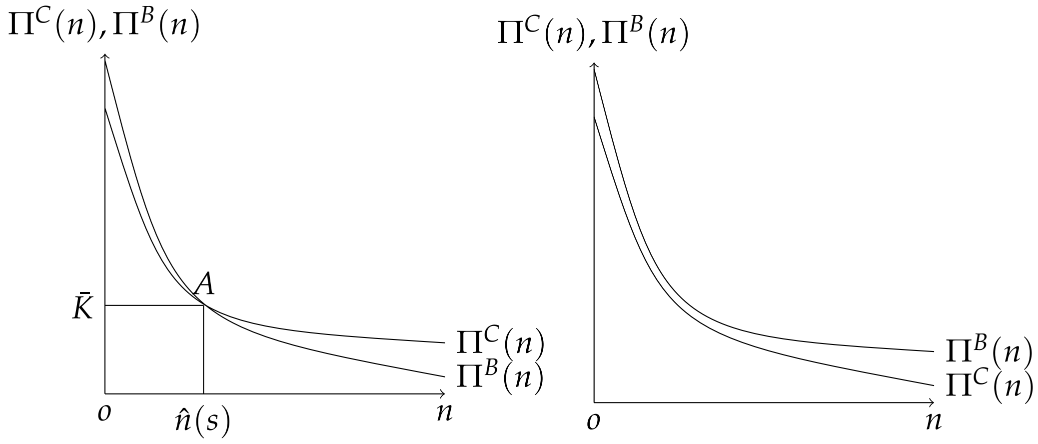


Figure 3.3: Region of reversal.

In Lemma 3.2 we show that when competition increases, Firm 0 becomes more protective of it's own downstream division under Γ_n^B by charging higher input price where as under Γ_n^C as competition increases Firm 0 charges lower input price. Hence, given any s and after a certain level of competition, the cost of outsourcing under Γ_n^B is significantly more than that under Γ_n^C. As a result profit ranking gets reversed. Figure 3.4 helps in explaining Proposition 3.1.



(a) For $s \in (0, 8/9)$ (b) For $s \in [8/9, (a-m)/(a-m_0))$

Figure 3.4: Behavior of downstream profit.

In Proposition 3.2 that follows we show that as the degree of competition increases, there is a loss in profit for the vertically integrated firm due to new entry and there is also a gain in profit for the vertically integrated firm due to increase in sales of input to these new entrants. The resulting aggregate profit of the vertically integrated firm goes up since the loss from the downstream market is less than the gain from the upstream market.

PROPOSITION **3.2** In case of both Γ_n^B and Γ_n^C, the equilibrium profit of Firm 0 in increasing in n.

Proposition 3.2 is a consequence of two effects. First, new entry enhances profit at the pre-entry input price (since the profit curve shifts upwards at the relevant stretch). Second, new entry will allow Firm 0 to increase its profit even further by adjusting the input price optimally (that is, due to optimal shift along the new profit curve).

Under Γ_n^B, as n goes to $(n+1)$, in equilibrium $z^B(n)$ *increases* to $z^B(n+1)$. As a consequence the revenue that results for Firm 0 from the upstream market more than compensates for the loss from downstream market (due to more entry) thereby increasing the aggregate profit of Firm 0. Unlike Γ_n^B, in Γ_n^C, as n goes to $(n+1)$, in equilibrium $z^C(n)$ *decreases* to $z^C(n+1)$. However, despite the fall in the per-unit price of inputs there is a more than offsetting rise in the total input sold and as a result the upsteam market revenue earned by Firm 0 increases so as to outweigh the loss in profit from the downstream market thereby increasing the aggregate profit of Firm 0. This is the qualitative difference in the profit rise of Firm 0 under Γ_n^B and Γ_n^C.

3.4 The long run

In this section, we analyze the long run interaction amongst the firms. By long run we mean that any firm in the downstream market can enter by incurring a fixed entry cost $K > 0$. Like the short run analysis, we restrict our analysis to two very fundamental market games: First, where all firms compete in quantity in the downstream market which we call downstream Cournot competition with entry and we denote it by Γ_K^C. Second, where all firms compete in prices in the downstream market which we call downstream Bertrand competition with entry and we denote it by Γ_K^B. Although Γ_K^C and Γ_K^B differ in terms of the strategy space, they share the same stages as defined below:

Stage-I In this stage the numbers of downstream rivals are determine using zero profit condition.

Stage-II In this stage there is interaction in the upstream market. Specifically, Firm 0 decides the common price at which it will sell the necessary input to all it's downstream rivals.

Stage-III In this stage firms compete in the downstream market. Specifically, for Γ_K^C (Γ_K^B) all firms compete with quantity (price).

Before getting into the details of our results, we briefly discuss all the stages of both the downstream Cournot competition with free entry and downstream Bertrand competition with free entry. Observe that the final two stages of Γ^C_K (Γ^B_K) are identical to Γ^C_n (Γ^B_n). Only difference is the analysis of the Stage-I for which we requires a zero profit condition for both Γ^C_K and Γ^B_K. To find this zero profit condition one needs to substitute (3.13) in (3.12) which gives the equilibrium profit that any firm $D_i \in N_D$ earns under Stage-II of Γ^C_K and it is given by

$$\Pi^C(n) := \pi^C(\boldsymbol{q}^C(z^C(n), n)) = \frac{4\bar{G}^2}{[4(2+(n-1)s) - 3ns^2]^2}. \tag{3.21}$$

Similarly, by substituting (3.20) in (3.19) we get the equilibrium profit of any firm $D_i \in N_D$ earns under Stage-II of Γ^B_K and it is given by

$$\Pi^B(n) := \pi^B(\boldsymbol{p}^B(z^B(n), n)) = \frac{(1+(n-1)s)[2\,(1+(n-1)s)^2 + ns^2]^2\bar{G}^2}{(1-s)(1+ns)[4\,(1+(n-1)s)^2\,(2+(n-1)s) + ns^2]^2}. \tag{3.22}$$

Suppose n^C and n^B respectively denote the equilibrium number of firms under Γ^C_K and Γ^B_K. Then the zero profit condition can be written as

$$\Pi^C(n^C) = K = \Pi^B(n^B). \tag{3.23}$$

By substituting the expression of $\Pi^C(n^C)$ and $\Pi^B(n^B)$ in (3.23) and then solving for n^C one obtains the following relation between n^C and n^B:

$$n^C(n^B) = -\frac{4(2-s)}{s(4-3s)} + \frac{2A(s,n^B)\left[4(1+(n^B-1)s)^2(2+(n^B-1)s) + n^Bs^2\right]}{s(4-3s)\left[2(1+(n^B-1)s)^2 + n^Bs^2\right]} \tag{3.24}$$

where $A(s,n^B) = \sqrt{[(1-s)(1+n^Bs)]/[(1+(n^B-1)s)]}$. Observe that $dn^C/dn^B = (\partial\Pi^B(z^B(n),n)/\partial n)/(\partial\Pi^C(z^C(n),n)/\partial n) > 0$ since $\partial\Pi^B(z^B(n),n)/\partial n < 0$ and $\partial\Pi^C(z^C(n),n)/\partial n < 0$, that is, n^C is positively related to n^B and one can also show that n^C increases at an increasing rate with n^B. For the remaining analysis we restrict ourselves to situation where in equilibrium at least one firm survives. This means that we put an upper bound over entry cost K such that under both Cournot and Bertrand competitions there is at least one down stream rival firm. Specifically, we restrict our attention to the set $S_{n^B} = \{(s,n^B) \mid n^B \geq 1, 0 < s < 1, n^C(n^B) \geq 1\}$ which is shown in the Figure 3.5.

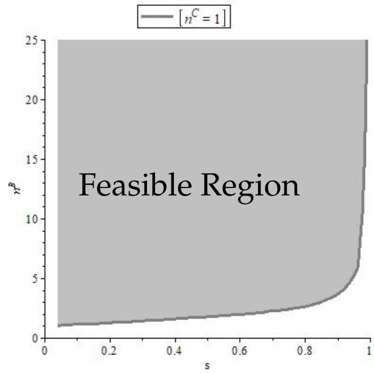


Figure 3.5: Feasible values of n^B for Long Run.

3.4.1 The long run results

LEMMA **3.3** Following can be said about the relation between n^B and n^C:

(i) If either $(a-m)/(a-m_0) < 8/9$ holds or if, $(a-m)/(a-m_0) \geq 8/9$ and $s \in (0, 8/9)$, holds, then there always exist a $\bar{K}$ such that for all $K > \bar{K}$ we have $n^B > n^C$ and for all $K < \bar{K}$ we have $n^B < n^C$.

(ii) If $(a-m)/(a-m_0) \geq 8/9$ and $s \in [8/9, (a-m)/(a-m_0))$ holds, then for all $K > 0$ we have $n^B > n^C$.

The explanation of Lemma 3.3 (i) follows from Figure 3.4 (a) where we have added a horizontal line representing the fixed cost of magnitude $O\bar{K}$ that represents the case $n^B = n^C$. For any fixed cost $K > \bar{K}$, we have $n^B > n^C$ and for any fixed cost $K < \bar{K}$, we have $n^B < n^C$. The explanation for Lemma 3.3 (ii) follows from Figure 3.4 (b) where no matter what fixed cost we take, $n^B > n^C$.

PROPOSITION **3.3** If for both Γ_K^C and Γ_K^B, at least one firm can survive in equilibrium, then we have following:

(i) Firm 0 always earns higher aggregate profit under Γ_K^B than under Γ_K^C.

(ii) If $m = m_0$, then there exists a range of (s, n^B) such that Γ_K^B yields higher welfare than Γ_K^C.

(iii) In case of both Γ_K^C and Γ_K^B we have excess entry in the long run than the optimal numbers of entry that maximizes the social welfare.

The first part of Proposition 3.3 (i) is an extension of Arya et al. (2008b) and Proposition 3.3 (ii) suggests a reversal of Arya et al. (2008b)'s result for welfare ranking. Proposition 3.3 (i) is an extension of Arya et al. Arya et al. (2008b) by allowing free entry. From Proposition 3.2 we know that the equilibrium profit of VIP is increasing in n for both Γ_n^C and Γ_n^B. Moreover, for any n, profit of VIP under Γ_n^B is more than that under Γ_n^C. Therefore, when entry cost is large enough the equilibrium profit of VIP is higher under Γ_K^B than under Γ_K^C (using using Lemma 3.3). Even when the entry cost is small, profit of VIP under Γ_K^B continues to be larger than that of the VIP under Γ_K^C. The reason being that even if the equilibrium number of firms under Γ_K^C is larger than that under Γ_K^B, it fails to generate large enough input selling profit such that aggregate equilibrium profit of VIP under Γ_K^C can dominate the aggregate equilibrium profit of VIP under Γ_K^B. Proposition 3.3 (ii) is a reversal of Arya et al. (2008b)'s result in terms of welfare rankings when entry cost is large enough. Using Lemma 3.3 we can conclude that if the entry cost is large enough then the number of firm that survives under Γ_K^B is higher than that under Γ_K^C. Therefore, given entry cost is large enough, we have larger aggregate equilibrium output and consumer surplus under Γ_K^B than under Γ_K^C. Furthermore, the increase in equilibrium consumer surplus dominates the decrease in profit of VIP under Γ_K^B in comparison to that under Γ_K^C. Hence, aggregate welfare is higher under Γ_K^B than under Γ_K^C. Finally, one can show that the set of equilibrium outcomes under Γ_n^C and Γ_n^B satisfies all the assumptions of Mankiw and Whinston (1986). Consequently, we have negative business-stealing effect and hence excess entry under free entry equilibrium.

3.5 Conclusion

In this paper, we have identified a fundamental difference between Cournot and Bertrand competition in the presence of a vertically integrated firm that uses the uniform input pricing strategy. In case of the Cournot competition, input price charged by the vertically integrated firm decreases with the number of downstream firms. However, in case of Bertrand competition the optimal input price is increasing with the number of downstream firms only after a threshold level. This difference actually determines the ranking reversal of profit of each downstream rival between these two market competition relative to Arya et al. (2008b) when number of downstream firms is large enough. Given the above mentioned ranking reversal, we have interesting long-run consequences. There exists a critical entry cost level below (above) which we have more (less) equilibrium number of firm under Γ_K^C than under Γ_K^B. It is precisely due to this aspect that we have streches of ranking reversal as well as non-reversal relative to Arya et al. (2008b) in terms of social welfare.

3.6 Appendix

Proof of the Lemma 3.1: By substituting (3.13) in (3.10) we obtained the equilibrium quantity of output,

$$q^C(z^C(n), n) = \frac{2\bar{G}}{[4(2 + (n-1)s) - 3ns^2]},$$

that any rival firm $D_i \in N_D$ produces under Γ_n^C. Since $4(2 + (n-1)s) - 3ns^2 > 0$ for all $n \geq 0$ and $s \in (0, 1)$, for $q^C(z^C(n), n)$ to be positive it is necessary that $\bar{G} > 0$. Hence, for Γ_n^C non-foreclosure condition is $\bar{G} > 0$. Similarly, by substituting (3.20) in (3.17) we get the equilibrium quantity of output,

$$q^B(z^B(n), n) = \frac{(1 + (n-1)s)[2(1 + (n-1)s)^2 + ns^2]\bar{G}}{(1-s)(1+ns)[4(1 + (n-1)s)^2(2 + (n-1)s) + ns^2]},$$

that any rival firm $D_i \in N_D$ produces under Γ_n^B. Again, since for all $n \geq 0$ and for all $s \in (0,1)$, (a) $(1+(n-1)s)/(1-s)(1+ns) > 0$ and (b) the denominator $4(1+(n-1)s)^2(2+(n-1)s)+ns^2 > 0$, from (a) and (b) we can conclude that $q^B(z^B(n),n) > 0$ implies that $\bar{G} > 0$. Therefore the non-foreclosure condition for both Γ_n^C and Γ_n^B is $\bar{G} > 0$. Hence the result.

Proof of Lemma 3.2: *Proof of (i):* To prove the result, we differentiate the equation (3.13) with respect to n to get,

$$\partial_n z^C(n) = -\frac{2s^2(2-s)\bar{G}}{[4(2+(n-1)) - 3ns^2]^2}. \tag{3.25}$$

Clearly, the right hand side of the equation (3.25) is negative, therefore we have $\partial_n z^C(n) < 0$ implying that $z^C(n)$ is monotonically decreasing in n. Finally from equation (3.13) we have

$$z_\infty^C = \frac{a-m}{2} - \lim_{n\to\infty} \frac{s^2\bar{G}}{2\left[\frac{4(2-s)}{n} + 4s - 3s^2\right]} = \frac{a-m}{2} - \frac{s\bar{G}}{8-6s}.$$

Proof of (ii): To prove the result we differentiate equation (3.20) with respect to n we get

$$\partial_n z^B(n) = -\frac{2s^2(1+(n-1)s)\left[(1-s)(2-s) - s(2-s)n - 2n^2s^2\right]\bar{G}}{[4(1+(n-1)s)^2(2+(n-1)s)+ns^2]^2}. \tag{3.26}$$

Given (3.26), $\partial_n z^B(n) > 0$ if and only if $(1-s)(2-s) - s(2-s)n - 2n^2s^2 < 0$ implying $n > \hat{n}(s) = [-(2-s) + \sqrt{(2-s)(10-9s)}]/4s$. Therefore, if $\max\{1, \hat{n}(s)\} = 1$, then $n \geq \hat{n}(s)$ (since $n \geq 1$) and hence $z^B(n)$ is increasing in n. If $\max\{1, \hat{n}(s)\} > 1$, then for all $1 \leq n < \hat{n}(s)$, we have $z^B(n)$ is decreasing in n and for all $n > \hat{n}(s)$, we have $z^B(n)$ is increasing in n. Again form condition (3.20) one can show that

$$z_\infty^B = \frac{a-m}{2} - \frac{s^2\bar{G}}{2\left[\lim\limits_{n\to\infty} 4(1+(n-1)s)^2\left(\frac{2}{n} + (1-\frac{1}{n})s\right) + s^2\right]} = \frac{a-m}{2}.$$

Proof of the Proposition 3.1: Given non-foreclosure condition (NF), both $\Pi^C(n)$ and $\Pi^B(n)$ are decreasing in n (see conditions (3.21) and (3.22)). Consider the difference

$\Delta(n) := \Pi^C(n) - \Pi^B(n)$. Then

$$\Delta(n) = \frac{n\left[\sum_{i=0}^{6} C_i(s)n^i\right]\bar{G}^2}{(1-s)(1+ns)[4(1+(n-1)s)^2(2+(n-1)s)+ns^2]^2[4(2+(n-1)s)-3ns^2]^2}, \tag{3.27}$$

where $C_0(s) := 32s^2(2-s)(4-2s-s^2)(1-s)^3 > 0$ for all $s \in (0,1)$, $C_1(s) := 4s^3(1-s)(9s^5 - 60s^4 + 42s^3 + 276s^2 - 520s + 256) > 0$ for all $s \in (0,1)$, $C_2(s) := 4s^4(36s^5 - 183s^4 + 177s^3 + 386s^2 - 796s + 384) > 0$ for all $s \in (0,1)$, $C_3(s) := s^5(-261s^4 + 985s^3 - 708s^2 - 936s + 992) > 0$ for all $s \in (0,1)$, $C_4(s) := s^6(261s^3 - 732s^2 + 408s + 160) > 0$ for all $s \in (0,1)$, $C_5(s) := -12s^7(12s^2 - 23s + 8) \gtreqless 0$ for all $s \gtreqless \frac{23-\sqrt{145}}{24}$, and, $C_6(s) := 4s^8(8-9s) \gtreqless 0$ for all $s \gtreqless 8/9$. The denominator of condition (3.27) is positive and all $C_i(s)$ are finite for any given $s \in (0,1)$. Therefore, we have to deal with the following three intervals on the substitution parameter s:

Case-I: For $s \in [8/9, 1)$, $C_i(s) > 0$ for all $i = 0, \ldots, 5$ and $C_6(s) \geq 0$. Therefore, given $n \geq 1$, $\lambda(n,s) = \sum_{i=0}^{6} C_i(s)n^i > 0$ and we have $\Delta(n) > 0$.

Case-II For $s \in [(23-\sqrt{145})/24, 8/9)$, $C_i(s) > 0$ for all $i = 0, \ldots, 5$ though $C_6(s) < 0$. Therefore for any $n \geq 1$ we must have $\sum_{i=0}^{5} C_i(s)n^i > 0$ but $C_6(s)n^6 < 0$. Moreover, for sufficiently large n, the absolute value of $C_6(s)n^6$ will dominates the absolute value of $\sum_{i=0}^{5} C_i(s)n^i$ implying that $\Delta(n) < 0$ for sufficiently large n.

Case-III For $s \in (0, (23-\sqrt{145})/24)$, $C_i(s) > 0$ for all $i = 0, \ldots, 4$ though $C_5(s) < 0$ and $C_6(s) < 0$. Therefore for any $n \geq 1$ we must have $\sum_{i=0}^{4} C_i(s)n^i > 0$ but $C_5(s)n^5 + C_6(s)n^6 < 0$. Therefore, for sufficiently large n, the absolute value of $C_5(s)n^5 + C_6(s)n^6$ will dominates the absolute value of $\sum_{i=0}^{4} C_i(s)n^i$ implying that $\Delta(n) < 0$ for sufficiently large n.

From the non-foreclosure condition (NF) we know that $s \in (0, (a-m)/(a-m_0))$. Therefore, given Case-I, Case-II and Case-III, we have the following possibilities:

Possibility-A: If $((a-m)/(a-m_0)) \geq 8/9$, then for all $s \in (0, 8/9)$, there exist a $\tilde{n}(s)$ such that given any $n > \tilde{n}(s)$, $\sum_{i=0}^{6} C_i(s)n^i < 0$. Hence, for n large enough $\Delta(n) < 0$. However, for all $s \in [8/9, ((a-m)/(a-m_0)))$, we always have $\Delta(n) > 0$.

Possibility-B: If $((a-m)/(a-m_0)) < 8/9$, then, for all $s \in (0, ((a-m)/(a-m_0)))$, there exists $\tilde{n}(s)$ such that for all $n > \tilde{n}(s)$, we have $\sum_{i=0}^{6} C_i(s)n^i < 0$. Hence, for n large enough $\Delta(n) < 0$.

Proof of Proposition 3.2: By substituting (3.13) in (3.11), the equilibrium profit of Firm 0 is under Γ_n^C is

$$\Pi_0^C(n) := \pi_0^C(\boldsymbol{q}^C(z^C(n), n)) = \frac{(a-m_0)^2}{4} + \frac{n\bar{G}^2}{4(2+(n-1)s) - 3ns^2}.$$

Similarly by substituting (3.20) in (3.18) the equilibrium profit of Firm 0 under Γ_n^B is

$$\begin{aligned}\Pi_0^B(n) &:= \pi_0^B(\boldsymbol{p}^B(z^B(n), n)) \\ &= \frac{(a-m_0)^2}{4} + \frac{n(1+(n-1)s)^3\bar{G}^2}{(1-s)(1+ns)\left[4(1+(n-1)s)^2(2+(n-1)s)+ns^2\right]}.\end{aligned}$$

Moreover,

$$\partial_n \Pi_0^C(n) = \frac{8\bar{G}^2\,(s-2)\,s\,(3\,s-4)}{\left(3\,ns^2 - 4\,sn + 4\,s - 8\right)^2} > 0,$$

and,

$$\partial_n \Pi_0^B(n) = \frac{\bar{G}^2(1+(n-1)s)^2 \left[\begin{array}{c} 8s^4n^4 + 2s^3(16-11s)n^3 + s^2(48-74s+29s^2)n^2 \\ +16s(2-s)(1-s)^2 n + 4(2-s)(1-s)^3 \end{array}\right]}{(1-s)(1+ns)^2\left[8+4(n-1)^3s^3+(16-31n+16n^2)s^2+20(n-1)s\right]^2} > 0.$$

Hence the result follows.

Proof of the Proposition 3.3: *Proof of (i):* The long run difference in profit for Firm 0, that is, $\Delta(\Pi_0) := \Pi_0^B(n^B) - \Pi_0^C(n^C)$ gives

$$\Delta(\Pi_0) = \frac{\left[\begin{array}{c} 4n^B(1+(n^B-1)s)^2\left[4(1+(n^B-1)s)^2(2+(n^B-1)s)+n^Bs^2\right] \\ -n^C\left[2(1+(n^B-1)s)^2+n^Bs^2\right]^2\left[4\left(1+(n^C-1)s\right)-3n^Cs^2\right] \end{array}\right]K}{4\left[2(1+(n^B-1)s)^2+n^Bs^2\right]^2} \tag{3.28}$$

Substituting (3.24) in (3.28) one can show that

$$\Delta(\Pi_0) = \frac{n^B s^2}{1+(n^B-1)s} P(s,n^B) + 2(2-s)h(s,n^B)\left[\sqrt{A(s,n^B)} - A(s,n^B)\right]$$

where $P(s,n^B) = (1+(n^B-1)s)^2\left[n^B s - 3(1-s)\right] + (1-s)(4-3s)(1+n^B)$ and $h(s,n^B) = 2(1+(n^B-1)s)^2 + n^B s^2$. Given, $s \in (0,1)$, and, $h(s,n^B) > 0$ for every $s \in (0,1)$ and $n \geq 1$, $2(2-s)h(s,n^B)[\sqrt{A(s,n^B)} - A(s,n^B)] > 0$. Therefore, if we can show that $P(s,n^B) > 0$, then we will have $\Delta(\Pi_0) > 0$. Observe that $P(s,0) = (1-s)(1+2s-2s^2) + s^3 > 0$ and for all $s \geq (-1+\sqrt{13})/6$,

$$\partial_{n^B} P(s,n^B) = s\left[3(n^B-1)^2 s^2 + 2(4s-1)(n^B-1)s + (3s^2+s-1)\right] > 0.$$

However, if $s < (-1+\sqrt{13})/6$, then $P(s,n^B) \geq P(s,n^*) > 0$ where $P(s,n^*)$ is the minimum of $P(s,n^B)$ for any $s < (-1+\sqrt{13})/6$. Hence, $P(s,n^B)$ is always positive which implies that $\Delta(\Pi_0) > 0$.

Proof of (ii): To prove this consider the difference $\Delta(W) := \mathcal{W}^C(n^C) - \mathcal{W}^B(n^B)$ (long run welfare difference between downstream Cournot competition with free entry and downstream Bertrand competition with free entry) when there is no cost difference then $m_0 = m$. It follows that

$$\Delta(W) = \left[n^C + 2G_1(n^B)\right]\frac{s\sqrt{K}}{2}\frac{a-m}{2} + \left[\frac{n^C\left(u(n^C)-4\right)}{8} - n^B G_2(n^B)\right]K, \quad (3.29)$$

$G_1(n^B) = (2n^B(1-s)(1+n^B s)g(s,n^B)/2h(n^B))$, $G_2(n^B) = (1+(n^B-1)s)^2[12(1+(n^B-1)s)^3 + 8(1+(n^B-1)s)^2 + 4n^B s^2(1+(n^B-1)s) + 3n^B s^2]/2\{h(s,n^B)\}^2$, $g(s,n^B) = \sqrt{(1+(n^B-1)s)/(1-s)(1+n^B s)}$ and $u(s,n^C) = 3\left[4(2+(n^C-1)s - 3n^C s^2)\right]$. Using $\Pi^B(n^B) = K$ we get

$$\frac{a-m}{2} = \sqrt{\frac{1+n^B s}{(1-s)(1+(n^B-1)s)}\frac{v(s,n^B)}{h(s,n^B)}}\sqrt{K}, \quad (3.30)$$

where $v(s,n^B) = 2(2-s)h(n^B) + n^B s\left[4(1+(n^B-1)s)^2 - s(4-3s)\right]$. Using conditions (3.24) and (3.30) in (3.29) we get

$$\Delta(W) = \frac{\mathcal{P}(n^B, s)K}{2s(4-3s)(1+(n^B-1)s)\{h(s,n^B)\}^2}. \tag{3.31}$$

Therefore, if $\mathcal{P}(n^B, s) \gtreqless 0$, then $\mathcal{W}^C(n^C) \gtreqless \mathcal{W}^B(n^B)$. From the implicit plot $\mathcal{P}(n^B, s) = 0$ one can show that $\mathcal{P}(n^B, s) < 0$ for (n^B, s) below the U-shaped curve (see Figure 3.6). Hence the result.

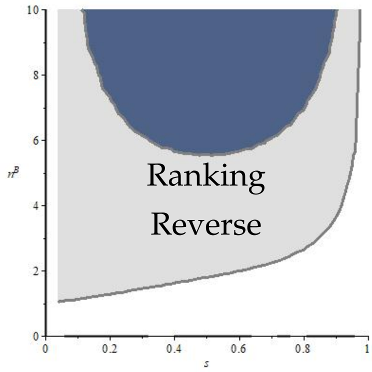


Figure 3.6: Welfare ranking.

Table 3.1: Few evidences of long-run results

Entry Cost (K)	n^B	n^C	Δ_W
5.54	2	1	-4.5
4.208	3	2	-2.9
3.3	4	3	-1.67
2.2	6	5	0.5
1.35	9	9	2.67
0.5	18	19	6.26

Table 3.1 provides evidence of all kinds of possibilities in terms of welfare difference obtain by assuming $a = 20$, $s = 0.5$ and $m = m_0 = 0$.

Proof of (iii) The derivative of the long run welfare function with respect to the number of firms is

$$\partial_n \mathcal{W}^C(n) = \Pi^C(n) - K + n(p^C - m)\partial_n \hat{q}^C + (p_0^C - m_0)\partial_n \hat{q}_0^C - (p^C - z^C(n))q^C(z^C(n), n)$$

where $p^C = P_i(\boldsymbol{q}^C(z^C(n), n))$, $p_0^C = P_0(\boldsymbol{q}^C(z^C(n), n))$, $\partial_n \hat{q}^C = \partial_n q^C(z^C(n), n) + \partial_z q^C(z^C(n), n)\partial_n z^C(n)$ and $\partial_n \hat{q}_0^C = \partial_n q_0^C(z^C(n), n) + \partial_z q_0^C(z^C(n), n)\partial_n z^C(n)$. Given $\partial_n \hat{q}^C < 0$, $\partial_n \hat{q}_0^C < 0$ and each downstream firm produces positive output and earns nonnegative profit, we have the business-stealing effect given by

$$B(n) := n(p^C - m)\partial_n \hat{q}^C + (p_0^C - m_0)\partial_n \hat{q}_0^C - (p^C - z^C(n))q^C(z^C(n), n) < 0.$$

Hence, at the free entry equilibrium we have $\partial_n \mathcal{W}^C(n^C) = B(n^C) < 0$. Finally, using $\partial_n \Pi^C(n) < 0$ we conclude that $n^C > n^{C*}$ where $\partial_n \mathcal{W}^C(n^{C*}) = 0$. The proof of the Bertrand competition is similar and hence omitted.

Chapter 4

A strong equivalence result with evolutionary stable conjectural variations

4.1 Introduction

Equilibrium outcomes in oligopoly depends on action space of firms (for example, whether the firm is a price-setter or quantity-setter) as well as on conjectural variations (each firm's conjectures regarding responses of its rival firms). For example, under zero conjectural variation where each firm anticipates that rivals do not respond to their actions, profits are higher and welfare is lower when all firms choose quantity rather than price (Singh and Vives (1984); Vives (1985); Cheng (1985)). This finding, obtained in a symmetric oligopoly environment, underpins the widely held notion that Bertrand is less profitable but more efficient than Cournot competition.[1]

We consider a differentiated oligopoly market comprise of n profit maximizing firm. Each firm can be either price chooser or quantity chooser. Each firm posses a linear conjectural variation to the other firms behavior with respect to his strate-

[1] Several authors have shown that this finding may not necessarily hold in different contexts like in the presence of cost asymmetry (Häckner (2000); Zanchettin (2006)); in the presence of endogenous R & D possibilities (Qiu (1997)), in the context of licensing (Fauli-Oller and Sandonis (2002)) and in mixed markets (Ghosh and Mitra (2009).

gic variation. Our objective is to capture the long run interaction among the firms and we want to understand what type of conjecture will survive or has better fit in the long run. For this purpose we assume that each firm's conjecture is subject to an evolutionary selection. Like Müller and Normann (2005), we use static concept of evolutionary stability. Given this framework with a linear demand specification we establish a strong equivalence result in terms of market outcome that illustrates the fact that in a differentiated oligopoly model, the equilibrium outcomes are same for all combinations of price-setting and quantity-setting firms including the two extremes: Bertrand-where all firms are price-setters - and Cournot-where all firms are quantity-setters. Maintaining zero conjectural variation, Miller and Pazgal (2001) show that Bertrand and Cournot outcomes are the same in the presence of managerial delegation if owners have access to a rich choice set for managerial contracts. Our paper provides a complementary reasoning for equivalence- namely, evolutionary stable selection of conjectural variation.

In reality we find many example where firms compete for a very long period and after a certain point of time they charge almost same price consistently for a long period of time. For example, in soft-drink market the price difference between the Cocacola and Pepsi are almost zero. In case of health drink market we also observe the stability of price difference for Complan and Horlicks. Our result shows why such stability in price difference persists in the long run.

The literature on conjectural variations mainly focuses on consistent conjectures (Bresnahan (1981)), where each firm rightly anticipates rival firms' reaction. Kamien and Schwartz (1983) show that Bertrand and Cournot outcomes are identical in a linear duopoly under consistent conjectures. In a similar setting, Müller and Normann (2005) show that consistent conjectures are also evolutionarily stable, that is, loosely speaking, conjectural variations implied by consistent conjectures constitute best response. Our model is in the spirit of Müller and Normann (2005) as we also focus on best responses in conjectural variations space though we go beyond duopoly. In addition, our model allows for co-existence of both price-setters and quantity-setters.

The chapter 4 is organized as follows. In the next section the basic framework. Then in Section 4.3, we discuss the conjectural variation equilibrium. After that in Section 4.4 we make a discussion of evolutionary stability and obtained the evolutionary stable solution. In Section 4.5 we present our equivalence result. Finally we end the chapter with discussion (Section 4.6) of our results followed by an appendix section (Section 4.7) where we provide the proofs of all the results.

4.2 Preliminaries

We consider an imperfectly competitive industry where a set of $N = \{1, \ldots, n\}$ profit maximizing firms operate. For each $i \in N$, let p_i and q_i denote firm i's price and quantity respectively. The inverse demand function for firm $i \in N$ is given by

$$p_i(q) = a - q_i - s \sum_{k \in N \setminus \{i\}} q_k, \;\; i \in N, \tag{4.1}$$

where a is the intercept and $s \in (0,1)$ is the substitution parameter.[2] Each firm has a constant marginal cost m and zero fixed cost. Profit of a firm $i \in N$ is $\pi_i \equiv (p_i - m)q_i$.

4.2.1 Modes of competition

Suppose a set $B \subseteq N$ of firms select price-setting strategy and the complement set $C = N \setminus B$ of firms select quantity-setting strategy. We assume that b and c respectively denote the numbers of firm in B and C. Let $Q = \sum_{k \in N} q_i$ be the total output and for $i \in N$ and let $Q_{-i} = \sum_{k \in N \setminus \{i\}} q_k$ be the total output of all firms but i. Let $Q^B = \sum_{\rho \in B} q_\rho$ and let $Q^C = \sum_{\tau \in C} q_\tau$. For $r \in B$, let $Q_{-r} = \sum_{\rho \in B \setminus \{r\}} q_\rho$, and, for $t \in C$, let $Q_{-t} = \sum_{\tau \in C \setminus \{t\}} q_\tau$. Summing all the the inverse demand functions (given by the condition

[2]This inverse demand function can be derived from the quadratic utility function given by $U(q) = a \sum_{k \in N} q_k - \frac{1}{2} \left(\sum_{k \in N} q_k^2 + s \sum_{k \in N} \sum_{k' \in N \setminus \{k\}} q_k q_{k'} \right)$.

(4.1)) of the firms in the set B and then solving for Q^B we get

$$Q^B = \frac{ba - \sum\limits_{k\in B} p_k - sbQ^C}{[1+(b-1)s]} \tag{4.2}$$

Then by substituting in the inverse demand function of any firm $t \in C$ we get the demand function that any firm $t \in C$ faces,

$$D_t^C\left(\boldsymbol{q}^C, \boldsymbol{p}^B, b\right) = \frac{(1-s)a - (1-s)(1+bs)q_t - s(1-s)Q_{-t}^C + s\sum\limits_{\rho\in B} p_\rho}{1+(b-1)s}, \tag{4.3}$$

where $\boldsymbol{q}^C = (q_\tau)_{\tau\in C}$ and $\boldsymbol{p}^B = \left(p_\rho\right)_{\rho\in B}$. The demand function that any firm $r \in B$ faces,

$$D_r^B\left(\boldsymbol{q}^C, \boldsymbol{p}^B, b\right) = \frac{(1-s)a - (1+(b-2)s)\,p_r - s(1-s)Q^C + s\sum\limits_{\rho\in B\setminus\{r\}} p_\rho}{(1-s)\,(1+(b-1)s)}. \tag{4.4}$$

The profit function of any $t \in C$ is given by

$$\pi_t^C\left(\boldsymbol{q}^C, \boldsymbol{p}^B, b\right) = \left(D_t^C\left(\boldsymbol{q}^C, \boldsymbol{p}^B, b\right) - m\right) q_t. \tag{4.5}$$

Similarly, the profit function of any $r \in B$ given by

$$\pi_r^B\left(\boldsymbol{q}^C, \boldsymbol{p}^B, b\right) = (p_r - m)D_r\left(\boldsymbol{q}^C, \boldsymbol{p}^B, b\right). \tag{4.6}$$

4.2.2 Conjectural variations

Conjectural variation captures any firm i's conjecture regarding the reaction of each of its rival firm $j \in N \setminus \{i\}$ (in terms of changing the value of j's strategic variable) due to a unit change in the value of firm i's own strategic variable. Given any partition b, for any $t, \tau \in C$, $v_{t\tau}^{CC} := \frac{dq_\tau}{dq_t}$ is the conjectural variation of firm t about firm τ's output. Also for any $t \in C$ and $\rho \in B$, $v_{t\rho}^{CB} := \frac{dp_\rho}{dq_t}$ is the conjectural variation of firm t about firm ρ's price. Similarly given any partition b for any $r, \rho \in B$, $v_{r\rho}^{BB} := \frac{dp_\rho}{dp_r}$ is the conjectural variation of firm r about firm ρ's price. Also for any $r \in B$ and $\tau \in C$, $v_{r\tau}^{BC} := \frac{dq_\tau}{dp_r}$ is the conjectural variation of firm r about firm τ's output. The vector of conjectural variation parameters for any $t \in C$ is denoted by $\boldsymbol{v}_t^C = ((v_{t\tau}^{CC})_{\tau \in C\setminus\{t\}}, (v_{t\rho}^{CB})_{\rho \in B}) = (\boldsymbol{v}_t^{CC}, \boldsymbol{v}_t^{CB})$ and that of any $r \in B$ is denoted by $\boldsymbol{v}_r^B = ((v_{r\tau}^{BC})_{\tau \in C}, (v_{r\rho}^{BB})_{\rho \in B\setminus\{r\}}) = (\boldsymbol{v}_r^{BC}, \boldsymbol{v}_r^{BB})$. Let $\boldsymbol{v} = ((\boldsymbol{v}_\tau^C)_{\tau \in C}, (\boldsymbol{v}_\rho^B)_{\rho \in B}) = (\boldsymbol{v}^C, \boldsymbol{v}^B)$ denote the vector of all conjectural variation parameters with $\boldsymbol{v}^C$ and $\boldsymbol{v}^B$ are respectively the vector of conjectural variation parameters for quantity setters and price setters. We also denote for any firm $t \in C$, $\boldsymbol{v}_{-t}^C = (\boldsymbol{v}_\tau)_{\tau \in C\setminus\{t\}}$ be the vector of conjectural variation parameters of all firms in the set C but not t and for any firm $r \in B$, $\boldsymbol{v}_{-r}^B = (\boldsymbol{v}_\rho)_{\rho \in B\setminus\{r\}}$ be the vector of conjectural variation parameters of all firms in the set B but not r.

4.3 Conjectural variations equilibrium (CVE)

Given any $\boldsymbol{v}$, the optimum strategy choice vector of this oligopoly industry, $(\boldsymbol{q}^C(\boldsymbol{v}), \boldsymbol{p}^B(\boldsymbol{v})) := ((q_\tau^C(\boldsymbol{v}))_{\tau \in C}, (p_\rho^B(\boldsymbol{v}))_{\rho \in B})$ is obtained by simultaneously solving the set of n conditions

$$\left(\left(\frac{d\pi_\rho^B(\boldsymbol{q}^C(\boldsymbol{v}), \boldsymbol{p}^B(\boldsymbol{v}), b|\boldsymbol{v}_\rho^B)}{dp_\rho} = 0 \right)_{\rho \in B}, \left(\frac{d\pi_t^C(\boldsymbol{q}^C(\boldsymbol{v}), \boldsymbol{p}^B(\boldsymbol{v}), b|\boldsymbol{v}_\tau^C)}{dq_\tau} = 0 \right)_{\tau \in C} \right).$$

Specifically, in equilibrium, output of a quantity-setting firm and price of a price-setting firm respectively are:

$$q_t^C(\boldsymbol{v}) = \frac{(1-s)(a-m)}{X_t(\boldsymbol{v}_t)[1 + s(1-s)H^C(\boldsymbol{v^C}) - sH^B(\boldsymbol{v^B})]}, \quad \forall t \in C, \tag{4.7}$$

$$p_r^B(\boldsymbol{v}) = m + \frac{(1-s)(a-m)}{Y_r(\boldsymbol{v}_r)[1 + s(1-s)H^C(\boldsymbol{v^C}) - sH^B(\boldsymbol{v^B})]}, \quad \forall r \in B, \tag{4.8}$$

where

$$Y_r(\boldsymbol{v}_r) = 2(1 + (b-2)s) + s(1-s)\sum_{\tau \in C} v_{r\tau}^{BC}(b) - s \sum_{\rho \in B\setminus\{r\}} v_{r\rho}^{BB(b)} + s,$$

$$X_t(\boldsymbol{v}_t) = 2(1-s)(1+bs) + s(1-s)\sum_{\tau\in C\setminus\{t\}} v_{t\tau}^{CC}(b) - s\sum_{\rho\in B} v_{t\rho}^{CB}(b) - s(1-s),$$

and $H^C(\boldsymbol{v}^C) = \sum_{\tau\in C}(1/X_\tau(\boldsymbol{v}_\tau))$, $H^B(\boldsymbol{v}^B) = \sum_{\rho\in B}(1/Y_\rho(\boldsymbol{v}_\rho))$.

The resulting profit of any firm $r \in B$ is

$$\Pi_r^B(\boldsymbol{v}) := \pi_r^B(\boldsymbol{q}^C(\boldsymbol{v}), \boldsymbol{p}^B(\boldsymbol{v})) = \left(\frac{Y_r(\boldsymbol{v}_r) - (1+(b-1)s)}{(1-s)(1+(b-1)s)}\right)\left(p_r^B(\boldsymbol{v}) - m\right)^2, \quad (4.9)$$

and that of any firm $t \in C$ is

$$\Pi_t^C(\boldsymbol{v}) := \pi_t^C(\boldsymbol{q}^C(\boldsymbol{v}), \boldsymbol{p}^B(\boldsymbol{v})) = \left(\frac{X_t(\boldsymbol{v}_t) - (1-s)(1+(b-1)s)}{1+(b-1)s}\right)\left(q_t^C(\boldsymbol{v})\right)^2. \quad (4.10)$$

LEMMA **4.1** If Stage 1 choice of conjectural variation vector is $\boldsymbol{v}$ and if Stage 2 optimal choice vector $(\boldsymbol{q}^C(\boldsymbol{v}), \boldsymbol{p}^B(\boldsymbol{v}))$ satisfies the second order conditions, then

(i) $X_t(\boldsymbol{v}_t) > (1-s)(1+(b-1)s)$, and

(ii) $Y_r(\boldsymbol{v}_r) > (1+(b-1)s)$.[3]

OBSERVATION **4.1** Observe that the conjectural variation equilibrium choice of all firms, given by the set of equations (4.7), depends only on the aggregate value vector $((Y_\rho(\boldsymbol{v}_\rho^B))_{\rho\in B}, (X_\tau(\boldsymbol{v}_\tau^C))_{\tau\in C})$. In particular, if the conjectural variation vectors $\boldsymbol{v}^1 = ((\boldsymbol{v}_\rho^{B1})_{\rho\in B}, (\boldsymbol{v}_\tau^{C1})_{\tau\in C})$ and $\boldsymbol{v}^2 = ((\boldsymbol{v}_\rho^{B2})_{\rho\in B}, (\boldsymbol{v}_\tau^{C2})_{\tau\in C})$ are such that the aggregateted numbers $X_t(\boldsymbol{v}_t^{C1})$ and $X_t(\boldsymbol{v}_t^{C2})$ are equal for all $t \in C$ and the aggregateted numbers $Y_r(\boldsymbol{v}_r^{C1})$ and $Y_r(\boldsymbol{v}_r^{C2})$ are also equal for all $r \in B$, then both $\boldsymbol{v}^1$ and $\boldsymbol{v}^2$ yields same conjectural variation equilibrium. Hence the effectiveness of any conjectural variation vector $\boldsymbol{v}$, in case of determining the conjectural variation equilibrium, depends on the resultant aggregate value vector $((Y_\rho(\boldsymbol{v}_\rho^B))_{\rho\in B}, (X_\tau(\boldsymbol{v}_\tau^C))_{\tau\in C})$. Again these aggregates (that is, $Y_r(\boldsymbol{v}_r^B)$ for all $r \in B$ and $X_t(\boldsymbol{v}_t^C)$ for all $t \in C$) depends only on each firm's own conjectural variations vector. Hence in determining the conjectural variations equilibrium and profit of each firm, only the role of $((Y_\rho(\boldsymbol{v}_\rho^B))_{\rho\in B}, (X_\tau(\boldsymbol{v}_\tau^C))_{\tau\in C})$ which we call *effective conjecture* is important for our analysis. For brevity, we write $((Y_\rho)_{\rho\in B}, (X_\tau)_{\tau\in C})$ instead of $((Y_\rho(\boldsymbol{v}_\rho^B))_{\rho\in B}, (X_\tau(\boldsymbol{v}_\tau^C))_{\tau\in C})$.

[3]See Appendix 2.

4.4 Evolutionary stability

Suppose that the effective conjecture of each firm as its type and is subject to evolutionary selection. Under evolutionary selection of firm's effective conjecture (derived from their actual conjecture), the effective conjectures are not a resultant of consciously selection but is either inherited behavior from their forebears or are assigned through mutation. Therefore, we are focusing on the long run interaction across firms. The effective conjectures that survive through this evolutionary selection is supposedly a better fit for the society. We first determine firms' choice given their effective conjectures. Since their effective conjectures determine profits (success value), they also determine reproductive success, and we can study the evolutionary selection of the effective conjectures in a next step. The underlying assumption is that if firms differ in evolutionary success, the individual characteristics of more successful firm will spread within the population more quickly than the characteristics of the less successful ones. This leads to a dynamic process that determines distribution of individual characteristics within an economy. Therefore to obtain the evolutionary stability of the effective conjecture we have to consider each firm's resulting profits or their success function given by the conditions (4.5) and (4.6) as the function of this aggregates, that is on $(Y_\rho)_{\rho \in B}$ and $(X_\tau)_{\tau \in C}$. Given Observation 4.1, the set of all actual conjecture vectors for which the resultant aggregates are identical, has same reproductive success since the profits (or success value) are identical. Therefore, each success function is now in terms of aggregates only. These success functions are the following: For any $r \in B$,

$$\bar{\Pi}_r^B(\boldsymbol{Y}, \boldsymbol{X}) = \frac{(1-s)(Y_r - (1+(b-1)s))(a-m)^2}{(1+(b-1)s)Y_r^2\left[1+s(1-s)\bar{H}^C - s\bar{H}^B\right]^2}, \tag{4.11}$$

and, for any $t \in C$,

$$\bar{\Pi}_t^C(\boldsymbol{Y}, \boldsymbol{X}) = \frac{(1-s)^2(X_t - (1-s)(1+(b-1)s))(a-m)^2}{(1+(b-1)s)X_t^2\left[1+s(1-s)\bar{H}^C - s\bar{H}^B\right]^2}, \tag{4.12}$$

where $\bar{H}^C = (1/\sum_{\tau\in C} X_\tau)$ and $\bar{H}^B = (1/\sum_{\rho\in B} Y_\rho)$. Given (4.7) for some $(\boldsymbol{Y},\boldsymbol{X})$, the equilibrium choices can be written as

$$\bar{q}_t^C(\boldsymbol{Y},\boldsymbol{X}) = \frac{(1-s)(a-m)}{X_t[1+s(1-s)\bar{H}^C - s\bar{H}^B]}, \quad \forall t \in C, \tag{4.13}$$

$$\bar{p}_r^B(\boldsymbol{Y},\boldsymbol{X}) = m + \frac{(1-s)(a-m)}{Y_r[1+s(1-s)\bar{H}^C - s\bar{H}^B]}, \quad \forall r \in B, \tag{4.14}$$

In general, the evolutionary stable solution defined for symmetric game is the following. Consider a game $\Gamma = \langle \mathcal{N}, \{S_i\}_{i\in\mathcal{N}}, \{u_i : \Pi_{i\in\mathcal{N}} S_i \to \Re\}_{i\in\mathcal{N}}\rangle$ where $\mathcal{N}$ is the set of players, for each $i \in \mathcal{N}$, S_i is the set of strategies available to player i and $u_i(.)$ is the pay-off function of player i. The game Γ is symmetric if $S_i = S$ and $u_i = u$ for all $i \in \mathcal{N}$.

DEFINITION **4.1** A strategy profile $(s^*,\ldots,s^*)$ for a symmetric game Γ is an evolutionary stable solution (ESS) if

(i) either $u(s^*,\underbrace{s^*,\ldots,s^*}_{n-1}) > u(s,\underbrace{s^*,\ldots,s^*}_{n-1})$ for any $s \neq s^*$.

(ii) or when $u(s^*,\underbrace{s^*,\ldots,s^*}_{n-1}) = u(s,\underbrace{s^*,\ldots,s^*}_{n-1})$, then there exist $k^* \in \{0,\ldots,n-1\}$ such that for all $k \in \{k^*+1,\ldots,n-1\}$, if we have $u(s^*,\underbrace{s^*,\ldots,s^*}_{k},\underbrace{s,\ldots,s}_{n-k-1}) = u(s,\underbrace{s^*,\ldots,s^*}_{k},\underbrace{s,\ldots,s}_{n-k-1})$, then $u(s^*,\underbrace{s^*,\ldots,s^*}_{k^*},\underbrace{s,\ldots,s}_{n-k^*-1}) > u(s,\underbrace{s^*,\ldots,s^*}_{k^*},\underbrace{s,\ldots,s}_{n-k^*-1})$.

Definition 4.1 implies that any evolutionary stable solution is either a strict Nash equilibrium (condition (i)) or if there exists another strategy $s \neq s^*$ of any firm which yields the same payoff as s^* given the strategy s^* for all other firms, then s is not a mutation against s^*. In our context, it is natural to require that for any evolutionary stable effective conjectural variation vector the resulting choice of strategic variable (that is, prices and quantities) must be a conjectural variation equilibrium choice (see Müller and Normann (2005)). In our context, given the success function this is always true. Moreover, Definition 4.1 is applicable only when the game is symmetric, that is, when $b = 0$ (Cournot competition) and when $b = n$ (Bertrand competition). Though for all other intermediate cases ($b = 1,\ldots,n-1$) we cannot use Definition 4.1, to determine the evolutionary stable selection of effective conjecture for these intermediate cases we can consider Nash equilibrium in terms of effective conjectures which is a necessary condition to guarantee evolutionary stability. The following Lemma 4.2 gives a complete characterization of the Nash equilibrium choice when firms are allowed to choose their effective conjecture.

LEMMA **4.2** If each $r \in B$ is allowed to choose its effective conjecture Y_r and each $t \in C$ is allowed to choose its effective conjecture X_t with their respective success functions (given by (4.11) and (4.12)) and if the vector $(\boldsymbol{Y}^*, \boldsymbol{X}^*) = ((Y^*_\rho)_{\rho \in B}(X_\tau)_{\tau \in C})$ is the resulting Nash equilibrium choices, then we have the following:

(i) $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ must be partition symmetric, that is, $Y^*_r = Y^*$ for all $r \in B$ and $X^*_t = X^*$ for all $t \in C$.

(ii) $(1 + (b-1)s)(X^* + (1-s)Y^*) = X^* Y^*$,

(iii) The Nash equilibrium choice vector $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ is unique strict Nash equilibrium with $X^* = (1 + (b-1)s)F(n,s)$ and $Y^* = (1 + (b-1)s)G(n,s)$ with $F(n,s) := [(2-ns) + \sqrt{(2-ns)^2 + 8s(1-s)(n-1)}]/2$ and $G(n,s) := F(n,s)/[F(n,s) - (1-s)]$.

(iv) If $s \in (0, 0.9]$, then $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ is also stable.

PROPOSITION **4.1** Fix any $b \in \{0, 1, \ldots, n\}$. The selection of effective conjecture vector $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$, such that $Y^*_r = Y^*$ for all $r \in B$ and $X^*_t = X^*$ for all $t \in C$, is evolutionary stable solution for partition b.

Proof of Proposition 4.1: By Lemma 4.2 (iii), the conjecture vector $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ with the propety that $Y^*_r = Y^*$ for all $r \in B$ and $X^*_t = X^*$ for all $t \in C$ is unique strict Nash equilibrium under conjecture selection. Therefore, by condition (i) of Definition 4.1, the effective conjecture vector $\boldsymbol{X}^*(0) := (X^*_i(0))_{i \in N}$ such that $X^*_i(0) := (1-s)F(n,s)$, and, the effective conjecture vector $\boldsymbol{Y}^*(n) := (Y^*_i(n))_{i \in N}$ such that $Y^*_i(n) = (1 + (n-1)s)G(n,s)$, are respectively evolutionary stable solution for partition 0 and partition n. Now consider any intermediate partition $b \in \{1, \ldots, n-1\}$

and suppose that the effective conjecture vector $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ is not a evolutionary stable selection for this partition. Then, either there exist $r \in B$ such that $Y_r \neq Y^*$ is a best reply against $(\boldsymbol{Y}^*_{-r}, \boldsymbol{X}^*)$ and this firm act with its Y_r acts as a mutant against Y^*, or there exist $t \in C$ such that $X_t \neq X^*$ is a best reply against $(\boldsymbol{Y}^*, \boldsymbol{X}^*_{-t})$ and this firm with X_t acts as a mutant against X^*. Given $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ is a strict Nash equilibrium such mutations are ruled out.

4.5 Equivalence result

THEOREM **4.1** Fix any $b \in \{0, 1, \ldots, n\}$. Under the evolutionary stable solution (ESS), all firms produce same quantities, charge same prices and earn same profits. Further, this evolutionary stable outcome is invariant across all $b \in \{0, 1, \ldots, n\}$.

Proof Of Theorem 4.1: One can show that in Stage 1, the equilibrium quantity of any $t \in C$ is

$$\bar{q}^C(\boldsymbol{Y}^*, \boldsymbol{X}^*) = \frac{(1-s)(a-m)}{X^* \mathcal{D}^E}$$

where $\mathcal{D}^E := 1 + (s(1-s)(n-b)/X^*) - (sb/Y^*)$. The equilibrium quantity of any $r \in B$ is

$$\bar{q}^B(\boldsymbol{Y}^*, \boldsymbol{X}^*) = \frac{(Y^* - (1 + (b-1)s))\,(1-s)(a-m)}{(1-s)(1+(b-1)s)Y^* \mathcal{D}^E}.$$

Therefore, the difference in these two equilibrium quantities is

$$\bar{q}^C(\boldsymbol{Y}^*, \boldsymbol{X}^*) - \bar{q}^B(\boldsymbol{Y}^*, \boldsymbol{X}^*) = \frac{(1-s)(a-m)\left[(1+(b-1)s)\left\{X^* + (1-s)Y^*\right\} - X^*Y^*\right]}{(1-s)(1+(b-1)s)\mathcal{D}^E}. \tag{4.15}$$

Using Lemma 4.1(i) in (4.15) it follows that $\bar{q}^C(\boldsymbol{Y}^*, \boldsymbol{X}^*) = \bar{q}^B(\boldsymbol{Y}^*, \boldsymbol{X}^*)$, that is, firms playing price strategy produce the same equilibrium quantity as firms playing quantity strategy. Thus from the demand system we can also conclude that each firm in the industry charges the same price and earns the same profit. One can also show that the specific form of the equilibrium profit $\bar{\Pi}^C(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ of any firm $t \in C$ is

$$\Pi^C(\boldsymbol{Y}^*, \boldsymbol{X}^*) = \frac{(X^* - (1-s)(1+(b-1)s))\,(1-s)^2(a-m)^2}{(1+(b-1)s)\{X^*\}^2\,\{\mathcal{D}^E\}^2}$$

and the specific form of the equilibrium profit $\Pi^B(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ of any firm $r \in B$ is

$$\Pi^B(\boldsymbol{Y}^*, \boldsymbol{X}^*) = \frac{(Y^* - (1+(b-1)s))\,(1-s)^2(a-m)^2}{(1-s)(1+(b-1)s)\{Y^*\}^2\,\{\mathcal{D}^E\}^2}.$$

Using X^* and Y^* from Lemma 4.2(iii) in the profit expressions one can show that

$$\bar{\Pi}^C(\boldsymbol{Y}^*, \boldsymbol{X}^*) = \frac{[F(n,s) - (1-s)]\,\{G(n,s)\}^2(a-m)^2}{[F(n,s) + (1+ns)G(n,s)]^2}, \tag{4.16}$$

and

$$\bar{\Pi}^B(\boldsymbol{Y}^*, \boldsymbol{X}^*) = \frac{[G(n,s) - 1]\,\{F(n,s)\}^2(a-m)^2}{(1-s)\,[F(n,s) + (1+ns)G(n,s)]^2}. \tag{4.17}$$

Since the profit expressions (4.16) and (4.17) are independent of b (that is, independent of the number of price-setting firms and hence is also independent of the number of quantity-setting firms) it follows that the market outcomes corresponds to evolutionary stable selection are invariant across b.

4.6 Discussions

Our approach is in the spirit with Müller and Normann (2005) but there are two differences:

1. First, unlike Müller and Normann (2005), we consider the evolutionary stability of the aggregates not the actual conjecture. There exists a lots of research papers that deals with such indirect conjecture under the aggregative-games and semi-aggregative-games (see Possajennikov (2015), Possajennikov (2016)). In our approach, we also use this indirect conjecture approach, but not assuming conjecture on firm's aggregate or their personalize aggregate directly, rather we try to give some foundation such that the link between two approaches can be ex-

plained. According to our result corresponding to aggregate which is evolutionary stable we have infinite number of actual conjecture vectors which can weakly invade each others but no invasion can be strict.[4] Therefore, all the actual conjecture vector of a firm that have the same resultant evolutionary stable aggregate are neutrally stable solution under actual conjecture selection.

2. Second, we consider the asymmetric case also. In general, the evolutionary stable solution of any asymmetric game is determined by strict Nash equilibrium only (see Possajennikov (2016)). Here we also use the stability issue to capture the out of equilibrium convergence of conjecture selection.

Interpretation of the aggregates: Fix any $b \in \{0, 1, \dots, n\}$ and consider the total change of any firm's resulting market variable with respect to his own strategic variable given his actual conjecture. For any $r \in B$ it gives

$$\alpha_r^B := \frac{dD_r^B(\boldsymbol{p}^B, \boldsymbol{q}^C, b)}{dp_r}\bigg|_{\boldsymbol{v}_r^B} = -\frac{-(1+(b-1)s) + Y_r(\boldsymbol{v}_r^B)}{(1-s)(1+(b-1)s)}, \tag{4.18}$$

and for any $t \in C$ it gives

$$\alpha_t^C := \frac{dD_t^C(\boldsymbol{p}^B, \boldsymbol{q}^C, b)}{dq_t}\bigg|_{\boldsymbol{v}_t^C} = -\frac{-(1-s)(1+(b-1)s) + X_t(\boldsymbol{v}_t^C)}{(1+(b-1)s)}. \tag{4.19}$$

Condition (4.18) implies that α_r^B is one-to-one with Y_r and condition (4.19) implies that α_t^C is one-to-one with X_t. Therefore, evolutionary stable selection of aggregates is nothing but evolutionary selection of these conjecture over firms resulting market variables.

[4]One can show that at $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ we also have

$$\left(\left(\frac{\partial \Pi_r^B(\boldsymbol{v}^*)}{\partial v_{r\tau}^{BC}} = 0 \right)_{\tau \in C}, \left(\frac{\partial \Pi_r^B(\boldsymbol{v}^*)}{\partial v_{r\rho}^{BB}} = 0 \right)_{\rho \in B \setminus \{r\}} \right),$$

for all $r \in B$ and

$$\left(\left(\frac{\partial \Pi_t^C(\boldsymbol{v}^*)}{\partial v_{t\tau}^{CC}} = 0 \right)_{\tau \in C \setminus \{t\}}, \left(\frac{\partial \Pi_t^C(\boldsymbol{v}_t^{ES}, \boldsymbol{v}_{-t})}{\partial v_{t\rho}^{CB}} = 0 \right)_{\rho \in B} \right).$$

for all $t \in C$ where $\boldsymbol{v}^*$ is some actual vector of conjectural variation for which the resultant aggregate vector is $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$

4.7 Appendix

Proof of Lemma 4.1: *Proof of (i)*: If Stage 2 choice vector $(\boldsymbol{q}^C(\boldsymbol{v}), \boldsymbol{p}^B(\boldsymbol{v}))$ satisfies the second order conditions, then, for any firm $t \in C$

$$\frac{d^2\Pi_t^C(\boldsymbol{q}^C(\boldsymbol{v}), \boldsymbol{p}^B(\boldsymbol{v}))}{dq_t^2} = -\frac{2\left[(1-s)(1+bs) + s(1-s)\sum\limits_{\tau \in C\setminus\{t\}} \alpha_{t\tau} - s\sum\limits_{\rho \in B} \lambda_{t\rho}\right]}{1+(b-1)s} < 0.$$

Hence, $(1-s)(1+bs) + s(1-s)\sum_{\tau \in C\setminus\{t\}} \alpha_{t\tau} - s\sum_{\rho \in B} \lambda_{t\tau} > 0$ which in turn implies $X_t(\boldsymbol{v}_t) - (1-s)(1+(b-1)s) > 0$ and the result follows.

Proof of (ii): If Stage 2 choice vector $(\boldsymbol{q}^C(\boldsymbol{v}), \boldsymbol{p}^B(\boldsymbol{v}))$ satisfies the second order conditions, then, for any firm $r \in B$

$$\frac{d^2\Pi_r^B(\boldsymbol{q}^C(\boldsymbol{v}), \boldsymbol{p}^B(\boldsymbol{v}))}{dp_r^2} = -\frac{2\left[(1+(b-2)s) + s(1-s)\sum\limits_{\tau \in C} \mu_{r\tau} - s\sum\limits_{\rho \in B\setminus\{r\}} \sigma_{r\rho}\right]}{(1-s)(1+(b-1)s)} < 0.$$

Hence, we have $(1+(b-2)s) + s(1-s)\sum_{\tau \in C} \mu_{r\tau} - s\sum_{\rho \in B\setminus\{r\}} \sigma_{r\rho} > 0$ from which it follows that $Y_r(\boldsymbol{v}_r) - (1+(b-1)s) > 0$ and the we get the result.

Proof of Lemma 4.2: If $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ is Nash equilibrium choice vector then for any firm r, Y_r^* can obtained by solving the problem

$$\max_{Y_r} \, \bar{\Pi}_r^B(Y_r, \boldsymbol{Y}_{-r}^*, \boldsymbol{X}^*)$$

and similarly for any firm t, X_t^* can obtained by solving the problem

$$\max_{X_t} \, \bar{\Pi}_t^C(\boldsymbol{Y}^*, X_t, \boldsymbol{X}_{-t}^*)$$

Then $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ must satisfy the following set of n first order conditions:

$$\left(\left(\partial_\rho \bar{\Pi}_\rho^B(\boldsymbol{Y}^*, \boldsymbol{X}^*) = 0\right)_{\rho \in B}, \left(\partial_\tau \bar{\Pi}_\tau^C(\boldsymbol{Y}^*, \boldsymbol{X}^*) = 0\right)_{\tau \in C}\right),$$

The first order condition of any Firm $r \in B$ can be reduced to the following:

$$\left[1 + s(1-s)\bar{H}^{C*} - s\bar{H}^{B*}_{-r}\right][Y^*_r - 2(1+(b-1)s)] = -s, \quad (4.20)$$

where $\bar{H}^C = (1/\sum_{\tau \in C})$ $\bar{H}^{B*}_{-r} = \sum_{\rho \in B\setminus\{r\}}(1/Y^*_\rho)$. Similarly, the first order condition of any Firm $r \in B$ can be reduced to the following:

$$\left[1 + s(1-s)\bar{H}^{C*}_{-t} - s\bar{H}^{B*}\right][X^*_t - 2(1-s)(1+(b-1)s)] = s(1-s), \quad (4.21)$$

where $\bar{H}^{C*}_{-t} = \sum_{\tau \in C\setminus\{t\}}(1/X^*_\tau)$ and $\bar{H}^{B*}_{-r} = \sum_{\rho \in B}(1/Y^*_\rho)$.

Proof of (i): Taking equation (4.21) for firms $t', t'' \in C$ and then subtracting these equations we get

$$(X^*_{t'} - X^*_{t''})\left[1 + s(1-s)\bar{H}^{C*} - s\bar{H}^{B*} - \frac{2s(1-s)^2(1+(b-1)s)}{X^*_{t'}X^*_{t''}}\right] = 0. \quad (4.22)$$

If $X^*_{t'} \neq X^*_{t''}$, then (4.22) implies

$$1 + s(1-s)\bar{H}^{C*} - s\bar{H}^{B*} = \frac{2s(1-s)^2(1+(b-1)s)}{X^*_{t'}X^*_{t''}}. \quad (4.23)$$

Substituting (4.23) in (4.21) for firm t' and then simplifying it we get

$$\{X^*_{t'} - \mathcal{A}\}\{X^*_{t''} - \mathcal{A}\} = -(\mathcal{A})^2. \quad (4.24)$$

where $\mathcal{A} := (1-s)(1+(b-1)s) > 0$. Therefore, the right hand side of (4.24) is negative but, given Lemma 4.1 (i) (that requires $X^*_t > \mathcal{A}$ for all $t \in C$), the left hand side of (4.24) is positive. This is not possible and hence we must have $X^*_{t'} = X^*_{t''}$ implying $X^*_t = X^*$ for all $t \in C$. Similarly, using (4.20) and Lemma 4.1(ii) one can also show that $Y^*_r = Y^*$ for all $r \in B$.

Proof of (ii) Simplifying (4.21) and (4.20) we get

$$1 + s(1-s)\bar{H}^{C*} - s\bar{H}^{B*} = \frac{2s(1-s)[X^*_t - (1-s)(1+(b-1)s)]}{X^*_t[X^*_t - 2(1-s)(1+(b-1)s)]}, \quad (4.25)$$

and

$$1 + s(1-s)\bar{H}^{C*} - s\bar{H}^{B*} = -\frac{2s\left[Y_r^* - (1+(b-1)s)\right]}{Y_r^*\left[Y_r^* - 2(1+(b-1)s)\right]}, \tag{4.26}$$

respectively. Given part (i) of this Lemma $X_t^* = X^*$ for all $t \in C$ and $Y_r^* = Y^*$ for all $r \in B$, (4.25) and (4.26) yields

$$\left[X^*Y^* - (1+(b-1)s)\left\{X^* + (1-s)Y^*\right\}\right]\left[X^* + (1-s)Y^* - 2(1-s)(1+(b-1)s)\right] = 0. \tag{4.27}$$

From the Lemma 4.1 we know that (A) $X^* - (1-s)(1+(b-1)s) > 0$ and we also know that (B) $Y^* - (1+(b-1)s) > 0$. Conditions (A) and (B) gives (C) $X^* + (1-s)Y^* - 2(1-s)(1+(b-1)s) > 0$. Using (C) in (4.27) we get $X^*Y^* - (1+(b-1)s)\{X^* + (1-s)Y^*\} = 0$.

Proof of (iii): Substituting $X_t^* = X^*$ for all $t \in C$ and $Y_r^* = Y^*$ for all $r \in B$ in (4.20) and (4.21) we get two equations in two unknowns X^* and Y^*. We get two real roots for both X^* and Y^*. However, the second order condition for optimization holds only for $X^* = (1+(b-1)s)F(n,s)$ and $Y^* = (1+(b-1)s)G(n,s)$. Hence the result.

Proof of (iv) To check the stability of the conjecture selection $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ we need to check the negative definiteness of the matrix

$$H = \begin{bmatrix} \boldsymbol{\partial_Y \bar{\Pi}^B} & \boldsymbol{\partial_X \bar{\Pi}^B} \\ \boldsymbol{\partial_Y \bar{\Pi}^C} & \boldsymbol{\partial_X \bar{\Pi}^C} \end{bmatrix}$$

at $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ where $\boldsymbol{\partial_Y \bar{\Pi}^B} = [\partial_{r\rho}\bar{\Pi}_r^B]_{b\times b}$, $\boldsymbol{\partial_X \bar{\Pi}^B} = [\partial_{r\tau}\bar{\Pi}_r^B]_{b\times(n-b)}$, $\boldsymbol{\partial_Y \bar{\Pi}^C} = [\partial_{t\rho}\bar{\Pi}_t^B]_{(n-b)\times b}$ and $\boldsymbol{\partial_X \bar{\Pi}^C} = [\partial_{t\tau}\bar{\Pi}_t^C]_{(n-b)\times(n-b)}$. One can show that for each $r \in B$,

$$\partial_{rr}\bar{\Pi}_r^B = -\frac{\left[F(n,s) + s(1-s)(n-1)\right]\left[F(n,s) - (1-s)\right]^3 (a-m)^2}{(1-s)^2(1+(b-1)s)^2 F(n,s)\left[F(n,s) + ns\right]^3} < 0,$$

for each pair $(r,\rho) \in B^2$,

$$\partial_{r\rho}\bar{\Pi}_r^B = \frac{s\left[F(n,s) - 2(1-s)\right]\left[F(n,s) - (1-s)\right]^4 (a-m)^2}{(1-s)^2(1+(b-1)s)^2\{F(n,s)\}^2\left[F(n,s) + ns\right]^3} > 0,$$

and for any $(r,\tau) \in B \times C$, we have

$$\partial_{r\tau}\bar{\Pi}_r^B = -\frac{s\left[F(n,s)-2(1-s)\right]\left[F(n,s)-(1-s)\right]^2(a-m)^2}{(1-s)(1+(b-1)s)^2\{F(n,s)\}^2\left[F(n,s)+ns\right]^3} < 0.$$

One can show that for each $t \in C$,

$$\partial_{tt}\bar{\Pi}_t^C = -\frac{\left[F(n,s)+s(n-1)\right](a-m)^2}{(1+(b-1)s)^2 F(n,s)\left[F(n,s)+ns\right]^3} < 0,$$

for each pair $(t,\tau) \in C^2$ we have

$$\partial_{t\tau}\bar{\Pi}_t^C = \frac{s\left[F(n,s)-2(1-s)\right](a-m)^2}{(1+(b-1)s)^2\{F(n,s)\}^2\left[F(n,s)+ns\right]^3} > 0,$$

and for any $(t,\rho) \in C \times B$, we have

$$\partial_{t\rho}\bar{\Pi}_t^C = -\frac{s\left[F(n,s)-2(1-s)\right]\left[F(n,s)-(1-s)\right]^2(a-m)^2}{(1-s)(1+(b-1)s)^2\{F(n,s)\}^2\left[F(n,s)+ns\right]^3} < 0.$$

Therefore, the partition matrix $\boldsymbol{\partial_Y\bar{\Pi}^B}$ and $\boldsymbol{\partial_X\bar{\Pi}^C}$ are symmetric and $\boldsymbol{\partial_X\bar{\Pi}^B} = \boldsymbol{\partial_Y\bar{\Pi}^C}$. Hence H is a symmetric matrix. Since H have all negative diagonal elements therefore it is sufficient to check H is diagonally dominated. Given $F(n,s)-2(1-s) > 0$, if $(1-s)-\left[F(n,s)-(1-s)\right]^2 > 0$, then we have $|\partial_{r\rho}\bar{\Pi}_r^B| < |\partial_{r\tau}\bar{\Pi}_r^B| = |\partial_{t\rho}\bar{\Pi}_t^C| < |\partial_{t\tau}\bar{\Pi}_t^C|$. If $s \in (0, 0.9]$, then $|\partial_{rr}\bar{\Pi}_r^B| - (n-1)|\partial_{r\tau}\bar{\Pi}_r^B| > 0$ and $|\partial_{tt}\bar{\Pi}_t^C| - (n-1)|\partial_{t\tau}\bar{\Pi}_t^C| > 0$ holds and the selection is a stable one. Moreover, given $F(n,s)-2(1-s) > 0$, if $(1-s)-\left[F(n,s)-(1-s)\right]^2 < 0$, then it follows that $|\partial_{r\rho}\bar{\Pi}_r^B| > |\partial_{r\tau}\bar{\Pi}_r^B| = |\partial_{t\rho}\bar{\Pi}_t^C| > |\partial_{t\tau}\bar{\Pi}_t^C|$. Again, if $s \in (0, 0.9]$, then $|\partial_{rr}\bar{\Pi}_r^B| - (n-1)|\partial_{r\rho}\bar{\Pi}_r^B| > 0$ and $|\partial_{tt}\bar{\Pi}_t^C| - (n-1)|\partial_{t\rho}\bar{\Pi}_t^C| > 0$ holds and we get stability in this case as well.

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