CONTRIBUTIONS TO
ANALYSIS OF CONSUMER EXPENDITURE

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PREFACE AND ACKNOWLEDGEMENTS

THIS WORK grew out of a series of studies conducted at the Indian Statistical Institute (Planning Division) during the last four or five years. These studies are all related, either because they use common methods or they represent empirical attempts to verify the basic assumptions implicit in those common methods. All these studies, however, form a part of studies in consumer behaviour in India based on the available National Sample Survey data.

A new method of estimating the Engel elasticity from the usually available grouped data is indicated here. This method is simple, convenient and has certain advantages over the time-consuming weighted least squares method. It is quicker and probably less expensive, especially where the concentration curves or fractile tabulated data are already available. What is perhaps more important, it meets the case of zero observations in a more objective manner and the method is free from the bias present in the standard method where one uses the logarithms of the group means and not the group means of the logarithms.

Does the above method, which is apparently so simple, yield in some sense a better estimate of the Engel elasticity than those provided by customary regression analyses ? We give an answer in the affirmative by deriving a few statistical properties such as consistency and asymptotic efficiency for our estimates. Some generalisations have also been made dropping out the restrictive assumptions here and there, either about the income (total expenditure) distribution or the form of the Engel relationship, or both. It is established that the regression estimate obtained from grouped arithmetic means is biased, and the magnitude of bias has been calculated in a few cases.

In view of the relative advantage of our method, and the great need for estimates of expenditure elasticity for a large number of items of consumer demand in India, we have worked out expenditure elasticities for a large number of items of consumer expenditure, separately for rural and urban sectors of India. A practical use of these elasticities has been suggested in a problem involving commodity classification of all consumer items into some new categories, e.g., bare essentials, other essentials (semi-luxuries) and non-essentials (luxuries), which have recently drawn the attention of the Government of India.

We make use of our method in estimating what is called the quality elasticity, and such elasticities have been calculated for a few selected groups of commodities. An interesting aspect of the rural-urban differences in consumption patterns has been studied by using these elasticities. We have also considered the use of Engel elasticity in an important problem, viz., the problem of estimating increase in per capita demand. A method is outlined for evaluating the effect of re-distribution of personal income of households on their demand for consumer goods and services. A simple expression has been worked out for the expected change in demand in terms of intended rates of change in the average income and the inequality of income distribution.

On the empirical side, we have made an extensive study of the inter-temporal and inter-regional variation in the inequality of total per capita consumer expenditure in India. As a by-product, this study shows that to a high degree of approximation the National Sample Survey expenditure data conform to the lognormal hypothesis. In a follow-up study, we have examined a special problem in the context of comparing the current price distributions of consumer expenditure over time. The main object here is to study the effect of existence of differentials in consumer price index on the Lorenz measure of inequality. In another study, we consider both problems of absolute levels of mean consumption and inequality of consumption expenditures.

An important feature of this study is that it provides for the first time some new data on differential price movements. This type of differential price deflators is of great importance in intertemporal comparisons of expenditure distributions, in real terms. The recently advanced Fractile Graphical Analysis has been used as a general method in our investigations as also for testing the significance of shifts in the distributions. With the specially constructed fractile data, some other calculations have also been made following our methods.

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All the errors are, of course, mine.

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Chapter 0

SUMMARY OF CHAPTERS 1-8

0.0. A brief summary of our work is given in this chapter. The basic ideas of chapters 1-8 are summarised in a connected manner in eight corresponding Sections 0.1-0.8, and the important numerical results are indicated. Some uniform notations have been employed in this Chapter; these may vary in a few cases from the notations used in the following Chapters, but the context in which they appear is clearly explained in each Chapter.

0.1. A method of computing Engel elasticities from concentration curves: Expenditure elasticities are generally obtained by assuming certain forms such as the semi-log, double-log, or log-probit relation for the Engel curve and then estimating them from family budget data by using the method of least squares. This procedure does not seem to be correct when grouped data are available. In this Chapter, an alternative method is proposed for estimating the Engel elasticity based on the use of concentration curves, which may be easily constructed for any given size-distribution data.

Usually two types of concentration curves are distinguished: (i) the Lorenz curve which relates the proportion (q_x) of aggregative total expenditure to the proportion (p_x) of persons spending up to a given level (x) of total expenditure per capita, and (ii) the specific concentration curve which similarly relates the proportion (Q_x) of aggregative consumption of a specific commodity to the same proportion of persons.

The method proposed here makes use of the ordinates of the Lorenz curve of total expenditure and the concentration curve of specific commodity, corresponding to median per capita total expenditure. Let these ordinates be denoted by q and Q respectively, and their corresponding standard normal deviates[1)] by t_q and t_Q. The Engel elasticity (ε) of the specific commodity is then given by the ratio

$$\varepsilon = \frac{t_Q}{t_q} \qquad (0.1)$$

This method, however, is based on two crucial assumptions, viz., (a) that the Engel curve of specific commodity is of the form

$$\psi(x) = E(y \mid x) = \Lambda x^{\varepsilon} \qquad (0.2)$$

where y and x represent respectively the per capita expenditure on the specific commodity and total per capita expenditure on all items; E stands for expectation, Λ and ε being constants to be estimated; (b) that the distribution of x is lognormal with the parameters θ and λ. That is to say, the random variable x has the density function

1) $\varphi(t_p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t_p} \exp\left(-\frac{u^2}{2}\right) du, \qquad 0 < p < 1.$

$$g(x) = \frac{1}{x\lambda\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\log x - \theta}{\lambda}\right)^2\right], \quad x > 0 \qquad (0.3)$$

Under these two assumptions, which have been fairly well established for the National Sample Survey (NSS) consumer expenditure data used in our studies, the equations of the Lorenz curve and the specific concentration curve take the form

$$t_q = t_p - \lambda \qquad (0.4)$$

$$t_Q = t_p - \lambda\varepsilon \qquad (0.5)$$

If in equations (0.4) and (0.5) p is set equal to one-half, i.e., corresponding to the median of x, the value of t_p will be zero, and hence equation (0.1) for the Engel elasticity.

Expenditure elasticities have been worked out using our method for some of the selected items of consumer expenditure, and these are compared with other estimates obtained by conventional methods of regression.

0.2. Estimation of Engel curves from grouped survey data. In this Chapter we ask ourselves the question: Does the method of concentration curves developed in Chapter 1, which is apparently so simple and perhaps less expensive where such curves have already been drawn for other reasons, yield in some sense a better estimate of the Engel elasticity than those provided by customary regression analyses. This

Chapter provides an answer in the affirmative by deriving a few statistical properties such as consistency and asymptotic efficiency for our estimates. Some generalisations have been made by dropping out the restrictive assumptions here and there, either about the income (total expenditure) distribution or the form of Engel relationship, or both. It is also established that the regression estimate computed from grouped arithmetic means under the double-log hypothesis is biased, and the magnitude of bias has been calculated in a few cases.

The advantages of fractile data have been fully utilised in the methodological development of this Chapter. Special graphical tests are suggested for empirically verifying the joint assumptions of log-normality and constancy of Engel elasticity from such data. The necessary test criteria are provided by the equations (0.4) and (0.5) which also ensure the symmetry of the underlying concentration curves. Besides graphical tests, some simple analytical tests are also considered for assessing the goodness of fit. These tests make use of interpenetrating sub-samples, and are based on the concept of distance between two populations.

The remaining parameters λ, θ and Λ in our basic equations (0.2) and (0.3) are estimated as follows :

$$\lambda = -t_q$$

$$\Theta = \log_e \bar{x} - \frac{1}{2} t_q^2$$

$$A = \bar{y}(\bar{x})^{-t_Q/t_q} \exp\left[\frac{1}{2} t_Q(t_q - t_Q)\right] \qquad (0.6)$$

All these estimates, like the estimate of the elasticity, are consistent for their respective parameters. The consistency of these estimates has been proved by considering certain new properties of ordered statistics.

The concept of ordering is an integral part of all size-distribution data, particularly when they are formed according to fixed fractile groups. This point is not explicitly taken into account in conventional methods of regression. The use of grouped survey data leads to further complications in the regression analysis. Such difficulties arise from the possible presence of serial correlation and errors in the variables. In view of these and other difficulties, the regression estimates will not only be biased but also inconsistent. This point has been carefully examined in the context of our method of estimation of Engel curves.

Given the same basic assumptions (0.2) and (0.3), there is another possibility of obtaining the Engel elasticity. That is to use the expressions of the Lorenz and the specific concentration coefficients. Denoting these by L_o and L_s respectively, we have

$$L_o = 2\,\phi\left(\frac{\lambda}{\sqrt{2}}\right) - 1 \tag{0.7}$$

$$L_s = 2\,\phi\left(\frac{\lambda}{\sqrt{2}}\right) - 1 \tag{0.8}$$

so that the elasticity is given by

$$\varepsilon = \frac{t_{\frac{1}{2}}(1 + L_s)}{t_{\frac{1}{2}}(1 + L_o)} \tag{0.9}$$

In this form the elasticity may easily be estimated by replacing L_o and L_s by their respective estimates L_o and L_s.

Perhaps, a still simpler method is to take the ratio of L_s to L_o as an estimate of the Engel elasticity. But this ratio does not provide a consistent estimate if the distribution of x is not log-normal; it does, however, give a consistent estimate if the distribution of x is log-logistic, i.e., if the cumulative distribution function $F(x)$ satisfies the following relationship :

$$\log \frac{F(x)}{1 - F(x)} = a + b \log x \tag{0.10}$$

The case of semi-log Engel curve is also examined in the light of our methods. For the semi-log case we have

$$E(y \mid x) = \gamma + \delta \log x \tag{0.11}$$

in which γ and δ are parameters. The 'variable' elasticity $\varepsilon(x)$ is given by

$$\varepsilon\ (x) = \frac{\delta}{\gamma + \delta \log x} \tag{0.12}$$

Under the lognormal hypothesis, the Engel elasticity computed at the median income, $C = \exp(\theta)$, is given by

$$\varepsilon(C) = \frac{\delta}{\gamma + \delta\theta} \tag{0.13}$$

A consistent procedure for estimating the parameters (γ, δ), also based on concentration curves, is indicated in this Chapter by deriving the equation of the specific concentration curve. In the semi-log case, the specific concentration curve has the equation

$$Q_x = p_x - \varepsilon\ (C)\ Z\ (t_{p_x}) \ . \tag{0.14}$$

where $Z(t_{p_x})$ is the ordinate of the standard normal curve at t_{p_x}. The median elasticity is estimated by

$$\varepsilon(C) = \frac{0.5 - Q}{\lambda\ Z(0)} \tag{0.15}$$

where $Z(0) = \frac{1}{\sqrt{2\pi}}$. Other parameters are at once easily estimated. Finally an interesting inequality is derived showing that the mean, median and constant elasticities are in ascending order.

0.3. <u>Some estimates of expenditure elasticity.</u> In India at present there is a great need for estimates of income (or expenditure) elasticity for a large number of items of consumer expenditure. Many practical uses of such elasticities are known: A major use perhaps is in the classification of commodities according as whether they are essential or luxuries. This information is needed in the determination of commodity taxes as well as in the estimation of tax yields. If one wants to compile consumer price indices by some new categories of consumption, e.g., bare essentials, other essentials (semi-luxuries), and non-essentials (luxuries), some objective classificatory criterion is needed, and this is provided by the Engel elasticity. In problems of demand projections the role of expenditure elasticities is well known. The objective of this Chapter is to provide estimates of expenditure elasticities for as many items of expenditure as are possible from the presently available data on consumer expenditure in India.

The method adopted here for estimating the Engel elasticities and the assumptions underlying its use have already been discussed in Chapters 1 and 2. This method is convenient and has certain positive advantages over the standard weighted least squares method of estimation based on grouped family budget data. It is quicker, especially when the concentration curves are already available. What is perhaps more important, it meets the cases of zero observations ($\bar{y}$' s) in a more objective manner and the method is free from the small biases present in the standard method where one uses the logarithms of the group means $(\bar{x}_i, \bar{y}_i)$ and <u>not</u> the group means of the logarithms.

Expenditure elasticities have been extensively worked out for the United Kingdom and other advanced countries. Among the notable contributions in this area are those of Stone (1954), Prais and Houthakker ((1955)[3]. In all these studies the conventional method of multivariate regression has been employed. For most part, Stone's analysis is based on aggregative time series data, while Prais and Houthakker's study is entirely based on family budget data. The time series approach to demand analysis, though more appropriate in problems of prediction, is perhaps not suitable for India at present for the simple reason that the required time series data are not available or, if available, they are inadequate for all analytical purposes, both quantitatively and qualitatively. For detailed analysis of consumer expenditure in India, one has to rely on the available National Sample Survey data which are generally grouped in size classes of total per capita expenditure. The method of regression is not quite appropriate for such data, particularly when estimating convex or concave Engel curves .

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3) See Richard Stone, The Measurement of Consumer's Expenditure and Behaviour in the United Kingdom: 1920-1938. Cambridge University Press, 1954; H. S. Houthakker, The Analysis of Family Budgets, Cambridge University Press, 1955.

Using our own methods, we present in this Chapter a comprehensive table showing expenditure elasticities for as many as, seventy items of consumption[2]. The estimates are given separately for rural and urban India, and are based on the extensively tabulated NSS tenth round data. The materials used here have been taken from a mimeographed paper by Lydall and Ahmad (1960), and as these data do not provide for sub-samples, the value of our estimates is perhaps slightly diminished since we have not considered the question of margins of sampling and non-sampling errors of the survey data. However, for a number of items, sub-samplewise estimates are compiled but these estimates were computed by the method of least squares whose limitations have already been discussed in the earlier Chapters. Alternative formulae of Chapter 2 have also been used side by side for the estimation of elasticities. A practical application of our estimates is considered towards the end in some detail in the context of an attempt to compile consumer price indices by some new and important categories of consumption.

0.4. Quality elasticities for certain commodities. This Chapter is a preliminary attempt to examine the extent of quality preferences in rural and urban areas of India. The concept of quality elasticity is

2) Some of these, however, are sub-totals.

used, and such elasticities are obtained by making use of the technique of concentration curves for a few selected commodities. In some cases, however, weighted regression is used. These studies are based on grouped consumption data provided by the NSS.

The NSS household expenditure data are generally provided for items which are not homogeneous, or of uniform quality, but represent groups of sub-items which are closely related substitutes. Consider, for instance, the item 'cereals'. Here the sub-items are rice, wheat jowar, bajra, etc., plus the products like muri, chira obtained by drying, parching, powdering etc., there are also coarse, medium and fine varieties. All these are included under 'cereals'. For a few of such composite items, the NSS gives estimates of average item demand in both quantitative (Q) and value (V) terms, so that one can calculate the Engel elasticity of either with respect to the total consumer expenditure per capita (E). The two elasticities may be called 'quantity' and 'value' elasticities and are represented by ε_Q and ε_V respectively. These two types of elasticities have been calculated in this Chapter for the following composite items: cereals, foodgrains (i.e. cereals plus cereals substitutes), milk, salt, sugar, and milk for which estimates are available for both quantitative and value demand.

When both value and quantity elasticities are available, one may study their differences and consider it as the 'quality' elasticity. This has been actually done in this Chapter. It has been shown that in general the value elasticity is somewhat larger than the quantity elasticity. Now the positive difference, the 'quality' elasticity (ε_p) is the elasticity of the average price paid by consumers at different levels of living (E). Ignoring the possibility that this elasticity may be spurious, and may arise due to seasonal or regional variation in prices, or due to some kind of price discriminatory phenomena, if any, this elasticity of price must obviously be attributed to the shift of consumers' preference to the finer or superior sub-items of the item group as E rises. (e.g. from coarser cereals like jowar to finer cereals like wheat). So the quality elasticity indicates sensitiveness of consumers to qualities of sub-items.

Our calculations show that the quantity and value elasticities in rural areas are, for all items considered except salt, higher than the corresponding figures in urban areas. For example, sugar appears somewhat to be a luxury for the rural population but this is perhaps not true to the same extent in urban areas. The commodities sugar and milk are in the nature of luxuries with their elasticities exceeding unity and the rest belong to the necessary group.

The positive signs for quality elasticities for all the commodities irrespective of the regional differences suggest that the consumers generally tend to pay higher prices for oxtensibly similar items, that is, to move for better qualities within the commodity group as their standard of living improves. Moreover, the relative luxuries, sugar and milk, are associated with higher values of quality elasticity. The present analysis goes to confirm the general notion that in rural areas of India the consumption habits are relatively more regid than in urban areas.

From another consideration, it is observed that the changes in physical demand for specific commodities are mostly explained by changes in the level of living, i.e., per capita total expenditure, whereas quality considerations are relatively of minor importance. The high values of partial price elasticities for sugar and milk in urban areas indicate that these commodities have comparatively high degree of substitutability. There is also some suggestion that the rural population usually prefers coarser varieties of food grains at low prices.

0.5. <u>A problem of estimating increase in demand</u>: This Chapter briefly outlines a method of evaluating the effect of redistribution of personal income (total expenditure) of households on their demand for consumer goods and services. It is shown, for instance, that a reduction of inequality of incomes leads to an increase in demand for certain types of commodities whether or not there is an increase in the average income. A simple expression is worked out for the expected change in demand in terms of intended rates of change in the average income and income inequality. In obtaining this expression, the same basic assumptions of Chapter I were made regarding the shape of the income distribution and the form of the demand-income relationship.

Denoting by x the overall expenditure per capita (proxy for income) and by y the expenditure per capita on a specific item, let us write the marginal distribution of x by $g(x)$, and the conditional expectation of y for a given value of x by $E(y \mid x) = \psi(x)$. Now suppose that the we intend to alter the marginal distribution of x from $g(x)$ to $g^*(x)$. Then under the assumption that ψ remains invariant, the relative increase in the average expenditure per capita is given by

$$I = \frac{E(\psi \mid g^*)}{E(\psi \mid g)} - 1 \tag{0.16}$$

Here,

$$E(\psi \mid g) = \int_0^\infty \psi(x)\, g(x)\, dx \tag{0.17}$$

is the average expenditure per capita on the particular item when the overall expenditure distribution is $g(x)$. Similarly $E(\Psi \mid g^*)$ is defined.

From our basic assumptions 1.2 and 1.3 of Chapter 1, the relative change in the average per capita demand for the specific commodity becomes

$$I = \exp\left[(\theta^* - \theta) + \frac{1}{2}(\lambda^{*2} - \lambda^2) \right] - 1, \qquad (0.18)$$

the starred parameters referring to the intended distribution $g^*(x)$. There is also an implicit assumption that the Engel curve is absolutely invariant.

Suppose now that a change in the distribution is intended both in respect of the average income (μ) (total expenditure per capita of households) and the Lorenz measure (L) of inequality. Let α and β respectively denote the proportionate increase and <u>decrease</u> in the value of parameters μ and L over a given period of time. At the end of the period, we should have:

$$\mu^* = \mu(1 + \alpha) \qquad (0.19)$$

$$L^* = L(1 - \beta) \qquad (0.20)$$

These target values are supposed to be predetermined. Using these values the unknown parameters θ^* and λ^* in (0.18) may be easily worked. Expression (0.18) may be written

$$I_{\alpha,\beta}(\varepsilon) = C(\beta,\varepsilon)(1+\alpha)^{\varepsilon} - 1 \qquad (0.21)$$

where

$$C(\beta,\varepsilon) = \exp\left[\frac{1}{2}\varepsilon(1-\varepsilon)(\lambda^2 - \lambda^{*2})\right] \qquad (0.22)$$

$$\lambda^* = \sqrt{2}\; t_{\frac{1}{2}(1+L^*)} \qquad (0.23)$$

Two interesting special cases are :

Case (i): When $\beta = 0$, i.e. when the inequality remains unchanged during the period while the average income increases, i.e. $\lambda^* = \lambda$ so that $C(\beta,\varepsilon) = 1$ and $\alpha > 0$.

In this case we have the most common form

$$I_{\alpha,o}(\varepsilon) = (1+\alpha)^{\varepsilon} - 1 \qquad (0.24)$$

which is positive for all $\varepsilon > 0$.

Case (ii). When $\alpha = 0$, i.e. when there is reduction in the inequality but no change in the average income level, then $\beta > 0$ and $\mu^* = \mu$.

Consequently,

$$I_{0,\beta}(\varepsilon) = C(\beta,\varepsilon) - 1 \qquad (0.25)$$

which is positive for all $\varepsilon < 1$, but negative for $\varepsilon > 1$.

Similar results are obtained under the semi-log hypothesis, which may give a better fit for necessary goods and services whose expenditure elasticity is not very high. The various results are finally numerically illustrated from the National Sample Survey data, making use, of course, of our elasticity estimates of Chapter I.

Now assuming that the average income (total expenditure) per capita increases by 15 per cent and the concentration of incomes becomes lower by 5 per cent, the increase in demand per person for foodgrains, clothing and medicine are found to be 9.0, 21.9 and 26.1 percent respectively. If one did not consider the effect of changes in the Lorenz ratio one would get 8.4, 24.2 and 33.3 percent for the same items. In the latter case, the demand for clothing and medicine is to some extent exaggerated (by 3 and 7 percent respectively) while the demand for foodgrains is slightly underestimated (to the extent of 0.6 percent).

If we do not consider the possibility of a rise in the agerage level of income per capita, even then the demand for necessaries increases under a more even distribution of incomes. For instance, in the case of consumption of food the demand increases by 1.5 percent if the level of concentration is brought down by 20 percent (through taxation or appropriate policies of the Government). This latter possibility, however, does not seem to be very realistic. The results of this Chapter are purely of methodological interest based as they are on hypothetical rates of change in the distribution parameters. In the following Chapter we have made an extensive study of the available NSS expenditure distributions over a number of rounds in order to see if they changed significantly over time in respect of the inequality parameter.

0.6. <u>Intertemporal and inter-regional variation in expenditure inequality in India.</u> This chapter embodies the results of an extensive empirical study of the National Sample Survey (NSS) data relating to consumer expenditure distribution in India. The investigation here covers both intertemporal and inter-regional comparisons. In the intertemporal analysis, NSS second to fourteenth round data covering approximately a period of 9 years have been used whereas the inter-state comparisons are confined only to thirteenth round. Throughout the study, rural and urban sectors have been separately treated and the entire analysis is carried out by two independent and interpenetrating sub-samples, which are a special feature of the Indian National Sample Survey. This Chapter also provides a systematic survey of Indian data on consumer expenditure distribution.

This study leaves aside the problem of comparing the means of consumer expenditure over different rounds inspite of the growing importance of that problem. What is attempted, instead , is to compare those distributions in respect of what is vaguely known as inequality. Various measures of concentration have been used for this purpose; they include the customary relative shares in the aggregate consumption according to various groups of population, e.g., bottom 10 percent, arranged according to their levels of living, as well as the summary index, the Lorenz ratio, which we have already used in the foregoing chapters.

Graphical tests have been made to compare in their entirely the distributions over the fourteen rounds covering the period October 1950 - June 1959, after bringing them to common means. For all these rounds, and for all the State distributions of the thirteenth round, the hypothesis of lognormality has been graphically tested by means of the log-probit diagrams; but the estimates of the lognormal were obtained by our methods of chapter 2 instead of from the fitted log-probit straight lines.

The present analysis not only covers rural and urban areas separately but also extends to all India for which the expenditure distributions were specially constructed. The various indices of inequality were plotted against the rounds and suitable diagrams were constructed keeping in view the margin of uncertainty of those graphs provided by the two interpenetrating sub-samples of the NSS.

Certain tentative but important conclusions have been drawn from our analysis. The overall finding seems to be that the expenditure distributions, which conform to a high degree of approximation to the lognormal hypothesis, have remained more or less unchanged over the 9 year period under study in both rural and urban areas of India. The small shifts from round to round were mostly self-correcting fluctuations with no discernible trend, except for slight increase in consumption of certain classes of population. It appears from our analysis that the share in total consumption of the bottom 10 percent, or 25 percent of the population tended to increase, though very slightly, for the rural sector, while in the urban areas the corresponding shares remained practically constant between the third and twelfth rounds. The position is somewhat reversed in the case of the top (or the richest) 10 percent, 25 percent of the population in the two sectors. The top 10 percent (or 25 percent) of the urban population seems to have increased their share of total urban expenditure, (mostly after the tenth round), whereas the comparable stratum of rural population has more or less maintained its share.

The above observations must be taken with great caution, particularly those involving the top 10 percent of the population in either sector, as these are subject to wider margins of errors as shown by the divergence between the sub-sample results. The differences observed are quite small, and may not be significant in the statistical sense, leaving apart the problem of non-sampling errors.

The top 5 percent of the urban population forms in a way the richest class of the country's population, their share in the total consumption of the country seems to have increased at a much faster rate.

There is also a suggestion that the State distributions varied appreciably from one another in respect of inequality during the thirteenth round, and in all cases the urban inequality was considerably greater than the rural inequality.

This Chapter highlights some of the special difficulties, both statistical and conceptual, that have to be encountered in intertemporal studies of expenditure distributions, with special reference to the National Sample Survey data. A major conceptual difficulty arises from possible variation in prices over time ; this variation in prices may not be uniform for all groups of households at different levels of living. The effect of differentials in consumer price indices on comparisons based on the Lorenz measure of inequality is examined in the next Chapter.

0.7. The effect of differentials in consumer price index on Lorenz measure of inequality. For comparing the inequalities of two expenditure distributions differing in time and space, one should bring the latter distribution to the prices of the earlier. This need not arise if the consumer price index, for the latter period with the earlier as base, does not change with the level of expenditure ; but where this index increases (or decreases) monotonically with expenditure the current price distribution for the later period shows greater (or less) inequality, as measured by the Lorenz ratio, than the corresponding constant price distribution. Analogous results have been proved in this Chapter for the concentration curves of expenditure on specific commodities. These results are numerically illustrated from the National Sample Survey data, which were specially tabulated from the available household schedules according to the principle of fractile graphical analysis. All calculations reported in this Chapter were made by two independent and interpenetrating sub-samples.

Concentration ratios were calculated separately for food items, non-food items, and for all items, for the distributions of 1952-53 and 1957-58. Use was made of two sets of appropriate price deflators, which were worked out entirely from the NSS data, for twenty fractile groups from the two sub-samples, in adjusting the 1957-58 distributions for differential price variation.

The method employed by us in the construction of consumer price indices by levels of living is fully discussed in the following Chapter. This study suggests that the overall consumer price index is a slightly decreasing function of the level of living. The food index follows the same pattern, even more prominantly, whereas the non-food index manifests a small increasing trend. The effect of differential price adjustments on the concentration may be seen from the following table. The results in Table 0.1 are well in conformity with our theoretical conclusions ; they indicate that the effect of price adjustment on the Lorenz measure of inequality is not quite negligible.

Table 0.1. Concentration ratio of expenditure distributions.

item	1952-53			1957-58 unadjusted			1957-58 adjusted		
	s.s.1	s.s.2	pooled	s.s.1	s.s.2	pooled	s.s.1	s.s.2	pooled
food	0.2435	0.2306	0.2370	0.1997	0.1943	0.1970	0.2225	0.2073	0.2149
non-food	0.4730	0.3896	0.4313	0.4463	0.3477	0.3970	0.4687	0.3352	0.4020
all items	0.3333	0.2899	0.3116	0.2676	0.2259	0.2468	0.2842	0.2370	0.2606

0.8. A study of differential price movements : An application of fractile graphical analysis : Recently several studies on intertemporal and inter-regional shifts in consumer expenditure distribution in India have been reported. Some of them are primarily concerned with inequality (Bhattacharya and Iyengar, 1961 ; Murti and Pillai, 1960 ; Roy and Dhar 1961) while others (Mahalanobis, 1958 , 1960 ; 1961) cover both aspects of the distribution, viz., inequality and average consumption levels. In the latter studies, use has been made of fractile graphical analysis in order to compare the average consumption levels between two or more National

National Sample Survey rounds. All the studies, however, employ concentration curves as a general tool to measure disparties in the levels of living However, the conclusions drawn from these studies do not seem to be valid in any strict sense because they are invariably based on value comparisons. In a recent study by Professor P.C. Mahalanobis[4], quantitative comparisons have been made of consumption of cereals between 8th and 13th rounds of the NSS, but not of total consumption. This important limitation was noted in Chapter 6, and the effect of differentials in consumer price indices on measures of inequality was theoretically examined in the last Chapter.

In this Chapter, a preliminary attempt has been made to provide data on prices in a form appropriate for intertemporal analyses of consumer expenditure distributions, that is, in the form of a series of price deflators relating to different levels of living.

As a starting point of our analysis, one such index series in presented for rural areas of West Bengal for some twenty fractile groups. Entirely based on NSS household budget data, this series represents the movement of relative prices in 1957-58 taking the base as 1952-53. Two sub-samples were used throughout the calculations. For convenience, the following major groups of consumer items were taken: food, non-food, and

4) Mahalanobis (1961)

and all items. For each of these groups, Laspeyres type of indices were constructed both fractile groupwise and sub-samplewise. Using these differential price deflators, current price distributions of 1957-58 were converted to 'constant' prices of 1952-53. Fractile graphs have been drawn for all the base distributions of 1952-53, as well as for both undeflated and properly deflated distributions of 1957-58.

The next stage was to test for the 'real' separation $A_{*}^{(1,2)}$ between the 1952-53 distributions and those of 1957-58. For this purpose, sub-sample errors, $\Lambda_{1,2}^{(1)}$ and $\Lambda_{1,2}^{(2)}$, were calculated by using the well known formulae for the areas between the various fractile graphs.

Separations were likewise computed, with the combined estimates which were obtained by taking simple averages of the corresponding sub-sample estimates. Finally, these separations were divided by their respective total errors

$$E_{*}^{(1,2)} = \sqrt{[\Lambda_{1,2}^{(1)}]^2 + [\Lambda_{1,2}^{(2)}]^2} \qquad (0.26)$$

The main results are shown in Table 0.2 . These results and other associated graphs appear to show that in real terms the distributions of total expenditure, as well as food expenditure, have indeed fafourably changed in the villages of West Bengal during the five-year period 1952-53 to 1957-58. The inequality of these distributions has appreciably declined simultaneously with rise in the levels of real expenditure. Percentage shares in total consumption, food and non-food consumption were also calculated for the bottom 10 percent and top 10 percent of the rural population. These results are shown in Table 0.3. These results should give an indication of the effect of ignoring the price differentials in intertemporal studies of expenditure distributions.

The hypothesis of lognormality and constant elasticity have also been examined in the light of our new tests given in Chapter 2. Semi-log curves were also considered, and the 'variable' elasticity worked out for all the twenty fractile groups and plotted, giving rise to markedly declining fractile groups. Finally, the effect of price differential on lognormal parameters as well as on Engel elasticity has also been expirically examined.

Table 0.2 : Separation-Error ratio.

items	$\Lambda_*^{(1,2)} / E_*^{(1,2)}$	$\Lambda_*^{(1,2^a)} / E_*^{(1,2^a)}$
(0)	(1)	(2)
food	12.7901 **	6.5863 **
non-food	1.6779	1.3319
all items	7.9586 **	4.2163 **

a) price-adjusted
** significant

Table 0.3. Percentage share in total consumption of food, non-food and all items by bottom and top 10 percent of population, Rural West Bengal.

percentage population		share in consumption p.c.								
		1952-53			1957-58			1957-58[a]		
		s.s.1	s.s.2	pooled	s.s.1	s.s.2	pooled	s.s.1	s.s.2	pooled
					all items					
bottom	10	3.24	3.65	3.44	4.04	5.00	4.52	3.79	4.62	4.20
top	10	26.92	22.45	24.68	21.85	18.92	20.38	21.45	19.14	20.30
					food items					
bottom	10	3.79	4.32	4.06	4.72	5.46	5.09	4.48	5.04	4.76
top	10	19.84	18.89	19.02	16.89	16.83	16.86	16.75	17.12	16.94
					non-food items					
bottom	10	2.38	2.52	2.45	1.82	3.62	2.72	1.76	3.23	2.50
top	10	39.03	28.46	33.74	35.32	26.83	31.08	35.49	25.81	30.65

a) price-adjusted

Chapter 1

A METHOD OF COMPUTING ENGEL ELASTICITIES FROM CONCENTRATION CURVES[1]

1.1 Expenditure elasticities are generally obtained by assuming certain forms such as the semi-log, double-log or probit relation for the Engel curves and then estimating them from family budget data by the conventional method of least squares. This procedure is by no means the best, particularly when grouped data are available. An alternative method is proposed here for estimating the Engel elasticities from two types of concentration curves which neatly summarise the average consumption pattern of the community.

This chapter presents, on the assumption of lognormality and constant elasticity, a simple graphical method of deriving the Engel elasticities for various items of consumer expenditure from what are known as concentration curves. Usually two types of such curves are distinguished: (a) the Lorenz curve which relates the proportion of total expenditure to the proportion of persons spending up to a given level of total expenditure per capita and (b) the specific concentration curve which relates the proportion of total consumption of a

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1. This chapter is substantially based on a paper which was read at the Section of Statistics of the 47th Session of the Indian Science Congress, Bombay, 1960, and which appeared subsequently in <u>Econometrica</u> (October, 1960) [3].

specific commodity to the proportion of persons spending up to a given level of total expenditure per capita. We shall briefly indicate a method of using these curves to calculate the Engel elasticities and shall present some numerical results for a few important items of consumer expenditure. These estimates are compared with other estimates obtained by the conventional method of least squares.

1.2. **The method**: For purposes of simplicity we shall assume a bivariate lognormal distribution for per capita total expenditure and per capita expenditure on the specific item, to be designated by x and y respectively. This, by definition, implies that their natural logarithms, $\log x$ and $\log y$, follow a bivariate normal distribution with certain parameters. Moreover, the marginal distribution of $\log x$ is normal with mean and standard deviation, θ and λ, say, and the conditional distribution of $\log y$ for a given value of x is also normal with mean, $\alpha + \varepsilon \log x$, and standard deviation, σ_o, independent of x. Of course α and ε are functions of the parameters of the parent population of (x, y). It is easy to show that

$$E(y \mid x) = \Lambda x^{\varepsilon} \qquad (1.1)$$

where

$$\Lambda = e^{\alpha + \frac{1}{2}\sigma_o^2}.$$

on observing that, since the distribution of y given x is lognormal, the following holds:

$$E(y \mid x) = E(y \mid \log x) = e^{E(\log y|x) + \frac{1}{2} Var(\log y|x)} \tag{1.2}$$

It follows from (1.1) that the distribution of $E(y|x)$ is log-normal.

The parameter σ_o represents one essential aspect of consumer behaviour, namely that individuals spend different amounts on consumption even if their spendable incomes are equal. In fact, equation (1.1) is an average relationship between total expenditure and specific expenditure in which ε is the Engel elasticity of the specific item of expenditure.

Now the proportion of population having a per capita total expenditure c or less is given by

$$p_c = \int_0^c g(x)\, dx \tag{1.3}$$

where $g(x)$, the marginal probability density function of x, is given by

$$g(x) = \begin{cases} \dfrac{1}{\lambda x \sqrt{2\pi}}\, e^{-\frac{1}{2}\left[\frac{\log x - \theta}{\lambda}\right]^2} & \text{if } x > 0, \\ 0 & \text{if } x \leq 0. \end{cases}$$

Expression (1.3) can be simplified by direct integration or by following a simpler argument. Now, since $\log x$ is $N(\theta, \lambda)$,

$$p_c = \text{Prob}\left\{ \log x \leq \log c \right\} = \varphi\left(\frac{\log c - \theta}{\lambda} \right) \qquad (1.4)$$

where φ is defined by the relation

$$\varphi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} \, du. \qquad (1.5)$$

The proportion of the total expenditure spent by these persons is given by

$$q_c = \frac{\int_0^c x\, g(x)\, dx}{\int_0^\infty x\, g(x)\, dx} \qquad (1.6)$$

which simplifies to

$$q_c = \varphi\left(\frac{\log c - \theta}{\lambda} - \lambda \right). \qquad (1.7)$$

The Lorenz curve is parametrically defined by $\left\{ p_c, q_c \right\}$. Elimination of c between (1.4) and (1.7) therefore gives the equation of the Lorenz curve:

$$t_q = t_p - \lambda \qquad (1.8)$$

where t is defined by the relation

$$k = \varphi(t_k), \quad 0 \leq k \leq 1. \qquad (1.9)$$

Similarly the proportion, Q_c, of the total consumption of a specific commodity by persons whose **per** capita total expenditure is c

or less is given by

$$Q_c = \frac{\int_0^c E(y \mid x)\, g(x)\, dx}{\int_0^\infty E(y \mid x)\, g(x)\, dx}. \tag{1.10}$$

The expression in (1.10) may be simplified by using the relation (1.1) and the properties that powers and multiples of lognormal variates are lognormal variates and, also, that moment distributions of lognormal variates are lognormal [1]. The denominator is simply $E(\Lambda x^{\varepsilon})$ which has the value $\Lambda e^{\varepsilon\theta + \frac{1}{2}\varepsilon^2\lambda^2}$. The numerator reduces on simple integration to

$$\Lambda\, e^{\varepsilon\theta + \frac{1}{2}\varepsilon^2\lambda^2}\, \varphi\left(\frac{\log c - \theta}{\lambda} - \lambda\varepsilon\right).$$

Consequently, equation (1.10) becomes

$$Q_c = \varphi\left(\frac{\log c - \theta}{\lambda} - \lambda\varepsilon\right). \tag{1.11}$$

The specific concentration curve[2] is defined parametrically by

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2 Our definition of a specific concentration curve is conceptually different from that of the familiar concentration curve for consumption relating the proportion of the total commodity consumed by persons consuming up to a given level of the commodity. The latter could, on the other hand, be regarded as the Lorenz curve for the given commodity. This presupposes the availability of data in size classes of commodity consumption rather than total expenditure. But in most published family budget data the classifying criterion is the total per capita expenditure thus necessitating the concept of a specific concentration curve used in the present paper.

$(p_c^{\gamma}, Q_c^{\gamma})$ and after elimination of c between (1.4) and (1.11) we get the equation for the specific concentration curve :

$$t_Q = t_p - \lambda^{\varepsilon} \qquad (1.12)$$

We notice that this curve involves the Engel elasticity as well as the inequality parameter of the distribution of total expenditure while the Lorenz curve involves only the latter. We shall use the Lorenz curve and specific concentration curves together to calculate the Engel elasticities of specific expenditures. To so-called egalitarian line is defined by $(p_c = q_c = Q_c)$.

These curves are symmetric about the diagonal joining the points (1,0) and (0, 1). The symmetry can be proved by observing that, corresponding to any point (p, q) on the Lorenz curve there is a point (1 - q, 1 - p) on the curve. The same is true of the specific concentration curves. The above equations also indicate that these curves fall well below the egalitarian line unless λ is very small. Moreover, the specific concentration curve lies between the egalitarian line and the Lorenz curve if the elasticity of the specific item falls between zero and unity. If the elasticity is greater than unity the specific curve falls below the Lorenz curve (luxuries). On the other hand, if the elasticity is negative the specific curve lies above the egalitarian line (inferior goods). These curves may, therefore, be used to classify items into necessities or luxuries.

It should, however, be noted that the symmetry of the concentration curves is a necessary but not sufficient condition for lognormality. One test that is generally used in applied work to test lognormality is the well-known probit test.

It is easy to show from (1.4) that if c be replaced by the mean per capita total expenditure we have

$$p = \varphi\left(\frac{\lambda}{2}\right), \qquad q = \varphi\left(-\frac{\lambda}{2}\right), \tag{1.13}$$

so that $p + q = 1$. Therefore, corresponding to the value of p, say, p_o, where the Lorenz curve cuts the diagonal, $p + q = 1$, we have

$$\lambda = 2\,t_{p_o}. \tag{1.14}$$

For this value of p_o, the value of Q is given by

$$Q_o = \varphi\left(\frac{1 - 2\varepsilon}{2}\lambda\right) \tag{1.15}$$

and hence

$$\lambda(1 - 2\varepsilon) = 2\,t_{Q_o}. \tag{1.16}$$

Using equations (1.14) and (1.16) we may eliminate λ and get an expression for ε in terms of the t's which can be calculated from the concentration curves. In fact, the elasticity is given by

$$\varepsilon = \frac{1}{2}\left\{1 - \frac{t_{Q_o}}{t_{p_o}}\right\} \tag{1.17}$$

A simple expression can however be obtained from the standard normal deviates t_Q and t_q corresponding to the value $p = \frac{1}{2}$. That is,

$$\varepsilon = \frac{t_{Q \mid p = \frac{1}{2}}}{t_{q \mid p = \frac{1}{2}}}, \tag{1.18}$$

a result which immediately follows from equations (1.8) and (1.12) in which t_p vanishes on putting $p = \frac{1}{2}$.

1.3. Applications. Before applying the foregoing method to actual data it is of course necessary to ensure that the underlying assumptions are, at least approximately, valid. No explicit attempt has been made in this chapter to verify the assumption of lognormality and constant elasticity, but what, instead, is done is to use the values of the parameters as obtained empirically by the above method and draw the theoretical concentration curves and compare them with the actual concentration curves. In what follows we consider, besides the Lorenz curve for total expenditure, the specific concentration curves for food-grains and clothing marked 0, 1 and 3 respectively, and present the two sets of curves in Figure 1.1. A brief indication of the type of data that are employed in the present analysis is given below, in Table 1.1.

1.

Table 1.1. Average consumer expenditure in rupees per person per month: All-India (Rural and Urban): 1955-56.

Item	Expenditure classes in Rupees per person per month												
	0-8	8-11	11-13	13-15	15-18	18-21	21-24	24-28	28-34	34-43	43-55	55 and over	all classes
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Food grains	3.49	4.69	5.89	6.35	6.93	7.58	8.26	8.52	8.71	9.33	10.08	13.98	6.61
Milk and Milk products	0.15	0.43	0.66	0.92	1.11	1.78	2.15	2.69	3.69	4.71	9.98	8.56	1.75
All food	4.95	7.17	9.14	10.36	11.74	13.42	15.37	16.79	19.12	21.18	28.95	37.29	12.26
Clothing	0.21	0.53	0.79	1.03	1.43	1.92	2.19	2.60	3.86	4.93	4.33	10.24	1.77
Health	0.01	0.09	0.11	0.19	0.27	0.37	0.46	0.51	0.84	1.21	1.73	3.90	0.43
All items	6.26	9.41	11.98	13.96	16.49	19.54	22.51	25.79	30.68	37.72	47.24	83.29	18.74
Percent of persons	14.01	17.31	11.74	9.14	11.55	8.21	7.04	5.53	5.17	4.85	2.59	2.86	100.00
Cumulative percentage	14.01	31.32	43.06	52.20	63.75	71.96	79.00	84.53	89.70	94.55	97.14	100.00	

Table 1.2: Engel elasticities and concentration ratios for selected items.

Items	Engel Elasticity			Concentration Ratio
	Our method	Least Squares Ordinary	Least Squares Weighted	
Food grains	0.58	0.49	0.54	0.21
Milk and Milk products	1.67	1.67	1.79	0.57
All-Food	0.76	0.79	0.81	0.28
Clothing	1.55	1.09	1.41	0.53
Health	1.95	2.11	2.31	0.64
Total (All Items)				0.36

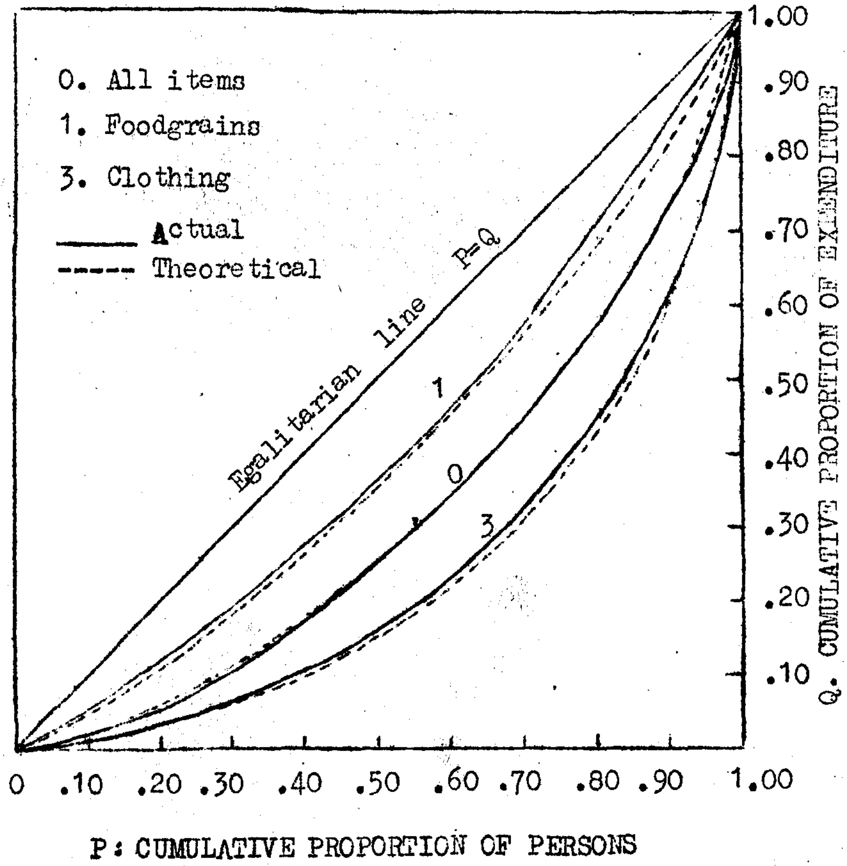

Figure 1.1. Some actual and theoretical concentration curves based on Table 1.1.

From the graphs in Figure 1.1 it is clear that the theoretical and the actual Lorenz curves (marked 0) exhibit remarkable agreement showing thereby that the assumption of lognormality is approximately valid. However, the specific concentration curves for foodgrains, unlike those for clothing, show some discrepancy, the theoretical curve being consistently below the empirical curve. This apparent discrepancy possibly indicates a departure from the constant elasticity hypothesis, which is reasonable in this case because food grains might be expected to show a definite decline in elasticity, particularly at higher incomes. From Figures 1.2 and 1.3 described below, this tendency also appears, if not prominently, in the case of health, especially in the medium range of incomes. For other items, however, it is found that the assumption of constant elasticity is fairly reasonable. The theoretical curves

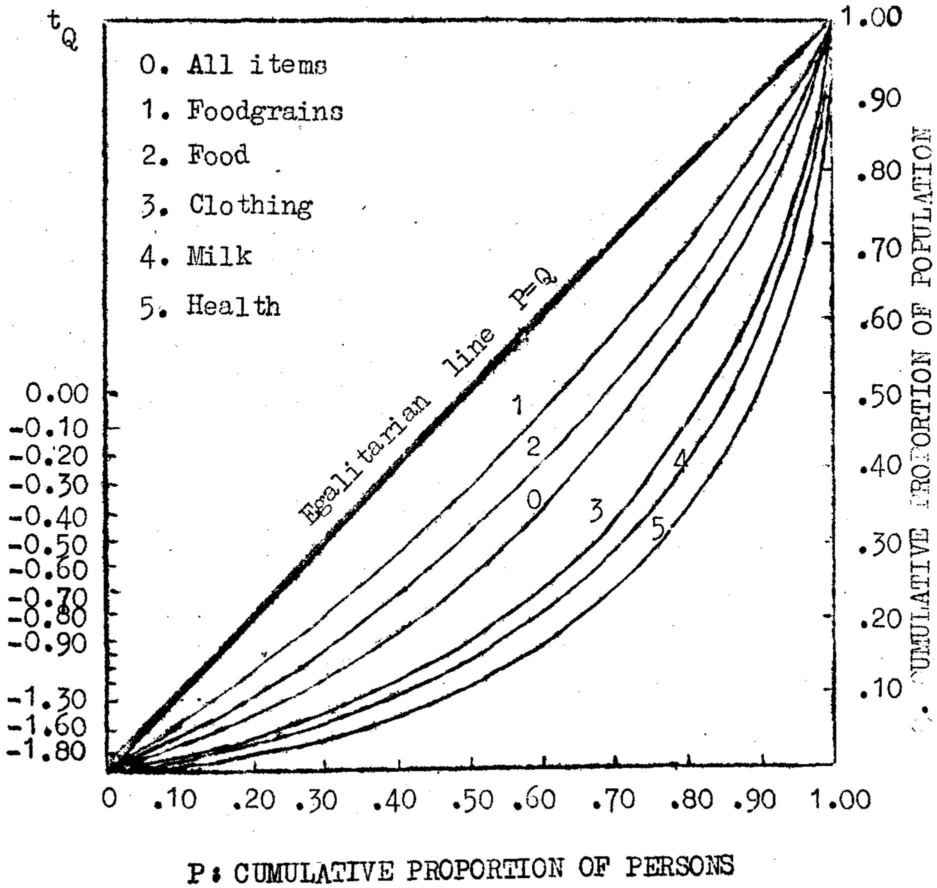

Figure 1.2 : Empirical concentration curves based on Table 1.1.

are together presented in Figure 1.3 so that one can see the adequacy of the underlying assumptions by actually comparing them with the observed curves given in Eigure 1.2. A more conclusive graphical test for the joint assumption of lognormality and constant elasticity is suggested in Chapter 2 and this has been employed in our empirical studies in Chapter 3.

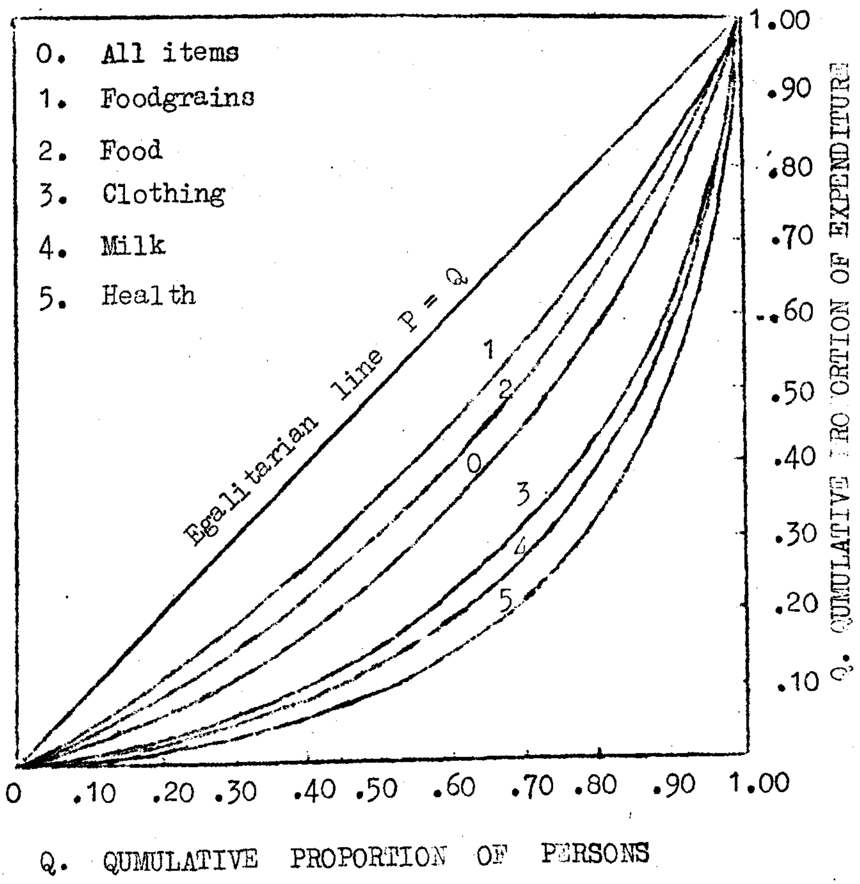

Figure 1.3 : Theoretical Concentration Curves Based on Table 1.1

To illustrate the use of the method given in Section 1.2 we have utilised the statistical material given in Table 1.1 based on the Indian National Sample Survey: Tenth Round [4].

The cumulative proportion of persons is given in the last line of Table 1.1 and those of total expenditure and specific expenditure can be derived by using the first six lines. The corresponding concentration curves together with the Lorenz curve are shown in Figure 1.2 in which the numbers 1 to 5 refer to the following five composite commodities: 1. Foodgrains, 2. All-Food, 3. Clothing, 4. Milk and Milk Products, and 5. Health. The Lorenz curve of total expenditure is of course designated by 0. All these curves display approximate symmetry about the diagonal opposite to the egalitarian line.

Expenditure on health possesses the highest concentration and milk and milk products come only next to that. Clothing has a more egalitarian distfibution than either of the above items. On the other hand, Food grains is most evenly distributed, but food taken as a whole has a greater concentration than Food grains taken alone. This is to be expected since 'all-food' includes not only food-grains but also other relatively luxurious items of expenditure.

We shall measure concentration by what is generally known as Gini's concentration ratio, which is defined as twice the area between the concentration curve and the egalitarian line. Under the assumption of lognormality and constant elasticity this ratio can be computed by a simple formula involving the parameters in the concentration curve. More specifically, the concentration ratio for total expenditure is $2\,\varphi(\lambda/\sqrt{2}) - 1$, while for the i-th specified item it is given by $2\,\varphi(\lambda\varepsilon_i/\sqrt{2}) - 1$, where ε_i is the Engel elasticity of the i-th item, which may be obtained by application of the method given in Section 1.2. Table 1.2 gives the elasticities as well as concentration ratios for the items considered in Table 1.1. The elasticities computed by least squares on the assumption of constant elasticity are also presented to facilitate comparison. The proportion of persons in the twelve per capita expenditure classes are used as weights in the case of weighted least squares estimates.

Table 1.3: Engel elasticities and concentration ratios for selected items of consumer expenditure for Rural India, 1955.

Item	Engel Elasticity	Concentration Ratio
Sugar	1.33	0.43
Education	2.43	0.70
Medical services	2.62	0.74
Total expenditure		0.33

In Figure 1.2 a special scale has been constructed on the left so that the normal deviate can be quickly computed corresponding to the areas given on the right. This serves well to get the elasticities approximately by mere visual inspection.

Following the same method as in Section 1.2 the Engel elasticities are given in Table 1.3 for three more items of consumer expenditure for rural India based on similar data [5]. Some of our results are used elsewhere [2] to study the effect of changes in concentration of income distribution on the projection of future levels of consumption.

1.4. Some concluding remarks: Our method is obviously not applicable in cases where the concentration curves are not symmetric. Moreover, the method may not be the simplest graphical method for estimating the elasticities from such data as we have used in our analysis. When lognormality is present, however, our method affords an easy way to obtain the estimates of the constant Engel elasticities from concentration curves which may have been drawn for other reasons, and it is particularly useful in the context of the practice in the Indian Statistical Institute of making use of size distributions and concentration curves as general analytical tools.

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Chapter 2

ESTIMATION OF ENGEL CURVES FROM GROUPED SURVEY DATA[1]

2.1. In an earlier paper [9] described in chapter 1, a simple graphical method was developed for computing Engel elasticities from concentration curves. This method, which has since been used in some of the Indian studies [10, 11] on consumer behaviour rests on two basic assumptions - the log-normality of the income (or total expenditure) distribution and the constancy of Engel elasticity - which admit empirical testing. Our method has been used with advantage in empirical work involving the calculation of a large number of elasticities from the available National Sample Survey data which usually provide aggregate consumption patterns either in fixed size classes of income (total consumer expenditure per capita) or in fixed fractile classes.[2] The latter type of tabulation, which has certain advantages in economic analyses [14] involving inter-temporal or international comparisons of levels of living, provides the primary data for the application of our method.

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1 The studies reported in this chapter were initiated at the Indian Statistical Institute and completed in March, 1963 at the Harvard Economic Research Project under a Rockefeller Foundation Fellowship.

2 Some of those studies are reported in subsequent chapters.

Perhaps the question that has not been adequately examined is the following: Does this method, apparently so simple and probably less expensive, yield in any statistical sense, a better estimate of the Engel curve than the one provided by the regression method under similar assumptions? In this chapter we attempt to answer this question and show that our procedure is consistent. An expression for the asymptotic variance of our estimate, which may be computed from the given data, is also worked out for the slope of the double-log Engel curve. On the other hand, the regression estimate of the elasticity computed from group means under the double-log hypothesis is shown to be asymptotically biased with the bias increasing with the 'true' elasticity. Under certain conditions, which arise in actual practice, it is shown that our method yields asymptotically more efficient estimate than Wald's in the double-log case, at least for relative luxuries.

In Section 2.2, the basic notations are developed and some empirical tests are proposed for verifying the basic assumptions. An alternative procedure of estimation is considered in Section 2.3 which is based on the concepts of specific and Lorenz concentration ratios. The classical method of least squares is compared with our method in Section 2.4. We consider alternative hypotheses for the distribution of income and the Engel curve and suggest appropriate estimation procedures in Section 2.5 and finally make a few general remarks in the last Section.

tables and charts are given in Appendix A.2.

2.2. Some Methodological Considerations: This section deals with the estimation of the parameters of the double-log Engel curve defined by

$$\psi(x) = E(y \mid x) = A x^{\varepsilon} \tag{2.1}$$

in which y and x represent respectively the household expenditure on the specific commodity and income (or total expenditure). The latter is supposed to be log-normally distributed with the parameters (θ, λ). That is to say, the random variable x has the density function g(x) given by

$$g(x) = \frac{1}{x\lambda\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\log x - \theta}{\lambda}\right)^2\right\}, \quad x > 0 \tag{2.2}$$

Equations (2.1) and (2.2) constitute our basic assumptions which may be easily tested given grouped survey data; this has been extensively done at the Indian Statistical Institute using the National Sample Survey data [3].

Under the assumptions stated in (2.1) and (2.2), we propose the following procedure: Let

$$q = \frac{\bar{x}_1}{\bar{x}_1 + \bar{x}_2} \quad \text{and} \quad Q = \frac{\bar{y}_1}{\bar{y}_1 + \bar{y}_2} \tag{2.3}$$

In (2.3) the $\bar{x}$'s represent the mean incomes, and $\bar{y}$'s the mean specific expenditures corresponding to two fractile classes [14] of income.

In other words, we divide the households into two equal groups on the basis of income and compute the proportionate shares of total income and consumption accuring to the lower income group. Such proportions may be obtained directly from the fractile data. But, in fixed interval case, these have to be computed by interpolation from concentration curves. The elasticity ε is estimated by [9]

$$\hat{\varepsilon} = \frac{t_Q}{t_q} \tag{2.4}$$

where t denotes the standard normal deviate defined by

$$\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} \exp\left(-\frac{t^2}{2}\right) d\,t \tag{2.5}$$

The remaining parameters in the equations (2.1) and (2.2) may be estimated as follows :

$$\hat{\lambda} = -\,t_q$$

$$\hat{\theta} = \log \bar{x} - \frac{1}{2}\, t_q^2$$

$$\hat{\Lambda} = \bar{y}(\bar{x})^{-t_Q/t_q} \exp\left\{\frac{1}{2} t_Q \left(t_q - t_Q\right)\right\} \tag{2.6}$$

where $\bar{x}$ and $\bar{y}$ are the observed mean income and mean specific expenditure of households for both income groups combined.

It now remains to show that the above estimates are consistent for their respective parameters. In order to do this, we shall use a basic property [21] concerning the asymptotic joint distribution of the fractile means.

Let $(y_1, x_1), (y_2, x_2), \ldots, (y_n, x_n)$ be n independent observations on the random variable (y, x) defined in (2.1) and let us rearrange the sample as follows :

$$\{y_{(1)}, x_{(1)}\}, \{y_{(2)}, x_{(2)}\}, \ldots, \{y_{(n)}, x_{(n)}\}$$
$$x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$$

Assuming $n = 2m$, we define the sample fractile means thus :

$$\begin{aligned} \bar{x}_1 &= \frac{1}{m}(x_{(1)} + \cdots + x_{(m)}), \quad \bar{x}_2 = \frac{1}{m}(x_{(m+1)} + \cdots + x_{(n)}) \\ \bar{y}_1 &= \frac{1}{m}(y_{(1)} + \cdots + y_{(m)}), \quad \bar{y}_2 = \frac{1}{m}(y_{(m+1)} + \cdots + y_{(n)}) \end{aligned} \tag{2.7}$$

Let us normalise the above fractile means and write

$$\begin{aligned} u_i &= \sqrt{m}\,(\bar{x}_i - \mu_i) \\ & \qquad\qquad (i = 1, 2) \\ v_i &= \sqrt{m}\,(\bar{y}_i - \nu_i) \end{aligned} \tag{2.8}$$

where the corresponding parameters (μ_i, ν_i) are defined as the truncated means,

$$\begin{aligned} \mu_1 &= E(x \mid x \leq C), \quad \mu_2 = E(x \mid x > C) \\ \nu_1 &= E(y \mid x \leq C), \quad \nu_2 = E(y \mid x > C), \end{aligned} \tag{2.9}$$

C being the median of x. It may be shown that the joint distribution of $\{u_1, u_2 \,\vdots\, v_1, v_2\}$ tends to a multivariate normal distribution with zero mean and variance covariance matrix

$$\left(\begin{array}{c|c} \Sigma & E \\ \hline & T \end{array} \right) \tag{2.10}$$

The partitioned variance-covariance matrices are systematically evaluated and presented below:

$$\begin{aligned}
&\mu = E(x) = \exp\left(\theta + \tfrac{1}{2}\lambda^2\right) \\
&\nu = E(y) = \underset{X}{E}\, E\,(y \mid x) = \Lambda \exp\left(\varepsilon\,\theta + \tfrac{1}{2}\lambda^2\varepsilon^2\right) \\
&\mu_1 = 2\mu\,\Phi(-\lambda),\quad \mu_2 = 2\mu\,\Phi(\lambda) \\
&C = \text{median} = \exp(\theta) \\
&\nu_1 = 2\;\Phi(-\lambda\varepsilon),\quad \nu_2 = 2\;\Phi(\lambda\,\varepsilon) \\
&\xi = \Psi(C) = \Lambda C^{\varepsilon}
\end{aligned} \tag{2.11}$$

Let us now introduce a set of deviations of the fractile means from their respective median positions.

$$\begin{aligned}
&M_1 = M_1^o = C - \mu_1 \qquad && M_2 = M_2^o = \mu_2 - C \\
&N_1 = N_1^o = \xi - \nu_1 \qquad && N_2 = N_2^o = \nu_2 - \xi
\end{aligned} \tag{2.12}$$

The next step will be to derive the truncated variances and covariances in the given fractile classes.

$$\begin{aligned}
\sigma_1^2 &= \mathrm{Var}\;(x \mid x \leq C) = 2\mu^2 \exp\;(\lambda^2)\;\Phi(-2\lambda) - \mu_1^2 \\
\sigma_2^2 &= \mathrm{Var}\;(x \mid x > C) = 2\mu^2 \exp\;(\lambda^2)\;\Phi(2\lambda) - \mu_2^2 \\
\tau_1^2 &= \mathrm{Var}\;(y \mid x \leq C) = 2\nu^2 \exp\;(\lambda^2\varepsilon^2)\;\Phi(-2\lambda\varepsilon) - \nu_1^2 \\
\tau_2^2 &= \mathrm{Var}\;(y \mid x > C) = 2\nu^{\lambda} \exp\;(\lambda^2\varepsilon^2)\;\Phi(2\lambda\varepsilon) - \nu_2^2 \\
\rho_1\sigma_1\tau_1 &= \mathrm{Cov}\;(x,y \mid x \leq C) = 2\mu\nu \exp\;(\lambda^2\varepsilon)\;\Phi(-\lambda-\lambda\varepsilon) - \mu_1\nu_1 \\
\rho_2\sigma_2\tau_2 &= \mathrm{Cov}\;(x,y \mid x > C) = 2\mu\nu \exp\;(\lambda^2\varepsilon)\;\Phi(\lambda+\lambda\varepsilon) - \mu_2\nu_2
\end{aligned} \tag{2.18}$$

The elements of the matrix (2.10) are obtained from (2.12) and (2.13). We have

$$\Sigma = \begin{bmatrix} \Sigma_{11} \Sigma_{12} \\ \Sigma_{21} \Sigma_{22} \end{bmatrix} = \begin{bmatrix} \sigma_1^2 + \frac{1}{2} M_1 M_1^o & \frac{1}{2} M_1 \; M_2^o \\ \frac{1}{2} M_1 \; M_2^o & \sigma_2^2 + \frac{1}{2} M_2 M_2^o \end{bmatrix}$$

$$E = \begin{bmatrix} E_{11} \; E_{12} \\ E_{21} \; E_{22} \end{bmatrix} = \begin{bmatrix} \varrho_1 \sigma_1 \quad + \frac{1}{2} M_1 N_1^o & \frac{1}{2} M_1 \; N_2^o \\ \frac{1}{2} M_2 \; N_1^o & \varrho_2 \sigma_2 \quad + \frac{1}{2} M_2 N_2^o \end{bmatrix}$$

$$T = \begin{bmatrix} T_{11} \; T_{12} \\ T_{21} \; T_{22} \end{bmatrix} = \begin{bmatrix} + \frac{1}{2} N_1 \; N_1^o & \frac{1}{2} N_1 \; N_2^o \\ \frac{1}{2} N_1 \; N_2^o & + \frac{1}{2} N_2 \; N_2^o \end{bmatrix} \tag{2.14}$$

Formulae (2.11) to (2.14) are involved in the asymptotic variance of our estimates of the parameters, particularly ε .

The consistency of the estimate (2.4) may be easily proved by observing that it is primarily a function of the observed fractile means which, in large samples, tend to their respective population values :

$$q = \frac{\bar{\bar{x}}_1}{\bar{\bar{x}}_1 + \bar{\bar{x}}_2} \sim \frac{\mu_1}{\mu_1 + \mu_2} = \frac{2\mu \, \Phi(-\lambda)}{2\mu} = \Phi(-\lambda)$$

$$Q = \frac{\bar{\bar{y}}_1}{\bar{\bar{y}}_1 + \bar{\bar{y}}_2} \sim \frac{\nu_1}{\nu_1 + \nu_2} = \frac{2\nu \, \Phi(-\lambda \, \varepsilon)}{2\nu} = \Phi(-\lambda \, \varepsilon)$$

$$\hat{\varepsilon} = \frac{t_Q}{t_q} \sim \frac{t_\phi(-\varepsilon \; \lambda)}{t_\phi(-\lambda)} = \varepsilon \tag{2.15}$$

It should be noted that the denominator involved in our estimate cannot be zero for, by definition, λ is positive. This implies that $\hat{\varepsilon}$ is a continuous function of the observed means and therefore consistent. By similar argument we may establish the consistency of the estimates (2.6) of the remaining parameters.

The asymptotic variance of the estimate of elasticity is found, after some routine mathematical drill, to be

$$m \text{ Var } (\varepsilon) \sim \frac{1}{\lambda^2}\left\{ \frac{\text{Var }(Q)}{Z^2(\lambda\varepsilon)} - 2\varepsilon\frac{\text{Cov }(q,Q)}{Z(\lambda)Z(\lambda\varepsilon)} + \varepsilon^2\frac{\text{Var }(q)}{Z^2(\lambda)}\right\} \quad (2.16)$$

where

$$Z(t) = Z(-t) = \Phi'(t) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right)$$

$$\text{Var}(q) = \frac{1}{16\mu^2}\left\{ \left(\frac{\mu_2}{\mu}\right)^2 \Sigma_{11} - 2\left(\frac{\mu_2}{\mu}\right)\left(\frac{\mu_1}{\mu}\right)\Sigma_{12} + \left(\frac{\mu_1}{\mu}\right)^2\Sigma_{22}\right\}$$

$$\text{Var}(Q) = \frac{1}{16\nu^2}\left\{ \left(\frac{\nu_2}{\nu}\right)^2 T_{11} - 2\left(\frac{\nu_2}{\nu}\right)\left(\frac{\nu_1}{\nu}\right)T_{12} + \left(\frac{\nu_1}{\nu}\right)^2 T_{22}\right\}$$

$$\text{Cov}(q,Q) = \frac{1}{16\mu\nu}\left\{ \left(\frac{\mu_2}{\mu}\right)\left(\frac{\nu_2}{\nu}\right)E_{11} - \left(\frac{\mu_2}{\mu}\right)\left(\frac{\nu_1}{\nu}\right)E_{12} - \left(\frac{\mu_1}{\mu}\right)\left(\frac{\nu_2}{\nu}\right)E_{21} + \left(\frac{\mu_1}{\mu}\right)\left(\frac{\nu_1}{\nu}\right)E_{22}\right\}$$

(2.17)

A consistent estimate of Var $(\hat{\varepsilon})$ may be obtained by replacing the population values in (2.16) by their respective estimates. Once this has been done, the usual large sample statistical tests of significance may be devised for the elasticity. We shall not venture to work out variances for all the other parameters found in equations

(2.1) and (2.2) except for inequality parameter λ. The latter plays an important role in interregional, intertemporal and international studies on income distribution; it is directly related to the Lorenz measure of inequality L by the equation [1]

$$L = 2\,\Phi\left(\frac{\lambda}{\sqrt{2}}\right) - 1 \qquad (2.18)$$

As noted earlier in (2.6), the inequality parameter is estimated by $\hat{\lambda} = -t_q$ which has the asymptotic variance

$$\mathrm{Var}\,(\hat{\lambda}) = \frac{\mathrm{Var}\,(q)}{Z^2(\lambda)} \qquad (2.19)$$

The above expression, which incidentally appears in the formula for the asymptotic variance of $\hat{\varepsilon}$, may also be computed either directly from the group means or from quantities which may be derived therefrom.

It might be appropriate at this stage to indicate certain procedures to test the underlying basis assumptions. A simple graphical test for log-normality of income distribution and constant income elasticity of demand is provided in terms of the following proportions computed for variable levels of income:

p: proportion of households (or individuals) earning a given income (x) or less,

q: proportion of aggregate income earned by the above stratum of households (or individuals), and

Q: proportion of aggregate consumption accruing to the above stratum of households (or individuals).

The test for the log-normality of income distribution is that the standard normal deviates, t_q and t_p, as defined in (2.5), are linearly related by the equation $t_q = t_p - \lambda$ where λ is the inequality parameter. For constant income elasticity besides log-normality, t_Q and t_p are likewise related by the equation $t_Q = t_p - \lambda\varepsilon$ where represents the income elasticity of demand[3]. These conditions indeed ensure symmetry of the corresponding concentration curves relating q and Q with p [see [9], p. 884]. The above tests, though necessary, need not be sufficient just as symmetry of the Lorenz curve does not ensure log-normality.

The tests proposed above are perhaps stronger than the customary linear log-probit test and are easily adaptable to both fixed-interval and fractile forms of grouped data. It may be noted that a log-probit graphical test can be performed with the usual frequency distribution in fixed classes of income. However, while one has size distribution data giving both distribution of households as well as of total income in income brackets, such as in the case of income tax statistics or the family budget data, the log-probit test does not fully utilize the latter information and is therefore weaker than our tests.

3. These equations readily follow from the definition of p, q, and Q. If the distribution of income (x) is log-normal and the Engel curve is of the form $\overline{(\underline{+})}(X) = \Lambda x^{\eta}$, then $p = \phi(t)$, $q = \phi(t-\lambda)$ and $Q = \phi(t-\lambda\varepsilon)$ where $t = (\log x - \theta)/\lambda$, is $N(0, 1)$.

Analytical tests such as the use of frequency chi-square statistic, $\chi^2 = \Sigma\, (O - E)^2/E$ where O = observed and E = expected frequency is inappropriate when the sampling scheme is other than simple (stratified, multistage, pps, etc.). If the sampling scheme is better than the simple in some sense, the χ^2-statistic would on an average be smaller than the actual χ^2 and therefore underestimate the significance. If there are g classes and $\hat{\pi}_i$ is the sample estimate of the relative frequency in the i-th class, an appropriate statistic for examining the goodness of fit can be given in terms of a consistent estimate $((d_{ij}))$ of the dispersion matrix of the $\hat{\pi}_i$'s. However, since the cost of computing $((d_{ij}))$ is often much higher than the cost of computing the $\hat{\pi}_i$'s, the dispersion matrix is seldom computed.

Roy and Dhar [19] have performed some tests based on the concept of distance between two populations. Their approach may be summed up as follows: Let $d(\underset{\sim}{\pi}, \underset{\sim}{\pi}^*)$ be a distance function between two discrete probability distributions $\underset{\sim}{\pi} = \{\pi_i\}$ and $\underset{\sim}{\pi}^* = \{\pi_i^*\}$ in g classes. Let Ω be a family of distributions of which $\underset{\sim}{\pi}^*$ is a typical member. The distance between $\underset{\sim}{\pi}$ and Ω is defined as

$$\Delta = \inf_{\underset{\sim}{\pi}^*} d\,(\underset{\sim}{\pi}, \underset{\sim}{\pi}^*) \qquad (2.20)$$

If $\hat{\underset{\sim}{\pi}} = \{\hat{\pi}_i\}$ be an estimate of $\underset{\sim}{\pi}$, then $D = \inf_{\pi^*} d(\hat{\underset{\sim}{\pi}}, \underset{\sim}{\pi}^*)$ is taken as an estimate of Δ. Under certain conditions, a normalising constant a_n

depending on the sample size n can be chosen so that $a_n(D - \Delta)$ is asymptotically distributed as $N(0, 1)$ provided that $\Delta \neq 0$. If two inter-penetrating sub-samples each of size n are available, as in the National Sample Survey of India, there will be two independent estimates D_1 and D_2 of Δ and asymptotically $(D_1 + D_2 - 2\Delta)/ |D_1 - D_2|$ has a t-distribution with one degree of freedom. From this a confidence interval for Δ can be built up.

The measure of consistency used by them is defined by

$C = \sum_{i=1}^{g} \sqrt{\pi_i \pi_i^*}$, which is equivalent to using Bhattacharya's distance [2] function

$$d(\underset{\sim}{\pi}, \underset{\sim}{\pi}^*) = \cos^{-1} \left(\sum_{i=1}^{g} \sqrt{\pi_i \pi_i^*} \right) \tag{2.21}$$

Using the estimates of log-normal parameters, θ and λ, computed from two sub-samples in each round of the National Sample Survey separately for rural and urban India by Bhattacharya and Iyengar [3], the measure of consistency in each case was computed. In most cases C has been found to be of the order of 0.98 confirming thereby the findings of earlier studies [3, 20].

The test, though admittedly inefficient, is a valid procedure particularly in the context of a complex sampling design in the National Sample Survey which renders customary statistical tests somewhat inapplicable [15].

2.3. An alternative procedure: In this section we shall briefly indicate another possibility of obtaining Engel elasticities using grouped expenditure data. One may argue that our estimate $\hat{\varepsilon}$ developed in the previous section, though consistent, need not necessarily be the best in the sense of minimum variance, especially when one has more than two fractile classes. The National Sample Survey of India, for instance, provides consumer expenditure data for certain commodities by ten or twenty fractile groups [16, 17]. Our method, if applied to such data, possibly ignores much of the inter-group variation by combining the given classes into two median classes and may probably yield inefficient estimates. Under these circumstances, a somewhat different but intuitively satisfactory procedure was proposed by the author in an earlier paper [10]. We shall presently show that the alternative estimate, say $\hat{\varepsilon}_g$, which makes use of all the g pairs of group means $(\bar{y}_i, \bar{x}_i)$, is asymptotically unbiased for large values of g. The alternative procedure consists of the following steps:

First, let us compute the cumulative proportions $(\hat{q}_i, \hat{Q}_i)$ of total income and specific expenditure corresponding to the i-th fractile class.

$$\hat{q}_i = \frac{\bar{x}_1 + \dots + \bar{x}_i}{\bar{x}_1 + \dots + \bar{x}_g} \qquad (i = 1, 2, \dots, g) \qquad (2.22)$$

$$\hat{Q}_i = \frac{\bar{y}_1 + \dots + \bar{y}_i}{\bar{y}_1 + \dots + \bar{y}_g}$$

Secondly, we calculate the Lorenz ratio $\hat{L}_o$ and the specific concentration ratio $\hat{L}_s$ (For definition of these concepts, see [7], p. 884) using the cumulative proportions (2.22):

$$\hat{L}_o = 1 - \frac{1}{g} \sum_{i=1}^{g} (\hat{q}_i + \hat{q}_{i-1})$$
$$\hat{L}_s = 1 - \frac{1}{g} \sum_{i=1}^{g} (\hat{Q}_i + \hat{Q}_{i-1}) \qquad (2.23)$$

where $(\hat{q}_0, \hat{Q}_0) = (0, 0)$ and $(\hat{q}_g, \hat{Q}_g) = (1, 1)$.

Lastly, we compute the Engel elasticity by using the formula:

$$\hat{\varepsilon}_g = \frac{t_{\frac{1}{2}(1+L_s)}}{t_{\frac{1}{2}(1+L_o)}} \qquad (2.24)$$

where t is the standard normal deviate defined in (2.5). It will be noted that the proportions $\frac{1}{2}(1+L_o)$ and $\frac{1}{2}(1+L_s)$ are respectively nothing but the areas above the Lorenz curve and the specific concentration curve contained in the unit square.

This procedure is applicable also for grouped data classified according to fixed class-intervals in which case, of course, the definitions (2.22) and (2.23) will have to be slightly generalised (see [10], p. 385)

We shall next examine whether (2.24) is consistent for ε. In order to do this let g be fixed. With a given $g(> 2)$ it is easy to verify the following statement, in view of our assumptions (2.1) and (2.2).

$$\hat{L}_o \sim 1 - \frac{1}{g} \sum_{i=1}^{g} [\Phi(t_i - \lambda) + \Phi(t_{i-1} - \lambda)]$$

$$\hat{L}_s \sim 1 - \frac{1}{g} \sum_{i=1}^{g} [\Phi(t_i - \lambda\varepsilon) + \Phi(t_{i-1} - \lambda\varepsilon)] \qquad (2.25)$$

Thus for fixed g, however, the estimate (2.24) is <u>not</u> unbiased, but is negatively biased for items for which ε exceeds unity. The magnitude of the bias is not small enough to ignore when g is small.

We shall now show that as $g \to \infty$, this estimate approaches the true elasticity. For this purpose, let us consider the sequence $\{a_g\}$ where

$$a_g = \frac{1}{g} \sum_{i=1}^{g} \Phi(t_i - \lambda \ ) \qquad (2.26)$$

Applying the law of large numbers to the above sequence, which is permissible under our assumptions, we see that as $g \to \infty$

$$a_g \to E_t \Phi(t - \lambda \ ) \qquad (2.27)$$

where E denotes the expected value. The right hand side of (2.27) may be easily shown to be equal to the probability that any two independent standard normal variates do not exceed each other by $-\lambda\varepsilon$ [12]. This probability is simply $\Phi(-\frac{\lambda\varepsilon}{\sqrt{2}})$. Therefore, as $g \to \infty$.

$$\hat{L}_s \sim 1 - 2\Phi\left(-\frac{\lambda\varepsilon}{\sqrt{2}}\right) \tag{2.28}$$

Similarly, $\hat{L}_o$, being a special case of (2.28) in which ε is set equal to unity, tends to $1 - 2\Phi(-\frac{\lambda}{\sqrt{2}})$. It follows at once that as $g \to \infty$

$$\frac{1}{2}(1 + \hat{L}_s) \sim \Phi\left(\frac{\lambda\varepsilon}{\sqrt{2}}\right)$$

$$\frac{1}{2}(1 + \hat{L}_o) \sim \Phi\left(\frac{\lambda}{\sqrt{2}}\right) \tag{2.29}$$

so that

$$\hat{\varepsilon}_g \doteq \frac{t\,\Phi\left(\frac{\lambda}{\sqrt{2}}\right)}{t\,\Phi\left(\frac{\lambda}{\sqrt{2}}\right)} = \tag{2.30}$$

This establishes the asymptotic unbiasedness of our estimate (2.24).

Perhaps it should be possible to derive an expression for the asymptotic variance of $\hat{\varepsilon}_g$. In fact, when g is fixed, this can be worked out using the elements of the generalised matrix (2.10). This problem and other related problems, however, are being investigated. It should be emphasised that the alternative procedure does not yield consistent estimates for moderately low values of g and it is conceivable that it may not be superior to the conventional method of least squares. We shall return to this problem in Section 2.4.

Without sacrificing any information, we may use our method discussed in Section 2.2 for obtaining consistent estimate of the Engel elasticity. Let $g = 2k$ where k is a positive integer. Now we extend the definition of q and Q:

$$q = \frac{\bar{x}_1 + \cdots + \bar{x}_k}{\bar{x}_1 + \cdots + \bar{x}_g}, \quad Q = \frac{\bar{y}_1 + \cdots + \bar{y}_k}{\bar{y}_1 + \cdots + \bar{y}_g} \qquad (2.31)$$

It is easy to verify that, for a given g, as $m \to \infty$, q and Q tend respectively $\Phi(-\lambda)$ and $\Phi(-\lambda\varepsilon)$. This is because

$\bar{x}_i \sim g\mu\left\{\Phi(t_i-\lambda) - \Phi(t_{i-1}-\lambda)\right\}$ and $\bar{y}_i \sim g\nu\left\{\Phi(t_i-\lambda\varepsilon) - \Phi(t_{i-1}-\lambda\varepsilon)\right\}$

with $t_0 = -\infty$, $t_k = 0$ and $t_g = +\infty$.

The estimate of the elasticity is as before given by $\tilde{\varepsilon} = \frac{t_Q}{t_q}$ where the q and Q are as defined in (2.31). The asymptotic variance of this estimate takes the form

$$m\mathrm{Var}(\tilde{\varepsilon}) \sim \frac{1}{\lambda^2}\left[R_{11}\,\varepsilon^2 - 2R_{12}\,\varepsilon + R_{22}\right] \qquad (2.32)$$

where

$$R_{11} = \frac{1}{16\mu^2 Z^2(\lambda)}\left\{\left(\frac{\mu_2}{\mu}\right)^2 S_{uu} - 2\left(\frac{\mu_2}{\mu}\right)\left(\frac{\mu_1}{\mu}\right)S_{uu^*} + \left(\frac{\mu_1}{\mu}\right)S_{u^*u^*}\right\}$$

$$R_{12} = \frac{1}{16\mu\nu Z(\lambda)Z(\lambda\varepsilon)}\left\{\left(\frac{\mu_2}{\mu}\right)\left(\frac{\nu_2}{\nu}\right)S_{uv} - \left(\frac{\mu_2}{\mu}\right)\left(\frac{\nu_1}{\nu}\right)S_{uv^*} - \left(\frac{\mu_1}{\mu}\right)\left(\frac{\nu_2}{\nu}\right)S_{u^*v} + \left(\frac{\mu_1}{\mu}\right)\left(\frac{\nu_1}{\nu}\right)S_{u^*v^*}\right\}$$

$$R_{22} = \frac{1}{16\nu^2 Z^2(\lambda\varepsilon)}\left\{\left(\frac{\nu_2}{\nu}\right)^2 S_{vv} - 2\left(\frac{\nu_2}{\nu}\right)\left(\frac{\nu_1}{\nu}\right)S_{vv^*} + \left(\frac{\nu_1}{\nu}\right)^2 S_{v^*v^*}\right\} \qquad (2.33)$$

The S's involved in (2.33) may be obtained from the generalised matrix (2.10):

$$S_{uu} = \sum_{i=1}^{k} \sum_{j=1}^{k} \Sigma_{ij}$$

$$S_{u^*u} = S_{uu^*} = 2 \sum_{i=1}^{k} \sum_{j=k+1}^{g} \Sigma_{ij} = 2 \sum_{i=k+1}^{g} \sum_{j=1}^{k} \Sigma_{ij}$$

$$S_{u^*u^*} = \sum_{i=k+1}^{g} \sum_{j=k+1}^{g} \Sigma_{ij} \qquad (2.34)$$

We may note in passing that the S's in (2.33) are simply sums of elements of the matrix Σ equally partitioned into four quadrants. Similarly, expressions which appear in R_{12} and R_{22} of (2.32) can be derived in terms of the elements of the partitioned matrices of E and T respectively.

2.4. The Least Squares Estimate: In this section we return to answer the main question-whether or not the above method, apparently so simple and probably inexpensive, is better than the commonly used method of least squares, under the given assumptions. In what follows it will be shown that the method of least squares yields asymptotically biased estimates. We shall mainly focus our attention on the estimate of elasticity.

Suppose with the same data $(\bar{x}_1, \bar{x}_2, \bar{y}_1, \bar{y}_2)$ we compute the regression coefficient b_2 assuming the double-log form for the Engel curve.

Then

$$b_2 = \frac{\sum_{i=1}^{2} (\log \bar{x}_i - \overline{\log \bar{x}_i})(\log \bar{y}_i - \overline{\log \bar{y}_i})}{\sum_{i=1}^{2} (\log \bar{x}_i - \overline{\log \bar{x}_i})^2} \tag{2.35}$$

which simplifies to

$$\frac{\log \bar{y}_2 - \log \bar{y}_1}{\log \bar{x}_2 - \log \bar{x}_1} \tag{2.36}$$

This estimate obtained from linear double-log hypothesis is same as Wald's which was derived to estimate the slope of the linear regression in two variables when both the dependent and independent variables are subject to errors of measurement [23].

It is easy to see that b_2 is not consistent for ε , for we observe that b_2 is consistent for

$$\beta(\varepsilon) = \frac{\log \Phi(\lambda\varepsilon) - \log \Phi(-\lambda\varepsilon)}{\log \Phi(\lambda) - \log \Phi(-\lambda)} \tag{2.37}$$

which is in general not equal to ε except for the values -1, 0 and 1. In fact, the asymptotic value β is a monotonic function of ε with the asymptotic bias increasing, constant or decreasing according as whether

$$\frac{Z(\lambda\varepsilon)}{\Phi(\lambda\varepsilon)\,\Phi(-\lambda\varepsilon)} \gtrless \frac{1}{\lambda}[(\log \Phi(\lambda) - \log \Phi(-\lambda)] \qquad (2.38)$$

By means of numerical examples we may show that, for a given value of λ, the left hand side in (2.38) is smaller than the right hand side for some values of ε while for other values of ε the opposite holds. For illustrating this, we assume $\lambda = 0.6$ in which case the right hand side becomes 1.6215. Table 2.1 gives the values of the left hand side for $\lambda = 0.6$ and $\varepsilon = -1.0$, -0.5, 1.0, 1.5 and 2.0 as well as the percentage asymptotic bias.

Table 2.1

	-1.0	-0.5	0	0.5	1.0	1.5	2.0
LHS	-1.6739	-1.6154	$2/\sqrt{\frac{2}{\pi}}$	1.6154	1.6739	1.7715	1.9067
$\beta(\varepsilon)$	-1.0	-0.4940	0	0.6940	1.0	1.5303	2.1201
bias (%)	0	-1.2	0	1.2	0	2.02	6.00

From Table 2.1 above it is clear that the asymptotic bias in percentage terms is small within the range of elasticities we have considered. The bias, however, increases fast enough as we move along the ε - axis. The asymptotic bias regarded as a function of ε resembles the letter w intersecting the elasticity axis at $\varepsilon = -1.0$ and 1 (Figure 2.1). We should now examine whether the bias in the least squares estimate is accompanied by an increase or decrease in its variance.

The asymptotic variance of the regression estimate may be calculated from the variance-covariance matrix (2.10):

$$m \, Var(b_2) \sim \frac{1}{k^2}\left(S_{11}\beta^2 - 2S_{12}\beta + S_{22}\right) \tag{2.39}$$

where

$$k(\lambda) = \log \Phi(\lambda) - \log \Phi(-\lambda) \tag{2.40}$$

$$S_{11} = \frac{\Sigma_{11}}{\mu_1^2} - 2\frac{\Sigma_{12}}{\mu_1\mu_2} + \frac{\Sigma_{22}}{\mu_2^2}$$

$$S_{12} = \frac{E_{11}}{\mu_1\nu_1} - \frac{E_{12}}{\mu_1\nu_2} - \frac{E_{21}}{\mu_2\nu_1} + \frac{E_{22}}{\mu_2\nu_2}$$

$$S_{22} = \frac{T_{11}}{\nu_1^2} - 2\frac{T_{12}}{\nu_1\nu_2} + \frac{T_{22}}{\nu_2^2} \tag{2.41}$$

The comparison of the variance (2.16) and (2.39) is not quite straight forward; it involves three sets of comparisons of the coefficients of like terms. The direction of inequality of those coefficients depends on the magnitude and sign of the elements of the variance-covariance matrix.

The expression (2.16) for the variance of $\hat{\varepsilon}$ may be rewritten in the form

$$m \, Var \, \hat{\varepsilon} = \frac{1}{\lambda^2}\left(R_{11}\varepsilon^2 - 2R_{12}\varepsilon + R_{22}\right) \tag{2.42}$$

in which

$$R_{11} = \frac{1}{16\mu^2 Z^2(\lambda)} [(\frac{\mu_2}{\mu})^2 \Sigma_{11} - 2(\frac{\mu_2}{\mu})(\frac{\mu_1}{\mu}) \Sigma_{12} + (\frac{\mu_1}{\mu})^2 \Sigma_{22}]$$

$$R_{12} = \frac{1}{16\mu\nu Z(\lambda)Z(\lambda\varepsilon)} \{ (\frac{\mu_2}{\mu})(\frac{\nu_2}{\nu})E_{11} - (\frac{\mu_2}{\mu})(\frac{\nu_1}{\nu})E_{12} - (\frac{\mu_1}{\mu})(\frac{\nu_2}{\nu})E_{21} + (\frac{\mu_1}{\mu})(\frac{\nu_1}{\nu})E_{22} \}$$

$$R_{22} = \frac{1}{16\nu^2 Z^2(\lambda\varepsilon)} \{ (\frac{\nu_2}{\nu})^2 T_{11} - 2(\frac{\nu_2}{\nu})(\frac{\nu_1}{\nu})T_{12} + (\frac{\nu_1}{\nu})^2 T_{22} \}$$

(2.43)

Expression (2.42) is a special case of (2.32) in which there are only two fractile groups. In order, therefore, to show that $Var(\hat{\varepsilon}) < Var(b_2)$ we have to examine whether the following inequalities are simultaneously satisfied:

$$\frac{R_{11}}{S_{11}} < \frac{\lambda^2}{k^2} \qquad \frac{R_{12}}{S_{12}} > \frac{\lambda^2}{k^2} \qquad \frac{R_{22}}{S_{22}} < \frac{\lambda^2}{k^2} \qquad (2.44)$$

These inequalities may be numerically verified by choosing arbitrary values for the parameters involved but this is beyond the scope of our present study. A detailed investigation in this direction is still under way. We shall, however, consider a special case and show that our estimate has a smaller asymptotic variance.

Let us write the variance difference in the form

$$m[Var(\hat{\varepsilon}) - Var(b_2)] = a_{11} Var(q) - 2a_{12} \sqrt{Var(q)\ Var(Q)}\ \rho_{qQ} + a_{22} Var(Q)$$

(2.45)

where the a's are given by

$$a_{11} = \frac{\varepsilon^2}{\lambda^2 Z^2(\lambda)} - \frac{\beta^2}{k^2 \Phi^2(\lambda)\Phi^2(-\lambda)}$$

$$a_{12} = \frac{\varepsilon}{\lambda^2 Z(\lambda)Z(\lambda\varepsilon)} - \frac{\beta}{k^2 \Phi(\lambda)\Phi(-\lambda)\Phi(\lambda\varepsilon)\Phi(-\lambda\varepsilon)}$$

$$a_{22} = \frac{1}{\lambda^2 Z^2(\lambda\varepsilon)} - \frac{1}{k^2 \Phi^2(-\lambda\varepsilon)\Phi^2(\lambda\varepsilon)} \qquad (2.46)$$

We observe that the a's are all negative for $\varepsilon \geq 1$[4] by virtue of the following inequality:

<u>Theorem 1</u>: <u>For all non-negative λ and $\varepsilon \geq 1$,</u>

$$H(\lambda\varepsilon) = \frac{\Phi(\lambda\varepsilon)\,\Phi(-\lambda\varepsilon)}{Z(\lambda\varepsilon)} \leq \frac{\lambda}{k} \qquad (2.47)$$

<u>where k, as defined earlier in (2.40), is equal to $\log \Phi(\lambda) - \log \Phi(-\lambda)$, the equality holding when $\lambda \to 0$.</u>

The proof of the theorem consists in showing that $H(\lambda)$ is a monotonic decreasing function of λ and is less than $\frac{\lambda}{k}$ for all $\lambda > 0$. Differentiating k with respect to λ, we have

$$k' = \frac{Z(\lambda)}{\Phi(\lambda)\,\Phi(-\lambda)} \qquad (2.48)$$

which is simply the reciprocal of $H(\lambda)$. Thus it is enough to show that k' is monotonic increasing i.e. $k'' > 0$ and the elasticity of k with

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4. Computationally it is verified that this proposition is true for $\varepsilon > 0.7$.

respect to λ is larger than unity, i.e. $\lambda k' \geq k$. Both these propositions may be established if we prove a basic lemma concerning the normal distribution.

Lemma. If $\Phi(t)$ is the standard normal distribution function, $\Phi(t) = \int_{-\infty}^{t} Z(t)\,dt$ when $Z(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$ and $k(t) = \log_e \Phi(t) - \log_e \Phi(-t)$, then $k(t)$ is strictly convex in t for $t > 0$ and strictly concave in t for $t < 0$.

Proof: Noting that $\Phi''(t) = -tZ(t)$, the derivatives of $k(t)$ are

$$k'(t) = \frac{Z(t)}{\Phi(t)\,\Phi(-t)} \tag{2.49}$$

$$k''(t) = \frac{Z(t)}{\Phi^2(t)\,\Phi^2(-t)}\,\Lambda(t) \tag{2.50}$$

where $\Lambda(t) = [\,\phi(t) - \phi(-t)]\,Z(t) - t\phi(t)\phi(-t)$ (2.51)

Since $\lim_{n \to \infty} t^n \phi(-t) = 0$ we should have $0 = \Lambda(-\infty) = \Lambda(0) = \Lambda(\infty)$. We shall now prove by contradiction that $\Lambda(t) > 0$ for $t > 0$. Suppose this were not the case. Then there exists at least one point t^* in $(0, \infty)$ such that $\Lambda(t^*) \leq 0$, and, since $\Lambda(0) = \Lambda(\infty) = 0$, $\Lambda'(t^*) = 0$ i.e.

$$\Lambda'(t^*) = 2\,Z^2(t^*) - \Phi(t^*)\,\Phi(-t^*) = 0 \tag{2.52}$$

so that

$$\begin{aligned}\Lambda(t^*) &= [\,\Phi(t^*) - \Phi(-t^*)]\,Z(t^*) - 2t^*Z^2(t^*) \\ &= Z(t^*)\,B(t^*), \text{ say}\end{aligned} \tag{2.53}$$

where

$$B(t) = [\Phi(t) - \Phi(-t)] - 2t\,Z(t) \tag{2.54}$$

But $B(0) = 0$, $B(\infty) = 1$ and $B'(t) = 2t^2 Z(t) > 0$. Hence, $B(t^*) > 0$, a contradiction to our assumption. Therefore $A(t) > 0$, and consequently, for $t > 0$, $k''(t) > 0$, that is, $k(t)$ is strictly convex.

Since $\Phi(t) + \Phi(-t) = 1$, we have $k(-t) = -k(t)$ so that $k''(-t) = -k''(t) < 0$ showing that for $t < 0$ $k(t)$ is strictly concave.

Corollary. The function $w(t) = |k(t)|$ is strictly convex and $|\alpha k(t)| > |k(\alpha t)|$ for all $0 < \alpha < 1$. Equivalently, for a given t, $|k(\varepsilon t)/\varepsilon|$ is a strictly increasing function of ε for all $\varepsilon > 0$.

The proof of Theorem 1 follows directly from the above lemma. However, for purposes of comparing the magnitudes involved in either side of (2.47), tables and charts have been provided in the Appendix.

Now, since $a_{11} < 0$, the necessary and sufficient condition that $Var(\hat{\varepsilon}) \leq Var(b_2)$ is that the discriminant of the quadratic form (2.45)

$$\begin{vmatrix} a_{11} & a_{12}\rho_{qQ} \\ a_{12}\rho_{qQ} & a_{22} \end{vmatrix} \geq 0$$

or, in other words,

$$\rho^2_{qQ} \leq \frac{a_{11}a_{22}}{a^2_{12}} \tag{2.55}$$

As an illustration of this, let $\lambda = 0.6$ as before and $\varepsilon = 2.0$. From the available tables for the normal distribution we compute the a's and see that the condition (2.55) takes the form $\varrho^2_{qQ} \leq 0.9251$. As long as ϱ_{qQ}, the correlation between shares of total income and consumption possessed by the lower income class, does not exceed 0.96, our estimate will have a smaller asymptotic variance than Wald's estimate in the double-log case. The a's are, of course, functions of λ and ε, and the condition (2.55) varies numerically from one situation to another.

Theoretically it is possible to show that the right hand side of (2.55) is less than one and is exactly equal to unity for some values of λ and ε in which case

$$H(\lambda) = \frac{\beta}{\varepsilon} H(\lambda \varepsilon) \tag{2.56}$$

For a given level of income inequality there do exist some commodities whose income elasticity satisfies equation (2.56). This is because H is a monotonic decreasing function while the asymptotic regression β is larger than the true elasticity for luxuries and smaller for necessaries. In such situations, equation (2.55) is automatically satisfied since, by definition, the correlation cannot exceed unity implying that our estimate has greater asymptotic efficiency than Wald's estimate. In the appendix, the values of $H(\lambda)$ are tabulated in order that one may find the values of λ and ε that satisfy (2.56).

Regressions based on only two pairs of observations $(\bar{x}_i, \bar{y}_i)$ $(i = 1,2)$ do not seem realistic in actual practice. We shall, therefore, consider a situation in which we have g pairs of observed means $(\bar{x}_i, \bar{y}_i)$ corresponding to g given fractile groups. The regression coefficient in the double-log model is computed by using the earlier expression (2.35) in which the summation of squares and products of deviations is carried over g classes:

$$b_g = \frac{\sum_{i=1}^{g} (\log \bar{x}_i - \overline{\log \bar{x}_i})(\log \bar{y}_i - \overline{\log \bar{y}_i})}{\sum_{i=1}^{g} (\log \bar{x}_i - \overline{\log \bar{x}_i})^2} \tag{2.57}$$

By a generalisation of our argument, we may show that (2.57), for a given g, is asymptotically biased, the bias increasing for larger values of the elasticity. However, if in (2.57) the arithmetic means are replaced by the corresponding geometric means, the regression estimate preserves the desirable small sample properties such as unbiasedness and efficiency. But geometric means are not usually computed in practice due to presence of 'zero' observations apart from the computational inconvenience.

Let $C_0 = 0, C_1, C_2, \ldots, C_{g-1}, C_g = \infty$ be the $0, \frac{1}{g}, \frac{2}{g}, \ldots, \frac{g-1}{g}$, and last fractiles of the distribution of x, assumed to be log-normally distributed with parameters (θ, λ); let t_i be the standard normal deviate corresponding to C_i $(t_0 = -\infty,\ t_g = \infty)$, i.e.

$$C_i = \exp(\theta + \lambda t_i) \tag{2.58}$$

We have the following general expressions for the truncated means:

$$\mu_i = g\mu[\Phi(t_i - \lambda) - \Phi(t_{i-1} - \lambda)]$$

$$\nu_i = g\nu[\Phi(t_i - \lambda\varepsilon) - \Phi(t_{i-1} - \lambda\varepsilon)] \tag{2.59}$$

where μ, ν and Φ are already defined. In a somewhat similar manner it may be proved [22] that the joint distribution of $[u_1, \ldots, u_g \vdots v_1, \ldots, v_g]$ where $u_i = \sqrt{m}(\bar{x}_i - \mu_i)$ and $v_i = \sqrt{m}(\bar{y}_i - \nu_i)$ is asymptotically multivariate normal with zero mean and the generalised variance-covariance matrix (2.10). The elements of the generalised matrix are shown in the appendix. Applying the foregoing remark to b_g we observe that the latter is consistent for the expression (2.57) in which the observed means are replaced by their corresponding population means, but not for ε except for certain values of the elasticity. To evaluate the magnitude of the asymptotic bias in general terms is difficult but the numerical course is open to us. We assume plausible values for λ and ε and compute the asymptotic regression coefficient β taking $g = 5$ and $g = 10$.

Let us write

$$q_i = \Phi(t_i - \lambda) - \Phi(t_{i-1} - \lambda)$$

$$Q_i = \Phi(t_i - \lambda\varepsilon) - \Phi(t_{i-1} - \lambda\varepsilon) \tag{2.60}$$

The asymptotic regression coefficient β is given by

$$\beta = \frac{\sum_{i=1}^{g} (\log q_i - \overline{\log q_i})(\log Q_i - \overline{\log Q_i})}{\sum_{i=1}^{g} (\log q_i - \overline{\log q_i})^2} \qquad (2.61)$$

We shall assume the following values for λ and ε : $\lambda = 0.6$

ε = -1.0, -0.5, 0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5 and 4.0. The computational results are summarised in Table 2.2 given below.

Table 2.2

ε	$\beta(\varepsilon)$		bias (ϕ)	
	g = 5	g = 10	g = 5	g = 10
-1.0	-0.9975	-0.9988	-0.2499	-0.1212
-0.5	-0.4966	-0.4986	-0.6871	-0.2828
0	0	0	0	0
0.5	0.4972	0.4988	0.5632	0.2360
1.0	1	1	0	0
1.5	1.5137	1.5060	0.9145	0.4063
2.0	2.0440	2.0197	2.2005	0.9838
2.5	2.5970	2.5434	3.8797	1.7373
3.0	3.1791	3.0808	5.9698	2.6919
3.5	3.7967	3.6352	8.4772	3.8622
4.0	4.4555	4.2097	11.3884	5.2434

In the above computations sufficient number of decimal places were kept in view of the anticipated magnitude of bias. It is seen that the percentage bias does not appear very serious though for large values of elasticity it can not be ignored. The bias tends to diminish with increase in the number of fractile groups. Perhaps with $g = 20$ the bias may almost tend to be negligible. This fact alone is not a sufficient justification for choosing the regression method unless we have also explored the relative speed of convergence of the estimate to its population value as the number of groups increases. The asymptotic bias is plotted in Figure 2.1 for chosen values of g, namely, $g = 5$ and $g = 10$. The graph shows that as the bias is monotonic in the elasticity no uniform correction for the regression estimate can be suggested.

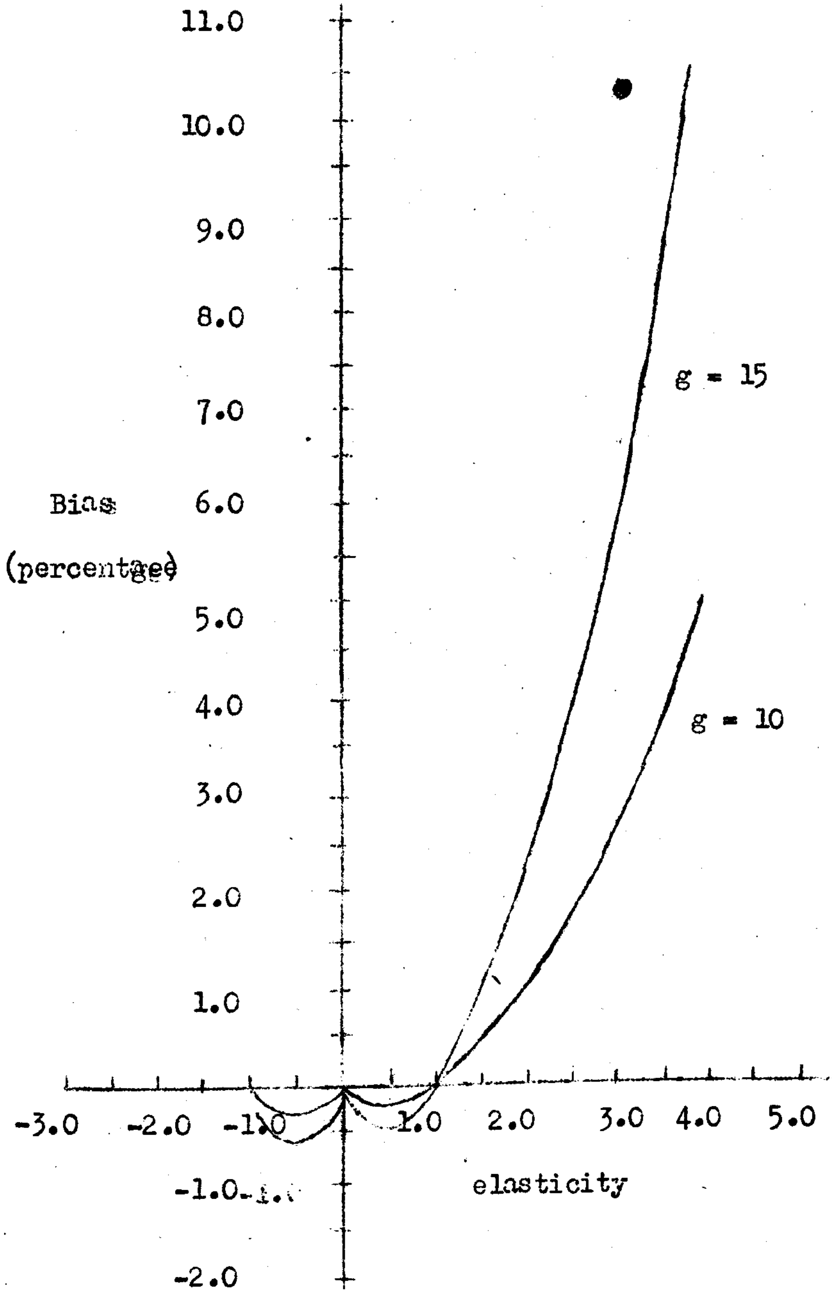

Figure 2.1 : Asymptotic bias of least squares estimate.

An expression for the asymptotic variance of the least squares estimate (2.57) can also be worked out although this involves some tedious algebra. Let us write for each i $(i = 1,2,\ldots, g)$

$$\ell_i = \frac{1}{2\mu d}\left[\left\{\frac{\log Q_i - \overline{\log Q_i}}{q_i} - \left(\overline{\frac{\log Q_i - \overline{\log Q_i}}{q_i}}\right)\right\} - 2\left\{\frac{\log q_i - \overline{\log q_i}}{q_i} - \left(\overline{\frac{\log q_i - \overline{\log q_i}}{q_i}}\right)\right\}\right]$$

$$m_i = \frac{1}{2\nu d}\left[\frac{\log q_i - \overline{\log q_i}}{Q_i} - \left(\overline{\frac{\log q_i - \overline{\log q_i}}{Q_i}}\right)\right]$$

$$d = \sum_{i=1}^{g} (\log q_i - \overline{\log q_i})^2 \qquad (2.62)$$

where the q's and Q's are as defined in (2.60). The asymptotic variance is given by

$$m \operatorname{Var}(b_g) \sim \underset{\sim}{\ell}\, \Sigma\, \underset{\sim}{\ell}' + \underset{\sim}{\ell}\, E\, \underset{\sim}{m}' + \underset{\sim}{m}\, T\, \underset{\sim}{m}' \qquad (2.63)$$

in which $\underset{\sim}{\ell} = (\ell_1, \ell_2, \ldots, \ell_g)$ and $\underset{\sim}{m} = (m_1, m_2, \ldots, m_g)$ are g - dimensional vectors.

The variance comparison between the least squares estimate and our estimate is quite difficult as has already been noted even in the simple case of $g = 2$. Numerical-graphical devices, however, are powerful aids in these circumstances though admittedly they lack in mathematical rigour.

2.5. Estimation in the Log-logistic Case: In this section we relax some of the assumptions of previous sections and assume alternative forms for the distribution of income and the demand relationships. First, we shall consider some plausible hypotheses for the income distribution retaining the constant elasticity assumption, and then proceed to consider an important case of 'variable' elasticity.

As before, let x denote household income and let us assume that the variable x has a log-logistic distribution [4] which is characterised by the equation,

$$\log \frac{F(x)}{1 - F(x)} = a + b \log x \tag{2.64}$$

where $F(x)$ is the cumulative distribution function. Further, let $E(y \mid x) = \Psi(x) = A x^{\varepsilon}$ as in (2.1). The problem then is to investigate whether we could still use the methods of Sections 2.2 and 2.3 for estimating the distributional parameters of (2.64) and those of the Engel curve. This examination would help us to see whether our methods can be applied to situations where the log-normal hypothesis is replaced by some other plausible alternative such as (2.64), which may in practice be verified from given size distribution data. We shall be concerned in the main with the estimation of Engel elasticity though other parameters are by themselves important.

For the log-logistic distribution, the Lorenz curve is given by the parametric equations

$$p_\xi = \frac{\xi}{1+\xi}$$

$$q_\xi = \frac{1}{B(\ell, m)} \int_0^\xi \frac{\tau^{\ell-1}}{(1+\tau)^{\ell+m}} d\tau \tag{2.66}$$

where $\xi = e^a x^b$; $\ell = 1 + \frac{1}{b}$, $m = 1 - \frac{1}{b}$ so that $\ell + m = 2$. Elimination of ξ between (2.65) and (2.66) yields the Lorenz curve in the form

$$q = \frac{1}{B(\ell, m)} \int_0^p \tau^{\ell-1}(1-\tau)^{m-1} d\tau \tag{2.67}$$

Similarly, the specific concentration curve for the given commodity takes the form

$$Q = \frac{1}{B(\ell^*, m^*)} \int_0^p \tau^{\ell^*-1}(1-\tau)^{m^*-1} d\tau \tag{2.68}$$

where $\ell^* = 1 + \frac{\varepsilon}{b}$ and $m^* = 1 - \frac{\varepsilon}{b}$ so that $\ell^* + m^* = 2$. It should be pointed out that the distribution of income assumed above should have $b > 1$ and $\varepsilon < b$ so that (2.67) and (2.68) are defined for all $0 < p < 1$.

It is a simple exercise to show that for the log-logistic distribution (5.1) the Lorenz measure of inequality is given by $\frac{1}{b}$ whereas the specific concentration ratio is $\frac{\varepsilon}{b}$.[5] That is, $L_s = \varepsilon\ L_0$, in the notation of (2.23). It appears intuitively that the ratio of specific concentration ratio to the Lorenz ratio gives in this case a consistent estimate of Engel elasticity. Also, the other distributional parameter 'a' can be computed by

$$\hat{a} = \frac{1}{\hat{L}_0} \log\left\{\frac{\Gamma(1+\hat{L}_0)\ \Gamma(1-\hat{L}_0)}{\bar{x}}\right\} .$$

Let us compute the proportions q and Q corresponding to $p = \frac{1}{2}$. i.e. $q = I_{0.5}(\ell, m)$ and $Q = I_{0.5}(\ell^*, m^*)$ where $I_p(\ell, m)$ is the incomplete B-function. The interesting problem then is to apply the formula (2.4) and determine the nature and magnitude of bias, if any. To do this, let $\frac{1}{b} = 0.4$ (which is roughly the position in the case of consumer expenditure distributions in urban India). That is, $\ell = 1.4$ and $m = 0.6$. For these values of (ℓ, m), the incomplete B-function corresponding to $p = 0.5$ works out to be $q = 0.2453,521$. Similarly, for the specific commodities with assumed values of elasticity, such

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5 This follows from (2.68) since $Q = I_p(\ell^*, m^*)$ may be expanded in an infinite series, $Q = \frac{1}{B(\ell^*, m^*)} \sum_0^\infty (-1)^r \binom{m-1}{r} \frac{p^{\ell^*+r}}{\ell^*+r}$, and $\int_0^1 Q\,dp = \int_0^1 Q\,dp = 1 - \frac{B(\ell^*+1, m^*)}{B(\ell^*, m^*)} = \frac{m^*}{\ell^* + m^*}$. The specific concentration ratio L_s becomes $L_s = 1 - 2\int_0^1 Q\,dp = \frac{\ell^* - m^*}{\ell^* + m^*} = \frac{\varepsilon}{b}$ since $\ell^* = 1 + \frac{\varepsilon}{b}$, $m^* = 1 - \frac{\varepsilon}{b}$.

proportions may be computed. For example, if $\varepsilon = -1$, then $* = 0.6$, $m^* = 1.4$ so that $Q = 0.7546,479$. In this manner, the proportions Q and their standard normal deviates are computed for $\varepsilon = -1.0, -0.5, 0.5, 1.0, 1.5$ and 2.0 and the main results are summarised in Table 2.3. Column (4) gives estimates of elasticity computed by using the formula (2.4) and the bias is shown in the last column.

Table 2.3

ε	Q	t_Q	$\hat{\varepsilon}$	% bias
-1.0	0.7546,479	0.6891,733	-1.0000	0
-0.5	0.6273,240	0.3247,748	-0.4712	-5.18
0.5	0.3726,760	-0.3247,748	0.4712	-5.18
1.0	0.2453,521	-0.6891,733	1.0000	0
1.5	0.1346,750	-1.1045,950	1.6028	6.8
2.0	0.0311,000	-1.8648,900	2.7060	35.3

The bias appears to be considerable and, again, increases with the 'true' elasticity. It is thus clear that our method does not necessarily possess all the desired properties of good estimators if the basic assumption of log-normality is changed. However, for the log-logistic case, the ratio of specific concentration ratio to the Lorenz ratio appears to be more logical.

At this stage it is worth considering the unconventional approach of Section 2.3 versus the least squares estimate in the log-logistic case, that is, to see the nature of bias that may arise due to our using the group 'arithmetic' means instead of geometric means. We shall briefly outline the procedure and leave out the computations for the present.

For the log-logistic distribution (2.64), the mean is found to be

$$E\ (x) = e^{-a/b}\, B(\ell\,,\ m) \tag{2.70}$$

If there are g fixed fractile classes $(C_i\,,\ C_{i+1})$, $(i = 0, \ldots, g-1)$, then the truncated means are given by

$$\mu_i = E\ (x \mid C_i \leq x \leq C_{i+1}) = g\, e^{-\frac{a}{b}}\, [I_{z_{i+1}}(\ell\,,\ m) - I_{z_i}(\ell\,,\ m)] \tag{2.71}$$

where $z = \frac{\zeta}{1+\zeta}$, and $I_z(\ell,\ m)$ is the incomplete B-function of the first type $(0 < z < 1)$; $z_1, \ldots, z_{g-1}$ are the $g-1$ fractiles of the B_1-variate with parameters $(\ell\,,\ m)$. Now since $E\ (y|x) = A x^{\varepsilon}$ by assumption, the truncated means of y are given by

$$\nu_i = g\, A\, e^{-\frac{a}{b}\varepsilon}\, [I_{z_{i+1}}(\ell^*,\ m^*) - L_{z_i}(\ell^*,\ m^*)] \tag{2.72}$$

An interesting problem will be to assign specific values to the parameters involved in (2.71) and (2.72), and compute the series $\{\mu_i\,,\ \nu_i\}$ for chosen values of g as, for example, $g = 5$ or 10 as before, and finally work out the regression of μ_i on ν_i. Computations on these lines are left out as they are expected to yield similar results as in Figure 2.1.

In exactly similar manner, we may work out the consequences of the wellknown Pareto hypothesis of income distribution. The Pareto distribution is characterised by the double-log linear relationship

$$\log [1 - F(x)] = -\alpha \log \frac{x}{x_0}, \; x > x_0 \tag{2.73}$$

where $F(x)$ is the cumulative distribution function; x_0 is the lower income limit and $\alpha > 1$ represents the inequality parameter related to the Lorenz measure by the equation

$$L_0 = \frac{1}{2\alpha - 1} \tag{2.74}$$

This, however, is omitted from our consideration as a trivial exercise.

Prais and Houthakker [18] in their monumental work The Analysis of Family Budgets have made use of five basic forms of Engel curve including the double-log case which gives constant income (expenditure) elasticity. Forms leading to variable elasticities are often found more realistic in economics [6]. The semi-log case, for example, falls in this category. Stated in symbols, the semi-log hypothesis takes the form with an implicit

6. These are indicated by the asymmetry of the specific concentration curve. (vide Figure 1.1 of Chapter 1).

additive error term distributed as $N(0, \sigma_0)$:

$$E(y \mid x) = \gamma + \delta \log x \tag{2.75}$$

Also implicit in this hypothesis is the assumption that the marginal propensity to consume is on an average inversely proportional to income. This hypothesis has found some empirical support especially for necessaries such as food articles [18, p. 96]. The 'variable' elasticity is given by

$$\varepsilon(x) = \frac{\delta}{\gamma + \delta \log x} \tag{2.76}$$

For purposes of projection, the elasticity is usually computed at the mean income by the principle of least squares.

We shall show below that the method of concentration curves can be used also in the semi-log case thus relaxing the constant elasticity stipulation of (2.1). The assumption of log-normality will, however, be retained for reasons of simplicity.

Under the log-normal hypothesis, the Engel elasticity computed at the median income C, is given by

$$\varepsilon(C) = \frac{\delta}{\gamma + \delta\theta} \tag{2.77}$$

A consistent procedure for estimating the parameters (γ, δ) again involves the use of concentration curves. As pointed out earlier the Lorenz curve of income distribution is given by $t_q = t_p - \lambda$ while, for the semi-log Engel curve (2.75), the specific concentration curve has the equation

$$Q = p - \varepsilon\ (C)\ \lambda\ Z(t_p) \qquad (2.78)$$

The specific concentration ratio is given by $\frac{\lambda \varepsilon (C)}{\sqrt{\pi}}$ so that the semi-log form becomes realistic as long as $\varepsilon (C) < \frac{\sqrt{\pi}}{\lambda}$.

At the median income, $p = 0.5$, $t_p = 0$ and $\hat{\lambda} = - t_{\hat{q}_{0.5}}$ so that the 'median' elasticity is estimated by

$$\hat{\varepsilon}\ (C) = \frac{0.5 - \hat{Q}_{0.5}}{\hat{\lambda}\ Z(0)} \qquad (2.79)$$

where $Z(0) = \frac{1}{\sqrt{2\pi}}$; $\hat{Q}_{0.5}$ and $\hat{q}_{0.5}$ are obtained directly from fractile data, or computed from concentration curves in the case of fixed interval data. Now, since the denominator in (2.76) is estimated by $\bar{y}$, the overall mean of specific expenditure, ∂ has the estimate $\hat{\varepsilon}(C)\ \bar{y}$. If the estimates of ∂ and θ are substituted again in (2.77), an estimate of γ can be obtained in terms of $\hat{\varepsilon}\ (C)$, $\hat{\partial}$ and $\hat{\theta}$. Finally, elasticities for the various fractile groups (income classes) can be obtained by substituting group means $\bar{x}_i$'s in (5.76), is desired.

On the other hand, the elasticity $\varepsilon\ (\mu)$ computed at the mean is obtained by

$$\varepsilon\ (\mu) = \frac{\partial}{\gamma + \partial \log\ \mu} \qquad (2.80)$$

which is related to ε (C) by the relation

$$\varepsilon(\mu) = \varepsilon(C) \frac{1}{1 + \varepsilon(C) \frac{\lambda^2}{2}} \qquad (2.81)$$

so that

$$\varepsilon(\mu) < \varepsilon(C)$$

provided the commodity in question is not 'inferior'; for inferior goods, the expression (2.79) becomes negative since in that case the specific concentration curve lies above the egalitarian line. Also, it will be noted that the 'constant' elasticity ε is larger than ε (C). This at once leads to the inequality

$$\varepsilon(\mu) < \varepsilon(C) < \varepsilon \qquad (2.83)$$

This inequality is empirically confirmed by Prais and Houthakker [18, p. 94] in respect of six food commodities in their analysis of British family budgets.

The standard errors of the above estimates are difficult to compute though not impossible, at least in large samples. Some of the empirical studies on the lines suggested in the foregoing sections will be reported in a subsequent note.

2.6 Some concluding remarks: The commonly used method of least squares has its general applications in estimating linear regressions in which the equations are subject to error. Among other restrictive assumptions are that the residuals are serially uncorrelated and are also uncorrelated with the explaining variables ; the latter are assumed to be free from observational errors. Also, data on individual units, be it households or individuals, are required for obtaining statistically best results. But in situations where we are required to estimate the Engel elasticity from grouped survey materials which are available in the form of grouped arithmetic means in size classes of income (or total expenditure) these assumptions are less likely to be true. Also, from grouped size distribution data with coarse and unequal class intervals, it is not possible to obtain very satisfactory estimates of income inequality though this if often the practice in empirical work [13].

Our method does not require that data on individual units be available. While estimating the Engel elasticity it explicity makes use of the knowledge of the distribution of income. The basic assumptions underlying our estimation procedure are easily testable. In the regression, however, the assumption of normality of the residual terms, taken additivity or multiplicatively, is often taken for granted.

The fixed class interval data with unequal frequencies has certain disadvantages such as heteroscedasticity from the estimational point of view. The method of fractile analysis seems to be a better method of analysing economic data, particularly in the context of our method of estimation, since it readily provides the basic raw material for our study.

The regression estimate in the present case is shown to biased; the bias, which arises due to aggregation, increases with the true value of the elasticity and does not tend to vanish even in large samples. The method of fractiles on the other hand provides consistent estimates of the Engel curve; in this the problem of 'zero' entries does not seriously arise [8, p. 26]. Asymptotic variances are provided for our estimates which can be estimated in large samples as well as subjected to usual tests of significance. Approximate tests of significance may be readily devised given two interpenetrating sub-samples by using the fractile error [14].

It is not suggested, however, that our method is a substitute for the general method of least squares which can be used in a variety of situations involving several variables. But where we have a priori knowledge about the distribution of income and the nature of demand relationships, it may be appropriate to devise special methods which give consistent results such as we have proposed. It is generally agreed that the choice of a particular method is dictated by the type of data that are easily and readily available.

An immediate generalisation of our method to cases involving more than two variables seems possible. In that case it may be possible to apply this method for estimating, for instance, the well-known Cobb - Douglas production function or the household demand relationships involving family income and family size [7, 4]. Such possible generalisations are still under investigation.

It seems also possible to apply our method to estimate the Engel curve in the additive logarithmic form [6] as well as extend it to other well known forms [24].

A few important and difficult statistical problems remain open. The estimates of standard errors, etc., are all based on random sampling assumptions. It is therefore necessary to build up a satisfactory theory of estimation, at least in large samples, of the demand curve when the sampling has been done by a multi-stage design. Also, in some family budget surveys the households are classified according to per capita total expenditure, and not according to household income; this must be pointed out. Moreover, in the framework of general equilibrium, the additivity of the Engel curves and the simultaneous character of the system are also important econometric problems that require some consideration, but these complications are not considered here.

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Appendix 2.1

1. Let t_i be the $\frac{i}{g}$ th quantile of the standard normal distribution and C_i the corresponding quantiles of the distribution of x which is assumed to be log-normal with parameters (θ, λ). Then we have

$$C_i = \exp(\theta + \lambda t_i) \qquad i = 1, 2, \ldots, g-1$$

with $t_0 = -\infty$ and $t_g = +\infty$.

2. Let $\xi_i = E(y|x = C_i) = \Lambda C_i^{\varepsilon}$

$$\mu_i = E(x|C_{i-1} \leq x \leq C_i) = g\mu[\Phi(t_i - \lambda) - \Phi(t_{i-1} - \lambda)]$$

where $\mu = E(x) = \exp(\Phi + \frac{1}{2}\lambda^2)$. Similarly, let

$$\nu_i = E(y|C_{i-1} \leq x \leq C_i) = g\nu[\Phi(t_i - \lambda\varepsilon) - \Phi(t_{i-1} - \lambda\varepsilon)]$$

where $\nu = \Lambda \exp(\varepsilon\theta + \frac{1}{2}\lambda^2\varepsilon^2)$.

3. Let $\sigma_i^2 = \mathrm{Var}(x|C_{i-1} \leq x \leq C_i)$

$$= \mu^2[ge^{\lambda^2}\Phi(t_i - 2\lambda) - \Phi(t_{i-1} - 2\lambda)] - \mu_i^2$$

$$\tau_i^2 = \mathrm{Var}(y|C_{i-1} \leq x \leq C_i)$$

$$= \nu^2[ge^{\lambda^2\varepsilon^2}\Phi(t_i - 2\lambda\varepsilon) - \Phi(t_{i-1} - 2\lambda\varepsilon)] - \nu_i^2$$

$$\rho_i\sigma_i\tau_i = \mathrm{Cov}(x, y|C_{i-1} \leq x \leq C_i)$$

$$= \mu\nu[ge^{\lambda^2\varepsilon}\Phi(t_i - \lambda\overline{1+\varepsilon}) - \Phi(t_{i-1} - \lambda\overline{1+\varepsilon})] - \mu_i\nu_i$$

4. Let us define the following

$$M_i = i(C_i - \mu_i) - (i-1)(C_{i-1} - \mu_i), \quad (i = 2, \ldots, g-1)$$

$$M_1 = C_1 - \mu_1 ; \quad M_g = -(g-1)(C_{g-1} - \mu g)$$

$$M_i^o = (g-i)(C_i - \mu_i) - (g-i+1)(C_{i-1} - \mu_i), \quad (i = 2, \ldots, g-1)$$

$$M_1^o = (g-1)(C_1 - \mu_1) ; \quad M_g^o = -(C_{g-1} - \mu_g)$$

$$N_i = i(\xi_i - \nu_i) - (i-1)(\xi_{i-1} - \nu_i) \quad (i = 2, \ldots, g-1)$$

$$N_1 = (\xi_1 - \nu_1) ; \quad N_g = -(g-1)(\nu_{g-1} - \nu_g)$$

$$N_1^o = (g-i)(\xi_i - \nu_i) - (g-i+1)(\xi_{i-1} - \nu_i) \quad (i = 2, \ldots, g-1)$$

$$N_1^o = (g-1)(\xi_1 - \nu_1) ; \quad N_g^o = -(\xi_{g-1} - \nu_g).$$

5. We shall next define the variance-covariance matrices Σ, T and E.

The elements of Σ are given by

$$\Sigma_{ij} = \frac{1}{g} M_i M_j^o, \quad j > i$$

$$= \frac{1}{g} M_j M_i^o, \quad j < i$$

$$= \sigma_i^2 + \frac{1}{g} M_i M_i^o + (C_i - \mu_i)(C_{i-1} - \mu_i), \quad i = j \neq 1, g$$

$$= \sigma_1^2 + \frac{1}{g} M_1 M_1^o, \quad i = j = 1$$

$$= \sigma_g^2 + \frac{1}{g} M_g M_g^o, \quad i = j = g$$

Similarily the elements of T are defined replacing the σ's by γ'; and the M's by N's. Lastly, the elements of the E matrix are given by

$$
\begin{aligned}
E_{ij} &= \frac{1}{g} M_i N_j^o, & j > i \\
&= \frac{1}{g} N_i M_j^o, & j < i \\
&= \rho_i \sigma_i \gamma_i + \frac{1}{g} M_i N_i^o + (C_i - \mu_i)(\xi_{i-1} - \nu_i), & i = j \neq 1, g \\
&= \rho_1 \sigma_1 \gamma_1 + \frac{1}{g} M_1 N_1^o, & j = j = 1 \\
&= \rho_g \sigma_g \gamma_g + \frac{1}{g} M_g N_g^o, & i = j = g
\end{aligned}
$$

6. Then if $u_i = \sqrt{m}\,(\bar{x}_i - \mu_i)$ and $v_i = \sqrt{m}\,(\bar{y}_i - \nu_i)$, the theorem states that the distribution of $w = (u_1, \ldots, u_g ; v_1, \ldots, v_g)$ is asymptotically normal with mean zero and variance-covariance matrix given by

$$
\left[\begin{array}{c|c} \Sigma & E \\ \hline & T \end{array}\right]
$$

For proof of this theorem, see Sethuraman [6].

Appendix 2.2[7]

t	Z(t)	$\Phi(t)$	k(t)	H(t)	t ÷ k
(1)	(2)	(3)	(4)	(5)	(6)
0.	0.3989	0.5000	0	0.6267	0.6267
0.10	0.3970	0.5298	0.159569	0.6257	0.6267
0.20	0.3910	0.5793	0.319897	0.6233	0.6252
0.30	0.3814	0.6179	0.480571	0.6190	0.6242
0.40	0.3683	0.6554	0.642903	0.6131	0.6222
0.50	0.3521	0.6915	0.806916	0.6258	0.6196
0.60	0.3332	0.7257	0.973047	0.5975	0.6166
0.70	0.3123	0.7580	1.141665	0.5872	0.6131
0.80	0.2897	0.7881	1.313461	0.5764	0.6091
0.90	0.2661	0.8159	1.488848	0.5644	0.6045
1.00	0.2420	0.8413	1.667897	0.5516	0.5996
1.10	0.2179	0.8643	1.849870	0.5383	0.5946
1.20	0.1942	0.8849	2.039648	0.5242	0.5883
1.30	0.1714	0.9032	2.233341	0.5099	0.5821
1.40	0.1497	0.9192	2.431501	0.4963	0.5758
1.50	0.1295	0.9332	2.636915	0.4811	0.5688
1.60	0.1109	0.9452	2.847808	0.4671	0.5618
1.70	0.0940	0.9554	3.041294	0.4532	0.5590
1.80	0.0790	0.9641	3.290640	0.4380	0.5470
1.90	0.0656	0.9713	3.521635	0.4253	0.5395
2.00	0.0540	0.9772	3.757926	0.4130	0.5322
2.10	0.0440	0.9821	4.004946	0.4000	0.5244
2.20	0.0355	0.9861	4.261822	0.3859	0.5162
2.30	0.0283	0.9893	4.526757	0.3746	0.5081
2.40	0.0224	0.9918	4.794962	0.3616	0.5005
2.50	0.0175	0.9938	5.077028	0.3543	0.4924
2.60	0.0136	0.9953	5.355640	0.3456	0.4855
2.70	0.0104	0.9965	5.651429	0.3365	0.4778
2.80	0.0079	0.9974	5.988942	0.3291	0.4675
2.90	0.0060	0.9981	6.263962	0.3167	0.4630
3.00	0.0044	0.9987	6.644026	0.2954	0.4515

7. $Z(t) = \exp(-t^2/2)$; $\Phi(t) = \int_{-\infty}^{t} Z(t)\,dt$; $k(t) = \log \Phi(t) - \log \Phi(-t)$

$$H(t) = \Phi(t)\,\Phi(-t)/Z(t)$$

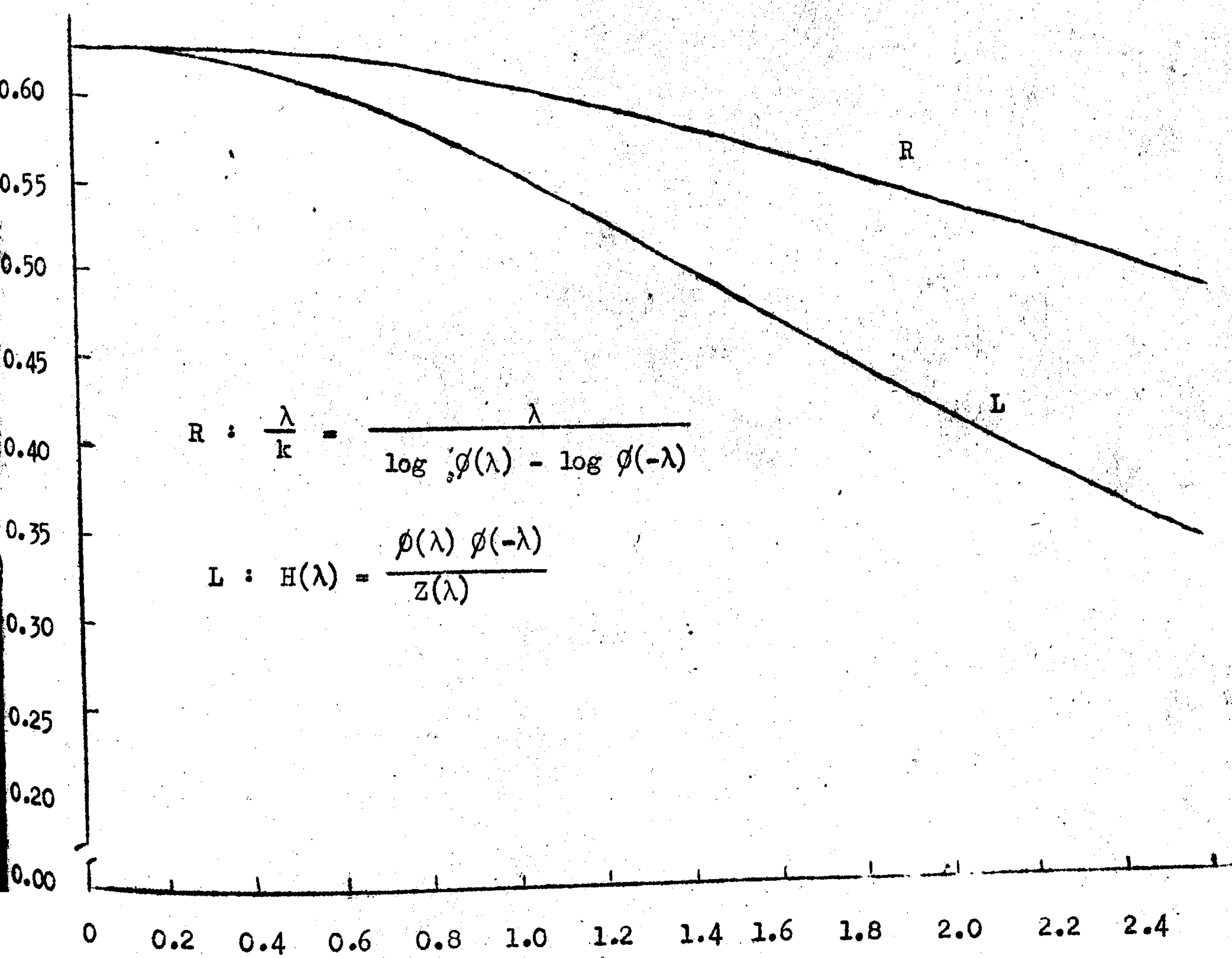

Figure A 2.1. Functions $H(\lambda)$ and $\frac{\lambda}{k}$ compared.

Chapter 3

SOME ESTIMATES OF ENGEL ELASTICITIES

3.1. The object of this study is to provide estimates of Engel elasticities for as many items of consumption as are possible from the available data on consumer expenditure in India. In India at present, there is a great need for estimates of income (or total expenditure) elasticity for a large number of items of consumer expenditure. Many practical uses of such elasticities are known: A major use perhaps is in the classification of commodities and services according as whether they are essential or luxuries. This information is needed in the determination of commodity taxes as well as in the assessment of taxation yields [22]. If one wants to compile consumer price indices by some new categories of consumption, e.g. bare essentials, other essentials (semi-luxuries) and non-essentials (luxuries), which have recently drawn the attention of the Government of India,[1] some objective classificatory criterion is need [24]; this is provided by the Engel elasticity [25]. In problems of demand projections the role of Engel elasticity is well-known [4, 5, 6, 12, 13, 14, 23, 28, 34]. Expenditure elasticities have also been employed in order to study the pattern of consumer expenditure in rural and urban areas of

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1. Some of our results were submitted to the Planning Commission in December 1961 in a paper entitled 'A preliminary note on estimates of expenditure elasticities'.

India [42,33].

The present study, however, leaves aside the examination of the standard as well as the recently advanced forms of Engel curves in the light of our data, although such an examination may be of considerable interest [1, 15, 22, 30, 37]. Since cross-tabulations of NSS expenditure data by households income (total expenditure) and size are not generally available, we have ignored the possibility of economies of scale in consumption in large households and made no adjustment for household age-sex composition [10, 30]. For the same reason, we have also neglected other systematic factors such as race, occupation, age and level of education of head of household, which may cause variation in the household demand patterns [11]. We have, instead, used the double-log hypothesis uniformly for all commodities. The linear double-log form meets most of the requirements of the Engel curve, except perhaps the additivity criterion. Recently, some additive logarithmic forms have been suggested by Leser [22] Houthakker [15, 16] and others. However, a detailed study of alternative forms of the Engel curve is beyond the scope of the present study.

In section 3.2 we give a brief review of some recent studies in India concerned with the estimation and use of Engel elasticities. The type of data presently used and the estimation procedure are outlined in section 3.3. Our main results are presented in section 3.4 together with some discussions followed by a practical application. A few concluding remarks are made in the last section.

3.2. The usual procedure. A variety of different mathematical functions of the form

$$y = \psi(x) \tag{3.1}$$

have been suggested by various workers from time to time; here, x represents total consumer expenditure of household per capita, and similarly, y represents per capita expenditure of households on a specific item of consumption. The choice of a particular form of Engel curve is guided by one or more of the following considerations: goodness of fit, computational simplicity, non-negativity, additivity and theoretical feasibility in the sense that the corresponding utility function is well defined. No single function, howeveer, seems to satisfy all the five criteria simultaneously and uniformly for all commodities. The decision on which function to apply in particular cases is largely a matter of personal judgment.

Recently in India, several studies have been reported by workers of the Indian Statistical Institute and others, dealing with the estimation and use of Engel elasticities. In most cases, the data used relate to fixed intervals of income (Total expenditure) of households per capita, and were obtained from the National Sample Survey (NSS) sources. The standard NSS tables provide estimates of (i) proportions (w_i) of persons belonging to some twelve different class intervals of monthly per capita total consumer expenditure (x), (ii) average per capita total consumer expenditure ($\bar{x}_i$), and (iii) average per capita item consumption ($\bar{y}_i$), usually in value terms, for these twelve class intervals. The graph showing $\bar{y}_i$ against $\bar{x}_i$ gives the expenditure-consumption curve. This is often called the Engel curve[2], although the terminology may not be strictly correct.

The determination of the shape of such curves from the observed points ($\bar{x}_i$, $\bar{y}_i$) is rendered somewhat difficult by the presence of heteroscedasticity which arises from grouping of households into size classes of x. These class intervals are rather coarse and include unequal number of households. Notwithstanding this major difficulty, most workers in India (and also abroad) have fitted the Engel curve

$$\bar{y} = \psi(\bar{x}) \qquad (3.2)$$

by using the method of weighted least squares.

---

2. Named after Ernst Engel, a German Statistician, who first drew attention to these relationships in 1857. The elasticity obtained from this curve is called the Engel elasticity; it differs from the income elasticity of demand by a multiplying factor, viz., the marginal propensity to consume.

Roy and Dhar [32] fitted straight lines by inspection of the graphs showing log $\bar{y}_1$ against log $\bar{x}_i$. A similar approach was made by Lydall and Ahmad [23], although no specific hypothesis was made regarding the Engel curve. In some studies, based on fractile data, unweighted least squares was used to estimate the slope of the double-log Engel curve [13], since the group weights w_i were equal.

The constant elasticity curve was generally found to be reasonably adequate [18, 32, 34, 35] at least for a number of items, although one study [32] suggested that Tornqvist's forms could be somewhat superior, in the sense of their having a smaller residual sum of squares. But the constant elasticity given by the double-log curve generally agreed with the elasticity calculated at the overall mean of the $\bar{x}_i$'s given by the Tornqvist's forms. Straight lines and second degree parabolas were also used [7, 36], besides semi-log forms [2, 20], and a pair of straight lines either after [34, 35] or without logarithmic transformation [36].

Projection formulae have been derived for some of the variable elasticity forms [2] but it has been observed that for moderate changes in the income distribution, projections based on constant elasticity curves would always be reasonably satisfactory. However, a variable elasticity form may give a greater insight into the nature of the item by showing the change in elasticity with increase in income, or by suggesting a threshold value of income, or a satiety level of consumption. As has already been seen in Chapter 2, the semi-log Engel curve possesses all these advantages, and its statistical estimation is not unduly difficult.

In a pioneering work, Roy and Laha [34] used total household expenditure, not per capita, on all items and on the specific item of consumption. Engel curves and the elasticities obtained therefrom are perhaps dubious, although the estimates of elasticities are generally of the same order as those obtained by the standard method from per capita data. This is because household size is highly correlated with total household expenditure, and the average household size shows marked variation between different levels of total household expenditure [32, 35]. In a recent paper by Datta Mazumdar [13], on the other hand, the households have been first grouped by total household expenditure but the subsequent calculations were based on per capita $\bar{x}_i$'s and $\bar{y}_i$'s for the different groups of households. It is perhaps difficult to judge the validity of these different procedures of estimation since in most cases the standard errors or multiple correlations are not available for those estimates. Such measures of reliability are not usually calculated on the ground that the probability design used in the National Sample Survey is complicated.

A few studies have also been made outside the Indian Statistical Institute, but they all employ the constant elasticity hypothesis and use the same source of data, viz., the National Sample Survey [8, 9, 28]. A notable exception, however, is a study by Murti [27], in which a few known forms of Engel curves [29] were fitted by using individual household data obtained from some 75 working class households in Vishakapatnam city. Apart from various obvious limitations, all these studies have one other common limitation, viz., they do not explicitly take into consideration the form of the distribution of income in the estimation of the Engel curve.

3.3. Data and procedure of estimation. The present study is based on some specially tabulated cross-section data on household expenditure drawn from the tenth round of the National Sample Survey, which covered the period December 1955 - May 1956. The urban sample consisted of 1326 households and the rural sample 1616 households. Details of the number of households in each total expenditure per capita group and of the estimated percentages of households (and persons) in these groups are shown in Table 3.1.

TABLE 3.1.

Table 3.1. Distribution of households and persons by size class of monthly total per capita expenditure: All India, 1955-56*.

monthly total per capita expenditure (x)	number of sample households		estimated percentage of persons in the population (w)		average monthly per capita total expenditure ($\bar{x}$)	
	rural	urban	rural	urban	rural	urban
(1)	(2)	(3)	(4)	(5)	(6)	(7)
under Rs. 8	218	47	15.7	4.3	Rs. 6.22	Rs. 6.33
Rs. 8 - 11	264	128	18.2	12.3	10.12	9.44
11 - 13	173	91	12.1	9.4	11.95	12.00
13 - 15	150	79	9.4	7.7	13.89	14.01
15 - 18	198	132	11.3	12.9	16.45	16.30
18 - 21	150	117	7.8	10.3	19.51	19.33
21 - 24	126	88	7.0	7.2	21.91	22.61
24 - 28	93	131	5.1	8.2	25.39	26.14
28 - 34	89	120	4.6	8.2	30.15	31.19
34 - 43	73	127	4.4	7.6	37.56	37.94
43 - 55	38	105	2.1	5.5	44.21	48.48
55 and above	44	161	2.3	6.4	76.84	87.96
all	1616	1326	100.0	100.0	17.40	25.40

Based on the Tenth Round of the National Sample Survey, December 1955 - May 1956.

The estimated percentage distributions were obtained by weighting the sample data by appropriate 'multipliers'[3]. Tabulations were also made of average per capita expenditure on 69 groups of consumer expenditure items of which 56 were independent sub-groups, covering the whole range of household expenditure. Such data were compiled separately for rural and urban areas of India and are given in Tables 3.2 and 3.3.

---

3. These will take into account the varying sampling proportions used in the Survey.

Table 3.2. Average monthly per capita consumption expenditure on various items by total per capita expenditure groups. All-India (Rural), 1955-56.

Sl. no.	Item of expenditure	Monthly total per capita expenditure (Rs.)												
		0 - 8	8-11	11-13	13-15	15-18	18-21	21-24	24-28	28-34	34-43	43-55	55-	all classes
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1.	rice	1.54	2.38	4.05	3.53	4.38	4.72	5.04	5.28	4.76	5.84	3.22	7.18	3.69
2.	wheat	0.14	0.32	0.49	0.59	0.87	1.22	1.23	1.17	2.09	2.01	4.49	5.79	0.94
3.	other cereals	1.71	1.96	1.49	2.26	1.84	1.85	2.33	2.46	2.46	2.33	3.75	3.93	2.05
4.	total cereals	3.39	4.66	6.03	6.38	7.09	7.79	8.60	8.91	9.31	10.18	11.46	16.90	6.68
5.	milk & milk products	0.14	0.42	0.66	0.91	1.01	1.77	2.11	2.71	3.72	4.65	10.95	7.74	1.58
6.	cereal substitutes	0.10	0.08	0.04	0.12	0.19	0.15	0.08	0.12	0.02	0.01	-	0.02	0.09
7.	pulses	0.30	0.48	0.61	0.70	0.75	0.87	0.92	1.04	1.55	1.11	1.61	2.27	0.75
8.	vanaspati	0.001	0.002	0.005	0.006	0.007	0.02	0.01	0.04	0.05	0.12	0.008	1.52	0.05
9.	other oil & oil seeds products	0.189	0.288	0.315	0.404	0.463	0.51	0.59	0.70	0.75	0.77	0.852	0.24	0.44
10.	total oil & oil seeds products	0.19	0.29	0.32	0.41	0.47	0.53	0.60	0.74	0.80	0.89	0.86	2.76	0.49
11.	vegetables	0.16	0.27	0.39	0.37	0.52	0.57	0.73	0.71	0.77	0.85	0.89	1.15	0.46
12.	fruits & nuts	0.06	0.05	0.06	0.08	0.14	0.17	0.14	0.22	0.34	0.28	0.28	0.74	0.13
13.	meat, fish, eggs, etc.	0.15	0.22	0.34	0.40	0.50	0.44	0.67	0.69	0.51	0.85	0.63	0.99	0.41
14.	sugar	0.01	0.06	0.06	0.12	0.13	0.17	0.20	0.33	0.36	0.65	0.75	1.08	0.18
15.	other sugar & gur	0.02	0.01	0.03	0.04	0.05	0.04	0.05	0.08	0.07	0.10	0.03	0.38	0.05
16.	gur & khandsari	0.06	0.13	0.14	0.21	0.21	0.31	0.36	0.34	0.62	0.37	0.71	1.15	0.24
17.	other sugar & gur	0.09	0.20	0.23	0.37	0.39	0.52	0.61	0.75	1.05	1.12	1.49	2.61	0.47
18.	salt	0.05	0.05	0.05	0.06	0.06	0.07	0.07	0.08	0.06	0.07	0.09	0.09	0.06
19.	spices	0.25	1.05	0.35	0.43	0.43	0.47	0.60	0.59	0.66	0.70	0.63	1.29	0.58
20.	tea (leaf)	0.02	0.05	0.05	0.10	0.08	0.09	0.16	0.16	0.26	0.26	0.30	0.31	0.10
21.	coffee (powder)	0.004	0.005	0.008	0.01	0.02	0.02	0.02	0.06	0.02	0.05	0.003	0.02	0.02
22.	other bev. & refreshments	0.026	0.085	0.072	0.10	0.15	0.15	0.15	0.17	0.18	0.32	0.697	0.59	0.13
23.	total bev. & refreshments	0.05	0.14	0.13	0.21	0.25	0.26	0.33	0.39	0.46	0.63	1.00	0.92	0.25
24.	pan, supari, etc.	0.07	0.09	0.14	0.12	0.17	0.19	0.17	0.26	0.22	0.25	0.26	0.60	0.16
25.	tobacco	0.140	0.24	0.25	0.35	0.39	0.44	0.50	0.52	0.50	0.62	0.63	1.34	0.36
26.	drugs etc.	0.007	0.03	0.06	0.04	0.03	0.03	0.02	0.03	0.007	0.28	0.05	0.24	0.04
27.	electricity	-	-	-	-	-	-	0.003	0.002	0.006	-	0.007	-	0.001
28.	kerosene	0.080	0.11	0.12	0.14	0.15	0.20	0.20	0.23	0.26	0.27	0.32	0.42	0.16
29.	other fuels & light	0.60	0.71	0.76	0.84	1.05	1.09	1.197	1.628	1.484	1.52	1.67	4.30	1.03
30.	total fuels & light	0.68	0.82	0.88	0.98	1.20	1.29	1.40	1.86	1.75	1.79	2.00	4.72	1.19
31.	cotton hand loan cloth	0.07	0.21	0.15	0.39	0.36	0.59	0.41	0.53	0.52	0.94	0.52	0.23	0.32
32.	cotton khadi	0.008	0.002	0.009	0.04	0.05	0.03	0.03	0.08	0.04	0.24	0.04	0.28	0.04

(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
33.	cotton mill cloth	0.11	0.27	0.53	0.53	0.85	1.08	1.40	1.31	2.78	3.11	2.50	6.41	0.99
34.	other cotton clothing	0.022	0.038	0.101	0.01	0.18	0.24	0.30	0.52	0.60	0.68	0.80	1.16	0.22
35.	total cotton clothing	0.21	0.52	0.79	0.97	1.44	1.94	2.14	2.44	3.94	4.97	3.86	8.08	1.57
36.	silk clothing	-	0.01	0.001	0.03	0.01	0.004	-	0.12	0.13	0.29	0.12	0.63	0.07
37.	woolen clothing	-	0.006	0.01	0.01	0.009	0.03	0.04	0.08	0.07	0.03	0.004	0.73	0.04
38.	bedding & upholstry	-	0.01	0.02	0.03	0.03	0.05	-	0.05	0.12	0.26	0.65	0.75	0.027
39.	cinema	0.003	0.005	0.004	0.02	0.02	0.01	0.02	0.02	0.07	0.19	0.10	0.14	0.03
40.	other amusements	-	0.015	0.006	0.05	0.02	0.04	0.06	0.12	0.02	0.14	0.09	0.23	0.03
41.	total amusements & sports	0.003	0.02	0.01	0.07	0.04	0.05	0.08	0.14	0.09	0.33	0.19	0.37	0.06
42.	school fees	-	0.005	0.01	0.02	0.02	0.05	0.07	0.05	0.12	0.06	0.03	0.05	0.03
43.	other educational expenditure	-	0.005	0.01	0.03	0.05	0.05	0.07	0.07	0.11	0.07	0.21	0.40	0.04
44.	total educational expenditure	-	0.01	0.02	0.05	0.07	0.10	0.14	0.12	0.23	0.13	0.24	0.45	0.07
45.	medicine	0.01	0.08	0.09	0.12	0.19	0.32	0.37	0.52	0.48	0.85	1.41	3.81	0.31
46.	soap (toilet)	0.003	0.007	0.01	0.006	0.02	0.03	0.04	0.04	0.06	0.08	0.08	0.15	0.02
47.	other toilets & cosmetics	0.037	0.073	0.09	0.09	0.09	0.11	0.09	0.18	0.15	0.20	0.14	0.32	0.10
48.	total toilets	0.04	0.08	0.10	0.10	0.11	0.14	0.13	0.22	0.21	0.28	0.22	0.47	0.12
49.	washing soap	0.02	0.02	0.04	0.06	0.07	0.08	0.07	0.10	0.14	0.20	0.21	1.53	0.09
50.	other sundry goods	0.02	0.03	0.06	0.04	0.05	0.08	0.10	0.05	0.07	0.12	0.20	0.62	0.07
51.	total sundry goods	0.04	0.05	0.07	0.10	0.12	0.16	0.17	0.15	0.21	0.32	0.41	2.15	0.16
52.	services	0.05	0.09	0.17	0.27	0.44	0.54	0.55	0.93	1.60	2.13	1.85	5.02	0.56
53.	railways	0.003	0.02	0.01	0.02	0.008	0.06	0.12	0.17	0.40	0.42	0.15	0.28	0.08
54.	bus	0.02	0.04	0.04	0.05	0.09	0.11	0.09	0.11	0.08	0.34	0.28	0.71	0.09
55.	cycle	0.001	-	-	0.002	0.004	0.004	0.004	0.04	0.001	-	0.01	0.09	0.006
56.	other conveyance	-	-	-	-	0.028	0.016	0.006	0.03	0.08	0.08	0.07	0.14	0.014
57.	total conveyance	0.02	0.06	0.05	0.07	0.13	0.19	0.22	0.35	0.56	0.84	0.51	1.22	0.19
58.	ceremonial expenditure	-	-	-	-	0.01	0.01	0.01	0.16	-	0.14	-	0.27	0.02
59.	residential house rent	-	0.008	0.002	-	0.02	0.02	0.01	0.04	0.02	0.22	0.08	0.008	0.02
60.	other rent	-	0.002	0.018	0.03	0.01	0.06	0.02	0.01	0.03		0.04	0.42	0.01
61.	total rent	-	0.01	0.02	0.03	0.03	0.08	0.03	0.05	0.05	0.22	0.12	0.43	0.03
62.	consumer taxes	-	0.04	-	-	0.04	0.05	0.03	0.04	0.03	0.26	0.51	0.16	0.04
63.	furniture	0.008	0.007	0.006	0.03	0.008	0.003	0.02	0.06	0.05	0.22	0.03	0.75	0.04
64.	ornaments	0.005	-	-	0.02	0.08	0.05	0.10	0.08	0.03	0.99	0.66	3.16	0.16
65.	domestic utensils	0.002	0.005	0.02	0.006	0.06	0.07	0.06	0.07	0.20	0.14	0.11	0.30	0.05
66.	footwear	0.006	0.04	0.02	0.04	0.04	0.10	0.15	0.12	0.35	0.46	0.48	0.82	0.11
67.	residential house & land	-	-	-	0.003	0.005	0.06	0.09	0.04	-	0.37	-	1.39	0.06
68.	other durables	-	-	-	0.011	-	-	-	-	0.04	0.01	-	0.50	0.01
69.	total durables	0.02	0.05	0.06	0.11	0.18	0.28	0.42	0.37	0.67	2.19	1.28	6.92	0.43
	all items	6.22	10.12	11.95	13.89	16.45	19.51	21.91	25.39	30.15	37.56	44.21	76.84	17.40

Table 3.3. Average monthly per capita consumption expenditure on various items by total per capita expenditure groups. All India (Urban), 1955-56.

Sl. no.	Item of expenditure	monthly total per capita expenditure (Rs.)												
		0 - 8	8-11	11-13	13-15	15-18	18-21	21-24	24-28	28-34	34-43	43-55	55-	all classes
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1.	rice	1.40	1.91	2.83	3.78	2.49	3.19	2.63	3.51	3.62	2.97	3.20	3.66	2.91
2.	wheat	0.27	0.62	0.48	0.87	1.24	1.53	1.64	1.55	1.59	1.90	1.95	2.71	1.31
3.	other cereals	1.46	1.74	1.31	0.66	1.38	1.27	1.58	1.60	1.45	1.63	1.91	1.63	1.63
4.	total cereals	3.13	4.27	4.62	5.31	5.11	5.99	5.85	6.66	6.66	6.50	7.06	8.00	5.85
5.	milk & milk prods.	0.21	0.55	0.69	1.02	1.60	1.80	2.36	2.61	3.59	4.91	7.88	10.22	2.75
6.	cereal substitutes	0.14	0.09	0.01	0.00	0.02	0.02	0.02	0.03	0.02	0.01	0.03	0.00	0.03
7.	pulses	0.29	0.40	0.51	0.52	0.59	0.73	0.84	0.87	1.06	0.98	1.16	1.54	0.76
8.	vanaspati	-	0.02	0.04	0.03	0.11	0.12	0.12	0.23	0.18	0.19	0.32	0.50	0.14
9.	other oil & oil seeds products	0.22	0.32	0.41	0.49	0.59	0.48	0.83	0.74	0.83	0.91	1.56	1.48	0.69
10.	total oil seeds prod.	0.22	0.34	0.45	0.52	0.70	0.60	0.95	0.97	1.05	1.10	1.88	1.98	0.83
11.	vegetables	0.21	0.32	0.54	0.49	0.57	0.61	0.82	1.02	1.23	1.35	1.59	2.64	0.88
12.	fruits & nuts	0.02	0.08	0.11	0.15	0.15	0.25	0.22	0.29	0.41	0.70	1.10	1.82	0.38
13.	meat, fish, egg etc.	0.16	0.22	0.45	0.54	0.64	0.65	0.81	1.00	1.10	1.41	1.63	2.95	0.88
14.	sugar	0.07	0.12	0.26	0.21	0.38	0.32	0.54	0.63	0.90	0.94	1.16	1.45	0.53
15.	gur & khandsari	0.05	0.06	0.07	0.07	0.18	0.14	0.29	0.15	0.13	0.18	0.20	0.46	0.16
16.	other sugar & gur	0.02	0.04	0.03	0.03	0.04	0.07	0.07	0.03	0.03	0.05	0.05	0.08	0.05
17.	total sugar & gur	0.14	0.22	0.36	0.31	0.60	0.53	0.90	0.81	1.06	1.18	1.41	1.99	0.74
18.	salt	0.03	0.04	0.04	0.04	0.04	0.06	0.04	0.06	0.05	0.05	0.06	0.08	0.05
19.	spices	0.26	0.36	0.41	0.45	0.47	0.60	0.64	0.59	0.65	0.65	0.91	1.35	0.58
20.	tea (leaf)	0.05	0.08	0.15	0.08	0.16	0.12	0.34	0.26	0.47	0.37	0.59	0.73	0.26
21.	coffee (powder)	0.01	0.01	0.01	0.04	0.02	0.06	0.05	0.07	0.15	0.12	0.13	0.46	0.08
22.	other beverages & refreshments	0.08	0.17	0.39	0.28	0.68	0.56	0.43	0.90	0.92	1.29	1.48	3.40	0.80
23.	total beverages & refreshments	0.14	0.26	0.55	0.40	0.86	0.74	0.82	1.23	1.54	1.78	2.20	4.59	1.14
24.	pan, supari etc.	0.08	0.07	0.16	0.19	0.21	0.26	0.23	0.26	0.35	0.45	0.61	0.80	0.28
25.	tobacco	0.12	0.19	0.29	0.28	0.37	0.44	0.49	0.63	0.56	0.79	0.79	1.38	0.50
26.	drugs etc.	-	0.00	0.02	0.10	0.10	0.10	0.21	0.10	0.11	0.06	0.15	0.33	0.10
27.	electricity	-	0.01	0.01	0.03	0.02	0.01	0.10	0.06	0.25	0.27	0.37	0.61	0.12
28.	kerosene	0.11	0.14	0.13	0.18	0.18	0.21	0.25	0.27	0.32	0.38	0.43	0.53	0.25
29.	other fuels & light	0.51	0.71	0.83	0.93	0.98	1.16	1.21	1.36	1.61	1.71	2.14	2.79	1.26

contd. ....

(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
30.	total fuels & light	0.62	0.86	0.97	1.14	1.18	1.38	1.56	1.69	2.18	2.36	2.94	3.93	1.63
31.	cotton handloom cloth	0.02	0.03	0.27	0.22	0.09	0.22	0.11	0.62	0.43	0.20	0.75	0.45	0.26
32.	cotton khadi	0.00	-	0.01	0.01	0.01	-	0.01	0.10	0.06	0.07	0.10	0.23	0.04
33.	cotton mill cloth	0.05	0.15	0.17	0.55	0.74	0.74	1.00	1.26	1.44	2.03	2.15	5.52	1.17
34.	other cotton clothing	0.01	0.03	0.03	0.05	0.16	0.24	0.21	0.19	0.19	0.16	0.32	0.93	0.20
35.	total cotton clothing	0.08	0.24	0.48	0.83	1.00	1.20	1.33	2.17	2.12	2.46	3.32	7.13	1.67
36.	silk clothing	-	-	-	-	0.01	-	0.02	-	0.17	0.06	0.06	0.52	0.06
37.	woolen clothing	-	-	-	0.00	0.06	0.06	0.16	0.07	-	0.07	0.14	0.44	0.07
38.	bedding & upholstry	0.00	-	0.00	0.00	0.02	0.15	0.09	0.05	0.05	0.08	0.08	0.22	0.06
39.	cinema	0.02	0.02	0.04	0.07	0.08	0.11	0.12	0.21	0.24	0.38	0.64	1.13	0.22
40.	other amusements	0.01	0.01	0.01	0.00	0.04	0.01	0.04	0.07	0.07	0.08	0.28	0.35	0.06
41.	total amusements & sports	0.03	0.03	0.05	0.07	0.12	0.12	0.16	0.28	0.31	0.46	0.92	1.48	0.28
42.	school fees	0.00	0.05	0.08	0.09	0.12	0.18	0.09	0.18	0.35	0.35	0.45	2.08	0.29
43.	other educational expenditure	0.00	0.01	0.10	0.03	0.09	0.10	0.07	0.20	0.25	0.34	0.42	1.94	0.25
44.	total educational expenditure	0.00	0.06	0.18	0.12	0.21	0.28	0.16	0.38	0.60	0.78	0.87	4.02	0.54
45.	medicine	0.02	0.03	0.06	0.35	0.12	0.33	0.55	0.32	0.67	1.45	1.04	2.67	0.56
46.	soap (toilet)	0.02	0.02	0.03	0.03	0.06	0.08	0.07	0.10	0.14	0.16	0.22	0.92	0.10
47.	other toilets & cosmetics	0.07	0.08	0.11	0.10	0.12	0.16	0.17	0.23	0.34	0.72	0.55	1.37	0.29
48.	total toilets	0.09	0.10	0.14	0.13	0.18	0.24	0.24	0.33	0.48	0.88	0.76	1.79	0.39
49.	washing soap	0.03	0.07	0.10	0.09	0.14	0.15	0.24	0.22	0.32	0.37	0.49	0.55	0.21
50.	other sundry goods	0.01	0.01	0.03	0.02	0.04	0.05	0.05	0.06	0.11	0.11	0.20	0.55	0.09
51.	total sundry goods	0.04	0.08	0.13	0.11	0.18	0.20	0.29	0.28	0.43	0.48	0.69	1.10	0.30
52.	services	0.08	0.23	0.25	0.46	0.42	0.82	0.87	1.22	2.02	2.64	3.27	5.83	1.31
53.	railways	0.01	0.04	0.01	0.06	0.05	0.19	0.27	0.25	0.32	0.24	1.05	1.66	0.29
54.	bus	0.01	0.01	0.04	0.07	0.09	0.12	0.20	0.26	0.32	0.24	0.84	1.00	0.23
55.	cycle	-	0.00	-	-	0.01	0.01	0.06	0.03	0.02	0.01	0.02	0.16	0.11
56.	other conveyance	0.00	0.01	0.01	0.01	0.01	0.02	0.05	0.04	0.16	0.11	0.16	0.17	0.11
57.	total conveyance	0.02	0.06	0.06	0.14	0.16	0.34	0.58	0.58	0.82	0.60	2.07	3.99	0.65
58.	ceremonial exp.	0.05	0.02	0.00	0.01	0.01	0.08	0.08	0.03	0.04	0.31	0.30	0.37	0.09
59.	residential house rent	0.02	0.18	0.42	0.21	0.37	0.45	0.89	0.85	1.13	2.06	2.27	4.99	1.00
60.	other rent	0.06	0.00	0.00	0.01	0.01	0.01	0.05	0.03	0.01	0.01	-	0.11	0.02
61.	total rent	0.08	0.18	0.42	0.22	0.38	0.46	0.94	0.88	1.14	2.07	2.27	5.10	1.02

contd. ....

Table 3.3. (contd.)

(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
62.	consumer taxes	0.07	0.02	0.01	0.07	0.02	0.01	0.08	0.03	0.08	0.42	0.22	0.75	0.13
63.	furniture	-	-	-	-	0.02	0.00	0.13	0.20	0.07	0.20	0.06	0.03	0.06
64.	ornaments	-	-	-	-	-	-	-	0.12	0.03	0.08	0.29	0.56	0.07
65.	domestic utensils	-	-	0.01	0.04	0.01	-	0.04	0.07	0.05	0.07	0.08	0.05	0.03
66.	Footwear	-	0.02	0.03	0.00	0.17	0.04	0.13	0.30	0.35	0.47	0.37	1.09	0.22
67.	residential house & land	-	-	-	-	-	0.08	-	0.00	0.10	0.04	0.00	5.20	0.35
68.	other durables	-	0.00	0.00	0.00	0.00	0.13	0.00	0.01	0.06	0.04	0.27	2.02	0.16
69.	total durables	-	0.02	0.04	0.04	0.20	0.25	0.30	0.70	0.66	0.90	1.07	8.95	0.89
	all items	6.33	9.44	12.00	14.01	16.30	19.33	22.61	26.14	31.19	37.94	48.48	87.96	25.40

As pointed out at the very outset, the procedure of estimation followed by us is based on the use of concentration curves, which are an important tool of demand analysis introduced by Mahalanobis [24] nearly a decade age in connection with researches on consumption behaviour in India. The specific concentration curve shows the inequality of item demand between the different x-classes. This is a generalization of the well-known Gini-Lorenz curve showing the inequality of income (here, its proxy, total consumer expenditure). A formal definition of these concepts has already been introduced in Chapter 1 .

The Lorenz ratio L_o gives an overall measure of the inequality of the x-distribution. Similarly, the specific concentration curve gives rise to the specific concentration ratio L_s, which provides an overall measure of inequality of item consumption between the different x-classes. Both types of curves and concentration coefficients can be constructed from the twelve-expenditure-class data tiven above in Tables 3.1 to 3.3 by purely arithmetical processes without making any assumptions regarding the distribution of x, i.e. the frequency density function g(x), or the Engel curve $E(y \mid x) = \psi(x)$.

Roy, Chakravarti and Laha [31] examined the properties of these curves in the general case, without making any restrictive assumptions about g(x) and $\psi(x)$. They derived a general formula for calculating the variable elasticity $\varepsilon(x)$ when both the specific concentration curve, $Q = Q(p)$, and the Lorenz curve, $q = q(p)$, are available[4]:

$$\varepsilon(x) = \frac{\frac{d^2Q}{dp^2}}{\frac{dQ}{dp}} \cdot x\frac{dp}{dx} \tag{3.3}$$

From the relative positions of the specific concentration curve and the Lorenz curve drawn in a unit square, one may easily see whether the commodity in question is a necessity or a luxury; the commodity is inferior if its specific concentration curve lies above the egalitarian line. While these conclusions follow at once from (3.3), it is not a convenient formula for obtaining the estimates of Engel elasticities. However, if it is assumed that within each total expenditure class the elasticity is more or less constant, we might obtain an approximate expression for the elasticity for the elasticity for a given expenditure class of households, but this will be no better than the customary arc elasticity which is completely distribution-free. For the ith expenditure class, the actual expression is:

$$\varepsilon_{(p_i, p_{i+1})} = \left[\log\left(\frac{dQ}{dp}\right)_{p_i+1} - \log\left(\frac{dQ}{dp}\right)_{p_i}\right] \Big/ \left[\log\left(\frac{dq}{dp}\right)_{p_i+1} - \log\left(\frac{dq}{dp}\right)_{p_i}\right] \tag{3.4}$$

---

4. By definition, $(x) = \frac{d\Psi}{\Psi} \Big/ \frac{dx}{x} = \frac{x}{\Psi}\frac{d\Psi}{dx}$

Being rather sensitive, the above formula also is not helpful as a general device for computing the variable elasticity, particularly for the bottom and top classes.

In the same paper [31], the authors considered a special case assuming a Pareto distribution for x with a Wolfe point $x_0 > 0$, and a constant elasticity Engel curve. The former assumption, however, does not seem particularly realistic, at least for consumer expenditure distributions in India. Some recent empirical studies [3, 33] have shown that it might be more realistic to consider situations where $g(x)$ is logarithmically normal and the Engel curve has a constant elasticity ε. These have been indeed our basic hypotheses in the foregoing Chapters, and we shall retain those assumptions for the purpose of our subsequent calculations.

The equation of the specific concentration curve, in the notation of Chapter 1, is given by

$$t_Q = t_p - \lambda \varepsilon \tag{3.5}$$

and the equation of the Lorenz curve by

$$t_q = t_p - \lambda \tag{3.6}$$

Here, t_Q, t_q, t_p represent the standard normal deviates corresponding to the proportions Q, q, p; λ stands for the standard deviation of log x, and ε for the constant elasticity.

The specific concentration curve has many properties, and some of them have already been listed earlier. When such curves have been constructed from empirical data, we may use the formula (1.18) of Chapter 1 to compute the Engel elasticity:

$$\varepsilon = \frac{t_{Q_{0.5}}}{t_{q_{0.5}}} \qquad (3.7)$$

This method has been actually used in the following Section. Earlier in Chapter 2, we also suggested another method depending on the expressions for the Lorenz and the specific concentration ratios. Though not consistent, the alternative estimator of the elasticity,

$$\varepsilon = \frac{t_{0.5}(1 + L_s)}{t_{0.5}(1 + L_o)} \qquad (3.8)$$

has certain practical advantages over (3.7), as will be seen in the following section. A still another set of estimates is provided by implicitly assuming that the distribution of x is log-logistic (see Chapter 2, p. 77) in which case the elasticity is obtained by dividing the specific concentration by the Lorenz rates, that is

$$\varepsilon = \frac{L_s}{L_o} \qquad (3.9)$$

Formulae (3.7), (3.8) and (3.9) are referred to in subsequent sections as Method I, Method II and Method III respectively.

3.4. <u>The main results</u>: As a first step, it was considered desirable to examine the relative adequacies of lognormal and log-logistic hypotheses for the observed distributions of expenditure in rural and urban India. Certain tests were therefore applied to our basic data of Table 3.1, and the results of our investigations are given in Table 3.4.

Table 3.4

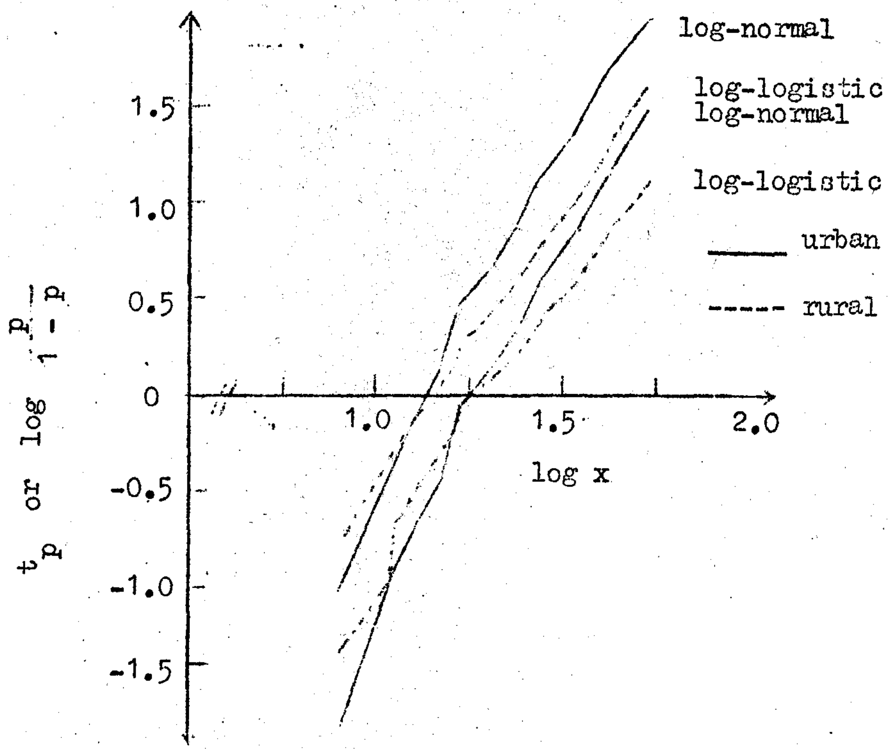
Rs.x	log x	Rural					
		p	p/(1-p)	log[p/(1-p)]	t_p	q	t_q
8	.90309	.147	.1862	- .75002	-1.0069	.056	-1.5893
11	1.04139	.339	.5129	- .28997	- .4152	.616	- .9904
13	1.11394	.460	.8418	- .06966	- .1004	.244	- .6935
15	1.17609	.554	1.2422	.09419	.1358	.318	- .4733
18	1.22527	.667	2.0030	.30168	.4316	.424	- .1917
21	1.32222	.745	2.9215	.46561	.6588	.511	.0276
24	1.38021	.815	4.4054	.64399	.8965	.599	.2508
28	1.44716	.866	6.4627	.81041	1.1077	.672	.4454
34	1.53148	.912	10.3636	1.01551	1.3532	.752	.6808
43	1.63347	.956	21.7273	1.33700	1.7060	.846	1.0194
55	1.74036	.977	42.4783	1.62816	1.9954	.899	1.2759
∞	∞	1.000			∞	1.000	∞

Table 3.4 (contd.)

Rs. x	log x	Urban					
		p	p/(1-p)	log[p/(1-p)]	t_p	q	t_q
8	.90309	.043	.0449	-1.34775	-1.7169	.011	-2.2904
11	1.04319	.166	.1990	- .70115	- .9701	.057	-1.5805
13	1.11394	.260	.3514	- .45420	- .6433	.102	-1.2702
15	1.17609	.337	.5083	- .29388	- .4207	.144	-1.0625
18	1.22527	.466	.8726	- .05918	- .0853	.228	- .7454
21	1.32222	.569	1.3202	.12064	.1738	.306	- .5072
24	1.38021	.641	1.7855	.25176	.3611	.371	- .3292
28	1.44716	.723	2.6101	.41666	.5918	.456	- .1105
34	1.53148	.805	4.1282	.61576	.8596	.557	.1434
43	1.63347	.881	7.4033	.86943	1.1800	.671	.4427
55	1.74036	.936	14.6250	1.16510	1.5220	.777	.7621
∞	∞	1.000			∞	1.000	∞

Cumulative proportions of estimated persons in the population as well as of total expenditure, denoted by p_i and q_i respectively, were calculated from Table 3.1 separately for rural and urban areas. Their standard normal deviates, t_{p_i} and t_{q_i}, as well as the ratios $p_i/(1 - p_i)$ were next calculated. Logarithms were taken for the latter ratios as well as for the x_i's which denote the end values of the given expenditure class-intervals ($i = 1, 2, \ldots, 12$). All these calculations are shown in Table 3.4.

Figure 3.1 : Log-probit and log-logit diagram for the distribution of Table 3.1



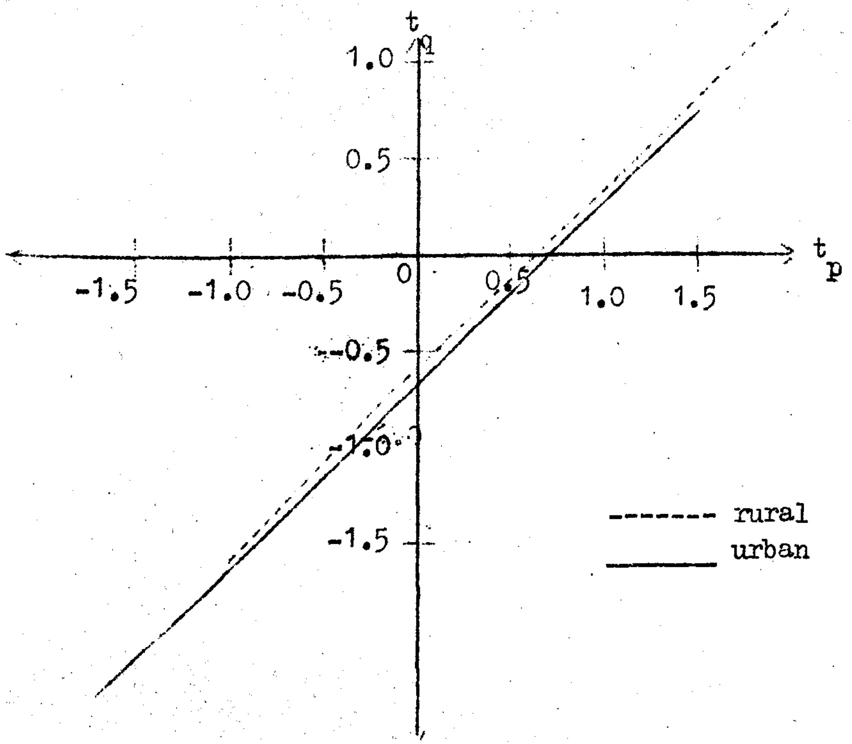


Figure 3.2 : Showing t_q against t_p for the distributions of Table 3.1

Appropriate graphs were then constructed from Table 3.4. As already pointed out, the graphical test for the lognormal hypothesis is provided by the linearity criterion (3.6), although the customary log-probit test is also frequently used in practice [3, 32]. Both these tests were made, although the former test is perhaps more appropriate in the present case [19] as pointed out in Chapter 2. Similarly, in the log-logistic case, a test is provided by the linearity of the graph showing $\log[p/(1-p)]$ against $\log x$. The log-probit and log-logit tests are shown in Figure 3.1, whereas the graphs showing t_q against t_p are given in Figure 3.2. The conclusion from these graphs is that both the hypotheses are plausible, but the graphs in Figure 3.2 strongly suggest in favour of lognormal hypothesis, for both rural and urban distributions.

The lognormal hypothesis having being verified for our total expenditure distributions, it remains now to see whether the remaining assumption, viz., the constant elasticity form for the Engel curve, is also true, at least approximately. For this purpose, we constructed a number of graphs using the basic data of Tables 3.2 and 3.3.. The appropriate test criterion is provided by equation (3.5), which holds if the distribution of total expenditure (x) is lognormal and the Engel curve has the constant elasticity form $\phi(x) = Ax^{\varepsilon}$. We have found from the graphs that for a number of commodities and services listed in Tables 3.2 and 3.2 the constant elasticity hypothesis is approximately valid. Such graphs, however, are not given here since they can be easily constructed from our basic data, if necessary. We now present the results of our calculations separately for rural and urban areas of India in Table 3.5.

Table 3.5. Estimates of Engel elasticity for selected commodities and services, All India : 1955-56

Sl. no.	Item of expenditure	Rural					Urban				
		$Q_{0.5}$	L_s	Elasticity η			$Q_{0.5}$	L_s	Elasticity η		
				I	II	III			I	II	III
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1.	Rice	.3534	.2226	.631	.633	.647	.4400	.0881	.227	.240	.239
2.	Wheat	.1726	.5063	1.576	1.536	1.472	.3123	.2569	.737	.682	.698
3.	Other cereals	.4373	.1106	.265	.311	.322	.4610	.0355	.147	.094	.096
4.	Total cereals	.3608	.2063	.595	.586	.600	.4158	.1133	.319	.299	.308
5.	Milk and milk products	.1358	.5571	1.838	1.726	1.619	.1771	.4820	1.394	1.350	1.310
6.	Cereals substitutes	.4272	-.0310	.308	-.084	-.090	.6932	.3780	-.758	-1.029	-1.027
7.	Pulses	.3145	.2676	.810	.769	.778	.3294	.2290	.666	.605	.622
8.	Vanaspati	.0298	.8641	3.146	3.347	2.512	.1911	.4480	1.314	1.242	1.217
9.	Other Oil & Oilseeds products	.3240	.2271	.764	.651	.560	.3149	.1990	.724	.529	.541
10.	Total Oil & Oilseeds products	.2802	.3243	.975	.938	.943	.2940	.3015	.814	.810	.819
11.	Vegetables	.2927	.2853	.911	.823	.829	.2625	.3422	.958	.924	.930
12.	Fruits and nuts	.2149	.3981	1.320	1.170	1.157	.1585	.5315	1.508	1.515	1.444
13.	Meat, egg, fish, etc.	.2928	.3954	.911	.883	.889	.2526	.3738	1.000	1.013	1.017
14.	Sugar	.1398	.5479	1.807	1.689	1.593	.2230	.3949	1.146	1.077	1.073
15.	Gur and Khandsari	.2366	.4124	1.198	1.216	1.199	.3090	.2705	.750	.721	.735
16.	Other sugar & gur	.2236	.4013	1.269	1.184	1.167	.4040	.1179	.365	.310	.320
17.	Total sugar & gur	.1991	.4456	1.414	1.329	1.295	.2519	.3512	1.005	.953	.954
18.	Salt	.4249	.0926	.316	.260	.269	.4111	.1099	.338	.289	.297
19.	Spices	.5034	.0418	-.013	.118	.122	.3618	.3051	.531	.821	.829
20.	Tea (leaf)	.2227	.4040	1.275	1.190	1.174	.2208	.3844	1.156	1.047	1.045
21.	Coffee (powder)	.1871	.3881	1.487	1.139	1.128	.1313	.5565	1.686	1.598	1.512
22.	Other beverages & refreshments	.2373	.3879	1.198	1.139	1.128	.2355	.4296	1.081	1.186	1.167
23.	Total beverages & refreshments	.2310	.3942	1.231	1.158	1.146	.2246	.4283	1.136	1.119	1.164
24.	Pan, supati etc.	.3164	.2642	.801	.757	.768	.2762	.3421	.894	.924	.970
25.	Tobacco	.3025	.2721	.868	.781	.781	.2828	.3116	.863	.839	.847
26.	Drug etc.	.3411	.3318	.685	.963	.965	.2616	.3454	.958	.936	.939
27.	Electricity	-	.7344	-	2.497	2.135	.0609	.6504	2.325	1.952	1.768
28.	Kerosene oil	.3316	.2378	.727	.680	.691	.3187	.2254	.707	.671	.686

Table 3.5. (contd.)

(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
29.	Other fuel and light	.3371	.2339	.704	.668	.680	.3300	.2352	.661	.627	.63
30.	Total fuel and light	.3363	.2346	.708	.668	.682	.3135	.2685	.733	.715	.73
31.	Cotton (Handloom)	.2517	.3150	1.118	.914	.916	.2637	.3042	.949	.816	.82
32.	Cotton (Khaddar)	.1074	.5545	2.079	1.171	1.612	-	1.612	-	1.856	1.70
33.	Cotton (mill-made)	.1537	.5211	1.705	1.593	1.515	.1704	.5035	1.435	1.422	1.36
34.	Other cotton clothing	.1094	.5773	2.061	1.803	1.678	.2061	.4475	1.234	1.242	1.21
35.	Total cotton clothing	.1672	.4706	1.616	1.410	1.368	.1868	.4687	1.337	1.305	1.07
36.	Silk clothing	.0651	.7466	2.533	2.561	2.170	-	.7675	-	2.496	2.08
37.	Woollen clothing	.0776	.1097	2.373	2.376	2.063	.1348	.5458	1.658	1.564	1.48
38.	Bedding and upholstry	.0840	.7163	2.306	2.406	2.031	.1344	.4539	1.666	1.261	1.23
39.	Cinema	.1041	.6985	2.106	2.318	2.082	.1234	.5832	1.744	1.698	1.58
40.	Other amusements	.1381	.5184	1.822	1.579	1.508	.1200	.6047	1.767	1.772	1.64
41.	Total amusements	.1265	.5600	1.916	1.734	1.629	.1222	.5881	1.752	1.713	1.59
42.	School fees	.1050	.5499	2.097	1.696	1.600	.1462	.5898	1.584	1.720	1.60
43.	Other educational expenditure	.0738	.6031	2.420	1.906	1.753	.1118	.6459	1.828	1.936	1.75
44.	Total educational expenditure	.0860	.5694	2.285	1.772	1.664	.1284	.6168	1.708	1.818	1.67
45.	Medicine	.1023	.6455	2.125	2.081	1.878	.1363	.5371	1.652	1.536	1.46
46.	Soap (toilet)	.1329	.5409	1.861	1.659	1.573	.1431	.5810	1.604	1.684	1.57
47.	Other toilets and cosmetics	.3785	.1431	.520	.408	.416	.1757	.4966	1.399	1.395	1.34
48.	Total toilets	.2990	.4711	.882	1.417	1.369	.1795	.4890	1.376	1.369	1.32
49.	Washing soap	.1467	.5808	1.755	1.810	1.688	.2330	.6262	1.096	1.855	1.70
50.	Other sundry goods	.2568	.4019	1.092	1.184	1.168	.1450	.5710	1.591	1.655	1.55
51.	Total sundry goods	.1742	.5231	1.570	1.600	1.521	.2072	.4319	1.228	1.192	1.17
52.	Services	.0981	.6144	2.163	1.946	1.786	.1315	.5414	1.679	1.550	1.42
53.	Railways	.0801	.6613	2.350	2.151	1.922	.0831	.6225	2.083	1.841	1.69
54.	Bus	.1892	.4691	1.475	1.410	1.364	.1175	.5650	1.782	1.627	1.53
55.	Cycle	.0294	.6748	3.171	2.205	1.962	.0715	.5880	2.197	1.713	1.60
56.	Other conveyance	-	.7066	-	2.356	2.054	.0901	.5418	2.016	1.584	1.50
57.	Total conveyance	.1200	.5765	1.966	1.795	1.676	.0873	.6258	2.044	1.856	1.70
58.	Ceremonial expenditure	-	.7737	-	2.718	2.250	.1053	.5696	1.885	1.648	1.55
59.	Residential house rent	.0777	.6176	2.385	1.963	1.795	.1416	.5408	1.611	1.543	1.47
60.	Other rent	.1528	.5484	1.712	1.689	1.594	.2466	.3030	1.028	.816	.82
61.	Total rent	.1188	.5886	1.974	1.842	1.711	.1437	.5360	1.598	1.529	1.45

Table 3.5 (contd.)

(0)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
62. Consumer taxes	-	.5531	-	1.711	1.608	.1205	.6075	1.767	1.787	1.653
63. Furniture	.1073	.6573	2.079	2.133	1.911	-	.4657	-	1.296	1.268
64. Ornaments	.0074	.8364	4.111	3.125	2.431	-	.7925	-	2.629	2.156
65. Domestic utensils	.0792	.6122	2.362	1.938	1.780	-	.4224	-	1.162	1.150
66. Footwear	.1122	.6096	2.034	1.930	1.772	.1299	.5674	1.694	1.641	1.544
67. Residential house and land	.0021	.8409	4.815	3.155	2.444	-	.9661	-	4.427	2.628
68. Other durables	.0294	.8804	3.171	3.491	2.599	.0267	.8334	2.897	2.892	2.267
69. Total durables	.0554	.7413	2.673	2.540	2.155	.0488	.7555	2.488	2.433	2.055
All items	.2751	.344				.2528				

As a first step, proportionate shares in total consumption of specific goods and services accruing to lower 50 per cent of the population were computed, that is, the values of Q corresponding to $p = 0.5$ Smoothed concentration curves were not drawn, however, for the purpose of interpolation. Instead, two successive cumulative proportions p^* and p^{**} of population were selected such that $p^* < 0.5 < p^{**}$, and the required values of $Q_{0.5}$ were calculated by linear interpolation:

$$Q_{0.5} = Q^* + \frac{Q^{**} - Q^*}{p^{**} - p^*} (0.5 - p^*) \qquad (3.10)$$

These proportions are given in columns (2) and (7). The next important intermediate stage was the calculation of the specific concentration ratios. This was done for all commodity sub-groups by employing the well-known formula[5]

$$L_s = 1 - \sum_{i=1}^{12} (p_i - p_{i-1})(Q_i + Q_{i-1}) \qquad (3.11)$$

The specific concentration coefficients are shown in columns (3) and (8). No new principles were involved in the calculation of q or L_o for the Lorenz curve, except that Q's in (3.10) and (3.11) were replaced by the corresponding q's. The main results, viz., estimates of expenditure elasticities are given in columns (4, 5, 6) for rural the sector and (9, 10, 11) for the urban sector as obtained by alternative procedures described in Section 3.3.

5. This formula has already been encountered in Chapters 1 and 2.

In four cases in rural and six cases in rural and six cases in urban sectors the elasticities were not obtained by method I since the $Q_{0.5}$'s could not be calculated; for these items there were no entries $(\bar{y}_i)$ in the lower expenditure groups (vide Tables 3.2 and 3.3) The presence of zero entries did not present any difficulty for application of methods II and III, based on specific concentration ratios which could be readily computed for all sub-groups of commodities.

The estimates of Engel elasticity are found to vary, though slightly, from method to method, the first giving generally higher values than other two methods. But the ordering of commodities on the elasticity scale seems to be approximately same in all methods. Since method II is known to yield asymptotically biased results (see Chapter 2, p. 59) and the log-logistic hypothesis is somewhat unrealistic as compared with the lognormal assumption (vide graphs in Figures 3.1 and 3.2), we feel inclined to prefer the first set of estimates, which are consistent (see Chapter 2, p. 50) under the assumptions we have made in their derivation, to other two sets.

We may broadly distinguish three groups of commodities and services, viz., necessary, semi-luxury and luxury groups, according as their expenditure elasticity is less than or equal to unity, greater than 1 and less than or equal to 2, and larger than 2. This was actually done using the estimates of elasticity obtained by method I for all sub-groups of commodities listed in Tables 3.2 and 3.3. Following this classification, Mukherjee and Chatterjee [26] computed consumer price indices from the National Sample Survey data for the year 1957-58 with 1952-53 as base, and their results are given in Table 3.6. It should be pointed out here that they actually used the second set of estimates and therefore their classification is somewhat different.

Table 3.6: Index number of consumer prices
All India (Rural and Urban), 1957-58

Base 1952-53=100

categories of consumption	weights		index numbers based on budget pattern of NSS*					
	4th round	13th round	4th round			13th round		
			s.s.1	s.s.2	comb.	s.s.1	s.s.2	comb.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				rural				
bare essentials ($o < \varepsilon \leq 1$)	58.57	60.65	99.80	94.90	95.38	98.88	90.36	94.61
other essentials ($1 < \varepsilon \leq 2$)	27.20	31.00	105.67	105.81	105.42	105.98	105.57	105.63
non-essentials ($\varepsilon > 2$)	14.23	8.35	127.39	114.49	119.66	126.77	114.52	120.06
total	100.00	100.00	105.32	100.66	101.57	102.91	96.36	99.59
				urban				
bare essentials ($o < \varepsilon \leq 1$)	38.40	44.32	98.56	95.94	96.72	93.71	87.77	92.56
other essentials ($1 < \varepsilon \leq 2$)	57.90	53.77	116.62	118.77	117.68	109.97	111.93	111.01
non-essentials ($\varepsilon > 2$)	3.70	1.91	108.71	115.42	111.92	95.10	116.01	106.71
total	100.00	100.00	109.39	109.88	109.42	101.84	99.81	101.93

* NSS 4th round represents 1952-53 and 13th round 1957-58, in a rough sense

Prais and Houthakker, in their now classic work, The Analysis of Family Budgets [30], have suggested the semi-log form for the necessary group of commodities and the double-log form of Engel curve for semi-luxury and luxury groups of commodities. Following their suggestion, we shall present some results in Table 3.7 assuming the semi-log form for all those items with elasticity less than or equal to unity. No regressions were used in our calculations but the mean, and median elasticities were obtained by using our own methods of Chapter 2. For the semi-log curve, which gives variable elasticity depending on total expenditure, the median elasticity $\varepsilon(C)$ is given by

$$\varepsilon\ (C) = \frac{0.5 - Q_{0.5}}{-\ Z(0)t_{q_{0.5}}} \tag{3.12}$$

and the mean elasticity $\varepsilon(\mu)$ by

$$\varepsilon\ (\mu) = \frac{\varepsilon\ (C)}{1 + 0.5\ \ \varepsilon(C)\ t^{2}_{q_{0.5}}} \tag{3.13}$$

Both these elasticities were computed for some 20 essential commodities separately for rural and urban sectors. These are shown in Table 3.7 along with the corresponding constant elasticities. It is interesting to observe that the median elasticity is greater than the mean elasticity but less than the constant elasticity. This fact was empirically first observed by Prais and Houthakker [30] and later proved by

Iyengar [19] under certain realistic assumptions (see also Chapter 2).

Table 3.7. Estimates of 'mean', 'median', and 'constant' elasticities for selected commodities. All India, 1955-56.

	Item of expenditure	'mean' elasticity		'median' elasticity		'constant' elasticity	
		rural	urban	rural	urban	rural	urban
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1*	Rice (1)	.554	.215	.615	.226	.631	.227
2.	Wheat (2)	-	.612	-	.707	-	.737
3.	Other cereals (3)	.251	.142	.263	.147	.265	.147
4.	Total cereals (4)	.529	.296	.584	.317	.595	.319
5.	Cereals substitutes (6)	.290	-.868	.305	-.728	.308	-.758
6.	Pulses (7)	.683	.563	.778	.643	.810	.666
7.	Other oil and oilseeds products (9)	.651	.604	.737	.698	.764	.724
8.	Total oil and oil seeds products (10)	.791	.663	.922	.774	.975	.814
9.	Vegetables (11)	.416	.747	.450	.895	.911	.958
10.	Meet, fish, eggs (13)	.752	.773	.869	.932	.911	1.000
11.	Gur and Kandsari (15)	-	.621	-	.720	-	.750
12.	Other sugar and gur (16)	-	.335	-	.362	-	.365
13.	Salt (18)	.298	.312	.315	.335	.316	.338
14.	Spices (19)	-.014	.467	-.014	.521	-.013	.531
15.	Pan, Supari, etc.(24)	.677	.711	.770	.844	.801	.894
16.	Tobacco (25)	.721	.693	.828	.819	.868	.863
17.	Drug, etc., (26)	.595	.705	.666	.898	.685	.958
18.	Kerosene oil (28)	.627	.594	.706	.683	.727	.707
19	Other fuel and light(29)	.609	.561	.683	.640	.704	.661
20.	Total fuel and light(30)	.611	.608	.686	.703	.708	.773
21.	Cotton (handloom) (31)	-	.744	-	.891	-	.949
22.	Other toilets and cosmetics (47)	.467	-	.510	-	.520	-
23.	Total toilets (48)	.732	-	.843	-	.882	-

* Numbers in brackets correspond to serial numbers of items in Table 3.2

The conventional weighted least squares was also used in order to obtain the expenditure elasticities from the double-log linear relationship between specific consumption and total expenditure. The amount of details which were later available permitted the sub-samplewise estimation of elasticity for a number of items. The method owing to serious limitations already mentioned, could not provide estimates of elasticity for all the items. For comparative purposes, however, we present in Table 3.8 whatever results were obtained by using the regression methods [38]. The regression estimates, of course, differ from our estimates to some extent but the observed differences are usually small.[see table 3.8].

3.5. Concluding remarks: While there are some practical advantages in using the method of concentration curves for estimation of Engel elasticities, there are important theoretical limitations. The double-log Engel curve is not additive, and the sum of finite number of log-normal variates is not lognormal,unlike the case of normal distribution. Recently, however, some additive logarithmic forms of Engel curve have been suggested [15, 16]. It may be possible to use those forms and yet devise regression-free methods for their estimation. The elasticities obtained here are not income elasticities of demand. Our estimates of expenditure elasticities should be multiplied by suitable estimates of

Table 3.8. Least Squares estimates of Expenditure elasticity, All India, 1955-56

name of item	urban India			rural India		
	s.s.1	s.s.2	combined	s.s.1	s.s.2	combined
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1. rice			0.28			0.65
2. wheat			0.81			1.55
3. total cereals	0.33	0.32	0.32			0.63
4. pulses	0.68	0.64	0.64	0.73	0.89	0.81
5. vegetables	0.89	1.04	0.96	0.91	1.00	0.91
6. spices	0.55	0.58	0.56	0.66	0.57	0.60
7. vanaspati			1.47			2.37
8. oil, oilseeds & products	0.88	0.84	0.84	0.97	0.83	0.91
9. salt	0.30	0.33	0.31			0.29
10. gur	0.57	0.86	0.80			1.17
11. sugar	0.74	1.26	1.23			2.15
12. sugar sub-total	1.04	1.11	1.06	1.39	1.31	1.38
13. milk & milk products	1.58	1.40	1.49	1.66	2.10	1.86
14. meat, fish, eggs	1.05	1.19	1.11	0.87	0.89	0.88
15. fruits and nuts	1.59	1.56	1.56	1.03	1.11	1.07
16. tea (leaf)	1.14	1.12	1.11			1.29
17. coffee (powder)	1.83	1.43	1.69			
18. beverage & refreshments sub-total	1.32	1.25	1.27	1.30	1.28	1.25
19. pan, supari etc.			1.00			0.78
20. tobacco	0.90	0.83	0.90	0.92	0.73	0.84
21. kerosene oil	0.67	0.72	0.68	0.72	0.68	0.70
22. electricity	2.20	2.39	2.22			
23. fuel & light sub-total	0.73	0.72	0.72	0.71	0.58	0.66
24. cotton (handloom)			1.04	1.22	.072	1.02
25. cotton (mill-made)	1.82	1.39	1.71	1.88	1.65	1.77
26. cotton (khaddar)			1.82			1.38
27. clothing cotton sub-total	1.91	1.32	1.56	1.76	1.48	1.61
28. footwear			2.02			1.86
29. washing soap	1.08	1.15	1.11	1.48	1.28	1.45
30. toilet soap			1.35			1.22
31. toilets	1.53	1.26	1.29	0.94	0.96	0.94
32. school fee			1.63			1.23
33. education			1.85			2.03
34. railway	2.08	1.82	2.06	2.51	1.69	1.79
35. bus			2.02	1.57	1.18	1.30
36. conveyance sub-total	2.05	2.21	2.05	1.98	1.32	1.82
37. medicine			1.71			2.28
38. cinema	1.51	2.07	1.79	1.61	1.48	1.53
39. amusement and sports			1.76			1.70
40. drugs and intoxicants			1.36			0.75
41. ceremonials			1.49			2.53
42. residential house rent	1.73	1.76	1.74			
43. services			1.68			2.03
44. domestic utensils	0.63	1.63	1.01	1.95	1.00	1.88
45. furniture				1.74	0.88	1.36
46. total durables	2.10	2.84	2.66			2.33

incremental marginal propensity of total consumption separately for rural and urban areas in order to obtain the income elasticities. From the NCAER study on savings, and also our own [21] it appears that the all-India overall marginal propensity to consume in India is about 80 percent. There may be significant rural-urban differences but we do not have sufficiently reliable estimates of consumption propensities by rural-urban breakdowns in India. We have already pointed out that there are important advantages if data are classified according to fixed fractile groups instead of in fixed total expenditure classes. For the fractile data, the estimation procedure is simpler since the problem of heteroscedasticity of variances does not arise. The latter type of data is particularly useful in the context of our methods since we do not have to use the crude interpolation formula (3.10) to obtain the $Q_{0.5}$'s ; from the fractile data these could be readily obtained. Although theoretically possible, we have not attempted to calculate standard errors for our estimates.

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Chapter 4

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QUALITY ELESTICITIES FOR SELECTED COMMODITIES

4.1. This chapter is concerned with an examination of the extent of quality preferences in rural and urban areas of India. The concept of 'quality elasticity' is used, and such elasticities are computed for selected commodities by making use of concentration curves. In some cases, however, weighted regression is used. These studies are based on consumption data provided by the National Sample Survey of India.

In surveys of household expenditures the items recorded in the schedules are not uniform homogeneous commodities but represent aggregates of more or less closely related substitutes. An item of expenditure in a household expenditure schedule is to be regarded as a composite item made up of a number of constituent items. These constituent items may in some cases be the different varieties of the same commodity, each of different quality and bought at different prices. The variation in prices paid for a commodity arises primarily from quality differences, but regional, seasonal and price discriminatory factors may also contribute to the variation. To explain this variation,

* This chapter is based on a slightly expanded version of a paper which was presented at the 48th Session of the Indian Science Congress, Roorkee, January 1961, and which subsequently appeared in Sankhyā, 25 (1 and 2), 1963 [7].

we use the concept of quality elasticity [11] which is analogous to, and complements the expenditure elasticity.

The notion of the quality of a commodity is essentially multidimensional and its complete specification is often very laborious. However, in so far as the differences in the quality of commodities are voluntarily chosen by the consumer in the market supposed to be in equilibrium, the multidimensional elements are brought under the measuring rod of money. If the variations in prices by regional, seasonal or price discriminatory factors are not of much importance, the average price paid for the commodity may be taken as an indicator of the quality of the commodity. The average prices are, of course, calculated by dividing expenditure on the commodity by the quantity brought and, to this extent, data regarding quantity and value of consumption are necessary in the analysis of quality sensitiveness.

The quality, which is reflected in the average price paid for the commodity, and the quantity of the commodity are supposed to depend on the standard of living of the household; that is, on the per capita total expenditure of the household. The quality sensitiveness is measured by the quality elasticity which represents, in relative terms, the increase in the average price paid by the consumer as a result of a unit rise in his total expenditure.

An attempt has been made in this study to estimate the quality elasticities for some important groups of consumer items by using the available cross-section materials. The method of estimation followed in this paper is somewhat different from the usual regression analysis and is therefore briefly described in Secton 4.2. Also certain broad tests of consistency of the new approach have been discussed. The main results are presented in Section 4.3, while a few concluding remarks are made in the last section.

4.2. The method of concentration curves. As pointed out in the foregoing chapters, the graphical or the method of least squares becomes complicated in view of the fact that the available cross-section data are generally grouped in size classes of per capita total expenditure of households. What one might possibly get from published sources of consumption materials, as are required for the present types of problems, are some weighted arithmetic averages of consumption (both quantity and value, although the latter is more readily available) of different groups of consumer items and the average total expenditure per capita in each of the size classes. Percentage distribution of population in these classes is also provided apart from other demographic or other classificatory characteristics. The selection of sample households in these surveys is often complicated, involving as it does two or more stages; at each stage the sampling units are selected with varying probabilities.

This is in fact the case with the National Sample Survey. The National Sample Survey of India has been collecting valuable data relating to household expenditure since 1951. These materials are published in the form of grouped data, the groups being formed according to monthly per capita expenditure of the household. Twelve groups have been distinguished - Rs.0-8, 8-11, 11-13, 13-15, 15-18, 18-21, 21-24, 24-28, 28-34, 34-43, 43-55 and 55 and above, and for each group, say, i (i = 1, 2,..., 12) the following details are given:

- $\bar{V}_i$, average value of consumption of the specific commodity (Rs.);
- $\bar{Q}_i$, average quantity of consumption of the specific commodity (e.g. seers);
- $\bar{E}_i$, average total expenditure on all commodities (Rs.);
- w_i, proportion of population; and lastly
- $\bar{P}_i$, average price paid for the commodity, which is simply the ratio $\frac{\bar{V}_i}{\bar{Q}_i}$.

The model assumed here for the quality relationship is of the form

$$\psi_P (E) = E\,(P|E) = \text{Const } E^{P} \qquad (4.1)$$

where P is the average price paid for the commodity in question by the households spending a total expenditure E; ε_P is the quality elasticity of the specific item. In order to estimate ε_P in (4.1), the conventional method of regression has been applied to National Sample Survey data by some workers [2, 3]; but, it appears that this method is not quite appropriate.

The alternative method adopted here is fully discussed elsewhere* by Iyengar [5, 6] in the context of computing the Engel elasticities from grouped data of the type mentioned earlier. This method is based on the use of concentration curves. These curves show proportions of consumption (value or quantity) (vertical axis) against various proportions of population (horizontal axis) spending up to a given level of total expenditure per capita. Two types of concentration curves may be distinguished: one describing the proportions of aggregate total expenditure and the other describing the proportions of aggregate consumption of specific commodities in terms of proportions of population. These two types are respectively called the <u>Lorenz curve</u> and the <u>specific concentration curve</u> [5]. These two types of curves may be easily constructed from the available National Sample Survey data. The method of concentration curves, which makes use of these two curves is based, however, on the assumptions of constant elasticity as in (4.1) and log-normality of the distribution of total expenditure. The validity of the latter assumptions has been examined in a number of places [1, 5, 12].

* Also, see Chapter 2, pp.

For purposes of computing the quality elasticity for specific commodities the following additional assumptions are made about the Engel curves:

$$\psi_V(E) = E(V \mid E) = \text{Const. } E^{\varepsilon_V} \qquad (4.2)$$

$$\psi_Q(E) = E(Q \mid E) = \text{Const. } E^{\varepsilon_Q} \qquad (4.3)$$

Equations (3.2) and (4.3) which represent respectively the Engel curves for the value and quantity of consumption of the specific commodity, imply again constant elasticities. The parameters ε_V and ε_P are the value elasticity and the quantity elasticity of the specific commodity. It follows at once that the value elasticity is the sum of the quantity and price elasticities. This is quite generally true even in the absence of constant elasticity assumptions. That is,

$$\varepsilon_P = \varepsilon_V - \varepsilon_Q \qquad (4.4)$$

Thus, in order to get the quality elasticity the following curves are needed: (a) the Lorenz curve for the distribution of E, (b) the specific concentration curve for the distribution of $\psi_V(E)$, and (c) the specific concentration curve for the distribution of $\psi_Q(E)$; these will provide estimates of ε_V and ε_Q and hence an estimate of ε_P. The elasticity (for value or quantity) is derived as the ratio of the standard normal deviate of the ordinate of the specific concentration curve to that of the ordinate of the Lorenz curve corresponding to the median total expenditure. In order to be sure that this method gives consistent results one may draw the concentration curve for $(\mu)_P(E)$ and derive independently an estimate of ε_P which may be compared with the difference $\varepsilon_V - \varepsilon_Q$. This, in fact, has been done in Section 4.3.

Quite frequently, econometricians are interested in building up demand relationships of the form,

$$E\,(Q \mid E,\, P) = \text{Const.}\ E^{\varepsilon_{QE}}\ {}^{\varepsilon_{QP}} \qquad (4.5)$$

which relates the quantity of consumption of the specific commodity to its own price and the standard of living of the household as measured by the total expenditure [13]. The parameters ε_{QE} and ε_{QP} are the partial demand elasticities with respect to total expenditure and the average price paid for the commodity. The method of weighted regression has been used, to estimate these elasticities in spite of known limitations of that approach. The partial elasticities are related to the quantity and quality in a simple manner. Now, taking expectations on (4.5) with respect to P one gets

$$\underset{P}{E}\,E\,(Q \mid E,\, P) = E\,(Q \mid E) = \text{Const.}\ E^{\varepsilon_{QE} + \varepsilon_P\, \varepsilon_{QP}} \qquad (4.6)$$

Since (4.6) is to be same as $\bar{\psi}_Q(E)$ it follows :

$$\varepsilon_Q = \varepsilon_{QE} + \varepsilon_P \, \varepsilon_{QP} \tag{4.7}$$

The partial elasticities computed by using the least squares approach are examined in the light of the identity (4.7) in the following Section.

4.3. Some Empirical Results. In estimating the different elasticities mentioned in Section 4.2 use has been made of a set of data relating to household expenditure collected from about 3,000 sample households in rural and urban areas of India in the 10th round of the National Sample Survey (December 1955 - May 1956)*. Among other details these data provide for each expenditure class i (i = 1,2,..., 12), W_i, $\bar{E}_i$, $\bar{V}_i$ and $\bar{Q}_i$. The averages are in monthly per capita terms. In the present study the following composite items have been considered by way of illustration :

i) cereals including rice, wheat, jowar, bajra, maize, barley small millets, ragi, Bengal gram, and their products ;

ii) food grains including cereals and substitutes for cereals like tapioca, etc,

* For definition, scope and design of the survey the reader may consult the National Sample Survey Report No.47 on Consumer Expenditure issued by the Cabinet Secretariat, Government of India, 1961 [4].

iii) <u>sugar</u> including cane sugar, gur (cane and others), sugar candy and others ;

iv) <u>salt</u> including sea salt, rock and other salts ;

v) <u>milk</u> including cow milk, buffalo milk, goat milk and other forms of liquid milk.

Most of these data have been taken from published sources [8,9] except the quantity figures, the latter being extracted from the tabulation prints available at the Indian Statistical Institute. Using the above data, cumulative proportions are first computed as under :

$$W_i = \sum_{j=1}^{i} w_j$$

$$W_i^E = \frac{\sum_{j=1}^{i} w_j \bar{E}_j}{\sum_{j=1}^{12} w_j \bar{E}_j}$$

$$W_i^V = \frac{\sum_{j=1}^{i} w_j \bar{V}_j}{\sum_{j=1}^{12} w_j \bar{V}_j}$$

$$W_i^Q = \frac{\sum_{j=1}^{i} w_j \bar{Q}_j}{\sum_{j=1}^{12} w_j \bar{Q}_j}$$

$$W_i^P = \frac{\sum_{j=1}^{i} w_j \bar{P}_j}{\sum_{j=1}^{12} w_j \bar{P}_j}$$

$(i = 1,2,\ldots, 12)$

The points $[W_i, W_i^E]$ give the Lorenz curve of total expenditure while other sets of points $[W_i, W_i^V]$, $[W_i, W_i^Q]$ and $[W_i, W_i^P]$ give respectively the specific concentration curves for value, quantity and price of the commodity. What is needed for purposes of estimation of the elasticities is not the entire curve in each case but a single point on the curve which corresponds to the median value of E, that is, the ordinate of the curve that corresponds to $W = 0.5$ (middle of the horizontal axis). These ordinates may be approximately obtained by simple interpolation between consecutive values of W which include $W = 0.5$. If such ordinates are denoted by W^{E*}, W^{V*}, W^{Q*} and W^{P*}, and their standard normal deviates by $t(W^{E*})$, $t(W^{V*})$, $t(W^{Q*})$, and $t(W^{P*})$ respectively, then the value, quantity and price elasticities are given by

$$\varepsilon_V = \frac{t(W^{V*})}{t(W^{E*})} \; ; \quad \varepsilon_Q = \frac{t(W^{Q*})}{t(W^{E*})} \; ; \quad \varepsilon_P = \frac{t(W^{P*})}{t(W^{E*})} \qquad (4.9)$$

The following table gives the important stages of computation:

Table 4.1. Quantity, Value and Quality Elasticities for Selected Groups of Commodities

description or magnitudes		rural					urban					
		cereals	food-grains	sugar	salt	all expenditure	cereals	food-grains	sugar	salt	milk	all expenditure
(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1.	W^{E*}					0.2628						0.2570
2.	W^{V*}	0.3579	0.3587	0.2004	0.4259		0.4159	0.4179	0.2530	0.4174	0.1771	
3.	W^{Q*}	0.3698	0.3724	0.2255	0.4380		0.4676	0.4688	0.2939	0.4342	0.2160	
4.	W^{P*}	0.4823	0.4799	0.4657	0.4877		0.4471	0.4482	0.4590	0.4831	0.4449	
5.	$t(W^{E*})$					-0.6341						-0.6526
6.	$t(W^{V*})$	-0.3638	-0.3611	-0.8416	-0.1866		-0.2121	-0.2070	-0.6651	-0.2096	-0.9269	
7.	$t(W^{Q*})$	-0.3319	-0.3266	-0.7554	-0.1560		-0.0'03	-0.0753	|0.5417	-0.1662	-0.7858	
8.	$t(W^{P*})$	-0.0451	-0.0502	-0.0853	-0.0301		-0.1332	-0.1307	-0.1030	-0.0426	-0.1383	
9.	ε_V	0.5737	0.5695	1.3272	0.2943		0.3250	0.3172	1.0192	0.3212	1.4203	
10.	ε_Q	0.5234	0.5151	1.1813	0.2460		0.1230	0.1154	0.8301	0.2547	1.2041	
11.	ε_P	0.0711	0.0793	0.1345	0.0475		0.2041	0.2003	0.1578	0.0653	0.2119	
12.	(9)-(10)	0.0503	0.0544	0.1359	0.0483		0.2020	0.201'	0.1891	0.0665	0.2162	

The last two rows of Table 4.1 are roughly of the same magnitude. It would appear therefore that the method of concentration curves is reasonably consistent in the sense of equation (4.4). From these figures one may draw some broad conclusions. In the first place the quantity and value elasticities in rural areas are, for almost all items except salt, higher than the corresponding figures in urban areas. For example, sugar appears somewhat to be a luxury for the rural population and this is perhaps not true to the same extent in urban areas. The commodities sugar and milk are in the nature of luxuries with their elasticities exceeding unity and the rest belong to the necessary group.

The last two rows, which are approximately of the same order, exhibit positive signs for quality elasticities for all the commodities irrespective of the regional differences. This would suggest that the consumers generally tend to pay higher prices for ostensibly similar items, that is, to move for better qualities within the commodity group as their standard of living improves. The degree of quality consciousness seems to be generally higher in urban areas. Moreover, the relative luxuries such as sugar or milk are associated with higher values of quality elasticity. The present analysis seems to confirm the general notion that in rural areas of India the consumption habits are relatively more rigid than in urban parts.

The partial elasticities ε_{QE} and ε_{QP} for each of the selected items are estimated by the method of weighted least squares by using the National Sample Survey materials and these are given in Table 4.2. Estimates of the quantity and quality elasticities, obtained by the method of concentration curves, are also given so that it is easy to verify the relationship (4.7) which connects the quantity and quality elasticities with the partial elasticities.

Table 4.2

description of the parameters	rural				urban				
	cereals	food-grains	sugar	salt	cereals	food-grains	sugar	salt	milk
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
ε_{QE}	0.5721	0.5826	1.3309	0.2540	0.2341	0.1893	1.2322	0.2443	1.6548
ε_{QP}	-0.7457	-0.8706	-0.5263	-0.7903	-0.5242	-0.3325	1.9702	-0.1140	-1.8807
ε_{P}	0.0711	0.0793	0.1345	0.0475	0.2041	0.2003	0.1578	0.0653	0.2119
$\varepsilon_{QE}+\varepsilon_{P}\,\varepsilon_{QP}$	0.5191	0.5126	1.2601	0.2165	0.1271	0.1227	0.9213	0.2369	1.2553
ε_{Q}	0.5234	0.5151	1.1913	0.2460	0.1230	0.1154	0.8301	0.2547	1.2041

The last two rows are approximately of the same order suggesting thereby that the method of concentration curves is failly consistent. It would also appear from the first two rows that the changes in physical demand are mostly explained by changes in the level of living, i.e. per capita total expenditure, whereas quality considerations are of minor importance. The negative sign of ε_{QP} probably indicates that households within a given per capita expenditure group would prefer to buy smaller quantities of goods when they are somewhat costly. This is in line with the interpretation of partial price elasticity of demand computed from time series data. The high values of partial price elasticities for sugar and milk in urban areas suggest that these commodities have relatively high degree of substitutability. There is also some suggestion that the rural population generally prefers coarser varieties of food grains at low prices.

4.4. Concluding Remarks: A few remarks may perhaps be made by way of conclusion. The method of concentration curves has been found quite useful for analysis of grouped size distribution data like those of the National Sample Survey for which some of the underlying assumptions have been shown to be plausible. This technique is consistent* and probably simpler than the weighted least squares method, especially when concentration curves have already been drawn for other reasons or when one has fractile-type data [10].

* Consistency and other statistical properties of our estimates are discussed in a recent paper [6]. See also Chapter 2.

The conclusions drawn from the above studies are rather preliminary in nature but are nonetheless encouraging in so far as they broadly substantiate some of the commonly held notions about the Indian consumption behaviour.

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Chapter 5

ON A PROBLEM OF ESTIMATING INCREASE IN CONSUMER DEMAND[1]

5.1. This chapter is concerned with a study of the effect of changes in concentration of income of individuals on the projection of future levels of consumption of both necessaries and luxuries. It is shown that, for example, if one does not take into account the changes in the income distribution in the direction of egalitarianism one is likely to underestimate the need for necessary goods and services. This and a few other results are derived under the assumptions of log-normality of the income distribution and the constancy of the Engel elasticity for all incomes. These results are also extended to the semi-log case and are numerically illustrated from the Indian National Sample Survey data.

Under a planned programme of governmental investment the real income of individuals is normally expected to rise. The rise in real income will, in turn, lead to increase in demand for consumer goods and services. Unless the productive forces in the country are mobilised to meet the increase in demand, even partially, prices may soar up and utilimately lead to serious inflation - a situation which is undesirable. For this and for other reasons estimates of expected increase in consumption of different items are quite useful in planning.

[1] A part of this Chapter was read at the Preliminary Conference of the Indian Members of the Econometric Society held at the Indian Statistical Institute, Calcutta, in January 1960 and subsequently published in Sankhyā [4].

If the pattern of expenditure by individuals at different levels of income is known and the assumption is made that when an individual moves on to a higher income group, his pattern of expenditure would tend to be the same, on an average, as that of an individual who already belongs to that higher income group, it is possible to estimate the expected increase in demand for any given change in the distribution of income.

In India, however, complete data on incomes are not available and consequently we are tempted to use data on overall consumer expenditure as proxy for income. This is to a considerable extent justified since in India the average per capita saving is negligible. There exists a wealth of data on consumer expenditure since the emergence of the National Sample Survey (NSS) in 1950-51.

A general formulation of this problem was given by Roy and Laha [11] who assumed constant elasticity of the Engel curve and Pareto's form for the expenditure distribution in estimating the relative increase in demand of certain goods using data collected from the fourth round of the NSS. Later, Roy and Dhar [10] tried, in addition to constant elasticity curves, several of Tornquist's forms for Engel curves and verified the log-normal hypothesis for the expenditure distribution; the relevant parameters were estimated from the seventh round materials of the National Sample Survey. They derived formulae for estimating the increase in demand due to infinitesimal changes in the log-normal parameters but did not derive any actual estimates. A method of computing Engel elasticities from concentration curves was derived by Iyengar [5] under the assumption of log-normality of the distribution of expenditure.

In this chapter we assume log-normality of the expenditure distribution and constant elasticity of the Engel curves, and present a formula for obtaining numerical estimates of the expected increase in demand for different items of consumer expenditure under a particular set of specifications about the changes desired in the expenditure distribution resulting from governmental investment plans. A similar expression is also derived under the semi-log hypothesis. The estimates of relevant parameters are taken for five important items from the National Sample Survey, tenth round (December 1955-May 1956).

5.2. Formulation of the general problem: If there exists a set of known relationships between overall expenditure per person and average expenditure per person on different items of the family budget then it may be easy to calculate the expected increase in expenditure on any particular item provided one can assume that, for any postulated distribution of the overall expenditure, such a relationship is invariant and the expenditure distribution remains unaltered in its form although undergoing some changes in its essential parameters. The analysis given below applies equally well to estimate demand in quantitative terms.

Denoting by x the overall expenditure per capita and by y the expenditure per capita on a particular item let us write the marginal distribution of x by $g(x)$ and the conditional expectation of y for a given value of x by $E(y \mid x) = \psi(x)$. Now suppose that the plan investment alters the marginal distribution of x from $g(x)$ to $g^*(x)$. Then under the assumption that the function ψ remains invariant, the percentage increase in the average expenditure per capita on the given item is given by

$$I = 100 \left[\frac{E(\psi \mid g^*)}{E(\psi \mid g)} - 1 \right] \qquad (5.1)$$

Here

$$E(\psi \mid g) = \int_0^\infty \psi(x) g(x) dx \qquad (5.2)$$

is the average expenditure per capita on the particular item when the overall expenditure distribution is $g(x)$. Similarly $E(\psi \mid g^*)$ is defined.

We shall set down that the autonomous plan investment will lead to 100α percent increase in the overall expenditure per capita of the community while at the same time the inequality of the overall expenditure distribution diminishes by 100β percent, the inequality being measured by the Lorenz ratio. Under these specifications it may be possible to calculate the new parameters in g^* in terms of the parameters of g and the given proportions α and β, and to evaluate I.

5.3. <u>The prediction formula</u>. Suppose first we adopt a constant elasticity hypothesis regarding the Engel curve. That is, we assume,

$$\psi(x) = \Lambda x^{\varepsilon} \tag{5.3}$$

where ε is the expenditure elasticity of the specific item under consideration. Let us further suppose that the expenditure distribution can be adequately represented by a log-normal distribution with parameters θ and λ (say) and it changes in such a manner that it remains log-normal but with different parameters θ^* and λ^*. In other words,

$$g(x) = \begin{cases} \dfrac{1}{x\lambda\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log x - \theta}{\lambda}\right)^2} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases} \tag{5.4}$$

This distribution has the mean, $\mu = E(x) = e^{\theta + \frac{1}{2}\lambda^2}$, and the Lorenz concentration ratio, $L = 2\phi\left(\frac{\lambda}{\sqrt{2}}\right) - 1$, [1] where ϕ is given by

$$\phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du. \tag{5.5}$$

Under these assumptions the expression (5.1) of the last section simplifies to

$$I = 100\ [e^{(\theta^* - \theta)\varepsilon + \frac{1}{2}(\lambda^{*2} - \lambda^2)\varepsilon^2} - 1]. \qquad (5.6)$$

This result can be established on observing that for the log-normal variable the powers and multiples are also log-normal, that is, $\psi(x) = \Lambda x^{\varepsilon}$ is also log-normal with the mean

$$E(\psi \mid g) = \Lambda e^{\varepsilon\theta + \frac{1}{2}\varepsilon^2\lambda^2} \qquad (5.7)$$

The expression in (5.6) involves two unknown parameters θ^* and λ^*. However, under the specifications in the last paragraph of Section 5.2 we have the following two relations giving θ^* and λ^* in terms of the original parameters and the proposed values of α and β.

$$e^{\theta^* + \frac{1}{2}\lambda^{*2}} = (1+\alpha)e^{\theta + \frac{1}{2}\lambda^2} \qquad (5.8)$$

$$L^* = (1-\beta)L \qquad (5.9)$$

The new parameter λ^* can be readily computed from the relation,

$$\lambda^* = \sqrt{2}\ t_{\frac{1}{2}(1+L^*)} \qquad (5.10)$$

since L^* is known as soon as β and L are known. Here t is given by the equation,

$$\phi(t_k) = k, \quad 0 \leq k \leq 1 \tag{5.11}$$

where ϕ is defined in (5.5) above. Using the result of (5.8) we can simplify (5.6)

$$I_{\alpha,\beta}(\varepsilon) = 100\,[C(\beta, \varepsilon)(1 + \alpha)^{\varepsilon} - 1] \tag{5.12}$$

where

$$C(\beta, \varepsilon) = e^{\frac{1}{2}\varepsilon(1 - \varepsilon)(\lambda^2 - \lambda^{*2})} \tag{5.13}$$

is ultimately a function of β and ε since λ^* involves β when the initial concentration is known.

As a particular case if we assume that the expenditure distribution has not changed apart from a proportionate increase in the per capita total expenditure, i.e., if $\beta = 0$, it follows from (5.13) that the C-coefficients reduce to unity irrespective of whether the commodity is a luxury item or a necessary item. In this case formula (5.12) becomes

$$I_{\alpha,o}(\varepsilon) = 100\,[(1 + \alpha)^{\varepsilon} - 1], \tag{5.14}$$

a form which does not require that the distribution of total expenditure be log-normal. This form is frequently used in applied work [8, 11]. It is easy to verify the following inequalities:

For $\beta > 0$,

$$I_{\alpha,0}(\varepsilon) < I_{\alpha,\beta}(\varepsilon) \text{ if } 0 < \varepsilon < 1$$

$$I_{\alpha,0}(\varepsilon) > I_{\alpha,\beta}(\varepsilon) \text{ if } \varepsilon > 1. \tag{5.15}$$

These inequalities as well as those which appear subsequently are easily proved by observing that by definition $C(\beta, \varepsilon)$, for $\beta > 0$, exceeds unity if $0 < \varepsilon \leq 1$ while it falls short of unity if $\varepsilon > 1$ (see Table 5.2).

On the other hand if one were to assume that the average income (total expenditure) is more or less stable through time and that the income distribution has slightly changed towards egalitarianism, one gets a positive increase in demand for necessary goods and a fall in demand for luxuries. In symbols,

$$I_{0,\beta}(\varepsilon) > 0 \text{ if } \beta > 0,\ 0 < \varepsilon < 1$$

$$I_{0,\beta}(\varepsilon) < 0 \text{ if } \beta > 0,\ \varepsilon > 1 \tag{5.16}$$

It is also interesting to note that

$$I_{0,\beta}(\varepsilon) < I_{\alpha,\beta}(\varepsilon) \text{ if } \alpha > 0 \tag{5.17}$$

irrespective of the nature of the commodity. This result holds even if the income distribution has tended to be more concentrated. The decrease in demand for luxuries varies with the degree to which they can be considered as luxuries. In other words, if s and s' are two items of luxury, i.e., $\varepsilon_s, \varepsilon_{s'} > 1$, then

$$I_{0,\beta}(\varepsilon_{s'}) < I_{0,\beta}(\varepsilon_s) \quad \text{if} \quad \varepsilon_{s'} > \varepsilon_s > 1. \qquad (5.18)$$

In the same manner we may obtain, for all necessary goods s and s' for which $0 < \varepsilon_s, \varepsilon_{s'} < 1$ the following:

$$I_{0,\beta}(\varepsilon_s) > I_{0,\beta}(\varepsilon_{s'}) \quad \text{if} \quad \varepsilon_s < \varepsilon_{s'} \qquad (5.19)$$

If, however, the inequality of the income distribution has increased as a result of planning (This is not generally welcomed by planners but nevertheless the possibility of such an occurrence cannot be ruled out), i.e., $\beta < 0$, the above conclusions will be reversed in most of the above inequalities. In that case we will have the following general inequalities:

$$I_{\alpha,-\beta}(\varepsilon) < I_{\alpha,0}(\varepsilon) < I_{\alpha,\beta}(\varepsilon) \quad \text{for all} \quad 0 < \varepsilon < 1$$
$$I_{\alpha,-\beta}(\varepsilon) > I_{\alpha,0}(\varepsilon) > I_{\alpha,\beta}(\varepsilon) \quad \text{for all} \quad \varepsilon > 1. \qquad (5.20)$$

If the changes in the concentration of income distribution were not taken into account and the assumption be made that the income of persons would increase by the same proportion and that the income elasticity is constant throughout the range of incomes we obtain a different formula which is used by some practical workers and which is approximate to (5.14). We shall distinguish this estimate and write I^* where I^* is given by

$$I^*_{\alpha,0}(\varepsilon) = 100\,\alpha\,\varepsilon \qquad (5.21)$$

Assuming that α is positive it is easy to obtain the following results [2]:

$$\begin{aligned} I_{\alpha,0}(\varepsilon) &< I^*_{\alpha,0}(\varepsilon) \quad \text{if } 0 < \varepsilon < 1 \\ I_{\alpha,0}(\varepsilon) &> I^*_{\alpha,0}(\varepsilon) \quad \text{if } \varepsilon > 1 \end{aligned} \qquad (5.22)$$

the equalities, however, holding when $\varepsilon = 1$.

It will be seen below from Figure 5.1 presented in Section 5.4 by way of illustration from the National Sample Survey, that although the estimates $I^*_{\alpha,0}$ deviate from $I_{\alpha,0}$ in the correct direction, i.e., towards $I_{\alpha,\beta}$, where β is positive, these $I^*_{\alpha,0}$ values cannot be taken as substitutes for $I_{\alpha,\beta}$.

5.4. The Semi-log Case: Engel curves leading to variable income elasticities are perhaps more realistic and are of great interest. Those are indeed suggested by the asymmetry of the specific concentration curve. The semi-log case falls in this category. Stated in symbols, the semi-log hypothesis takes the form[3]

$$\psi(x) = a + b \log x \tag{5.23}$$

where a, b are behavioural parameters to be estimated along with the distributional parameters θ, λ. The problem of estimation of these parameters is treated elsewhere [6 , see also Ch. 2].

The mean specific expenditure is given by $(a + b\theta)$ and, therefore, from (5.1), the expected change (I) in the average demand for the specific commodity is, in percentage terms,

$$I = 100\, \varepsilon_C (\theta^* - \theta). \tag{5.24}$$

Here ε_C represents the income elasticity of demand computed at the 'median' income, $C = e^{\theta}$. The 'median' elasticity may be easily shown to be [6, 7]

$$\varepsilon_C = \frac{b}{a + b\theta} \tag{5.25}$$

---

3 This form obviously restricts the values of x to the range $0 < \exp(-a/b) < x < \infty$. Hence, in the derivation of Lorenz curve as well as the specific concentration curve, the integration will have to be performed over the income range $x > \exp(-a/b)$. But since the proportion of incomes below the 'threshold' level is usually small and the concentration curves for most necessaries seem to rise above the horizontal axis right from the origin, the effect of ignoring the truncation may not be serious [2].

Now, in terms of the initial parameters and the stipulated rates of change in their values stated in (5.8)-(5.9), we may rewrite I in the form

$$I_{\alpha,\beta}(\varepsilon_C) = 100 \quad \varepsilon_C[\log (1 + \alpha) - \tfrac{1}{2}(\lambda^{*2} - \lambda^2)] \qquad (5.26)$$

The two special cases that are of interest are (i) when $\beta = 0$ and $\alpha > 0$, i.e., when the income inequality remains unchanged but the average income level rises, and (ii) when $\alpha = 0$ and $\beta > 0$, i.e., when there is a reduction in the inequality but no change in the average income level. In either case the demand tends to increase.

The semi-log hypothesis covers most of the consumer goods whose income elasticity is confined approximately to the range $(0, 1.8/\lambda)$[6]. For all necessary goods and services, the demand tends to increase following a reduction in the inequality of income distribution. Whether or not there is a rise in the average income level, a mere redistribution of personal income of households gives rise to additional demand for essential consumer goods. The same conclusions were reached in the last section under the double-log hypothesis. We shall therefore consider a numerical example from the National Sample Survey data to illustrate the double-log case.

5.5. <u>Some illustrative examples from National Sample Survey data</u>: We shall choose for purposes of illustration the following five important items of consumer expenditure which include necessaries as well as relative luxuries: (a) foodgrains, (b) all food, (c) clothing, (d) milk and milk products and (e) medicine and medical services. Their expenditure elasticities and specific concentration ratios are given in the following table.

Table 5.1: Estimates of engel elasticities and specific concentration ratios.

item (s)	Engel elasticity (ε_s)	specific concentration ratio (L_s)
foodgrains	0.58 (necessary)	0.21
all food	0.76 "	0.28
clothing	1.55 (luxury)	0.53
milk and milk products	1.67 "	0.57
medicine and medical services	1.95 "	0.64

These concentration ratios and elasticities were computed from the National Sample Survey Data on consumption collected in the tenth round covering the rural and urban areas of India and roughly correspond to the period Deceomber 1955-May 1956 and the method of computation of the Engel elasticities and of the concentration ratios is given in detail elsewhere [5,6]. The figures are to be taken only as illustrative and not as the NSS estimates. Our computations involved the use of a concentration map consisting of specific concentration curves and the Lorenz curve which together describe the average consumption pattern of the community. Lorenz curve is the one which relates the proportion of total expenditure to the proportion of persons spending up to a given level of total expenditure per capita while the specific concentration curve is defined as showing the proportions of total consumption (value) of the specific commodity against proportions of persons spending up to a given level of total expenditure per capita. The specific concentration ratios are defined, in analogy with the Lorenz ratio, as twice the areas bound by the specific concentration curves and the egalitarian line, and these ratios may be used in characterizing commodities according as whether they are necessaries or luxuries. Under the assumptions stated in Section 5.3, the specific concentration ratio L_s is given by $2\,\phi\left(\frac{\lambda\,\varepsilon_s}{\sqrt{2}}\right) - 1$ where ε_s is the Engel elasticity of the specific commodity s. The Lorenz ratio L has been defined earlier and equals $2\,\phi\left(\frac{\lambda}{\sqrt{2}}\right) - 1$. The item s is a necessary if $L_s < L$, and a luxury if $L_s > L$; in the former case, the specific concentration curve lies entirely above the Lorenz curve, and below in the latter case [9]. For instance, the first two items of Table 5.1 above are necessaries for which the concentration ratios are less than 0.36 and the last three items are luxuries since their concentration ratios exceed 0.36.

The above ratios may be empirically computed without making any assumptions regarding the distributions. Thus, if p_i be the proportion of population whose total expenditure per capita does not exceed x_i, the latter being the upper value of the i-th total expenditure per capita class, q_i and Q_i be the corresponding proportions of total expenditure and specific commodity consumption respectively by the above stratum of population, ($i = 1, 2, \ldots, m$, say) then the specific concentration ratio is approximately given by

$$L_s = 1 - \sum_{i=1}^{m} (p_i - p_{i-1})(Q_i + Q_{i-1}) \qquad (5.27)$$

where $(p_o, Q_o) = (0, 0)$ and $(p_m, Q_m) = (1, 1)$. The Lorenz ratio L is computed in the same manner using q_i in the place of Q_i. If we now make the assumptions of Section 5.3 we may define $k = \frac{1}{2}(1 + L)$ and $k_s = \frac{1}{2}(1 + L_s)$. Then $\frac{\lambda}{\sqrt{2}}$ is given by the standard normal abscissa corresponding to k, regarded as an incomplete probability integral, and the Engel elasticity ε_s of the specific commodity s is given by dividing the standard normal abscissa corresponding to k_s by $\frac{\lambda}{\sqrt{2}}$. This method is formally different from the one proposed elsewhere [5] and the latter has been used in this paper although the former method appears to be intuitively better.

The several assumptions made in Section 5.3 have approximately been shown elsewhere to be empirically true. (See [3, 5, 10]). In fact, it has been found from an analysis of data on consumer expenditure in rural and urban areas of India pertaining to the period December 1955 - May 1956 that the expenditure distribution can well be described by a log-normal hypothesis with a relative standard deviation (λ) of 0.66 which gives a Lorenz ratio of 36 percent. The constant elasticity assumptions have also been found to be realistic. This distribution is diagramatically represented in Figure 5.2(Appendix) corresponding to $\beta = 0$. The changes in the distribution have been indicated by shifts in the Lorenz curve brought about by assumed changes in the level of inequality of incomes (total expenditures). Six different alternatives are considered as the likely increase in the average expenditure per capita and five distinct values are assumed for the changes in the concentration of income distribution. The constants associated with the expression for the percentage increase in demand have been set out in Table 2.

Table 5.2. Values of $C(\beta, \varepsilon)$ for different values of β and ε.

item	elasticity	proportionate reduction in concentration ratio (β)				
		- 5 %	5 %	10 %	15 %	20 %
food-grains	0.58	0.9938	1.0053	1.0106	1.0154	1.0201
all food	0.76	0.9954	1.0040	1.0079	1.0116	1.0150
clothing	1.55	1.0217	0.9816	0.9639	0.9477	0.9329
milk	1.67	1.0286	0.9816	0.9639	0.9477	0.9329
health	1.95	1.0479	0.9604	0.9250	0.8898	0.8598

Using Table 5.2 we construct Table 5.3 giving percentage increase (or decrease) in demand per capita for the five items under different sets of specifications regarding changes in the average expenditure per capita and concentration.

Table 5.3. Percentage increase (or decrease) in demand per capita.

item	percentage increase in per capita total expenditure (100α)	percentage decrease in Lorenz ratio (100β)					
		-5	0	5	10	15	20
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
foodgrains	0	- 0.6	0.0	0.5	1.1	1.5	2.0
	5	2.2	2.9	3.4	4.0	4.5	4.9
	10	5.0	5.7	6.2	6.8	7.3	7.8
	15	7.8	8.4	9.0	9.6	10.1	10.6
	20	10.5	11.2	11.7	12.3	12.9	13.4
	25	13.1	13.8	14.4	15.0	15.6	16.1
all food	0	- 0.5	0.0	0.4	0.8	1.2	1.5
	5	3.3	3.8	4.2	4.6	5.0	5.3
	10	7.0	7.5	7.9	8.4	8.8	9.1
	15	10.7	11.2	11.6	12.1	12.5	12.9
	20	14.3	14.9	15.3	15.8	16.2	16.6
	25	17.9	18.4	18.9	19.4	19.9	20.3
clothing	0	2.2	0.0	- 1.8	- 3.6	- 5.2	- 6.7
	5	10.2	7.9	5.9	4.0	2.2	0.6
	10	18.4	15.9	13.8	11.7	9.9	8.1
	15	26.9	24.2	21.9	19.7	17.7	15.9
	20	35.5	32.7	30.2	27.9	25.7	23.8
	25	44.4	41.3	38.7	36.2	33.9	31.8

Table 5.3. (contd.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
milk and milk products	0	2.9	0.0	- 2.4	- 4.7	- 7.2	- 8.7
	5	11.6	8.5	5.9	3.4	0.7	- 1.0
	10	20.6	17.3	14.4	11.7	8.8	7.0
	15	29.8	26.2	23.1	20.2	17.1	15.2
	20	39.5	35.6	32.2	29.2	25.8	23.8
	25	49.3	45.2	41.6	38.3	34.7	32.5
medicine and medical services	0	4.8	0.0	- 4.0	- 7.5	-11.0	-14.0
	5	15.3	10.0	5.6	1.7	- 2.1	- 5.4
	10	26.2	20.4	15.7	11.4	7.2	3.5
	15	37.6	31.3	26.1	21.5	16.9	12.9
	20	49.5	42.7	37.1	32.0	27.0	22.7
	25	61.9	54.5	48.4	42.9	37.5	32.8

The interpretation of Table 5.3 is quite clear. For instance, assuming that the average income (total expenditure) per capita increase by 15 percent and the concentration of incomes becomes lower by 5 percent, the increase in demand per person for foodgrains, clothing and medicine are 9.0, 21.9 and 26.1 percent respectively. If one did not consider the effect of changes in the Lorenz ratio one would get 8.4, 24.2 and 31.3 percent for the same items. In the latter case, the demand for clothing and medicine is to some extent exaggerated (by 3 and 7 percent respectively) while the demand for foodgrains is slightly underestimated (to the extent of 0.6 percent). It is shown in Section 5.3 that, in general, if we do not take into consideration the changes (towards egalitarianism) in the distribution of incomes (total expenditures), the demand for necessary items would be underestimated and that for luxuries would be somewhat overestimated.

If we do not consider the possibilities of a rise in the average level of total per capita expenditure, even then it is possible that the demand for necessaries augments under a more even distribution of incomes. For example, in the case of consumption of food the demand increases by 1.5 percent if the level of concentration is brought down by 20 percent (through taxation or appropriate policies of the Government). However, in this situation the demand for luxuries including clothing, milk and milk products and medicine and medical services, would shrink to a far greater extent than would the demand for necessaries increase. Medicine and medical services, being the most luxurious item among the ones we have considered in this paper, will suffer a loss of demand to the extent of 14 percent if the inequality of total expenditure were to diminish by 20 percent.

The following table illustrates the difference between estimates of increase in demand obtained by using formulae (5.14) and (5.21) of Section 5.3. For the sake of simplicity, we shall assume $\beta = 0$. The

The estimates $I_{\alpha,0}(\varepsilon)$ are given outside the brackets, the $I^*_{\alpha,0}(\varepsilon)$ figures being inside.

Table 5.4: $I_{\alpha,0}(\varepsilon)$ & $I^*_{\alpha,0}(\varepsilon)$ compared

item	100_α : percentage increase in overall expenditure per capita				
	5	10	15	20	25
(1)	(2)	(3)	(4)	(5)	(6)
foodgrains	2.9(2.9)	5.7(5.8)	8.4(8.7)	11.2(11.6)	13.8(14.5)
food total	3.8(3.8)	7.5(7.6)	11.2(11.4)	14.9(15.2)	18.4(19.0)
clothing	7.9(7.8)	15.9(15.5)	24.2(23.2)	32.7(31.0)	41.3(38.8)
milk and milk products	8.5(8.4)	17.3(16.7)	26.2(25.0)	35.6(38.4)	45.2(41.8)
medicine and medical services	10.0(9.8)	20.4(19.5)	31.3(19.5)	42.7(39.0)	54.5(48.8)

For necessary goods the difference between these estimates is not serious for the lower values of α while the gap increases with the relative increase in the per capita income (total expenditure). A similar remark may be made regarding the relatively luxurious items, with the difference that the gaps in these cases are wider.

For purposes of interpretation we shall produce a few interesting graphs in Figure 5.1 below corresponding to $\beta = 0$, $\beta = -0.05$ and β=0.10 for just two items, one from the necessary group and another from the luxury group. These graphs show the percentage increase in demand per capita on the vertical axis and the percentage increase in income (total expenditure) per capita on the horizontal axis.

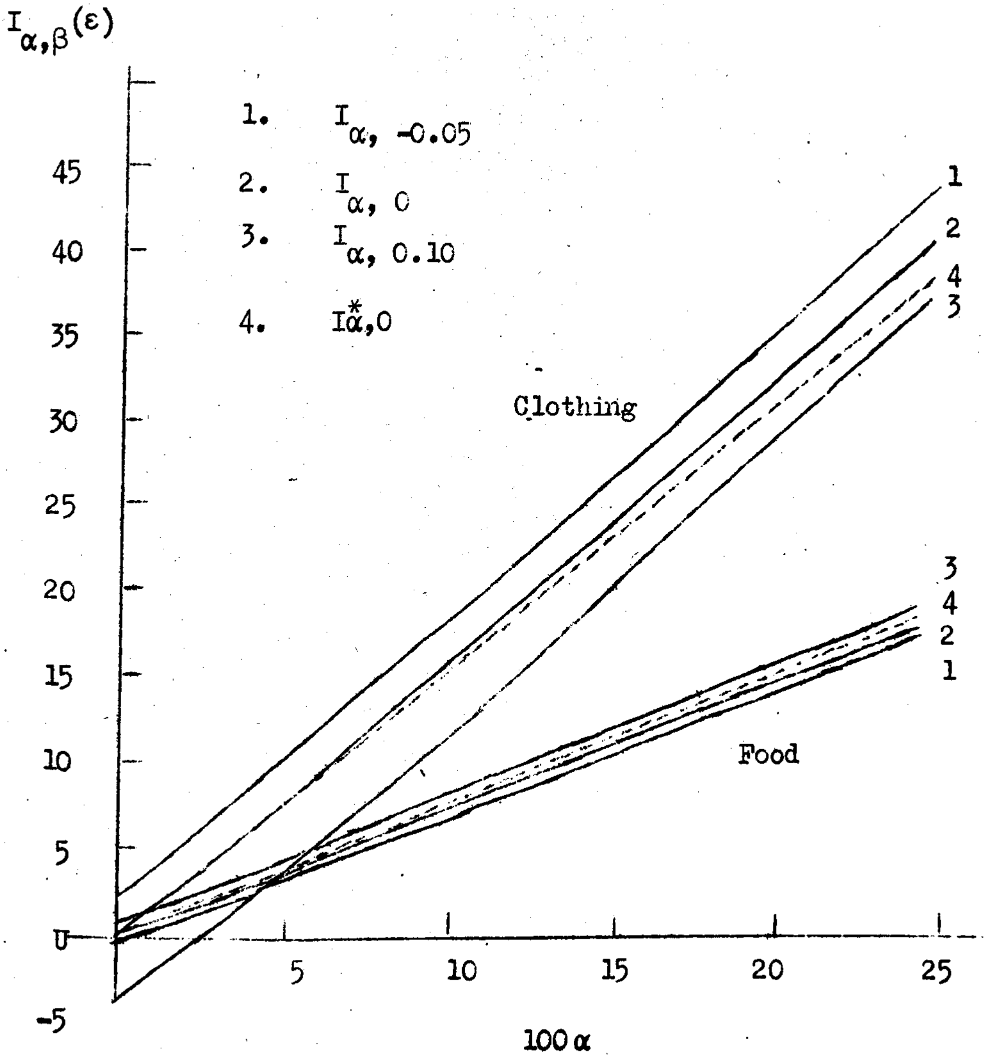

Figure 5.1. Percentage increase in demand per capita.

Appendix 5.1

In Section 5.3 the initial value of L is taken to be 0.36 which, when substituted in the relation, $L = 2\,\phi\left(\frac{\lambda}{\sqrt{2}}\right) - 1$, leads to $\lambda = 0.66$. The new values of the Lorenz ratios are computed from $L^* = (1 - \beta)L$. From these the λ^*'s are computed for $\beta = -0.05$, $\beta = 0$ and $\beta = 0.10$ and are presented in Table A.5.1 below.

Table A.5.1. Values of λ^* for different values of β

$100\,\beta$	L^*	λ^*
-5	0.378	0.6972
0	0.360	0.6600
10	0.324	0.5910

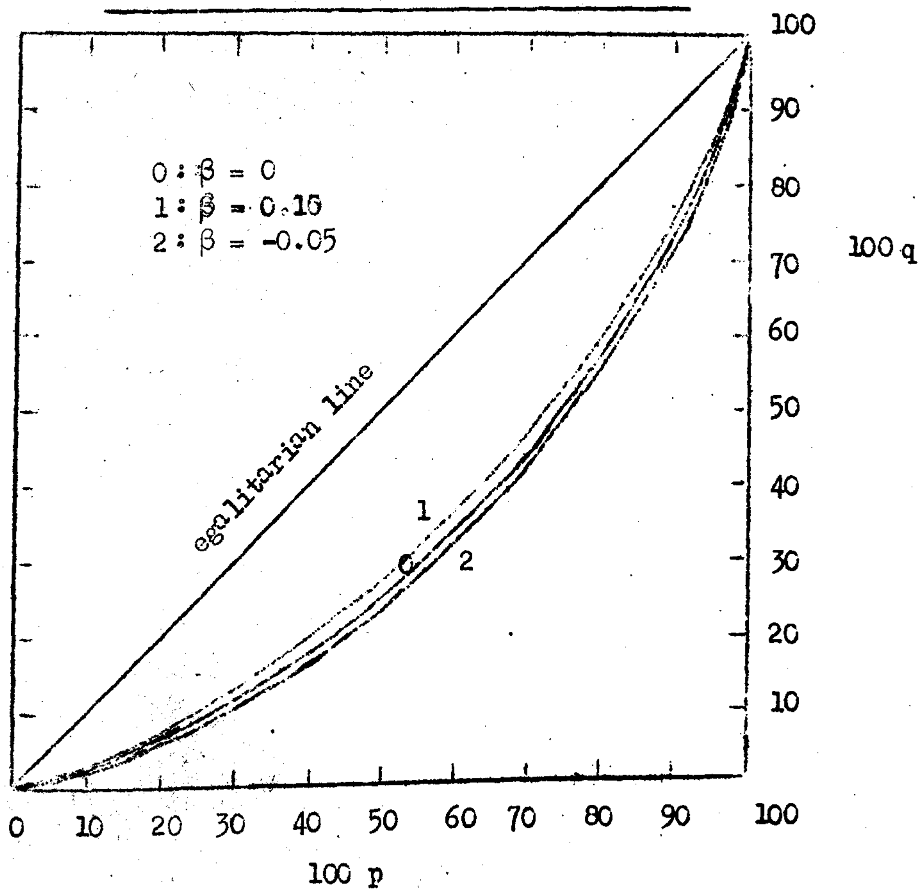

Figure A5.1. Lorenz curves of total expenditure.

Denoting by p the proportion of persons whose overall expenditure does not exceed a given amount, and by q the proportion of the total expenditure (of the community) spent by the these persons, the Lorenz curve is drawn by taking p on the horizontal axis and q on the vertical axis. However, when the distribution of overall expenditures is known a priori to be log-normal the equation to the Lorenz curve may be written as

$$t_q = t_p - \lambda^* \qquad (A.5.1)$$

where λ^* is the standard deviation of the logarithms of overall expenditures; t is, of course, defined in (5.11) of the text. The Lorenz curves of total expenditure are drawn in Figure A.5.1 corresponding to the three distinct values of λ^* given in Table A.5.1.

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Chapter 6

INTERTEMPORAL AND INTERREGIONAL VARIATION IN EXPENDITURE INEQUALITY IN INDIA*

6.1 In the foregoing Chapters we merely assumed lognormality for the expenditure distribution and derived appropriate techniques for estimating the Engel curve. Also, a few practical applications were considered to illustrate those techniques from the available cross-section data. In this Chapter, a systematic survey of Indian data on consumer expenditure distribution is attempted in the context of an intensive inter-temporal and inter-regional study of the expenditure distributions based on National Sample Survey (NSS) data.

The National Sample Survey (NSS)[1] has been collecting data on household budgets from sample households from the very first round. From the second round onwards, we have estimates for the distribution of population in a fixed set of twelve classes of monthly per capita total consumer expenditure. One is naturally tempted to compare these distributions and to study the changes, if any, which have taken place over the NSS rounds, i.e., over time. Such investigations could be of considerable importance. For instance, inter-temporal comparisons of the means between the rounds (of monthly per capita total consumer

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* The findings of the present study were reported first at a Research Seminar held in the Indian Statistical Institute in June 1960.

1. Some background information about the National Sample Survey (NSS) is given in Section 6.2.

expenditure[2]) might indicate changes in the average level of living. Such comparisons are, however, rather difficult, mainly because all expenditure figures are inevitably at current prices prevailing in the respective rounds; the lack of satisfactory cost of living indices of adequate coverage prove to be a serious handicap for such investigations. An attempt, however, has been made in Chapter 8 to compile a new series of differential price deflators, entirely from the National Sample Survey, at least for one region, viz., rural West Bengal involving two years, and to study the real changes in the levels of living over a period of five years.

For the present we leave aside the problem of comparing the means of consumer expenditure (after deflation) between different rounds, inspite of the obvious importance of that problem. What is attempted here is to compare the distributions of consumer expenditure (x) observed in different rounds in respect of what is commonly, but vaguely, called inequality[3]. There are various indices for measuring inequality of size distributions; and this investigation naturally started by calculating some of these measures for the NSS expenditure distributions. Use was made of the well-known Lorenz ratio[4], and of a

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2 For brevity, this variate, defined over the population of persons, will be subsequently referred to as consumer expenditure or, simply, as expenditure, and denoted by the symbol x.

3 This comparison is relatively un-affected by the variation in prices between rounds. (See, however, Section 6.7).

4 This is also sometimes called the Gini coefficient of concentration.

few other indicators of concentration, i.e., of inequality. All these measures have wide applicability in the measurement of inequality, and taken together, they should suffice for most quantitative studies on inequality. Theoretically, however, these indices are not completely satisfactory unless some assumptions can be made regarding the distributions of expenditure (x).

In view of this limitation inherent in all measures of inequality, and considering the importance of the expenditure distributions under investigation, an attempt was made to adopt a more general approach to the problem. It is perhaps reasonable to lay down, although it is not axiomatic, that any measure of inequality should be, like the coefficient of vatiation, concerned with the distribution of x relative to the median (C) or the mean (μ) or any other typical value t of x, that is to say, in symbols, with the distribution of x/C or x/μ or x/t. This principle is recognised by all the known measures of inequality, and by the Lorenz curve, which is, in fact, the distribution of x/μ except for some transformations. For obvious reasons, the distribution of x/μ underlies most of these techniques, in preference to the distributions of x/C etc. Given the distribution of x/μ, various indices can be constructed for measuring inequality, but none of these measures can convey the whole information regarding the inequality of the distribution of x/μ. The concept of inequality being inherently vague, like the concept of location or dispersion of frequency distributions, any measure of concentration will necessarily be somewhat arbitrary.

An attempt was therefore made to compare graphically the distributions of x/μ for different rounds, in their entirety, using, of course, the consistent estimate $\bar{x}$ in the place of μ. These distributions (to be subsequently referred to as the "adjusted" distributions) give a complete picture of inequality; and while, in general, it may be difficult to compare the inequalities of two distributions of x (or, equivalently, of x/μ) the problem would hardly arise if the corresponding distributions of x/μ be found to be in close agreement[5], in which case the two distributions of x/C or x/t would also be in close agreement.

An alternative approach was made by observing that the different measures of inequality assume special significance, and their limitations disappear, when the distribution of x/μ is uniparametric with changes in that parameter producing changes in inequality in an unambiguous manner. This is the case, for example, if the distribution of x is log-normal. For log-normal distributions, one single parameter, viz., the relative standard deviation, (i.e., the standard deviation of the logarithms of the original variate-values,) contains the entire information about inequality. Earlier workers [4, 6, 20] had shown

5 This was actually the case with the NSS expenditure distributions studied in this Chapter.

that the NSS expenditure distributions are approximately log-normal in character. These studies were, however, based on data for the 7th and the 10th rounds only. It was, therefore, decided to examine whether the log-normal hypothesis could provide a reasonable description for all distributions of x so far available from NSS, and, when the log-normal fit was actually found to be satisfactory in general, to compare the values of the relative standard deviation for different rounds.

The above-mentioned techniques were applied to all the NSS distributions of population in classes of x (i.e., size classes of expenditure) which were available at the Indian Statistical Institute, Calcutta. The study covered all rounds from the second to the fourteenth[6], for the rural sector, and from the third up to the twelfth, for the urban areas[7]. Rural and urban sectors of India were studied separately, but no zonal or state breakdowns were considered so extensively. Some work was also done on all-India (rural and urban combined) distributions of expenditure. Whenever necessary and wherever data permitted, use was made of the information from interpenetrating sub-samples (IPS).

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6 Fourteenth round data used in this paper actually relate to the first sub-round only of the fourteenth round. There were six bimonthly sub-rounds, and the first was carried out in July-August 1958.

7 The urban sector was not covered by the NSS until the third round.

Section 6.2 gives some background information about the NSS. The major findings are reported in Sections 6.3 to 6.6 below, which include a number of graphs illustrating important results. (In all these Sections, rural and urban sectors of all-India are treated separately). Section 6.3 is devoted to the Lorenz measure of concentration, and Section 6.4 to some other indices of inequality. Section 6.5 deals with the graphical comparison of the round-wise distributions of x/μ (actually of $x/\bar{x}$) over the different rounds. And lastly, Section 6.6 describes the results obtained through the log-normal approximations to distributions of x. The over-all finding seems to be that the distributions of x/μ have remained more or less stable over the NSS rounds for both rural and urban areas of India as a whole. The small shifts from round to round were mostly self-correcting fluctuations with no underlying trend, except for slight increases in the shares in consumption of certain classes of population (vide Section 6.4). Even these "slight increases" should not be regarded as established by the present study. There are some methodological limitations of the present study, besides those of the NSS consumer expenditure data. These limitations are discussed in Section 6.7. Section 6.8 outlines some future lines of work and presents some preliminary results about the all-India (rural and urban combined) distributions of expenditure. The basic material utilised in the study, regarding the NSS expenditure distributions, has been shown in Appendix 6.1, along with the relevant sample sizes[8]. Appendix 6.2 gives a table [extracted from Som and De [21]] showing population projections for India during 1951-60. Some of these projections were used for purposes of Sections 6.4 and 6.8 and one can find various other uses of this table.

6.2 Some background information about the NSS and the material :

Before discussing the actual results of the present investigation, it may be useful to give some background information about the NSS consumer expenditure data used in this analysis (vide Appendix 6.1). The NSS is a multipurpose sampling enquiry of all-India coverage, which collects data relating to various aspects of the Indian economy. The survey has a continuing programme, and is carried out in the form of successive rounds year after year. The first round, which was confined to rural areas only, was started in October 1950 and completed in March 1951. The second round, which likewise excluded the urban areas,

8 Appendix 6.1 also gives detailed references to the sources of such data.

was carried out during April 1951 to June 1951[9]. In the subsequent rounds, i.e., from the third round onwards, the geographical coverage was extended to include urban areas.

The above description does not apply to Jammu & Kashmir. This State (both rural and urban) was not covered by NSS before the ninth round (May-November 1955).

Consumer expenditure data, with which we are concerned, have been collected in all rounds of the NSS, but estimates for the distribution of persons according to expenditure[10] (monthly per capita total consumer expenditure) are not available for the first round. Expenditure distributions have been (or are being) estimated for all the subsequent reounds of NSS, and, at the time of carrying out this investigation, distributions of expenditure (x) were available, for the rural sector, for all rounds from the second up to the thirteenth, besides

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9 The following are the periods during which the different rounds of NSS were carried out :

NSS 1st round	.. October 1950-March 1951	NSS 10th round	.. Dec. 1955-May 1956
" 2nd "	.. April-June 1951	" 11th "	.. August 1956-February 1957
" 3rd "	.. August-Nov.1951	" 12th "	.. March-Aug.1957
" 4th "	.. April -Sep.1952	" 13th "	.. Sep.(middle)1957-May (middle)1958
" 5th "	.. Dec. 1952 - March 1953	" 14th "	.. June (last week) 1958-June 1959.
" 6th "	.. May - Sept.1953	" 15th "	.. July 1959-June 1960.
" 7th "	.. October 1953 - March 1954	NSS 16th round (in progress)	.. July 1960-
" 8th "	.. July 1954 - April 1955		
" 9th "	.. May-Nov. 1955		

10. For purpose of NSS, total consumer expenditure is taken as the value of total consumption (a) out of monetary purchases, (b) out of receipts in exchange of goods and services, (c) out of homegrown stock and (d) out of gifts, loans etc. Consumption under categories (b)-(d) are imputed at suitable prices.

for the first of the six sub-rounds of the fourteenth round carried out during July-August 1958. For the urban sector, on the other hand, these distributions were available up to the twelfth round beginning with the third, as the second round of NSS did not cover the urban areas[11].

It must be recorded here that the data for the twelfth round utilised in this study are somewhat provisional in nature. In particular, figures for the rural sector for two monthly per capita expenditure classes, viz., Rs.24-28 and Rs.28-34, are obviously subject to serious errors (vide Appendix 6.1).

Broadly, the method followed by NSS has been to collect budgets from sample households by the interview method. The concepts, definitions and procedures have remained more or less unchanged, from round to round, with one important exception : the <u>reference period</u>, to which the budgets relate, varied during the early rounds. This was largely by way of experimentation, as the question of optimum reference period had not been settled in the early days of the NSS. In the first round, which does not concern us here, a fixed reference period viz., July 1949 - June 1950, was used. In all the subsequent rounds, moving reference periods, either a week or a month (30 days) preceeding the date of interview, was used. The second, the third and the

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11 Vide Appendix 6.1 for detailed references to sources of data.

sixth rounds used the week reference period, while the fourth and the fifth rounds employed both 'week' and 'month', each in one part of the sample of households. From the seventh round onwards, the enquiry used only 'month' as the reference period.

Although in all cases, the estimates are so prepared as to relate to a month of 30 days, the estimates based on different reference periods are not strictly comparable. It has been observed that 'week' estimates are usually higher than 'month' estimates; but from the several studies on the problem of reference period [2, 3, 14, 15] it does not appear that the 'week' estimates differ from 'month' estimates simply by a multiplying factor. It would thus be unsafe to compare measures of concentration etc., (calculated from distributions of x,) based on 'month' and 'week' reference periods.

As the sample design adopted in NSS is probabilistic in nature, the NSS provides statistically valid estimates relating to consumer expenditure in India by various breakdowns. The breakdowns relevant for the present purpose are those by sectors (i.e., rural and urban) and by levels of living as shown by the monthly per capita total consumer expenditure of households [3, 12, 13]. Standard errors etc., are not usually computed for these estimates, as the necessary calculations would be quite expensive. There is instead the technique, followed from second round onwards, of presenting estimates separately for two (or more) independent and interpenetrating sub-samples (IPS). The comparison between the two sub-sample estimates gives a good deal of information about sampling and non-sampling errors of the (combined) estimates [8, 9, 10, 11].

6.3 The Lorenz ratio : The Lorenz curve and the associated Lorenz ratio arise very naturally in the measurement of inequality. They are extensively used in studies on income distributions and seem to be very meaningful in the present context, where we are concerned with expenditure distributions.

For the general definition and properties of the Lorenz curve and the Lorenz ratio, the reader is referred to Roy, Chakravarti and Laha[19]. It may, however, be emphasised here that the applicability of these tools does not depend on any *a priori* knowledge or assumptions regarding the nature of the underlying distributions, excepting that the variate in question should be (practically) continuous and non-negative. Secondly, the Lorenz curve, and hence the Lorenz ratio, are unaffected by scalar transformations of the variate[1].

In general, the Lorenz curve specifies the original distribution except for the mean, or in other words, it specifies what we referred to earlier as 'adjusted' distributions. Graphical comparisons based on Lorenz curves were not, however, carried out as essentially the same comparisons were made in two other ways including the one involving adjusted ogives (vide Sections 6.5 and 6.6). As regards numerical comparisons, the Lorenz curve/ratio will be utilised in this section section and the one following.

The Lorenz ratio was estimated for each of the expenditure distributions presented in the Appendix, by using the following expression

$$L = 1 - \sum_{i=1}^{12} (P_i - P_{i-1})(Q_i + Q_{i-1}), \quad (0 < L < 1) \qquad (6.1)$$

where P_i is the (estimated) proportion of population with per capita monthly total expenditure below the upper limit of the ith class interval for per capita monthly total expenditure, Q_i is the (estimated) proportion of total expenditure of all persons spent by the above stratum of population[12] $(i = 1, 2, \ldots\ldots, 12)$, and by convention, $(P_o, Q_o) = (0, 0)$. This formula gives only an approximation, being based on the broken Lorenz curve obtained by joining successively the points $(P_o, Q_o), (P_1, Q_1), \ldots\ldots, (P_{11}, Q_{11}), (P_{12}, Q_{12})$ without any smoothing.

The estimated Lorenz ratios are given in Table 6.1. The divergence between sub-sample estimates gives a rough idea of the margin of error.

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12 It p_i represents the proportion of persons in the ith class-interval and $\bar{x}_i$ the average of monthly per capita total expenditure for persons in this class,

then $P_i = \sum_{j=1}^{i} p_j$, and $Q_i = \left(\sum_{i=1}^{i} p_j \bar{x}_j \right) / \left(\sum_{j=1}^{12} p_j \bar{x}_j \right)$, where

$i = 1, 2, \ldots\ldots, 12$.

Table 6.1 : Lorenz ratios for the distributions of persons by monthly per capita total expenditure : NSS, All-India, 1951-1958

round	rural s.s.1	rural s.s.2	rural comb.	urban s.s.1	urban s.s.2	urban comb.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
			reference period : week			
2nd round	0.366	0.358	0.362	-	-	-
3rd "	0.330	0.337	0.334	0.394	0.372	0.384
4th "	0.320	0.340	0.330	0.350	0.362	0.352
5th "	0.304	0.315	0.310	0.423	0.376	0.394
6th "	0.331	0.328	0.330	0.387	0.338	0.364
			reference period : month			
4th round	0.337	0.342	0.340	0.372	0.357	0.365
5th "	0.337	0.322	0.330	0.426	0.350	0.397
7th "	0.341	0.326	0.334	0.377	0.366	0.371
8th "	0.360	0.340	0.350	0.363	0.405	0.390
*8th "	0.359	0.339	0.350	0.363	0.406	0.390
9th "	0.335	0.335	0.335	0.371	0.371	0.371
10th "	0.344	0.348	0.344	0.372	0.364	0.368
11th "	0.320	0.317	0.319	0.420	0.384	0.402
12th "	0.328	0.334	0.331	0.406	0.380	0.393
13th "	0.337	0.330	0.333	-	-	-
14th (first sub-round)	0.319	0.335	0.328	-	-	-

* For all rows previous to this, the geographical coverage did not include Jammu & Kashmir; for this row and for the subsequent rows, the coverage was all-India, including Jammu & Kashmir.

Figures 6.1 and 6.2 show the movement of these Lorenz ratios (of Table 6.1) over the different rounds of NSS, separately for urban and rural areas. Figure 6.2 is confined to data based on the week reference period; it shows the changes in the level of inequality between the 2nd and the 6th rounds. In Figure 6.1, however, Lorenz ratios based on 'week' have been plotted for the 2nd, the 3rd and the 6th rounds only; for the remaining rounds, Figure 6.1 shows the Lorenz ratios based on 'month'. (This distinction has been hinted at in the numbering of NSS rounds along the horizontal axis.) Such mixing was necessary for having a continuous series of ratios without any break, and since it can be verified from Table 6.1 that the reference period has little effect on the magnitude of the Lorenz ratio, the mixing of 'week' and 'month' ratios seems to be more or less justified. In each figure, the 'combined' ratios for consecutive rounds are joined by a thick line. There are also a pair of similar but broken lines enclosing the 'combined' line in each figure : these are obtained by joining the maximum (or the minimum) of the two 'sub-sample' ratios for successive rounds[13]. The divergence between these two (last-mentioned) lines gives a visual idea about the margins of error of the 'combined' Lorenz ratios. Thus, for example, Figures 6.1 & 6.2 show that the urban ratios are subject to wider margins of error than are the rural ones, which

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13 This point will recur in connection with many other figures presented in this paper. No attempt was made to join points, bearing the same sub-sample number (1 or 2), for consecutive rounds, as there had been little correspondence, in general, between sub-sample 1 (or 2) of different rounds.

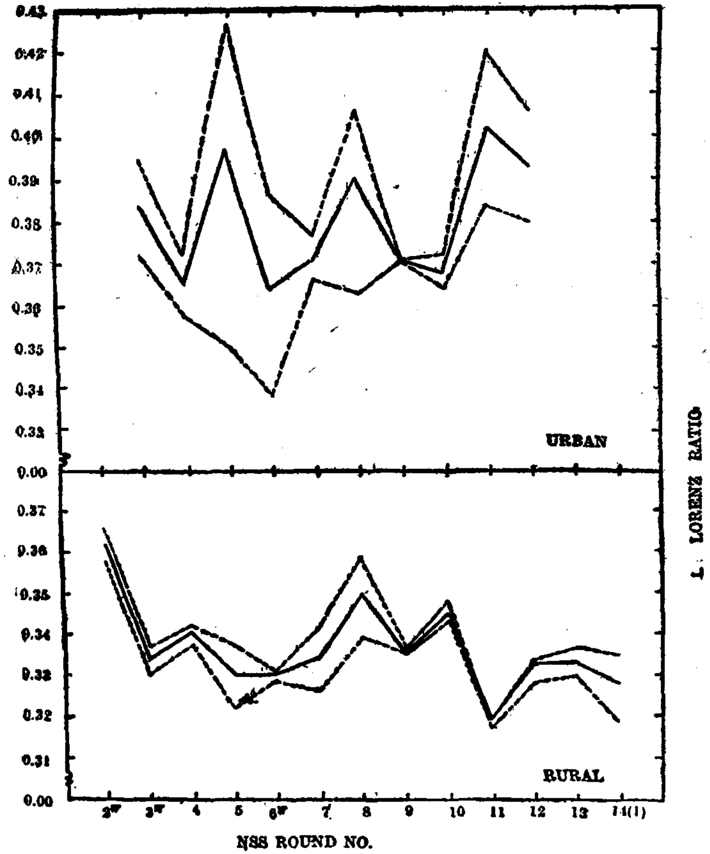

Figure 6.1. Lorenz ratio for the distributions of persons by monthly per capita expenditure based on month reference data

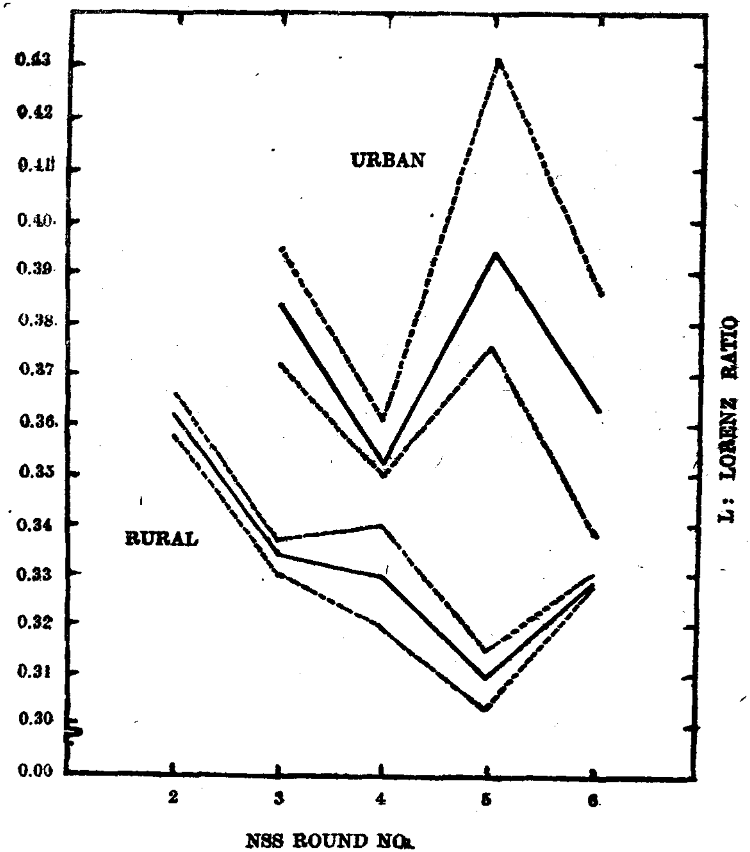

Figure 6.2. Lorenz ratio for the distributions of persons by monthly per capita expenditure based on week reference data

might be mainly due to the larger sample sizes for the rural sector. It may also be noted that, on the horizontal axis, the NSS rounds are shown at equal intervals, ignoring the unequal durations of the different rounds (vide footnote 9).

Figures 6.1 and 6.2 seem to show that for either sector there was no definite trend in the movement of concentration ratios over rounds, excepting that in the rural sector the concentration declined to some extent between the second and the third round. It seems unsafe, however, to consider this decline as reflecting a real change in the economy. It is unlikely that such a change occurred between the 2nd round (April-June 1951) and the third round (August-November 1951), while for the entire period following this no changes in inequality occurred, as shown by the up-and-down movement of the Lorenz ratio. We feel inclined to conclude that the 2nd round data are not dependable enough for us to accept this decline in inequality between the 2nd and the 3rd rounds.

It can also be seen that the inequality of expenditures, as measured by the Lorenz ratio, is somewhat greater in urban areas than in rural areas.

Similar conclusions were obtained by Murti and Pillai [16] who computed the Lorenz ratios based on "month reference period" for 4th to 11th rounds (excluding the 6th). These ratios were not calculated by sub-samples, but Murti and Pillai correctly concluded that the fluctuations in Lorenz ratio could be explained by sampling errors, although from the Lorenz curves presented by them, one would be tempted to conclude that between the 4th and the 11th rounds there was a fall in inequality in rural areas and some increase in concentration in the urban areas.

6.4 Some other indicators of concentration : In the general case, i.e., when no assumptions can be made regarding the nature of the expenditure distributions, the Lorenz curve is not completely specified by the Lorenz ratio. The family of Lorenz curves may include members which cross (and even re-cross) one another. In these circumstances, the Lorenz ratio is only an overall or summary index, measuring inequality, and is only one out of many measures which could be suggested.

In view of this limitation of the Lorenz ratio, it was considered necessary to supplement it by certain additional measures of concentration. These indicators are definable in terms of a few selected points on the Lorenz curve, and should reflect shifts in the Lorenz curve, if any, from round to round, in greater detail than can be done by the over-all Lorenz ratio.

Concretely, the ordinates Q of the broken Lorenz curve (defined in Section 6.3) were obtained by linear interpolation for six selected values of P, viz., P = 10 p.c. 25 p.c. 50 p.c. 90 p.c. and 95 p.c. These values (to be indicated by $Q_{0.10}$, $Q_{0.25}$, $Q_{0.50}$, $Q_{0.75}$, $Q_{0.90}$ and $Q_{0.95}$ respectively) are of considerable practical interest, being the

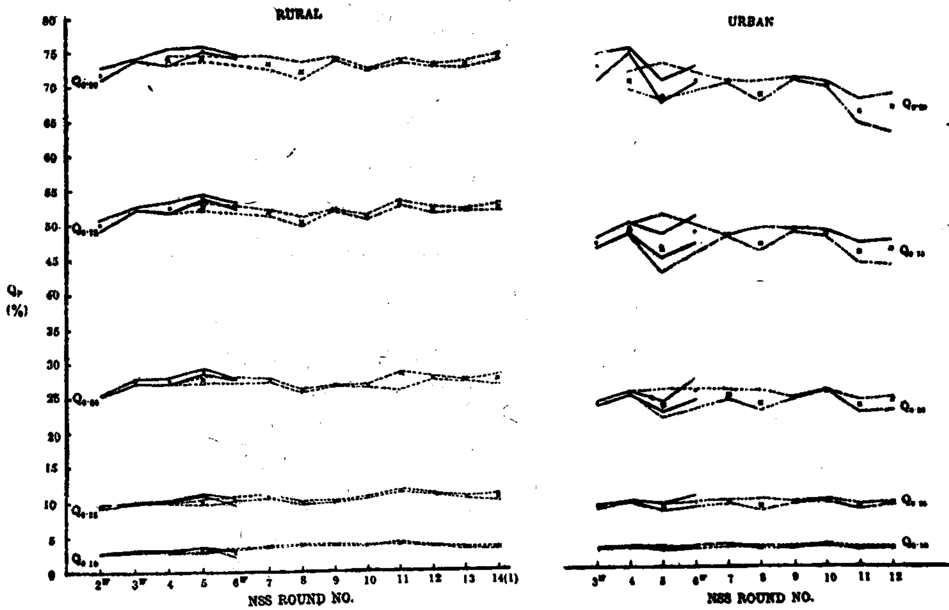

Figure 6.3. Movement of Q_P over NSS rounds for selected values of P

proportions of total expenditure spent by the poorest 10 p.c., 25 p.c., etc., of the population. The results of actual calculations are shown in Table 6.2

In Figure 6.3, some graphs are drawn, based on Table 6.2, showing the movement over NSS rounds of the percentage share in total consumption of the bottom (i.e., the poorest) 10 p.c., etc., of the population. Rural and urban sectors have, of course, been kept separate. For each sector, there are five sets of graphs drawn on the same scale : the set marked $Q_{0.10}$ shows the share of the bottom 10 p.c., the one marked $Q_{0.25}$ indicates the share of the poorest 25 p.c., and so on. Within each set of graphs, points based on 'week' reference period have been shown by dots to distinguish them from points based on 'month' reference period which are shown by crosses. As regards the joining of points for consecutive rounds, points for the 'combined' samples have not been joined at all (in the interest of neatness); for the subsamplewise points, thick lines were used for joining points based on 'week' and broken lines for joining points based on 'month'. Also, points representing the maximum (or the minimum) of the two subsamples were joined for successive rounds.

The points based on 'week' reference period in Figure 6.3 (rural sector) suggest that the share in total consumption of the poorest 10 p.c., (or 25 p.c., or etc.,) of the rural population

Table 6.2 : Proportions* Q_P of total consumer expenditure spent by the poorest P % of the population, for some selected values of P by rounds, sectors and sub-samples : NSS, all-India.

NSS round no.	half sample	rural						urban					
		$Q_{0.10}$	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	$Q_{0.90}$	$Q_{0.95}$	$Q_{0.10}$	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	$Q_{0.90}$	$Q_{0.95}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
						reference period : week							
2	1	2.84	9.58	25.34	49.32	71.08	84.68						
	2	2.65	9.08	25.56	50.85	72.87	83.84						
	comb.	2.74	9.27	25.50	50.12	71.98	84.30						
3	1	3.06	10.04	27.75	52.71	74.13	83.59	2.53	8.45	23.48	46.37	70.76	85.38
	2	2.89	9.98	27.12	52.34	74.02	83.77	2.69	9.98	24.03	47.96	74.91	87.45
	comb.	2.97	10.02	27.44	52.52	73.90	83.69	2.60	8.73	23.78	47.05	73.08	86.54
4	1	3.19	10.44	28.19	53.45	75.53	84.66	2.77	9.60	25.53	50.21	75.96	87.98
	2	2.93	10.06	27.02	51.89	73.17	82.81	2.92	9.45	25.11	48.58	75.02	87.51
	comb.	3.07	10.24	27.61	52.62	74.35	83.73	2.85	9.52	25.32	49.38	75.32	87.66
5	1	3.55	11.21	29.31	54.60	75.81	85.10	2.34	8.20	22.53	44.94	67.76	83.88
	2	3.53	11.06	28.51	53.88	75.11	84.28	2.72	9.05	24.07	48.65	71.04	84.99
	comb.	3.59	11.17	29.04	54.18	75.15	84.50	2.52	8.61	23.26	46.79	68.66	84.33
6	1	2.40	9.69	27.90	53.20	74.73	83.78	2.63	8.59	24.30	47.17	71.08	85.54
	2	3.24	10.77	27.91	52.50	74.25	84.32	2.90	10.35	27.49	51.06	73.11	85.36
	comb.	2.81	10.23	27.71	52.85	74.41	84.04	2.69	9.43	25.73	48.94	70.72	85.36

* Figures given in this table are really percentages ($100Q_p$) and not proportions Q_p .

Table 6.2 : contd.

NSS round no.	half-sample	rural						urban					
		$Q_{0.10}$	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	$Q_{0.90}$	$Q_{0.95}$	$Q_{0.10}$	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	$Q_{0.90}$	$Q_{0.95}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
							reference period : month						
4	1	2.77	9.84	27.06	52.16	74.55	83.94	2.84	9.25	25.17	48.62	69.66	84.27
	2	2.89	9.95	27.10	51.79	73.19	82.96	3.20	9.89	25.46	49.98	72.28	84.23
	comb.	2.82	9.89	27.07	51.98	73.86	83.45	3.01	9.54	25.27	49.13	70.88	84.15
5	1	3.00	9.94	27.21	52.21	73.84	83.65	2.38	8.00	21.69	42.98	68.19	84.10
	2	3.17	10.49	28.30	53.36	74.64	83.69	2.92	9.14	25.75	51.32	73.51	85.89
	comb.	2.99	10.22	27.77	52.83	74.22	83.69	2.60	8.46	23.37	46.39	68.51	84.25
7	1	3.50	10.63	27.39	51.47	72.53	82.05	2.84	9.08	24.36	48.31	71.00	83.13
	2	3.70	11.15	27.86	52.29	74.40	84.29	3.02	9.71	25.84	48.45	70.98	83.27
	comb.	3.59	10.89	27.63	51.83	73.46	83.16	2.94	9.40	25.10	48.34	71.01	83.20
8	1	3.91	9.77	25.87	49.78	71.03	81.37	2.99	9.83	25.74	49.58	71.22	81.58
	2	3.96	10.24	26.27	51.15	73.61	83.80	2.50	8.20	22.91	46.02	68.21	82.29
	comb.	2.93	9.82	26.33	50.35	72.30	82.55	2.73	8.97	23.94	47.12	69.19	81.45
8**	1	3.90	9.74	25.90	49.84	71.11	81.26	3.00	9.83	25.74	49.57	71.22	81.56
	2	3.96	10.27	26.83	51.24	73.67	83.75	2.49	8.20	22.88	46.00	68.20	82.33
	comb.	3.92	9.80	26.36	50.42	72.35	82.49	2.73	8.96	23.93	47.12	69.20	81.46

** For all rows previous to this, the geographical coverage was all-India excluding Jammu & Kashmir; for this row as well as for all the subsequent rows, the geographical coverage was all-India including Jammu & Kashmir.

Table 6.2 : contd.

NSS round no.	half-sample	rural						urban					
		$Q_{0.10}$	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	$Q_{0.90}$	$Q_{0.95}$	$Q_{0.10}$	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	$Q_{0.90}$	$Q_{0.95}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
						reference period : month							
9	1	3.86	9.93	26.97	52.17	74.36	84.21	2.89	9.22	24.66	49.45	71.41	81.75
	2	3.77	10.16	26.82	52.09	73.90	83.57	2.97	9.39	24.77	48.87	71.69	83.08
	comb.	3.82	10.05	26.91	52.12	74.13	83.81	2.94	9.32	24.70	49.15	71.54	82.43
10	1	3.60	10.71	27.05	51.55	72.73	82.65	3.15	9.55	25.26	48.37	70.44	82.34
	2	3.54	10.50	26.60	50.74	72.55	82.93	3.27	9.86	25.42	49.42	71.09	82.74
	comb.	3.56	10.58	26.79	51.03	72.60	82.76	3.21	9.67	25.35	48.89	70.76	82.52
11	1	4.04	11.78	28.69	52.70	73.57	82.71	2.50	8.48	22.67	44.68	64.99	79.18
	2	3.91	11.42	26.08	53.54	74.21	83.75	2.91	9.22	24.50	47.55	68.73	82.42
	comb.	3.97	11.59	28.62	53.07	73.84	83.18	2.71	8.86	23.60	46.02	66.84	80.83
12	1	3.63	11.08	28.20	52.70	73.48	82.21	2.99	9.12	23.99	44.31	63.83	75.64
	2	3.74	11.14	27.74	51.79	73.04	81.79	2.89	9.33	24.87	48.03	69.55	81.09
	comb.	3.68	11.11	27.99	52.24	73.24	81.98	2.94	9.16	24.43	46.69	67.53	79.91
13	1	3.22	10.40	27.45	52.14	72.95	82.54						
	2	3.41	10.67	27.64	52.30	73.94	83.82						
	comb.	3.32	10.56	27.56	52.23	73.44	83.18						
14	1	3.47	11.02	28.38	53.32	74.33	84.13						
(first	2	3.07	10.27	26.97	52.16	75.06	85.02						
sub-round)	comb.	3.24	10.65	27.69	52.63	74.69	84.23						

increased appreciably between the 2nd and the 3rd rounds. We have already expressed our misgivings about this apparent fall in inequality. Ignoring the points for the 2nd round for the rural sector, and considering 'week' and 'month' points together, it appears from Figure 6.3 that the share of the poorest 10 p.c., or 25 p.c., of the population tended to increase, though very slightly, for the rural sector, while in the urban areas the corresponding shares remained practically constant between the 3rd and the 12th rounds. The position is somewhat different in the case of the top (or the richest) 10 p.c., or 25 p.c., of the population in either sector. The top 10 p.c., (or 25 p.c.,) of the urban population seems to have increased their share of total urban expenditure, (mostly after the 10th round), whereas the corresponding stratum of the rural population has more or less retained its share. These observations must be taken with great caution, particularly those involving the top 10 p.c., of population in either sector. (The top graphs in Figure 6.3 are subject to wider margins of error than the bottom curves as shown by the divergence between subsample points. Also, they may be more seriously affected by the limitations of linear interpolation. The differences observed are quite small, and may not be significant in the statistical sense, leaving apart the problem of nonsampling errors.

For a more direct comparison we give Figure 6.4, which has been plotted in the same manner as Figure 6.3, and which is in fact a partial repetition of Figure 6.3. In Figure 6.4, the graphs marked $1 - Q_{0.90}$ show the movement of the share of the richest 10 p.c., of population in the total consumption of the sector (rural or urban), while the one marked $Q_{0.10}$ shows the corresponding picture for the poorest 10 p.c. There is, however, another set, marked $1 - Q_{0.95}$ showing the movement over NSS rounds of the share of the richest 5 p.c., of the population. Figure 6.4 naturally confirms our findings based on Figure 6.3 about the share of the richest or the poorest 10 p.c., of population in the two sectors. We find, moreover, that the increase in the share of the top 5 p.c., of the urban population was steeper and more systematic than for the top 10 p.c., of the same sector. For the rural sector, on the other hand, the share of the top 5 p.c., was as stable as that of the top 10 p.c.

As the top 5 p.c., of the urban population forms in a way the richest class of the country's population, it was considered interesting, at this stage, to study the share of this top 5 p.c., of the urban population in the total consumption of the country. Estimates of these shares for the different NSS rounds were made on the basis of the NSS roundwise (and subsamplewise) projections of population given by Som and De [21]. We may explain the calculation by considering a particular sub-sample (or the 'combined' sample) of a particular round, thus ignoring the round or subsample etc., in the notation. The total consumption of the top 5 p.c., of the urban population is given by $(1 - Q_{0.95})\, N_u \bar{x}_u$ where N_u is the estimated size of the urban population, and $\bar{x}_u$ the average monthly consumer expenditure per capita in the urban sector. (The quantity $Q_{0.95}$, of course, relates to the urban sector.) The total consumption of India as a whole would be simply $N_u \bar{x}_u + N_r \bar{x}_r$, where the subscript r stands for the rural sector, the symbols N_r and $\bar{x}_r$ being otherwise self-explanatory. Hence the proportionate share of the top 5 p.c., of the urban population in the country's total consumption is

$$\frac{N_u \bar{x}_u}{N_u \bar{x}_u + N_r \bar{x}_r} \left[1 - Q_{0.95}\right] \quad \ldots \quad (6.2)$$

This proportion has been presented in Table 6.3 for all the rounds by sub-samples and 'combined'. For the sake of comparisons, the table shows the corresponding proportions, that is, $\frac{N_u}{N_u + N_r}(0.05)$, which the top 5 p.c., of urban population formed of the total population of the country. Figure 6.5 is based on Table 6.3 and has been plotted on the same lines as Figures 6.3 and 6.4. (See, however, footnote to Table 6.3. Also, only the 'combined' figures have been plotted for the "proportion of population" as the subsample graphs would have been indistringuishable.) Figure 6.5 shows that the share of the top 5 p.c. of the urban population in the total consumption of the country increased at a faster rate than the proportion which the

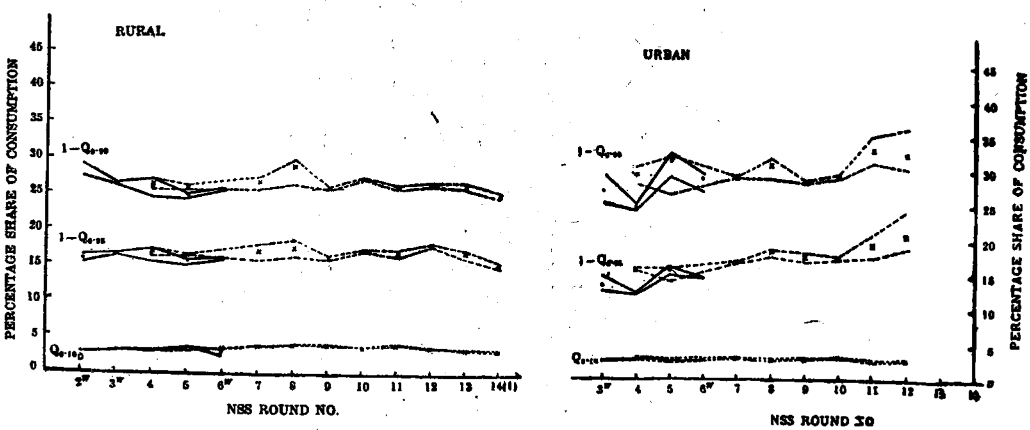

Figure 6.4. Trend in share of total consumption of the poorest 10 p. c., top 5. p. c. and top 10 p. c. of population

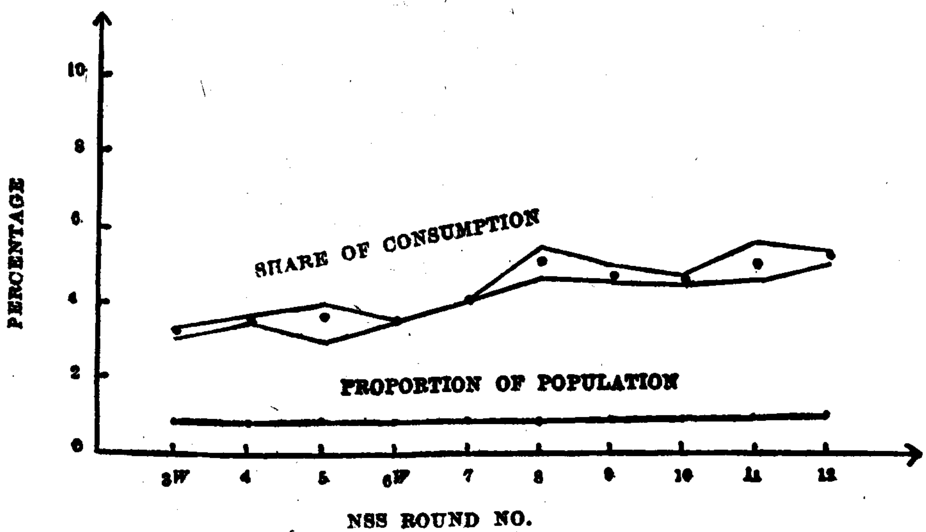

Figure 6.5. Size of top 5 p.c. of urban population as percentage of All-India population, and share of the same stratum in All-India consumption over NSS rounds

size of this top 5 p.c., (of the urban population) forms of the total size of the country's population.

Table 6.3 : Size of the top 5 p.c., of the urban population as percentage of the all-India population, and the share of the same stratum as percentage of total all-India consumption, by NSS rounds and sub-samples

NSS round*	proportion (%) of all-India population			share (%) in all-India consumption		
	sub-sample 1	sub-sample 2	combined	sub-sample 1	sub-sample 2	combined
(1)	(2)	(3)	(4)	(5)	(6)	(7)
3W	0.89	0.89	0.89	3.37	3.03	3.24
4	0.90	0.90	0.90	3.67	3.46	3.59
5	0.91	0.91	0.91	3.98	2.95	3.64
6W	0.91	0.91	0.91	3.52	3.49	3.53
7	0.92	0.92	0.92	4.05	4.02	4.00
8	0.93	0.96	0.93	4.62	5.42	5.09
9	0.94	0.97	0.94	4.94	4.54	4.67
10	0.95	0.98	0.95	4.45	4.54	4.67
11	0.96	0.99	0.96	5.52	4.57	4.96
12	0.97	1.00	0.97	5.31	5.01	5.30

* For the 3rd and the 6th rounds, cols. (5) to (7) are based on the "week" reference data only. For all other rounds, including the 4th and the 5th, these columns are based on 'month' reference data.

A second set of indicators of concentration was obtained as follows; Linear interpolation gave five selected fractiles of the distribution of persons by monthly per capita total consumer expenditure (x). The selected fractiles were : the first decile, the first quartile, the median, the third quartile and the ninth decile. Each fractile was expressed as fraction of $\bar{x}$ (proxy for μ), the arithmetic mean of the distribution referred to, and finally multiplied by Rs.20.00 or Rs.30.00, in the interest of ready comprehension, according as the distribution related to the rural or the urban sector. The value of these "adjusted" fractiles, denoted by $D_{0.10}$, $D_{0.25}$, $D_{0.50}$, $D_{0.75}$ and $D_{0.90}$ respectively, are shown in Table 6.4. These are in fact the fractiles of the "adjusted" distribution defined in Section 6.5.

It can be shown that apart from the multiplying factor Rs.20.00 or Rs.30.00, these fractions (i.e., fractiles divided by $\bar{x}$) five the slopes, $\frac{dQ}{dP}$ of the broken Lorenz curve at the points P = 0.10, 0.25, 0.50, 0.75, etc. [9] . This relation shows that, theoretically the set of all indices D_P presents the same picture as the set of all indices Q_P (i.e., the Lorenz curve); but in practice, with only a few values of D_P one cannot get a very accurate idea of the Lorenz curve, as the set of D_P's is independent of the expenditure distributions in between the fractiles involved. Under such circumstances, the set of D_P's might give slightly different pictures. If the Q_P's are regarded as the indices of immediate interest, the D_P's have to be considered as approximations to the Q_P's.

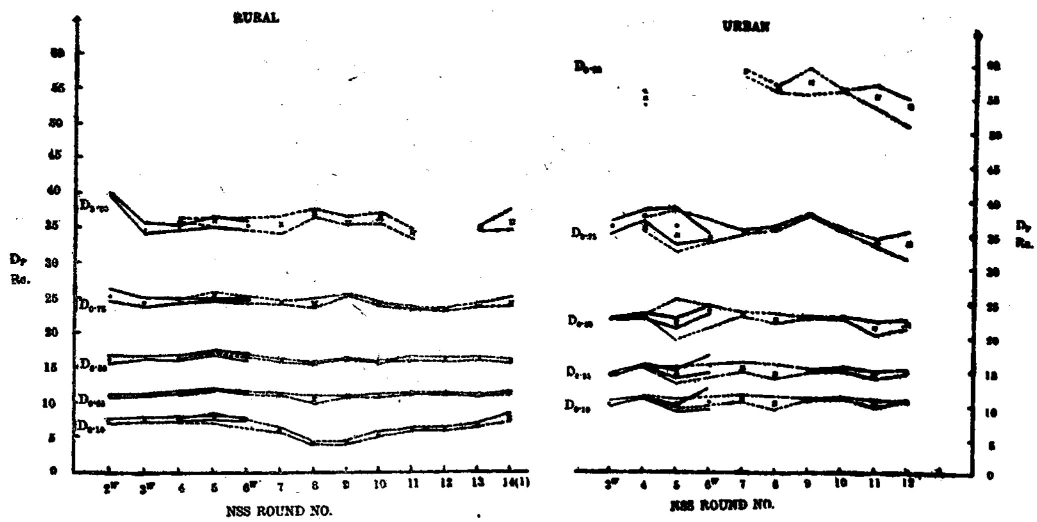

Figure 6.6. Movement of adjusted fractiles D_P over NSS rounds for selected values of P

Table 6.4 : Adjusted fractiles $D_{0.10}$, $D_{0.25}$, $D_{0.50}$, $D_{0.75}$ and $D_{0.90}$ by rounds, sectors and half-samples : NSS, all-India.

NSS round no.	half-sample	rural					urban				
		$D_{0.10}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.90}$	$D_{0.10}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.90}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
						reference period : week					
2	1	7.52	11.04	15.52	24.42	39.94					
	2	7.31	10.69	16.86	26.05	39.68					
	comb.	7.35	10.94	16.19	25.19	39.84					
3	1	7.60	11.01	16.74	23.56	34.33	9.68	13.91	22.35	34.39	-
	2	7.42	10.96	16.28	24.80	35.60	10.44	14.44	22.17	36.39	-
	comb.	7.50	10.98	16.50	24.29	34.75	9.92	14.28	22.19	35.64	-
4	1	7.91	11.52	16.91	24.82	34.77	10.80	15.57	23.03	37.97	-
	2	7.54	11.20	15.95	24.22	35.43	10.73	15.49	22.44	36.37	-
	comb.	7.71	11.37	16.50	24.47	35.05	10.91	15.52	22.85	37.18	-
5	1	8.26	11.93	17.51	24.50	35.30	8.96	13.84	20.90	32.99	-
	2	8.12	11.80	16.71	24.75	36.51	9.79	14.81	22.37	38.27	56.04
	comb.	8.18	11.85	17.07	24.61	35.90	9.39	14.31	21.60	35.67	-
6	1	7.49	11.57	17.06	24.83	34.85	9.22	14.69	23.13	33.57	-
	2	7.95	11.18	16.01	24.54	35.79	12.28	17.00	24.21	34.69	54.36
	comb.	7.73	11.35	16.63	24.82	35.48	10.26	15.82	23.60	34.00	-

Table 6.4 : contd.

NSS round no.	half-sample	rural					urban				
		$D_{0.10}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.90}$	$D_{0.10}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.90}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
						reference period : month					
4	1	7.24	11.18	16.47	24.75	36.07	10.62	15.07	22.63	34.76	53.31
	2	7.48	10.85	16.35	24.20	36.10	11.21	15.22	22.77	36.52	55.47
	comb.	7.37	10.99	16.40	24.51	36.14	10.87	15.08	22.95	35.41	54.27
5	1	7.17	11.49	17.26	24.44	36.05	9.25	12.97	19.37	31.90	-
	2	8.34	11.70	17.31	25.67	35.24	10.51	15.40	25.05	37.96	58.87
	comb.	8.07	11.60	17.30	25.10	35.71	9.69	14.04	21.73	34.43	-
7	1	5.96	10.97	16.22	24.03	34.19	10.47	14.63	22.92	35.23	58.60
	2	6.05	11.34	15.84	24.59	36.77	11.25	16.10	23.22	34.72	57.54
	comb.	6.00	11.15	16.14	24.28	35.50	10.82	15.40	23.04	34.95	58.27
8	1	4.00	9.99	15.73	23.58	36.19	11.19	15.74	23.14	35.43	56.50
	2	4.54	10.94	15.70	24.86	37.78	9.22	13.93	21.79	35.88	55.42
	comb.	4.26	10.64	15.73	24.11	37.08	10.08	14.49	22.43	35.23	56.41
8*	1	4.02	10.05	15.75	23.62	36.26	11.21	15.74	23.13	35.44	56.36
	2	4.55	10.96	15.73	24.86	37.76	9.22	13.89	21.80	35.91	55.51
	comb.	4.28	10.68	15.74	24.12	37.06	10.10	14.48	22.41	35.24	56.46

* For all rows previous this, the geographical coverage was all-India excluding Jammu & Kashmir; for this row as well as for all the subsequent rows, the geographical coverage was all-India including Jammu & Kashmir.

Table 6.4 : contd.

NSS round no.	half sample	rural					urban				
		$D_{0.10}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.90}$	$D_{0.10}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.90}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
reference period : month											
9	1	4.45	10.86	16.33	25.19	36.56	10.74	15.19	22.71	37.73	55.19
	2	4.75	11.10	16.18	25.56	35.13	10.87	14.65	22.54	37.69	58.98
	comb.	4.59	10.93	16.24	25.38	35.82	10.79	14.91	22.57	37.76	56.99
10	1	6.02	11.06	15.91	23.83	35.51	11.14	14.99	22.55	35.29	55.70
	2	5.66	10.68	15.63	24.29	37.28	11.18	15.47	22.72	35.71	55.99
	comb.	5.84	10.87	15.76	24.08	36.67	11.15	15.18	22.57	35.63	55.97
11	1	6.63	11.36	16.03	23.45	33.62	4.77	13.90	20.41	33.24	53.49
	2	6.07	11.44	16.68	23.58	34.59	10.78	14.77	22.25	34.15	56.57
	comb.	6.33	11.40	16.32	23.46	34.07	10.27	14.32	21.28	33.48	55.03
12	1	6.52	11.35	16.49	23.08	-	10.33	14.43	21.09	31.39	50.58
	2	6.07	11.06	15.78	23.54	-	10.60	15.15	22.37	35.30	54.76
	comb.	6.29	11.19	16.12	23.29	-	10.45	14.75	21.73	33.69	53.85
13	1	7.24	11.01	16.29	23.78	34.67					
	2	6.65	11.15	16.57	24.21	35.38					
	comb.	6.91	11.06	16.40	24.05	35.04					
14 ('first	1	8.73	11.62	16.06	24.22	35.08					
sub-	2	7.55	11.16	16.21	25.31	38.28					
round)	comb.	8.02	11.07	15.69	23.98	36.08					

Figure 6.6 is based on Table 6.4 in the same way as Figure 6.3 is derived from Table 6.2. The manner of plotting is precisely the same. We need only note that the values of $D_{0.90}$ could not be obtained for some of the urban expenditure distributions, as the highest class of monthly per capita expenditure, viz., Rs.55.00 and above, contained more than 10 p.c., of the population. Also, the value of $D_{0.90}$ was not calculated or plotted for the 12th Round (rural) : the effect of the inaccuracies mentioned in Section 6.2 was felt to be particularly serious for this index. (The same inaccuracies may have some small effect on the Lorenz ratio and on $Q_{0.75}$ and $Q_{0.90}$).

While examining Table 6.4 or Figure 6.6, it is necessary to remember two things : first, the limitations of 12th Round data mentioned in Section 6.2 and second the evarseness of the approximation given by linear interpolation when used for calculating $D_{0.10}$. The expenditures in the lowest class-interval of monthly per capita expenditure, viz., Rs.0-8, cannot be assumed to be evenly spread over this class-interval, so that for expenditure distributions having much more than 10 p.c., of population lying in this class-interval, linear interpolation gave very poor results. This may explain the fictitious trough in the graph of $D_{0.10}$ between the 6th and the 11th Rounds, for the rural sector, which is not confirmed by the graph for $Q_{0.10}$ (rural) in Figure 6.3.

Figure 6.6 confirms some of the observations based on Figure 6.3. The agreement is, however, only broad. The graph for $D_{0.10}$

(rural) cannot be depended upon, and that for $D_{0.25}$ (rural) indicates little change in the share of the poorest 25 % of the rural population. The "fall" in inequality in the rural areas after the 2nd round is apparent from the graph of $D_{0.90}$ only. Apart from this, the graphs for the rural sector show no trend. As regards the urban sector, one notices no definite trend in concentration if one excludes 12th round data. If 12th round data be included in the comparisons, there is a suggestion of a slight increase in the share in consumption of the top 10 p.c. of the urban population.

6.5 Graphical comparisons of adjusted distributions : Encouraged by the findings reported in Sections 6.3 and 6.4, we proceeded to compare all the expenditure distributions (for a given sector) in their entirety, after adjusting for variations in the arithmetic means for reasons stated in Section 6.1. The method adopted was purely graphical. The underlying theory is as follows : Suppose one has the distribution function $F(x)$ of a variate x, the mean of x being μ. Suppose one regards as a scale parameter, and wants to study the distribution function (d.f.) after a scalar transformation on the variate x such that the mean becomes μ'. Then one must transform to the variate $y = x\mu'/\mu$, and the d. f. of y would be $G(y) = F(y. \mu/\mu')$. Hence the ogive for y is obtained by plotting $G(y) = F(y \mu/\mu')$ against y, or, equivalently, $F(x)$ against $x \mu'/\mu$.

If now one has n distribution functions $F_1(x)$, $F_2(x)$, ..., $F_n(x)$ of the variate x, with means $\mu_1, \mu_2, \ldots, \mu_n$ respectively, one may adjust all the distributions in the above fashion, so that each adjusted d.f. leads to the same mean μ, and compare the resulting G(y)'s graphically.

In the present case, the variate x was chosen to represent the monthly per capita total consumer expenditure, $F_i(x)$ (i=1,2, etc.,) the d.fs. for the different distributions of persons given in Appendix 6.1, μ_i's the corresponding means, and a = Rs.20 for the rural sector and μ = Rs.30 for the urban sector, these choices being guided by considerations of convenience. The adjusted ogives were obtained by plotting the estimates P or $F_i(x)$ against $x\mu_i/\mu$, x taking in turn the values Rs.8, 11, ..., 55, the upper limits of the class-intervals of the NSS expenditure distributions.

Broadly speaking, all these adjusted ogives were found to be in close agreement for either of the two sectors. (The effect of reference period on these adjusted ogives was observed to be negligible.) There were differences between certain pairs of (adjusted) ogives in certain ranges, particularly, in the lower ranges; but sub-sample (adjusted) origes showed that most of these differences could easily be ascribed to the errors in the estimates.

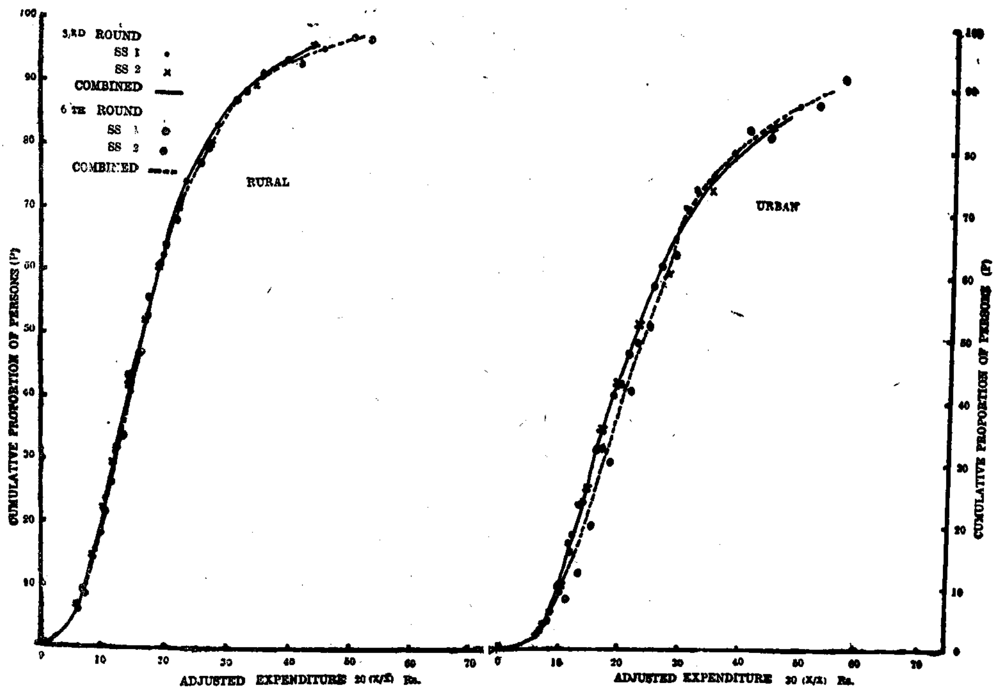

Figure 6.7. Comparison of adjusted ogives (for distributions of persons by monthly per capita expenditure) based on week reference data

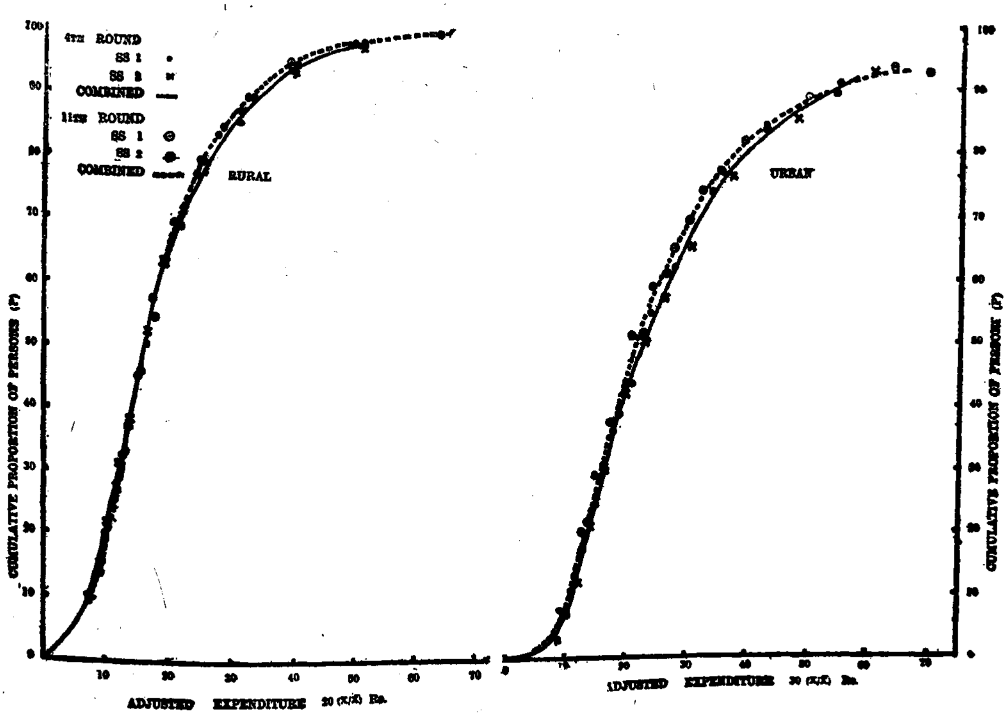

Figure 6.7(a). Comparison of adjusted ogives (for distributions of persons by monthly per capita expenditure) based on month reference data

By way of illustration, we present Figures 6.7 and 6.7(a) showing some typical comparisons. Figure 6.7(a) compares the adjusted distributions of the 4th and the 11th rounds, separately for the two sectors. For the purpose of Figure 6.7(a), month reference data of the 4th round were taken, so as to ensure comparability with 11th round data. Figure 6.7, on the other hand, compares the adjusted ogives for the 3rd and the 6th rounds, and is therefore based on week reference data. Here also rural and urban sectors have been distinguished. In both the figures, it was not possible to give the sub-samplewise (adjusted) ogives in the usual manner : the necessary points will be found in the figures, but they were not joined together by lines.

If one considers the 'combined' ogives only, Figures 6.7 and 6.7(a) will show some shifts in the adjusted distributions, at least for the urban sector. The 'subsample' points, however, suggest that these shifts are largely due to the errors of the estimates.

6.6 Comparisons based on the log-normal approximation : The investigation reported in this section has been briefly outlined Section 6.1. The work started with an examination of the goodness of fit of the log-normal distribution to all the 'combined' expenditure distributions presented in Appendix 6.1. All the distributions were represented by means of probit lines, i.e., actual, unadjusted ogives on log-probit scale. No tests of goodness of fit were attempted, but it was felt that all the ogives so plotted could be regarded as straight lines to a high degree of approximation.

To convey the actual position, we present Figure 6.8 showing some of these ogives on log-probit scale for both rural and urban sectors of India. Ogives for rounds 2(W), 4(W), 4(M), 13(M) and 14(1)(M) have been presented for the rural sector, and those for rounds 3(W), 4(W), 4(M) and 11(M) are presented for the urban sector. These probit lines have not been smoothed at all, since smoothing might vitiate the examination of the goodness of fit. It is, however, clear that the probit lines in Figure 6.8 are all approximately linear.

It is also found, when these ogives are superposed, as in Figure 6.8, that for either of the two sectors, all the probit lines are very nearly parallel to one another, and that the (common) slope is somewhat smaller for the urban sector than for the rural.

Theoretically, one should study the sub-sample ogives to see if the small deviations from perfect linearity or from perfect parallelism could be attributed to sampling and non-sampling errors. For various reasons, these deviations have been ignored in this study

Now, it is well-known that if (and only if) the underlying distribution of x be log-normal [with $E(\log x) = \theta$, and $Var(\log x) = \lambda^2$,] the ogive of x on log-probit scale will be a straight line,

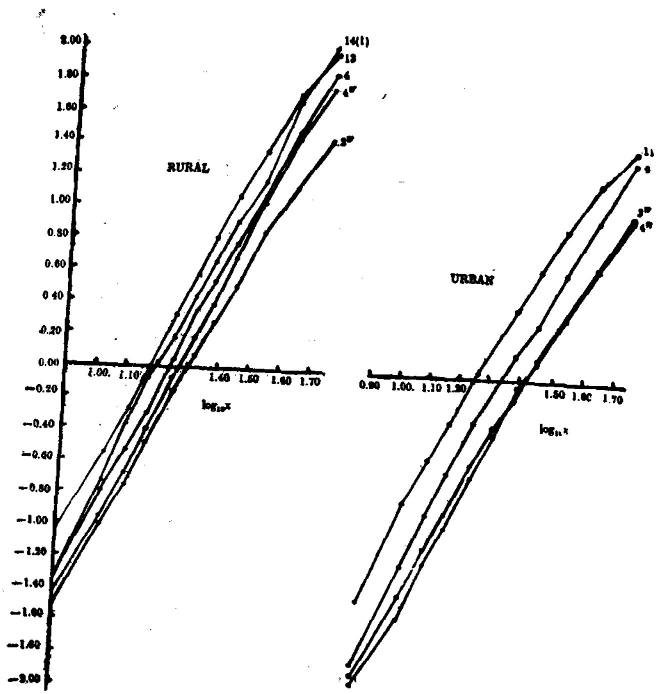

Figure 6.8. Some ogives on log-probit scale for distributions of persons by monthly per capita expenditure in Rs.(x)

(with the slope given by $1/\lambda$ and the intercept on the vertical axis by $-\frac{\theta}{\lambda}$). In the present case, therefore, the approximate linearity of the ogives on log-probit scale imply that all the underlying distributions are very nearly log-normal in character. Secondly, since all these ogives for a given sector are more or less parallel, the parameter λ, determining the slope, must have remained practically constant over the rounds. And lastly, from the comparison of the slopes, one can see that the value of λ was generally somewhat higher for the urban sector than for the rural.

As already stated in Section 6.1, in the case of log-normal distributions, this parameter λ alone summarise the entire information about the inequality of the distributions. The adjusted distribution and the Lorenz curve depend solely on λ, and the various measures of inequality are all, necessarily, monotone increasing functions of λ. The above observations therefore imply that (i) for either of the two sectors, the adjusted distributions remained broadly stationary over the NSS rounds, showing little changes in inequality, and that (ii) the inequality was somewhat greater in the urban sector than in the rural. The first conclusion was reached in Section 6.5 by another graphical approach, which, however, did not involve the hypothesis of log-normality.

As regards the intercepts on the vertical axis of the probit lines (vide Figure 6.8) it was found that these intercepts vary widely between the rounds, whereas, as already noted, the slopes are more or less the same. This was obviously due to fluctuations in the other parameter θ. These fluctuations cannot, however, be easily interpreted. Changes in θ are linked with changes in the mean $\bar{x}$, since we have, $\mu = \exp(\theta + \lambda^2/2)$, with λ practically constant. And the difficulties of studying the changes in $\bar{x}$ were mentioned in the very beginning of this Chapter, in Section 6.1.

Similar conclusions were reached by a more regorous investigation by Roy [16]. His work was, however, confined to data relating to 2nd to 7th rounds of NSS, covering the period April 1951 -March 1954.

Log-normality of the NSS expenditure distributions seems now to be fairly established. For the sake of interest, we give below the estimates of θ and λ for all the distributions of Appendix 6.1. These estimates were not obtained by refined methods of fitting log-normal distributions nor from the log-probit graphs. The relative standard deviation λ was estimated from the Lorenz ratios of Table 6.1, by means of the relation, $L = 2\,\phi(\lambda/\sqrt{2}) - 1$, where L is the Lorenz ratio and ϕ the normal probability integral. The parameter θ was next estimated by using the expression $\mu = \exp.(\theta + \lambda^2/2)$, substituting $\bar{x}$ for μ. Table 6.5 shows the results of these calculations.

Table 6.5 : Estimates of parameters of log-normal distributions fitted to NSS expenditure distributions by rounds, sectors and sub-samples*

NSS round no.	rural						urban					
	θ = mean (log x)			λ = s. d. (log x)			θ = mean (log x)			λ = s. d. (log x)		
	h.s.1	h.s.2	comb.	h.s.1	h.s.2	comb.	h.s.1	h.s.2	comb.	h.s.1	h.s.2	comb.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
						reference period : week						
2	3.036	2.944	2.990	0.673	0.658	0.665						
3	3.001	3.031	2.993	0.603	0.614	0.610	3.245	3.368	3.307	0.729	0.685	0.709
4	2.958	2.935	2.946	0.583	0.622	0.603	3.283	3.342	3.317	0.642	0.665	0.646
5	2.924	2.900	2.900	0.552	0.576	0.564	3.121	3.124	3.132	0.791	0.693	0.729
6	2.903	2.854	2.880	0.607	0.599	0.603	2.813	3.180	3.190	0.717	0.618	0.669
						reference period : month						
4	2.888	2.868	2.878	0.618	0.626	0.626	3.173	3.098	3.133	0.685	0.654	0.673
5	2.794	2.817	2.805	0.618	0.587	0.603	3.156	3.021	3.090	0.795	0.642	0.738
7	2.677	2.646	2.661	0.622	0.595	0.610	2.946	2.933	2.938	0.693	0.673	0.685
8	2.485	2.513	2.499	0.661	0.622	0.642	2.861	3.037	2.946	0.665	0.754	0.721
8**	2.483	2.513	2.497	0.661	0.622	0.642	2.862	3.034	2.947	0.665	0.754	0.721
9	2.523	2.544	2.535	0.614	0.610	0.610	2.948	2.916	2.933	0.681	0.681	0.681
10	2.652	2.682	2.670	0.630	0.638	0.630	2.977	3.023	2.999	0.685	0.669	0.677
11	2.673	2.653	2.661	0.583	0.576	0.583	2.955	2.921	2.940	0.783	0.709	0.746
12	2.676	2.660	2.667	0.599	0.610	0.607	3.050	2.972	3.013	0.754	0.701	0.725
13	2.760	2.715	2.739	0.614	0.603	0.607						
14 (first sub-round)	2.788	2.828	2.808	0.583	0.614	0.599						

* All logarithms are natural, that is, with respect to e = 2.7182818

** For this and the subsequent rows, the geographical coverage was all-India including Jammu & Kashmir; for all the preceding rows, however, the coverage was all-India excluding Jammu & Kashmir.

It may be argued that the estimates Q_P (of percentage shares in total consumption of different classes of people) presented in Table 6.2 are too rough, being based on unsmoothed data and on linear interpolation, which is known to be defective near the tails of the distribution. The estimates of the log-normal parameter λ given in Table 6.5 can be used for improving upon the estimates of Q_P given in Table 6.2. This can be done by an extremely simple formula. If Q_P is the share in total consumption of the bottom 100 P p.c., of the population in a given sector, than, under the assumption of log-normality, Q_P is related to P in the following way :

$$t_{Q_P} = t_P - \lambda \;, \qquad \ldots \qquad (6.3)$$

where t is the standard normal deviate corresponding to incomplete probability integral α $(0 \leq \alpha \leq 1)$, [5, 6] . A large number of the Q_P's were actually re-estimated by this method. Such estimates would not alter any of our conclusions based on Table 6.2 and are not therefore being presented in this Chapter.

6.7 Limitations of the present analysis : The conclusions reached in the earlier sections are not of mere academic interest, and considering their practical significance, it seems necessary to stress here the tentative nature of our conclusions. The present study suffers from a number of limitations, some of which would render refined comparisons either impossible or futile. The whole analysis should be regarded as a rough examination, which can detect only appreciable changes in the economy in a general way, so that all our conclusions are broad and provisional, subject to revisions and refinements as and when further data becomes available.

In the first place, it has to be noted that our study is concerned with consumer expenditure instead of with income. It is quite true that consumer expenditure is more closely related to the level of living than income, so that studies on consumer expenditure should have great bearing on the measurement of levels of living. But in discussions about economic inequality, it is income and not consumer expenditure which is very often considered. This limitation has nothing to do with the rough or tentative nature of our conclusions regarding the expenditure distributions, unless one regards the same conclusions as applicable to the corresponding income distributions, which, in our opinion, would be extremely unsafe.

It should also be remembered that monthly per capita total consumer expenditure cannot serve as a perfect indicator of the level of living. However, for the present purposes, the breakdowns of total consumer expenditure, such as expenditure on foodgrains, or milk, or education, or medicine and medical services, do not seem to be very important; and data on housing conditions etc., useful as they may be in studies on level of living, seem to be somewhat irrelevant in the present context, as such data are more indicative of past earnings than of current earnings. The one real limitation of total consumer expenditure is that it excludes public expenditure on commodities and services. This sector of national expenditure has obviously some influence on inequality, in its true sense : thus, inequality may be reduced appreciably if such expenditure is directed more for the benefit of the poorer classes. It is, however, difficult to state anything about how the benefits of such public expenditure were actually distributed over the different classes of population in India during the period under investigation.

Secondly, there are the limitations of NSS consumer expenditure data utilised for this study. These data were collected by the interview method for a moving reference period, either 'last week' or 'last month', and as mentioned in Section 6.2, the 'month' estimates are generally appreciably lower than the 'week' estimates. This is in keeping with a school of opinion that for a country like India, with a majority of informants illiterate and backward, the interview method cannot be relied upon to yield satisfactory family budget data : the effect of memory lapses and psychological biases is apt to be serious. Imputation of the value of home-consumption etc., (vide footnote 10) cannot always be very satisfactory. The large team of NSS investigators carry out this enquiry year after year in a very routine manner;

and the scale of the survey operations as also its routineness might affect the quality of field work. Again, while it is true that the concepts, definitions and procedures remained more or less unchanged over the NSS rounds, it is generally believed that the quality of field work gradually improved with experience. Even the '<u>Instructions to Field Workers</u>' became clearer and more explicit on many points as the rounds progressed. This is, of course, quite natural, but this implies the lack of perfect comparability between the different rounds. Actually, it is felt that the first few rounds, perhaps up to the third, were somewhat experimental in nature.

Moreover, the estimates obtained from the different rounds do not refer to complete financial or calendar years, or even to the same part of the year (vide footnote 9). The estimates are therefore affected by seasonality, which cannot be ignored in a predominantly agricultural country like India, especially for the rural sector. The comparison of concentration etc., of expenditure distributions relating to different 'seasons' is certainly not meaningless, but it would have been more meaningful to work with ("annual") distributions relating to full years.

There is, however, another and a more subtle effect of seasonality. The seventh round, for example, was carried out between October 1953 and March 1954, but the budgets collected from different sample households related to different parts of the six-month period. Such movement of the reference period may not matter for most tabulations on NSS data, but as regards the distributions of persons by levels of monthly per capita expenditure, seasonal variation exaggerates the variation between persons when the moving reference period is employed. Conceptually, the expenditure distributions would be more meaningful if the data collected from every sample household related to the same period.

Again, while the very approach of this study eliminates the effect of price variation between rounds[15], it cannot do so with the variation in prices within rounds, as different sample households were investigated in different parts of the survey period. There is, moreover, the effect of regional variation in prices, which has been ignored in this study.

Lastly, the use of monthly <u>per capita</u> expenditure as an indicator of level of living may be criticised on the grounds that it takes no account of the age-sex composition of households. Monthly total consumer expenditure <u>per adult or consumption unit</u> would probably have been an improvement.

It is perhaps too much to assume that all these factors have merely resulted in a proportional deviation of the expenditure distributions from 'ideal' distributions of monthly per adult total consumer expenditure.

15 That this elimination is not entirely satisfactory will be seen from the following paragraphs.

Before concluding, it may be worthwhile to reconsider some of the ideas of Section 6.1 and to show that the inequality measures used in this study do not completely answer the problem created by price variation between rounds. As indicated earlier, it is not axiomatic that the distribution of x/μ would give perfect description of inequality : theoretically, the actual values of x/μ do not have much significance in absolute terms. Thus, if x be some monthly per capita expenditure figure in the base period and μ, the mean of such figures, and if x' and μ' be the corresponding figures in another period with a different price set-up, it is in general found that $x/\mu \neq x'/\mu'$. The comparisons made in this study are meaningful in so far as a single cost of living index could set up the equivalence between expenditures in any pair of NSS rounds. As this is not possible in general, that is to say, as the true cost of living index would generally depend on the level of living, the expenditure distribution for each round should have undergone appropriate non-scalar transformations for being reduced to some common set of prices, (e.g., those prevailing in some base round,) before any measure of inequality was calculated. Such transformations would, however, have affected the measures of inequality. This point has been ignored in the present study which compared the inequalities of expenditure distributions for different NSS rounds without first bringing them to constant prices.[16]

It is sometimes stated that in recent years in India, the cost of living index has increased more rapidly for the poorer classes of the population. The different series of cost of living indices published by the States Statistical Bureau, West Bengal, seems to support this statement, although the differences are small. If this were true in general, one could say that the stationary pattern of the NSS expenditure distributions at current prices (after adjustment for means) really means an increasing inequality in the corresponding distributions at constant prices.[17] The idea is clearly of considerable importance; but data on cost of living indices by levels of living are scanty and/or unreliable, so that this line of thought could not be pursued successfully.

6.8 <u>Concluding observations</u> : Although for all practical purposes it suffices to study rural and urban sectors of India separately, it is of some interest to combine the two sectors and to examine the inequality of the all-India distribution of persons by monthly per capita total consumer expenditure. This can be done easily by utilising the projections (estimates) of population given in Som and De [21]

16 This points in fact to a general limitation of the Lorenz curve, the Lorenz ratio and most of the other statistical techniques as commonly employed for comparisons of inequality etc., over time or space.

17 This point is fully discussed in Chapter 7.

for the period covered by the different NSS rounds. These projections have been reproduced in Appendix 6.2 of this Chapter.

Let us consider any single round and denote the projected population in the rural and the urban areas by N_r and N_u respectively. Then the number of persons in the jth expenditure class will be $N_r p_{rj}$ and $N_u p_{uj}$ for the two sectors, and the corresponding contributions to the rural and urban consumption will be $N_r p_{rj} \bar{x}_{rj}$ and $N_u p_{uj} \bar{x}_{uj}$ respectively. (The notation is analogous to those introduced in Section 6.3, footnote 12; only the sector denoting suffix r or u has been added.) Thus, considering the two sectors together, there are $N_r p_{rj} + N_u p_{uj}$ persons in the jth expenditure class and their total consumption is $N_r p_{rj} \bar{x}_{rj} + N_u p_{uj} \bar{x}_{uj}$. From this, it is easy to compute the analogues of the quantities defined in Section 6.3 or footnote 12 for a given sector, and hence the different indices Q_p or D_p for inequality.

Such calculations were taken up for all the rounds, using week as well as month reference data. The all-India distribution was obtained by sub-samples as all the quantities mentioned in the last paragraph were available by sub-samples. The resulting distributions are shown in the Appendix tables. Tables 6.6 and 6.7 below present the various measures of inequality used in this Chapter for the all-India distributions, based on week and month reference period respectively.

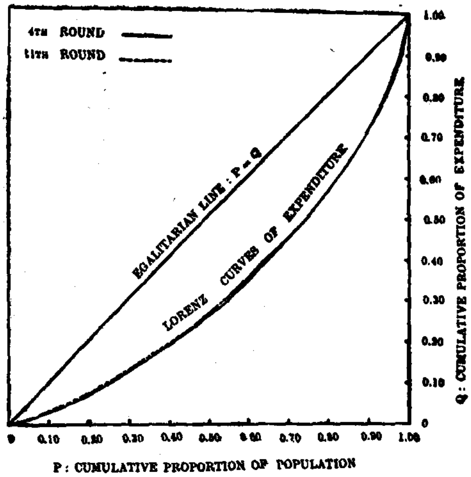

Figure. 6.9. Lorenz curves of total expenditure : All-India (rural and urban combined)

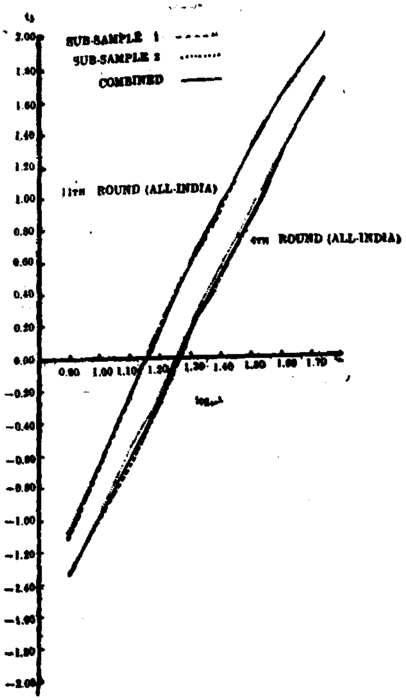

Figure 6.10. Ogives for distributions of persons by monthly per capita expenditure in Rs. (x) on log-probit scale: All-India (rural and urban combined)

Table 6.6 : Some indices of inequality for the all-India distributions based on week reference data

(reference period :week)

round	sub-sample	index or parameter							
		L	$Q_{0.10}$	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	$Q_{0.90}$	$Q_{0.95}$	$1-Q_{0.90}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
3	1	0.351	2.93	9.65	26.51	51.00	71.96	82.92	28.04
	2	0.354	2.80	9.62	26.04	50.63	72.60	83.57	27.49
	comb.	0.353	2.86	9.63	26.24	50.79	72.29	83.27	27.71
4	1	0.336	3.07	10.10	27.25	52.24	73.63	83.99	26.37
	2	0.358	2.84	9.69	26.06	50.30	71.45	82.72	28.55
	comb.	0.347	2.96	9.90	26.66	51.28	72.53	83.39	27.47
5	1	0.336	3.26	10.42	27.64	51.68	72.81	82.36	27.19
	2	0.334	3.34	10.49	27.33	52.02	73.40	83.05	26.60
	comb.	0.335	3.31	10.47	27.51	51.88	73.14	82.74	26.86
6	1	0.351	2.46	9.29	26.81	51.35	72.23	81.68	27.77
	2	0.337	3.15	10.40	27.15	51.75	73.33	83.60	26.67
	comb.	0.345	2.79	9.81	26.89	51.42	72.73	82.57	27.27

Table 6.6 : (contd.)

(reference period :week)

round	sub-sample	1-$Q_{0.95}$	$D^*_{0.10}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.90}$	θ	λ
(1)	(2)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
3	1	17.08	7.32	10.61	16.16	23.55	34.89	3.042	0.644
	2	16.43	7.17	10.61	15.72	24.41	36.13	3.089	0.650
	comb.	16.73	7.23	10.60	15.90	24.03	35.86	3.066	0.648
4	1	16.01	7.61	11.11	16.43	24.70	34.73	3.014	0.614
	2	17.28	7.23	10.69	15.71	23.83	36.50	3.006	0.657
	comb.	16.61	7.42	10.89	16.08	24.27	35.61	3.010	0.636
5	1	17.64	7.60	11.08	16.36	23.71	34.82	2.963	0.614
	2	16.95	7.67	11.28	16.01	23.98	36.53	2.938	0.612
	comb.	17.26	7.63	11.19	16.19	23.81	35.71	2.951	0.612
6	1	18.32	7.18	11.00	16.32	24.14	34.57	2.957	0.644
	2	16.40	7.62	11.00	15.94	24.47	37.15	2.915	0.616
	comb.	17.43	7.39	10.96	16.24	24.29	35.79	2.935	0.632

* D_p's correspond to a value of Rs.20 for the arithmetic mean.

Table 6.7 : Some indices of inequality for all-India distributions based on month reference data

(reference period:month)

round	sub-sample	L	$Q_{0.10}$	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	$Q_{0.90}$	$Q_{0.95}$	$1-Q_{0.90}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
4	1	0.351	2.76	9.63	26.52	50.76	72.81	82.40	27.19
	2	0.349	2.96	9.86	26.52	51.01	72.58	82.52	27.42
	comb.	0.350	2.86	9.73	26.47	50.90	72.75	82.50	27.25
5	1	0.371	2.71	9.33	25.47	49.22	70.33	80.27	29.67
	2	0.332	3.13	10.20	27.51	52.51	74.05	83.33	25.95
	comb.	0.353	2.87	9.73	26.42	50.74	72.01	81.62	27.99
7	1	0.355	3.28	10.19	26.35	50.31	71.29	81.19	28.71
	2	0.342	3.45	10.65	27.06	51.09	72.80	82.88	27.20
	comb.	0.349	3.36	10.42	26.71	50.69	72.01	81.91	27.99
8	1	0.370	3.62	9.36	25.31	49.01	70.46	80.91	29.54
	2	0.382	3.42	9.44	24.38	47.84	70.13	80.39	29.87
	comb.	0.376	3.52	9.40	24.88	48.48	70.40	80.77	29.60
8**	1	0.370	3.61	9.38	25.33	49.06	70.52	81.00	29.48
	2	0.381	3.41	9.45	24.42	47.92	70.19	80.44	29.81
	comb.	0.375	3.51	9.43	24.91	48.56	70.44	80.78	29.56
9	1	0.358	3.49	9.63	25.62	50.15	72.59	82.52	27.41
	2	0.357	3.44	9.87	25.69	50.24	72.11	82.38	27.89
	comb.	0.357	3.47	9.76	25.69	50.24	72.38	82.48	27.62
10	1	0.356	3.33	10.22	26.14	50.18	71.22	81.32	28.78
	2	0.360	3.26	10.07	25.75	49.70	71.76	82.17	28.24
	comb.	0.358	3.29	10.14	25.95	49.95	71.52	81.80	28.48
11	1	0.355	3.60	10.78	26.84	49.90	79.61	79.85	29.39
	2	0.342	3.56	10.75	27.22	51.23	72.15	81.80	27.85
	comb.	0.348	3.59	10.78	27.05	50.55	71.42	80.87	28.58
12	1	0.360	3.27	10.25	26.47	49.83	69.78	79.11	30.22
	2	0.355	3.42	10.48	26.55	50.12	70.83	80.62	29.17
	comb.	0.357	3.35	10.37	26.48	49.99	70.32	79.86	29.68

** For these and subsequent rows, the coverage is all-India, including Jammu & Kashmir, for the preceeding rows, the coverage excludes Jammu & Kashmir. Also, figures for the 12th round are provisional.

Table 6.7 : (contd.)

(reference period : month)

round	sub-sample	index or parameter							
		$1-Q_{0.95}$	$D^*_{0.10}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.90}$	θ	λ
(1)	(2)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
4	1	17.60	7.14	10.93	15.99	24.34	35.44	2.936	0.646
	2	17.48	7.40	10.66	15.90	24.22	36.32	2.911	0.638
	comb.	17.50	7.28	10.76	15.94	24.29	35.86	2.924	0.642
5	1	19.73	7.12	10.69	16.00	23.52	35.94	2.859	0.683
	2	16.67	8.02	11.37	16.97	25.45	36.56	2.853	0.607
	comb.	18.38	7.61	11.01	16.44	24.42	36.24	2.857	0.648
7	1	18.81	6.12	10.58	15.69	23.89	35.65	2.725	0.652
	2	17.12	6.26	11.03	15.76	24.06	36.70	2.698	0.626
	comb.	18.09	6.19	10.80	15.76	24.00	36.12	2.711	0.640
8	1	19.09	4.18	10.17	15.39	23.68	36.99	2.554	0.681
	2	19.61	4.47	9.94	14.74	24.06	37.32	2.609	0.705
	comb.	19.23	4.32	10.08	15.06	23.95	37.52	2.579	0.693
8**	1	19.00	4.20	10.18	15.39	23.70	37.05	2.555	0.681
	2	19.56	4.48	9.95	14.76	24.06	37.28	2.610	0.703
	comb.	19.22	4.34	10.10	15.07	23.95	37.48	2.581	0.691
9	1	17.48	4.58	10.40	15.71	24.73	37.07	2.603	0.657
	2	17.62	4.93	10.51	15.63	24.83	36.82	2.611	0.654
	comb.	17.52	4.75	10.46	15.68	24.80	36.93	2.607	0.655
10	1	18.68	6.41	10.69	15.57	23.54	36.43	2.716	0.654
	2	17.83	6.13	10.38	15.36	23.74	37.74	2.749	0.661
	comb.	18.20	6.25	10.53	15.47	23.75	37.06	2.731	0.657
11	1	20.15	6.58	10.67	15.20	22.98	33.71	2.724	0.654
	2	18.20	6.27	10.94	16.14	22.90	34.94	2.707	0.622
	comb.	19.13	6.41	10.82	15.64	22.89	34.31	2.714	0.638
12	1	20.89	6.74	10.67	15.62	22.30	31.37	2.749	0.661
	2	19.38	6.25	10.61	15.29	23.24	31.82	2.719	0.652
	comb.	20.14	6.47	10.63	15.44	22.77	31.45	2.733	0.655

* The D_p's correspond to a value of Rs.20 for the arithmetic mean.

** For these and subsequent rows, the coverage is all-India, including Jammu & Kashmir, for the preceeding rows, the coverage excludes Jammu & Kashmir. Also, figures for the 12th round are provisional.

As an illustration, Figure 6.9 represents the Lorenz curves of all-India expenditure distributions for 4th and 11th rounds. Figure 6.10 shows the ogives for the same distributions on log-probit scale. The ogive for the 4th round is very nearly linear, while that for the 11th round is only roughly so. However, taking a broad view of things, the log-normal parameters were estimated for all the rounds. These estimates will also be found in Tables 6.6 -6.7. The λ's are of the same order of magnitude, confirming what may be inferred from the other indices of inequality given in Tables 6.6 - 6.7.

It may be interesting to extend our calculations to individual states or zones of India within a particular sector (rural or urban). These are, however, made somewhat difficult by the re-organisation of States in 1956; also the sample sizes are often too small to yield useful results. Notwithstanding these limitations, we give in Table 6.8 some preliminary results statewise using the available 13th round NSS data; the inter regional study covers both rural and urban areas. From a glance at Table 6.8 it would appear that in five States (Kerala, Madhya Pradesh, Mysore, Rajasthan, and Union Territories) the concentration of total consumer expenditure exceeded the all-India level during the 13th round period. A detailed discussion of the inter regional variation in the distribution of consumer expenditure does not seem necessary here although its importance in the context of regional planning cannot be underestimated.

Table 6.8 · Some indicators of inequality of distribution of consumer expenditure - by states, sub-samples, National Sample Survey-13th round (September 1957 - May 1958)

sector	index	Jammu & Kashmir			Andhra Pradesh			Assam		
		s.s.1	s.s.2	comb.	s.s.1	s.s.2	comb.	s.s.1	s.s.2	comb.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
rural	L	0.2578	0.2706	0.2679	0.3102	0.3447	0.3267	0.2515	0.2381	0.2519
	$Q_{0.10}$	3.76	4.23	4.02	3.90	4.05	3.96	4.49	5.15	4.72
	$Q_{0.25}$	11.92	12.33	11.82	11.68	11.82	11.75	13.36	14.86	13.95
	$Q_{0.50}$	31.62	29.93	30.17	28.17	27.60	27.56	31.54	33.95	32.18
	$Q_{0.75}$	59.01	56.84	58.01	54.21	51.14	52.35	59.24	58.31	57.35
	$Q_{0.90}$	81.35	81.83	81.58	76.45	71.35	73.67	80.07	79.40	79.79
	$Q_{0.95}$	89.26	90.31	89.68	85.39	80.36	82.86	88.60	87.77	87.99
	$1-Q_{0.90}$	18.65	18.17	18.42	23.55	28.65	26.33	19.93	20.60	20.21
	$1-Q_{0.95}$	10.74	9.69	10.32	14.61	19.64	17.14	11.40	12.23	12.01
urban	L	0.2918	0.2762	0.2873	0.2720	0.3340	0.3090	0.3949	0.2089	0.2455
	$Q_{0.10}$	4.66	4.85	4.49	4.24	3.46	3.71	3.69	6.70	5.71
	$Q_{0.25}$	12.08	13.33	12.73	13.10	10.23	11.31	10.04	16.82	15.53
	$Q_{0.50}$	29.58	31.76	30.67	31.33	27.28	29.01	24.08	35.26	33.37
	$Q_{0.75}$	55.16	55.39	55.21	56.71	52.36	54.19	44.02	59.12	56.57
	$Q_{0.90}$	76.68	75.10	75.61	77.22	72.91	74.47	73.83	80.97	77.54
	$Q_{0.95}$	85.30	86.94	84.99	86.08	85.63	84.82	86.91	88.57	87.24
	$1-Q_{0.90}$	23.32	24.90	24.39	22.78	27.09	25.53	26.17	19.03	22.46
	$1-Q_{0.95}$	14.70	13.06	15.01	13.92	14.37	15.18	13.09	11.43	12.76

Table 6.8 : (contd.)

sector	index	Bihar			Bombay			Rajasthan		
		s.s.1	s.s.2	comb.	s.s.1	s.s.2	comb.	s.s.1	s.s.2	comb.
(1)	(2)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
rural	L	0.2950	0.3305	0.3125	0.2786	0.3061	0.2927	0.3957	0.3796	0.3926
	$Q_{0.10}$	3.69	3.14	3.17	3.83	2.80	3.68	2.22	3.10	2.60
	$Q_{0.25}$	11.94	9.42	10.70	11.72	10.20	11.28	8.24	10.01	9.03
	$Q_{0.50}$	29.94	26.81	28.40	30.73	30.00	29.79	24.04	25.58	24.25
	$Q_{0.75}$	55.35	53.31	54.39	56.60	54.48	55.53	46.30	46.72	46.48
	$Q_{0.90}$	76.44	76.36	76.45	77.82	76.41	77.13	68.08	69.20	67.72
	$Q_{0.95}$	85.16	85.84	85.39	87.00	86.19	86.56	84.04	81.35	82.84
	$1-Q_{0.90}$	23.56	23.64	23.55	22.18	23.59	22.87	31.92	30.80	32.28
	$1-Q_{0.95}$	14.84	14.16	14.61	13.00	13.81	13.44	15.96	18.65	17.16
urban	L	0.3029	0.4144	0.3656	0.3497	0.3437	0.3473	0.3183	0.3631	0.3430
	$Q_{0.10}$	4.73	3.09	3.76	3.00	2.99	3.00	4.33	2.73	3.42
	$Q_{0.25}$	12.56	8.74	10.28	9.61	9.85	9.72	12.34	9.41	10.81
	$Q_{0.50}$	29.00	22.34	24.92	25.91	26.83	26.31	28.76	26.01	27.26
	$Q_{0.75}$	53.15	46.54	49.09	50.58	50.63	50.60	52.00	49.32	50.65
	$Q_{0.90}$	75.64	69.44	71.31	73.52	73.12	73.34	73.03	69.88	71.28
	$Q_{0.95}$	86.58	83.93	82.33	85.65	85.81	85.72	83.73	84.28	83.96
	$1-Q_{0.90}$	24.36	30.56	28.69	26.48	26.88	26.66	26.97	30.12	28.72
	$1-Q_{0.95}$	13.42	16.07	17.67	14.35	14.19	14.28	16.27	15.72	16.04

Table 6.8 : (contd.)

sector	index	Kerala			Madhya Pradesh			Madras		
		s.s.1	s.s.2	comb.	s.s.1	s.s.2	comb.	s.s.1	s.s.2	comb.
(1)	(2)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)
rural	L	0.3567	0.3266	0.3449	0.4261	0.3689	0.4022	0.3128	0.3203	0.3160
	$Q_{0.10}$	3.55	3.89	3.71	2.86	3.52	3.16	3.99	4.10	4.03
	$Q_{0.25}$	10.18	11.36	10.72	8.95	10.62	9.71	11.31	10.72	11.00
	$Q_{0.50}$	25.20	28.11	26.51	22.59	26.55	24.29	28.71	37.11	28.27
	$Q_{0.75}$	50.26	51.06	50.48	43.36	48.48	45.63	54.19	66.91	53.51
	$Q_{0.90}$	72.59	74.28	71.59	64.34	67.65	65.48	75.30	93.50	79.44
	$Q_{0.95}$	84.08	84.42	81.63	77.90	77.96	76.28	84.56	96.74	84.39
	$1-Q_{0.90}$	27.41	25.72	28.41	35.66	32.35	34.52	24.70	6.50	20.56
	$1-Q_{0.95}$	15.92	15.58	18.37	22.10	22.04	23.72	15.46	3.26	15.61
urban	L	0.3440	0.3477	0.3488	0.3400	0.3387	0.3419	0.3894	0.4276	0.4049
	$Q_{0.10}$	3.30	3.44	3.37	3.67	2.90	3.04	2.77	2.75	2.80
	$Q_{0.25}$	9.43	10.42	9.91	10.81	8.81	9.72	9.14	8.64	8.94
	$Q_{0.50}$	26.25	27.20	26.32	26.63	25.93	25.95	24.39	21.92	23.01
	$Q_{0.75}$	51.77	49.17	50.71	50.64	53.88	52.18	48.33	42.90	45.57
	$Q_{0.90}$	73.53	72.71	73.09	74.25	75.69	74.42	69.72	65.14	67.41
	$Q_{0.95}$	86.56	83.67	85.07	83.51	83.30	83.36	82.48	81.56	81.90
	$1-Q_{0.90}$	26.47	27.29	26.91	25.75	24.31	25.58	30.28	34.86	32.59
	$1-Q_{0.95}$	13.44	16.33	14.93	16.49	16.70	16.64	17.52	18.44	18.10

Table 6.8 : (contd.)

sector	index	Mysore			Orissa			Punjab		
		s.s.1	s.s.2	comb.	s.s.1	s.s.2	comb.	s.s.1	s.s.2	comb.
(1)	(2)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)
	L	0.4101	0.3005	0.3737	0.3323	0.4630	0.3133	0.3369	0.2904	0.3166
	$Q_{0.10}$	2.53	3.64	3.02	4.15	3.37	4.39	3.16	3.46	3.24
	$Q_{0.25}$	7.99	11.71	9.68	10.38	8.51	10.98	10.87	10.72	10.80
	$Q_{0.50}$	22.19	29.32	25.38	26.87	22.17	28.76	27.63	28.34	27.93
rural	$Q_{0.75}$	44.72	55.18	48.97	52.60	39.80	53.92	50.81	57.37	53.56
	$Q_{0.90}$	65.15	75.40	68.65	73.84	53.94	74.15	72.81	79.73	75.48
	$Q_{0.95}$	81.57	84.03	78.47	84.16	88.01	83.34	86.17	87.99	85.72
	$1-Q_{0.90}$	34.85	24.60	31.35	26.16	46.06	25.85	27.19	21.27	24.52
	$1-Q_{0.95}$	18.43	15.97	21.53	15.84	11.99	16.66	13.83	12.01	14.28
	L	0.3021	0.2992	0.3022	0.3242	0.4268	0.3808	0.3423	0.3350	0.3351
	$Q_{0.10}$	3.35	3.33	3.33	4.38	3.67	4.01	3.84	3.31	3.58
	$Q_{0.25}$	11.15	10.67	10.90	12.12	9.17	10.05	11.26	10.40	10.44
urban	$Q_{0.50}$	29.02	28.42	28.74	29.35	22.00	24.83	27.59	27.31	26.93
	$Q_{0.75}$	54.81	56.88	55.10	51.45	44.35	47.07	52.59	51.55	51.99
	$Q_{0.90}$	76.69	78.77	77.81	70.52	67.18	66.57	74.18	72.82	73.34
	$Q_{0.95}$	86.64	87.54	87.12	81.29	83.59	82.36	84.02	85.65	84.26
	$1-Q_{0.90}$	23.31	21.23	22.19	29.48	32.82	33.43	25.82	27.18	26.66
	$1-Q_{0.95}$	13.36	12.46	12.88	18.71	16.41	17.64	15.98	14.35	15.74

Table 6.8 : (contd.)

sector	index	Uttar Pradesh			West Bengal			Union Territories		
		s.s.1	s.s.2	comb.	s.s.1	s.s.2	comb.	s.s.1	s.s.2	comb.
(1)	(2)	(39)	(40)	(41)	(42)	(43)	(44)	(45)	(46)	(47)
rural	L	0.2822	0.3027	0.2932	0.2463	0.2802	0.2649	0.4142	0.3177	0.3532
	$\Omega_{0.10}$	3.61	3.72	3.67	4.31	4.13	4.22	3.50	3.47	3.49
	$\Omega_{0.25}$	11.83	11.47	11.65	13.19	12.48	12.83	9.99	11.21	10.77
	$\Omega_{0.50}$	30.38	29.97	30.14	33.51	30.88	32.13	23.35	30.28	27.16
	$\Omega_{0.75}$	56.71	54.29	55.43	58.80	55.99	57.31	44.30	53.62	49.21
	$\Omega_{0.90}$	77.41	75.14	76.23	78.83	77.61	78.21	74.13	69.99	68.06
	$\Omega_{0.95}$	86.41	84.69	85.69	87.19	86.99	87.03	87.06	84.61	83.93
	$1-\Omega_{0.90}$	22.59	24.86	23.77	21.17	22.39	21.79	25.87	30.01	31.94
	$1-\Omega_{0.95}$	13.59	15.31	14.31	12.81	13.01	12.97	12.94	15.39	16.07
urban	L	0.3423	0.3350	0.3351	0.4054	0.3592	0.3847	0.3476	0.2982	0.3230
	$\Omega_{0.10}$	3.84	3.31	3.58	2.87	3.34	3.00	2.94	2.38	2.05
	$\Omega_{0.25}$	11.26	10.40	10.44	8.76	10.84	9.79	9.77	11.05	10.14
	$\Omega_{0.50}$	27.59	27.31	26.93	23.60	26.95	25.24	27.05	28.61	27.43
	$\Omega_{0.75}$	52.59	51.55	51.99	45.67	48.69	46.68	49.24	55.51	53.19
	$\Omega_{0.90}$	74.18	72.82	73.34	65.75	69.46	67.31	73.38	78.04	76.20
	$\Omega_{0.95}$	84.02	85.65	84.26	80.29	80.64	79.31	86.69	89.02	88.10
	$1-\Omega_{0.90}$	25.82	27.18	26.66	34.25	30.54	32.69	26.62	21.96	23.80
	$1-\Omega_{0.95}$	15.98	14.35	15.74	19.71	19.36	20.69	13.31	10.98	11.90

Also it would be interesting to combine distributions for consecutive rounds, e.g., 4th and 5th, or 11th and 12th, so as to obtain what might be called "annual" distributions. For purposes of such combination, the distributions of different rounds will have to be weighted in proportion to the durations of the respective rounds. After "annual" distributions are obtained, the comparisons described in this paper may perhaps be carried out more meaningfully.

Finally, it might be appropriate to use such annual distributions to derive the corresponding income distributions for India. This would of course involve precise idea of the distribution of personal savings by levels of living. Recently a few attempts have been made to obtain income distributions from the all-India NSS expenditure distributions given in our Appendix tables [7, 17] In either of these studies the assumption regarding savings is somewhat arbitrary, if not wholly unrealistic. It should now be possible to formulate more realistic savings functions when detailed results of the National Council of Applied Economic Research (NCAER) saving surveys are available.

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Appendix 6.1 : NSS expenditure distributions utilised in this study

We present in Tables A-6.2 - A.6.5 the basic material employed in our analysis, viz., the estimated distributions of persons by monthly per capita total consumer expenditure, thrown up by the different rounds of the NSS.

Some background information regarding NSS consumer expenditure data has been given in Section 6.2 of this Chapter. It may be necessary to refer to that Section for an understanding of Tables A.6.2 - A.6.5.

Tables A.6.2 - A.6.5 are more or less self-explanatory. The figures in cols. (3), (5), ..., (25) of these tables are the percentages of population falling in the twelve monthly per capita expenditure classes. The figures in cols. (4), (6).......,(26) are the corresponding averages of monthly per capita expenditure, for persons belonging to each of these twelve classes of monthly per capita expenditure. Col. (28) gives the over-all mean of monthly per capita expenditure.

The sample design adopted by the NSS is generally of a complicated nature. Broadly, however, it is of the multistage stratified type. For the rural sector, usually, the country is divided into a large number of geographical divisions (strata) and from each stratum, households are selected by a two - (or three-) stage process, villages and households forming the sample units (at the last two stages). The procedure is more or less parallel for the urban sector, with census blocks taking the place of villages : only, for obvious reasons, the strata were not always geographically compact. In all cases, there was provision for at least two independent and interpenetrating sub-samples.

Although the NSS samples deviated seriously from simple random samples of households, it would be useful to keep in view the sample sizes in Table A.6.1 while examining the distributions in the basic Tables A.6.2 - A.6.5. It may be noted, however, that the sample size for some of the earlier rounds (e.g., 4th) was actually much larger than reported in Table A.6.1 : the full material was not tabulated for obtaining the present type of distributions.

Table A.6.1 : Sample sizes underlying the distributions in Tables A.6.2 - A.6.5

NSS round no.	rural		urban	
	number of villages	number of households	number of census blocks	number of households
(1)	(2)	(3)	(4)	(5)
		reference period : week		
2nd round	1144	2210	-	-
3rd "	905	5103	490	1690
4th "	946	2393	406	1085
5th "	942	1359	405	615
6th "	955	1403	432	514

Table A.6.1 (contd.)

NSS round no.	rural		urban	
	number of villages	number of households	number of census blocks	number of households
(1)	(2)	(3)	(4)	(5)
		reference period : month		
4th round	946	2388	406	1074
5th "	942	1361	405	618
7th "	954	1413	441	558
*8th "	670	1798	442	1780
8th "	706	1869	466	1855
9th "	1624	1616	2102	2099
10th "	1624	1616	1328	1326
11th "	1840	7255	582	2840
12th "	1836	7248	584	2858
13th "	1848	6738	-	-
14th (first sub-round)	1265	1198		

* For all rows up to this, the geographical coverage was all-India excluding Jammu & Kashmir; for the rows subsequent to this, the coverage was all-India including Jammu & Kashmir.

The following are the sources of the data presented in this appendix.

(i) The NSS Report No.20 : Pattern of Consumer Expenditure : Second to Seventh Rounds, Cabinet Secretariat, Government of India.

(ii) The NSS Draft (Report) No.48 : Tables with Notes on Consumer Expenditure : Eighth Round, July 1954-March 1955.

(iii) The NSS Draft (Report) No.42 : Tables with Notes on Consumer Expenditure : Ninth Round, May - Nov. 1955.

(iv) The NSS Draft (Report) No.45 : Tables with Notes on Consumer Expenditure : Tenth Round, December, 1955 - May 1956.

(v) The NSS Draft (Report) No.72 : Tables with Notes on Consumer Expenditure : Eleventh Round, August 1956 - January 1957.

The publication numbered (i) is available in print, while those numbered (ii) - (v) are drafts submitted to Government of India by the Indian Statistical Institute. The data for the 12th round, the 13th round and the 1st sub-round of the 14th round were obtained from the NSS Department of the ISI : these data are yet to be submitted to Government in the form of draft reports.

Table A.6.2 : Estimated distribution of persons by monthly per capita total consumer expenditure over NSS rounds : All-India (rural)

reference period:week

round	sub-sample	monthly per capita expenditure classes in Rs.											
		0 - 8		8 - 11		11 - 13		13 - 15		15 - 18		18 - 21	
		100P	x̄	100P	x̄	100P	x̄	100P	x̄	100P	x̄	100P	x̄
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
2	1	5.23	5.70	8.00	9.33	5.46	11.00	9.24	13.57	13.86	15.86	11.16	18.57
	2	7.91	5.53	10.83	8.95	8.06	11.71	7.51	13.21	11.22	16.37	7.50	18.91
	comb.	6.56	5.61	9.41	9.09	6.75	11.44	8.38	13.41	12.54	16.10	9.34	18.70
3	1	6.13	6.11	9.54	9.41	8.03	12.04	7.43	14.08	9.82	16.56	11.79	19.65
	2	6.59	5.80	8.00	10.02	7.83	12.13	7.24	14.07	12.21	16.71	10.31	19.61
	comb.	6.37	5.94	8.75	9.69	7.93	12.11	7.33	14.07	11.05	16.63	11.02	19.67
4	1	6.93	6.15	9.04	9.81	8.37	11.68	8.91	13.82	11.70	16.39	11.61	19.67
	2	7.91	5.84	10.27	9.94	7.62	11.97	9.63	14.00	13.96	16.55	8.52	19.47
	comb.	7.43	6.02	9.66	9.89	7.99	11.82	9.27	13.92	12.85	16.49	10.04	19.59
5	1	6.98	6.68	10.75	10.09	8.21	12.10	7.81	14.06	13.55	16.13	10.99	19.71
	2	7.80	6.95	10.99	9.79	8.29	12.11	10.84	13.83	13.43	16.50	10.31	19.56
	comb.	7.39	6.34	10.87	9.94	8.25	12.10	9.33	13.93	13.49	16.33	10.65	19.64
6	1	9.42	5.00	8.86	9.58	8.07	11.99	7.36	14.12	13.09	16.55	13.98	19.50
	2	8.85	6.32	13.12	9.98	9.82	12.20	11.58	13.98	12.18	16.78	8.22	19.11
	comb.	9.13	5.64	10.99	9.82	8.95	12.11	9.48	14.01	12.63	16.68	11.09	19.35

Table A. 6.2:(contd.)

reference period : week

round	sub-sample	monthly per capita expenditure classes in Rs. 21 - 24		24 - 28		28 - 34		34 - 43		43 - 55		55 & above		all classes	
		100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$
(1)	(2)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
2	1	7.50	21.80	7.66	26.17	10.88	30.14	6.84	36.50	5.56	47.64	8.61	80.02	100.00	26.04
	2	8.72	21.36	8.38	24.98	11.34	29.41	6.88	37.25	5.52	44.02	6.13	76.58	100.00	23.48
	comb.	8.11	21.57	8.02	25.52	11.11	29.77	6.86	36.87	5.54	45.86	7.38	77.89	100.00	24.76
3	1	9.68	22.66	11.76	25.78	8.64	30.91	8.48	38.25	4.12	48.19	4.58	81.99	100.00	24.25
	2	8.31	22.40	9.45	25.80	10.05	31.30	9.16	37.64	6.63	48.79	4.22	87.13	100.00	25.02
	comb.	8.98	22.55	10.57	25.78	9.36	31.10	8.83	37.92	5.41	48.56	4.40	84.50	100.00	24.02
4	1	8.45	22.15	9.46	25.74	10.07	31.06	8.66	38.02	3.19	47.79	3.61	78.54	100.00	22.82
	2	7.51	22.26	10.46	25.79	9.55	30.51	6.37	37.70	4.08	47.65	4.12	85.13	100.00	22.84
	comb.	7.97	22.19	9.97	25.75	9.81	30.80	7.50	37.90	3.64	47.71	3.87	82.09	100.00	22.83
5	1	12.74	22.38	7.16	25.20	9.26	30.78	6.09	37.12	3.89	47.96	2.57	80.46	100.00	21.40
	2	7.17	22.30	11.15	25.98	6.51	30.63	6.78	36.03	3.23	45.71	3.50	76.73	100.00	21.18
	comb.	9.94	22.35	9.16	25.68	7.88	30.71	6.44	36.54	3.56	46.93	3.04	78.28	100.00	21.31
6	1	7.14	22.72	8.84	25.75	10.23	30.50	6.48	36.94	3.62	47.99	2.91	87.71	100.00	21.91
	2	8.63	22.43	6.94	25.80	9.14	30.94	4.29	37.51	4.03	47.28	3.20	75.17	100.00	20.78
	comb.	7.89	22.55	7.89	25.78	9.68	30.70	5.38	37.17	3.83	47.61	3.06	81.15	100.00	21.31

Table A.6.3 : Estimated distribution of persons by monthly per capita total consumer expenditure over NSS rounds : All-India (rural)

reference period : month

round	sub-sample	monthly per-capita expenditure classes in Rs.									
		0 - 8		8 - 11		11 - 13		13 - 15		15 - 18	
		100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
4	1	10.16	6.02	10.68	9.49	7.21	12.37	9.22	13.85	13.17	16.42
	2	9.97	6.18	12.16	9.58	9.36	12.13	6.84	14.07	14.03	16.48
	comb.	10.06	6.09	11.42	9.54	8.29	12.22	8.03	13.94	13.60	16.45
5	1	14.13	6.30	17.56	9.54	11.44	12.08	10.34	13.95	14.95	16.47
	2	15.29	6.42	19.17	9.16	12.26	12.13	11.19	14.03	10.88	16.58
	comb.	14.71	6.37	18.36	9.58	11.85	12.12	10.76	13.99	12.92	16.52
7	1	15.21	6.19	17.49	9.55	9.14	11.91	12.44	13.90	10.98	16.27
	2	15.73	6.22	18.09	9.67	14.85	11.91	8.24	14.07	10.69	16.15
	comb.	15.47	6.20	17.80	9.63	12.04	11.92	10.31	13.96	10.83	16.21
8	1	26.85	5.83	18.92	9.46	11.64	11.97	8.92	13.97	10.09	16.55
	2	25.56	5.92	22.33	9.48	11.00	11.97	8.23	14.05	8.50	16.44
	comb.	25.19	5.80	20.65	9.48	11.31	11.97	8.57	14.01	9.28	16.49
*8	1	26.67	5.83	18.89	9.46	11.71	11.98	8.86	13.98	10.11	16.55
	2	23.43	5.92	22.32	9.50	10.96	11.97	8.29	14.04	8.59	16.45
	comb.	25.03	5.80	20.63	9.48	11.33	11.98	8.57	14.00	9.34	16.50
9	1	23.90	5.82	18.67	8.54	11.44	12.03	8.60	14.10	9.90	16.41
	2	21.99	5.78	20.95	9.50	10.11	12.04	9.02	13.86	18.54	16.44
	comb.	22.95	5.81	19.82	9.52	10.77	12.04	8.81	13.96	9.22	16.42
10	1	15.35	6.23	18.42	9.29	12.63	11.95	9.51	14.00	11.24	16.63
	2	15.77	6.34	17.75	9.55	11.83	12.03	9.26	13.89	11.45	16.42
	comb.	15.57	6.27	18.07	9.41	12.23	11.97	9.38	13.96	11.35	16.52
11	1	14.07	6.83	18.76	9.56	12.54	12.96	12.14	14.13	12.15	16.48
	2	15.74	6.54	17.58	9.54	12.33	11.99	8.97	13.97	14.45	16.50
	comb.	14.89	6.74	18.20	9.54	12.43	11.97	10.59	14.06	13.27	16.51
12	1	11.29	5.96	11.97	9.15	9.62	11.43	7.71	12.66	13.61	16.35
	2	8.71	5.87	13.54	9.29	8.69	11.52	9.37	13.78	13.15	16.14
	comb.	10.01	5.91	12.74	9.22	9.16	11.47	8.54	13.28	13.37	16.26
13	1	11.58	6.15	16.02	9.49	9.54	12.06	10.65	14.03	12.09	16.45
	2	13.30	6.18	16.68	9.52	10.75	12.02	9.26	14.07	15.02	16.43
	comb.	12.44	6.17	16.35	9.52	10.14	12.03	9.96	14.04	13.55	26.47
14(1)	1	7.83	5.91	15.97	9.50	11.75	12.01	12.53	14.17	9.78	16.28
	2	10.30	6.28	12.08	9.44	13.02	12.03	9.92	13.90	10.65	16.30
	comb.	9.09	6.13	14.05	9.48	12.38	12.04	11.88	14.07	10.21	16.30

* For this row and for the subsequent rows, the geographical coverage was all-India including Jammu & Kashmir, for all the previous rows, the coverage was all-India excluding Jammu & Kashmir.

Table A.6.3 : (contd.)

reference period : month

round	sub-sample	monthly per-capita expenditure classes in Rs.							
		18-21		21-24		24-28		28-34	
		100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$
(1)	(2)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
4	1	12.62	19.60	6.00	22.88	8.20	25.95	7.94	30.82
	2	10.89	19.60	8.21	22.49	7.42	25.92	7.93	30.00
	comb.	11.76	19.62	7.10	22.65	7.81	25.93	7.93	30.42
5	1	9.64	19.80	5.94	22.74	6.10	26.05	3.48	30.37
	2	8.23	19.75	6.09	22.41	6.71	25.79	3.91	30.57
	comb.	8.94	19.76	6.02	22.57	6.41	25.91	3.65	30.48
7	1	9.22	19.04	7.38	22.07	6.67	26.25	4.07	30.72
	2	8.29	19.08	6.51	22.47	4.90	25.81	5.55	30.43
	comb.	8.75	19.06	6.94	22.26	5.77	26.06	4.82	30.53
8	1	6.52	19.52	4.56	22.39	3.36	25.64	3.42	30.21
	2	6.79	19.41	5.41	22.38	3.93	25.96	5.38	30.33
	comb.	6.65	19.45	4.99	22.38	3.65	25.81	4.42	30.28
8	1	6.62	19.53	4.56	22.40	3.36	25.65	3.44	30.25
	2	6.80	19.42	5.41	22.37	3.95	25.96	5.41	30.33
	comb.	6.71	19.46	4.99	22.37	3.66	25.82	4.44	30.30
9	1	7.70	19.29	5.36	22.56	5.02	25.57	4.97	30.27
	2	8.28	19.31	7.68	22.30	4.69	25.63	3.43	30.35
	comb.	7.99	19.33	6.53	22.41	4.85	25.60	4.19	30.31
10	1	9.04	19.61	5.57	22.28	6.49	25.67	3.86	29.94
	2	6.83	19.50	8.45	22.42	3.78	25.81	5.44	30.94
	comb.	7.92	19.59	7.03	22.36	5.12	25.73	4.66	30.54
11	1	7.51	19.75	6.08	22.41	5.98	25.97	5.39	30.78
	2	10.18	19.43	5.14	22.39	4.84	25.85	4.76	30.50
	comb.	8.81	19.58	5.62	22.40	5.42	25.92	5.08	30.68
12	1	12.35	19.03	8.12	22.42	7.83	25.48	6.51	30.61
	2	12.24	18.89	6.18	21.75	8.21	24.52	9.37	29.80
	comb.	12.30	18.96	7.16	22.13	8.02	24.98	7.93	30.13
13	1	10.32	19.47	8.45	22.32	6.42	25.72	5.81	30.53
	2	8.02	19.39	6.42	22.48	6.85	25.49	5.49	30.93
	comb.	9.17	19.41	7.44	22.40	6.63	25.82	5.65	30.72
14(1)	1	11.48	19.82	6.75	22.52	7.51	25.33	6.21	30.18
	2	8.61	19.01	7.92	22.98	7.23	25.63	6.03	29.24
		10.06	19.49	7.33	22.77	7.37	25.48	6.12	29.72

Table A.6.3 : (contd.)

reference period : month

round	sub-sample	monthly per-capita expenditure classes in Rs.							
		34-43		43-55		55 & above		all classes	
		100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$
(1)	(2)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
4	1	8.29	38.09	3.52	47.57	2.99	84.82	100.00	21.74
	2	6.17	37.64	3.79	48.03	3.23	86.72	100.00	21.41
	comb.	7.23	37.87	3.66	47.84	3.11	85.82	100.00	21.57
5	1	3.37	37.80	1.49	47.70	1.55	105.82	100.00	17.39
	2	2.31	37.33	1.91	48.31	2.05	89.22	100.00	17.21
	comb.	2.84	37.61	1.70	48.05	1.80	96.39	100.00	17.30
7	1	3.65	36.76	1.61	49.16	2.14	89.73	100.00	17.65
	2	3.81	37.01	2.32	48.89	1.02	87.67	100.00	16.82
	comb.	3.73	36.89	1.97	48.98	1.57	89.05	100.00	17.24
8	1	2.39	37.65	1.40	49.18	1.93	76.50	100.00	14.91
	2	2.90	37.68	0.60	47.73	1.37	74.04	100.00	14.97
	comb.	2.65	37.67	1.00	48.73	1.64	75.46	100.00	14.93
8	1	2.48	37.60	1.39	49.18	1.91	76.54	100.00	14.93
	2	2.88	37.69	0.60	47.73	1.36	74.04	100.00	14.98
	comb.	2.68	37.63	0.99	48.73	1.63	75.49	100.00	14.96
9	1	1.86	36.77	1.01	46.20	1.57	67.49	100.00	15.06
	2	2.29	37.92	1.57	46.44	1.45	71.69	100.00	15.33
	comb.	2.07	37.37	1.29	46.32	1.51	69.52	100.00	15.20
10	1	4.42	37.54	1.20	48.80	2.27	81.09	100.00	17.30
	2	4.28	37.72	2.94	45.30	2.22	80.99	100.00	17.91
	comb.	4.35	37.64	2.08	46.30	2.24	81.05	100.00	17.61
11	1	2.65	38.20	1.62	48.06	1.11	119.29	100.00	17.17
	2	3.27	37.87	1.74	47.90	1.00	103.38	100.00	16.75
	comb.	2.95	38.06	1.68	47.99	1.06	111.99	100.00	16.97
12	1	5.39	37.52	2.69	48.19	2.91	76.47	100.00	19.78
	2	4.70	34.71	2.39	41.90	3.45	75.09	100.00	19.88
	comb.	5.05	36.25	2.54	45.26	3.18	75.73	100.00	19.81
13	1	4.48	37.89	1.82	48.88	2.82	82.14	100.00	19.10
	2	4.53	38.49	1.70	47.43	1.98	81.45	100.00	18.11
	comb.	4.51	38.19	1.76	48.21	2.40	81.87	100.00	18.59
14	1	6.29	38.08	2.01	48.85	1.49	100.97	100.00	19.27
	2	8.97	37.78	3.22	51.10	2.87	68.66	100.00	20.43
	comb.	7.61	37.89	2.61	50.24	2.17	79.92	100.00	19.85

Table A.6.4 Estimated distribution of persons by monthly per capita total consumer expenditure over NSS rounds : All-India (urban)

reference period : week

round	sub-sample	monthly per capita expenditure classes in Rs.													
		0-8		8-11		11-13		13-15		15-18		18-21		21-24	
		100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
3	1	3.24	6.28	7.20	9.54	6.70	11.90	6.31	14.05	8.63	16.38	8.91	19.58	6.37	22.67
	2	2.51	6.46	3.73	9.98	4.26	12.15	5.18	14.09	10.53	16.26	9.17	19.54	7.32	22.47
	comb.	2.87	6.38	5.43	9.69	5.45	11.99	5.73	14.06	9.62	16.32	9.04	19.57	6.85	22.56
4	1	2.71	6.57	5.87	9.56	3.63	12.00	6.16	14.07	10.02	16.66	7.68	19.28	11.95	22.23
	2	2.28	6.17	1.59	9.69	7.57	11.97	4.90	14.16	7.84	16.30	11.53	19.41	8.43	22.77
	comb.	2.50	6.39	3.81	9.59	5.52	12.00	5.55	14.11	8.98	16.51	9.53	19.36	10.26	22.42
5	1	7.50	6.39	6.09	9.81	8.25	12.28	5.08	14.15	13.88	16.56	7.73	19.90	8.49	22.38
	2	5.28	5.88	9.82	10.00	3.59	12.10	9.88	13.91	10.93	16.45	9.62	19.06	4.71	22.25
	comb.	6.36	6.16	8.00	9.93	5.86	12.22	7.54	13.98	12.36	16.48	8.70	19.42	6.54	22.32
6	1	4.34	6.68	10.06	9.54	4.11	12.07	5.22	14.40	8.57	16.84	10.23	19.49	6.48	22.85
	2	4.89	6.40	3.34	9.58	3.87	11.97	7.85	13.94	10.19	16.34	11.45	19.34	10.19	22.96
	comb.	4.61	6.51	6.80	9.55	3.99	12.02	6.49	14.13	9.35	16.56	10.82	19.40	8.28	22.91

Table A.6.4 : (contd.)

reference period : week

round	sub-sample	monthly per capita expenditure classes in Rs. 24-28		28-34		34-43		43-55		55 & above		all classes	
		100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$
(1)	(2)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
3	1	10.87	25.97	12.33	31.05	9.04	38.30	7.66	48.60	12.74	97.88	100.00	33.52
	2	9.35	25.92	8.39	31.39	13.25	37.55	10.34	48.65	15.97	92.04	100.00	36.70
	comb.	10.09	25.93	10.32	31.20	11.19	37.81	9.03	48.63	14.40	94.56	100.00	35.13
4	1	7.03	25.98	10.49	30.68	11.46	37.52	7.90	46.44	15.10	78.72	100.00	32.73
	2	9.85	25.99	10.64	30.33	10.66	38.71	9.28	47.71	15.43	88.14	100.00	35.27
	comb.	8.38	25.98	10.56	30.48	11.08	38.06	8.56	47.09	15.27	83.29	100.00	33.93
5	1	6.37	26.30	11.68	31.84	8.10	39.41	5.79	48.59	11.05	99.88	100.00	30.89
	2	8.80	25.01	9.34	30.76	9.44	37.92	9.35	46.75	9.24	86.38	100.00	28.92
	comb.	7.62	25.54	10.48	31.36	8.79	38.59	7.62	47.44	10.12	93.56	100.00	29.92
6	1	12.43	25.88	12.17	30.87	9.68	37.56	4.13	49.74	12.58	91.22	100.00	31.54
	2	11.43	25.42	12.53	30.11	6.38	38.41	9.72	47.50	8.16	85.22	100.00	29.10
	comb.	11.95	25.66	12.35	30.50	8.08	37.88	6.84	48.20	10.44	88.94	100.00	30.35

Table A.6.5 : Estimated distribution of persons by monthly per capita total consumer expenditure over NSS rounds : All-India (urban)

reference period : month

round	sub-sample	monthly per capita expenditure classes in Rs.									
		0-8		8-11		11-13		13-15		15-18	
		100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
4	1	3.83	6.77	9.83	9.70	6.75	11.98	6.98	13.69	11.51	17.40
	2	2.97	6.64	9.39	9.59	8.62	11.97	8.81	14.37	12.17	16.51
	comb.	3.39	6.81	9.15	9.64	7.64	11.96	7.90	14.06	11.86	16.98
5	1	5.27	5.86	7.39	9.69	8.97	11.83	7.33	14.53	11.41	16.70
	2	7.21	6.64	10.04	9.21	8.01	11.62	5.53	13.98	11.09	16.16
	comb.	6.19	6.19	8.64	9.44	8.51	11.74	6.47	14.32	11.27	16.45
7	1	7.90	6.15	18.93	9.60	7.83	11.87	7.54	13.72	11.53	16.19
	2	7.14	6.36	10.17	9.00	9.33	11.94	11.13	14.19	11.18	16.37
	comb.	7.52	6.24	12.09	9.36	8.56	11.92	9.29	14.01	11.36	16.27
8	1	9.39	6.33	12.98	9.50	11.80	12.14	9.67	13.99	10.06	16.34
	2	8.10	6.32	10.79	9.57	6.46	12.00	9.32	14.05	8.13	16.34
	comb.	8.76	6.34	11.91	9.55	9.19	12.08	9.50	14.02	9.12	16.33
*8	1	9.34	6.33	13.03	9.48	11.78	12.12	9.69	13.99	10.14	16.35
	2	8.15	6.30	10.92	9.57	6.53	12.00	9.28	14.06	8.11	16.34
	comb.	8.76	6.33	11.99	9.53	9.21	12.09	9.49	14.02	9.15	16.32
9	1	7.59	6.25	11.83	9.20	9.46	11.78	8.18	13.92	12.48	16.28
	2	7.74	6.11	15.45	9.70	9.89	12.02	7.38	14.13	11.46	16.45
	comb.	7.66	6.18	13.61	9.48	9.67	11.91	7.79	14.03	11.98	16.35
10	1	4.75	6.24	12.90	9.27	10.45	11.97	6.57	14.15	12.72	16.43
	2	3.86	6.41	11.69	9.65	8.31	12.04	8.86	13.94	13.04	16.23
	comb.	4.33	6.33	12.31	9.44	9.41	12.01	7.68	14.02	12.87	16.32
11	1	7.94	5.75	12.24	9.62	8.81	12.03	9.43	14.01	12.65	16.42
	2	7.17	5.94	14.55	9.51	8.76	11.95	8.17	13.89	12.63	16.37
	comb.	7.54	5.85	13.43	9.56	8.79	11.99	8.78	13.95	12.65	16.40
12	1	4.25	6.99	10.41	9.41	8.43	12.06	7.85	13.98	12.54	16.45
	2	7.11	6.24	10.63	9.57	9.00	11.97	9.53	14.00	11.48	16.56
	comb.	5.72	6.51	10.52	9.49	8.72	12.01	8.72	13.99	12.00	16.51

* For this row and for the subsequent rows, the geographical coverage was all-India including Jammu & Kashmir; for all the previous rows, the coverage was all-India excluding Jammu & Kashmir.

Table A.6.5 : (contd.)

reference period : month

round	sub-sample	monthly per capita expenditure classes in Rs.							
		18-21		21-24		24-28		28-34	
		100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$
(1)	(2)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
4	1	7.51	18.78	11.19	22.63	7.28	25.97	12.04	31.02
	2	8.57	19.54	6.66	22.33	7.83	26.43	11.13	30.59
	comb.	8.04	19.17	8.91	22.55	7.56	26.22	11.58	30.81
5	1	10.41	19.92	5.61	22.57	7.29	25.89	11.13	31.16
	2	7.94	19.53	11.31	22.33	7.37	26.37	10.00	30.96
	comb.	9.24	19.75	8.31	22.39	7.33	26.12	10.59	31.08
7	1	7.60	19.27	8.19	22.89	10.01	25.68	5.32	30.42
	2	13.34	18.84	7.37	22.26	6.52	25.56	5.50	30.91
	comb.	10.44	18.98	7.79	22.59	8.32	25.64	5.41	30.67
8	1	9.18	19.24	8.34	22.49	8.03	25.82	5.71	30.47
	2	9.98	19.46	7.46	22.13	7.67	26.25	8.18	31.19
	comb.	9.57	19.32	7.91	22.32	7.85	26.02	6.92	30.90
8	1	9.14	19.21	8.31	22.50	8.01	25.83	5.70	30.46
	2	9.98	19.45	7.42	22.14	7.63	26.22	8.16	31.20
	comb.	9.55	19.34	7.88	22.33	7.82	26.02	6.90	30.88
9	1	6.49	19.41	8.64	22.26	6.39	25.91	10.50	30.82
	2	7.54	19.52	7.70	22.38	6.17	25.84	8.04	30.51
	comb.	7.01	19.47	8.18	22.28	6.28	25.86	9.28	30.67
10	1	11.79	19.54	6.84	22.50	7.57	25.99	7.00	31.02
	2	8.66	19.13	7.57	22.85	8.90	25.89	9.49	31.34
	comb.	10.28	19.38	7.19	22.70	8.21	25.94	8.20	31.18
11	1	7.64	19.42	5.89	22.44	9.20	26.03	7.95	30.49
	2	9.56	19.62	7.98	22.70	7.79	25.96	7.15	30.55
	comb.	8.63	19.53	6.96	22.60	8.47	26.00	7.54	30.52
12	1	11.36	19.57	8.49	22.50	8.72	25.71	8.66	30.74
	2	10.84	19.48	6.77	22.64	7.83	25.88	7.86	30.51
	comb.	11.09	19.55	7.60	22.56	8.26	25.80	8.25	30.63

Table A.6.5 : (contd.)

reference period : month

round	sub-sample	monthly per capita expenditure class in Rs.							
		34-43		43-55		55 & above		all classes	
		100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$
(1)	(2)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
4	1	10.19	37.74	6.67	49.46	9.26	95.05	100.00	30.20
	2	9.02	38.26	7.53	47.73	7.30	86.43	100.00	27.42
	comb.	9.60	37.98	7.10	48.51	8.27	91.21	100.00	28.79
5	1	7.47	38.22	4.89	49.69	12.83	102.33	100.00	32.19
	2	9.38	37.41	3.93	47.09	8.19	71.08	100.00	25.21
	comb.	8.38	37.79	4.43	48.60	10.64	90.93	100.00	28.86
7	1	8.17	37.77	5.29	47.93	6.60	81.74	100.00	24.21
	2	7.51	38.63	4.40	49.65	6.41	78.80	100.00	23.57
	comb.	7.85	38.15	4.86	48.70	6.51	80.33	100.00	23.88
8	1	6.25	37.93	3.75	47.96	4.84	81.56	100.00	21.83
	2	9.40	38.16	6.54	49.52	7.97	98.14	100.00	27.76
	comb.	7.79	38.06	5.11	48.94	6.37	91.71	100.00	24.72
8	1	6.26	37.93	3.76	47.97	4.84	81.47	100.00	21.82
	2	9.34	38.15	6.54	49.50	7.94	98.07	100.00	27.68
	comb.	7.77	38.05	5.12	48.92	6.36	91.52	100.00	24.69
9	1	7.88	37.98	5.33	47.96	5.23	87.89	100.00	24.06
	2	7.38	37.56	5.46	48.12	5.81	78.87	100.00	23.29
	comb.	7.64	37.78	5.39	48.04	5.51	83.21	100.00	23.69
10	1	8.13	38.41	4.95	48.72	6.33	87.76	100.00	24.83
	2	7.09	37.54	6.10	48.32	6.43	88.74	100.00	25.71
	comb.	7.63	38.00	5.51	48.48	6.38	88.13	100.00	25.24
11	1	7.06	37.76	4.06	48.35	7.13	108.65	100.00	26.09
	2	5.72	38.51	3.11	48.09	7.41	83.90	100.00	23.87
	comb.	6.37	38.11	3.57	48.23	7.27	95.66	100.00	24.98
12	1	6.07	38.51	6.21	48.43	7.01	117.13	100.00	28.05
	2	7.83	37.97	5.21	49.45	5.91	94.41	100.00	24.97
	comb.	6.98	38.20	5.70	49.02	6.44	106.36	100.00	26.46

Appendix 6.2 : Population estimates for mid-periods of NSS rounds

Table A.6.6 is reproduced from Som and De's paper entitled "Current population estimates for India", which was presented at the fifth session of the Indian Sociological Conference, Lucknow, 1960. It gives the population estimates for the mid-periods of the different NSS rounds, first to fourteenth, by the two independent, interpenetrating sub-samples, separately for the rural and the urban sectors of all-India (including Jammu & Kashmir). For the method of arriving at these projections, the reader is referred to Som and De's original paper. The estimates are presented here for ready use by interested readers : these were utilised in Section 6.4 and again in Section 6.8 for obtaining the all-India picture (rural-urban combined).

Table A.6.6 : Estimated population (in millions) for the mid-periods of the NSS rounds, first to fourteenth, in all-India rural and urban, by two independent, interpenetrating samples

NSS round	rural			urban		
	s.s.1	s.s.2	comb.	s.s.1	s.s.2	comb.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	297.4	297.4	297.4	63.4	63.5	63.5
2	298.8	298.7	298.8	64.2	64.2	64.2
3	300.2	300.0	300.1	65.0	65.0	65.0
4	303.2	302.8	302.9	66.5	66.4	66.5
5	305.6	305.0	305.2	67.8	67.7	67.8
6	307.0	306.3	406.6	68.5	68.4	68.4
7	309.5	308.6	309.0	69.8	69.6	69.7
8	313.3	312.1	312.6	71.8	74.5	71.7
9	316.7	315.3	315.9	73.6	76.2	73.4
10	319.3	317.7	318.4	74.9	77.5	74.7
11	322.5	320.7	321.4	76.6	79.1	76.3
12	324.8	322.8	323.6	77.7	80.2	77.5
13	328.7	326.4	327.4	79.7	82.2	79.4
14	333.9	331.2	332.3	82.3	84.7	81.9

Table A.6.7 : Estiated distributions of persons over monthly per capita expenditure classes by NSS Rounds : All-India, rural and urban combined, (excluding Jammu & Kashmir)

reference period : week

round	sub-sample	monthly per capita expenditure classes in Rs. 0-8		8-11		11-13		13-15		15-18		18-21		21-24	
		100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
3	1	5.62	6.13	9.12	9.43	7.79	12.02	7.23	14.08	9.61	16.53	11.28	19.64	9.09	22.66
	2	5.86	5.85	7.24	10.02	7.19	12.13	6.87	14.07	11.91	16.64	10.11	19.60	8.13	22.41
	comb.	5.75	5.98	8.16	9.69	7.49	12.09	7.04	14.07	10.79	16.58	10.66	19.65	8.60	22.55
4	1	6.17	6.18	8.47	9.78	7.52	11.71	8.42	13.85	11.40	16.43	10.90	19.62	9.08	22.17
	2	6.90	5.86	8.71	9.93	7.61	11.97	8.78	14.02	12.86	16.52	9.06	19.46	7.68	22.36
	comb.	6.54	6.05	8.61	9.87	7.55	11.84	8.60	13.94	12.15	16.49	9.95	19.55	8.38	22.24
5	1	7.07	6.62	9.90	10.06	8.22	12.13	7.31	14.07	13.61	16.21	10.40	19.74	11.97	22.38
	2	7.34	6.81	10.78	9.82	7.44	12.11	10.67	13.84	12.99	16.49	10.18	19.47	6.72	22.29
	comb.	7.20	6.73	10.35	9.94	7.82	12.12	9.00	13.94	13.28	16.36	10.30	19.61	9.32	22.35
6	1	8.49	5.16	9.08	9.57	7.35	12.00	6.97	14.16	12.27	16.59	13.30	19.50	7.02	22.74
	2	8.13	6.33	11.33	9.96	8.73	12.18	10.90	13.97	11.82	16.71	8.81	19.16	8.91	22.54
	comb.	8.31	5.73	10.23	9.79	8.05	12.10	8.93	14.03	12.02	16.66	11.04	19.36	7.96	22.62

Table A.6.7 : (contd.)

reference period : week

round	sub-sam-ple	monthly per capita expenditure classes in Rs.											
		24-28		28-34		34-43		43-55		55 & above		all classes	
		100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$	100P	$\bar{x}$
(1)	(2)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
3	1	11.60	25.81	9.30	30.94	8.58	38.26	4.75	48.31	6.03	87.96	100.00	25.78
	2	9.43	25.82	9.75	31.31	9.89	37.62	7.29	48.75	6.33	89.34	100.00	27.10
	comb.	10.48	25.81	9.53	31.12	9.25	37.90	6.05	48.58	6.19	88.67	100.00	26.46
4	1	9.02	25.77	10.15	30.99	9.16	37.91	4.04	47.31	5.67	78.63	100.00	24.60
	2	10.35	25.82	9.75	30.47	7.14	37.97	5.02	47.67	6.14	86.49	100.00	25.08
	comb.	9.68	25.79	9.94	30.74	8.14	37.94	4.53	47.50	5.93	82.65	100.00	24.84
5	1	7.02	25.38	9.70	31.01	6.45	37.64	4.24	48.12	4.11	89.94	100.00	23.38
	2	10.72	25.84	7.02	30.66	7.26	36.48	4.34	46.12	4.54	80.30	100.00	22.78
	comb.	8.88	25.66	8.35	30.86	6.87	37.02	4.30	47.09	4.33	84.78	100.00	23.08
6	1	9.50	25.78	10.58	30.58	7.06	37.09	3.71	48.35	4.67	89.43	100.00	23.68
	2	7.76	25.70	9.76	30.75	4.67	37.73	5.07	47.36	4.11	78.82	100.00	22.30
	comb.	8.63	25.75	10.17	30.66	5.87	37.35	4.38	47.78	4.41	84.52	100.00	22.99

Table A.6.8: Estimated distributions of persons over monthly per capita expenditure classes by NSS Rounds : all-India, rural and urban combined.

reference period : month

round	sub-sample	monthly per capita expenditure classes in Rs. 0-8 100P	0-8 $\bar{x}$	8-11 100P	8-11 $\bar{x}$	11-13 100P	11-13 $\bar{x}$	13-15 100P	13-15 $\bar{x}$	15-18 100P	15-18 $\bar{x}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
4	1	9.02	6.08	10.00	9.52	7.11	12.30	8.82	13.83	12.97	16.59
	2	8.71	6.22	11.66	9.58	9.23	12.10	7.19	14.14	13.71	16.48
	comb.	8.86	6.14	10.83	9.55	8.17	12.18	8.01	13.96	13.28	16.54
5	1	10.20	5.95	11.14	9.22	9.50	11.50	7.64	12.99	13.21	16.40
	2	8.44	5.99	12.90	9.28	8.57	11.54	8.67	13.80	12.78	16.14
	comb.	9.32	5.96	11.99	9.25	9.04	11.52	8.16	13.43	13.00	16.29
7	1	13.86	6.19	16.84	9.56	8.90	11.90	11.54	13.88	11.08	16.25
	2	14.15	6.23	16.63	9.59	13.83	11.91	8.77	14.10	10.78	16.19
	comb.	14.01	6.20	16.75	9.59	11.40	11.92	10.12	13.97	10.93	16.22
8	1	23.60	5.87	17.81	9.47	11.67	12.00	9.06	13.97	10.08	16.51
	2	20.58	5.95	20.11	9.49	10.13	11.97	8.44	14.05	8.43	16.42
	comb.	22.13	5.90	19.02	9.49	10.91	11.99	8.74	14.01	9.25	16.46
*8	1	23.44	5.87	17.80	9.46	11.72	12.01	9.01	13.98	10.12	16.51
	2	20.49	5.95	20.12	9.51	10.11	11.97	8.48	14.04	8.50	16.43
	comb.	22.00	5.89	19.02	9.49	10.93	12.00	8.74	14.00	9.30	16.47
9	1	20.82	5.85	17.38	9.50	11.07	11.99	8.52	14.07	10.39	16.38
	2	19.22	5.81	19.88	9.53	10.07	12.04	8.70	13.90	9.11	16.44
	comb.	20.07	5.84	18.65	9.51	10.56	12.02	8.62	13.97	9.74	16.40
10	1	13.34	6.23	17.37	9.29	12.22	14.02	8.95	11.95	11.52	16.59
	2	13.43	6.34	16.57	9.56	11.14	12.03	9.18	13.90	11.76	16.38
	comb.	13.43	6.27	16.98	9.41	11.69	11.98	9.06	13.97	11.64	16.48
11	1	12.89	6.79	17.51	9.57	11.82	11.97	11.62	14.11	12.25	16.47
	2	14.05	6.48	16.98	9.54	11.62	11.98	8.81	13.96	14.10	16.48
	comb.	13.48	6.64	17.28	9.54	11.73	11.97	10.24	14.04	13.15	16.49
12**	1	12.21	6.35	16.19	9.52	10.87	12.09	9.86	13.93	14.44	16.52
	2	13.66	6.42	17.45	9.61	11.61	12.12	10.86	14.02	11.00	16.58
	comb.	12.97	6.38	16.84	9.56	11.25	12.10	10.37	13.98	12.72	16.54

* For these and the subsequent rows, the coverage is all-India, including Jammu & Kashmir; for the preceding rows, the coverage excludes Jammu & Kashmir only.

** Figures for the twelfth round are provisional.

Table A.6.8 : (contd.)

reference period : month

round	sub-sample	monthly per capita expenditure classes in Rs. 18-21 100P	18-21 $\bar{x}$	21-24 100P	21-24 $\bar{x}$	24-28 100P	24-28 $\bar{x}$	28-34 100P	28-34 $\bar{x}$
(1)	(2)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
4	1	11.70	19.51	6.93	22.81	8.03	25.95	8.68	30.87
	2	10.47	19.59	7.93	22.47	7.49	26.02	8.51	30.14
	comb.	11.09	19.56	7.43	22.65	7.76	25.98	8.59	30.51
5	1	12.00	19.17	7.66	22.44	7.73	25.55	7.35	30.76
	2	11.46	18.97	7.11	21.92	8.06	24.83	9.48	30.02
	comb.	11.74	19.07	7.37	22.18	7.89	25.17	8.41	30.35
7	1	8.94	19.08	7.53	22.23	7.28	26.11	4.30	30.65
	2	9.22	19.02	6.67	22.43	5.20	25.75	5.54	30.52
	comb.	9.06	19.04	7.10	22.33	6.24	25.96	4.93	30.56
8	1	7.02	19.45	5.26	22.42	4.23	23.70	3.85	30.28
	2	7.40	19.42	5.81	22.52	4.65	26.05	5.92	30.56
	comb.	7.19	19.42	5.53	22.36	4.43	25.88	4.89	30.44
8	1	7.09	19.45	5.26	22.43	4.23	25.71	3.86	30.31
	2	7.41	19.43	5.80	22.31	4.66	26.04	5.94	30.56
	comb.	7.24	19.43	5.53	22.36	4.44	25.89	4.90	30.45
9	1	7.47	19.31	5.98	22.48	5.28	25.65	6.01	30.45
	2	8.14	19.35	7.68	22.32	4.98	25.68	4.33	30.41
	comb.	7.81	19.35	6.84	22.38	5.12	25.66	5.15	30.43
10	1	9.56	19.59	5.81	22.33	6.70	25.74	4.46	30.26
	2	7.19	19.41	8.28	22.50	4.78	25.84	6.23	31.06
	comb.	8.37	19.54	7.06	22.43	5.71	25.79	5.33	30.73
11	1	7.53	19.69	6.04	22.42	6.60	25.99	5.88	30.70
	2	10.06	19.47	5.70	22.48	5.42	25.88	5.23	30.51
	comb.	8.78	19.57	5.88	22.45	6.01	25.94	5.55	30.64
12	1	9.97	19.75	6.44	22.67	9.23	27.20	1.90	30.17
	2	8.75	19.67	6.23	22.46	9.91	27.25	1.72	30.49
	comb.	9.35	19.71	6.33	22.57	9.58	27.23	1.78	30.33

Table A.6.8:(contd.)

reference period : month

round	sub-sample	monthly per capita expenditure classes in Rs. 34-43 100P	34-43 $\bar{x}$	43-55 100P	43-55 $\bar{x}$	55 & above 100P	55 & above $\bar{x}$	all classes 100P	all classes $\bar{x}$
(1)	(2)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
4	1	8.63	38.02	4.09	48.12	4.12	88.96	100.00	23.27
	2	6.68	37.79	4.46	47.94	3.96	86.62	100.00	22.48
	comb.	7.66	37.89	4.28	48.04	4.04	87.81	100.00	22.88
5	1	5.77	37.68	3.09	48.62	4.71	89.26	100.00	22.02
	2	5.55	35.58	2.67	43.29	4.31	73.71	100.00	20.84
	comb.	5.66	36.66	2.88	46.19	4.54	82.21	100.00	21.47
7	1	4.48	37.10	2.29	48.64	2.96	86.45	100.00	18.86
	2	4.49	37.51	2.70	49.12	2.02	82.47	100.00	18.06
	comb.	4.49	37.30	2.50	48.88	2.47	84.84	100.00	18.46
8	1	3.11	37.75	1.84	48.72	2.47	78.35	100.00	16.21
	2	4.15	37.89	1.74	49.02	2.64	88.05	100.00	17.42
	comb.	3.61	37.83	1.77	48.84	2.53	83.12	100.00	16.76
8	1	3.18	37.72	1.83	48.72	2.46	78.35	100.00	16.23
	2	4.12	37.89	1.74	49.01	2.63	88.03	100.00	17.42
	comb.	3.63	37.80	1.76	48.83	2.51	83.11	100.00	16.77
9	1	3.00	37.37	1.82	47.17	2.26	76.39	100.00	16.77
	2	3.28	37.76	2.33	47.21	2.28	75.22	100.00	16.88
	comb.	3.12	37.56	2.06	47.17	2.26	75.80	100.00	16.80
10	1	5.12	37.80	1.91	48.76	3.04	83.73	100.00	18.73
	2	4.83	37.67	3.56	46.31	3.05	84.20	100.00	19.44
	comb.	4.97	37.74	2.73	47.14	3.03	83.89	100.00	19.06
11	1	3.50	38.03	2.09	48.17	2.27	112.86	100.00	18.87
	2	3.75	38.06	2.01	47.96	2.27	90.79	100.00	18.17
	comb.	3.61	38.08	2.04	48.07	2.25	101.87	100.00	18.50
12	1	3.89	38.01	2.40	48.06	2.60	111.69	100.00	19.44
	2	3.42	37.62	2.57	48.77	2.82	91.39	100.00	18.76
	comb.	3.64	37.83	2.47	48.48	2.70	100.99	100.00	19.07

Chapter 7

THE EFFECT OF DIFFERENTIALS IN CONSUMER PRICE INDEX ON LORENZ MEASURE OF INEQUALITY*

7.1. For comparing the inequalities of two expenditure distributions[1] differing in time[2], one should bring the later distribution to the prices of the earlier. This need does not arise if the consumer price index, for the later period with the earlier as base, does not change with the level of expenditure, but where this index increases (or decreases) monotonically with expenditure, then the current price distribution for the later period shows greater (or less) inequality than the corresponding constant price distribution. Analogous results have also been proved for the concentration curves[3] of expenditure on specific commodities. These results are numerically illustrated from the National Sample Survey data.

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* This chapter is based on an expanded version of a paper which was read at the Second Econometric Conference held at Waltair during 23-26 June 1961 [3].

1 Distributions of persons by total consumer expenditure per capita at current prices are being referred to as expenditure distributions.

2 The idea contained in this note is also applicable to the problems of comparing expenditure distributions differing in space.

3 For the difinition and properties of such curves, vide [5].

The present study arose out of an investigation [1] into the changes over time in the inequality of the all-India expenditure distributions thrown up by the different 'rounds' of the Indian National Sample Survey (NSS). It was felt that the comparison of inequalities of two expenditure distributions relating to different time-periods is conceptually somewhat difficult. Given two expenditure distributions, one for the 'base' period and the other for the 'current' period, it is necessary to bring both the distributions to some common set of prices, before any measure of inequality is calculated. The natural step would be to bring the current period distribution to the prices prevailing in the base period distribution remaining unchanged. This means finding out, with the help of consumer price indices, the (real) expenditure y at base period prices which is equivalent to each expenditure figure x involved in the current period distribution, and obtaining the distribution of the y's. The measures of inequality calculated for the base period distribution should be compared with those for this distribution of y's, and not with those for the x distribution. In other words, the comparison should be carried out <u>at constant prices</u>.

This adjustment, necessitated by price variations over time, is hardly ever carried out in practice. The function $\pi(y) = \frac{x}{y}$ defining the consumer price indices is not always known, even approximately, in the form of a series of consumer price indices, each index relating to a particular range of y. Most often the inequality measures calculated for the x-distribution are compared with those for the 'base' distribution.

Now, most of the known measures of inequality are unaffected by scalar transformations of the variate. Thus, if $\pi(y)$ were constant over the whole range of x or y, the distributions of x and y would be essentially the same for purposes of measurement of inequality. If, however, $\pi(y)$ varies with x or y, that is to say, if the consumer price index varies with the level of living, the inequality measures will be different for these two variates. In such cases, the measures for y should alone be compared with those for the base distribution.

It will be proved below that if $\pi(y)$ is a monotone increasing function of y, the distribution of x exaggerates the true extent of inequality (which is shown by the y-distribution); the Lorenz curve for x will lie uniformly below the Lorenz curve for y. If, on the other hand, $\pi(y)$ is a monotone decreasing function of y, the Lorenz curve for x will lie uniformly above the curve for y. Section 7.2

proves this result for the theoretical distributions of x and y, assuming only that x is a non-negative variate with a distribution of the continuous type. Section 7.3 proves the same results for the grouped case.

Section 7.4 considers the need of adjustments for concentration curves for individual commodities. No adjustment is found to be necessary for such curves based on quantitative consumption. For concentration curves of expenditure, however, results analogous to those for the Lorenz curve have been proved. Only, instead of $\pi(y)$, the function expressing the consumer price index for the commodity as a function of y, is involved.

Section 7.5 shows the results of such adjustments on some Lorenz and concentration curves obtained from the Indian National Sample Survey. Suitable deflators used were specially constructed for the purpose. The results of price adjustment do not seem to alter the basic conclusions substantially.

7.2. Ungrouped distributions of the continuous type. We first compare the inequalities of the ungrouped theoretical distributions of x and y which are assumed to be of the continuous type.

We start by examining a model, which, although simple, seems to be fairly illuminating. Let x be lognormally distributed with relative standard deviation[4] (inequality parameter) λ_x, and let $\pi(y) = \alpha y^{\beta}$, where α and β are constants. Then we have $x = y\pi(y) = \alpha y^{\beta+1}$ so that (i) $\beta > -1$, and (ii) y is also lognormally distributed, with its inequality parameter λ_y given by

$$\lambda_x = (\beta + 1)\lambda_y \tag{7.1}$$

Thus we have

$$\lambda_y < \lambda_x, \quad \text{if} \quad \beta > 0 \tag{7.2}$$

$$\lambda_y > \lambda_x, \quad \text{if} \quad -1 < \beta < 0 \tag{7.3}$$

Now the Lorenz curve for the log-normal expenditure distribution is defined by the equation

$$t_Q = t_P - \lambda \tag{7.4}$$

where Q is the proportion of total expenditure incurred by the poorest 100 P% of the population, and λ the relative standard deviation [2]. It follows from this equation that the Lorenz curve for a log-normal distribution having a greater value of λ is uniformly below the Lorenz curve for any log-normal distribution with a smaller value of λ. Obviously, all measures of inequality (e.g. the Lorenz ratio) would be increasing functions of λ. That the Lorenz ratio increases with λ can also be seen from the relation

4. Distributions of persons by total consumer expenditure at <u>current</u> prices are being referred to as expenditure distributions.

$$L = 2\,\phi\left(\frac{\lambda}{\sqrt{2}}\right) - 1 \qquad (7.5)$$

where ϕ denotes the normal probability integral.

Now case (7.2), with $\beta > 0$ corresponds to a situation where the consumer price index is higher for the rich than for the poor; in this case, $\lambda_y < \lambda_x$, so that the Lorenz curve of x (current expenditure) exaggerates the true extent of inequality (which is always given by the Lorenz curve of y.) In case (7.3), on the other hand, $\beta < 0$, which implies a higher consumer price index for the poor than for the rich; in this case, the Lorenz curve of x understates the true extent of inequality.

The above results can be shown to hold good even in the general case. It is sufficient to assume that x is a non-negative variate with a distribution of the continuous type, and that π is a monotonic function of y taking positive values. (For the sake of definiteness, we shall suppose that π is monotonic increasing). It follows that y is also non-negative and has a distribution of the continuous type.

Let $F_1(P)$ and $F_2(P)$ be the functions defining the Lorenz curves of x and y respectively, P being the incomplete probability integral (i.e. the abscissa). Clearly, these functions will be continuous and differentiable throughout (0,1).

The derivative of $F_1(P)$ with respect of P is given by

$$F_1'(P) = \frac{x_p}{E(x)} \qquad (7.6)$$

where x_P is the value of x having incomplete probability integral P, and E(x) denotes the expected value of x [5]. Similarly, the derivative of $F_2(P)$ with respect to P is

$$F_2'(P) = \frac{y_P}{E(y)} \tag{7.7}$$

the notation being perfectly analogous. We thus have

$$\frac{F_1'(P)}{F_2'(P)} = \frac{x_P}{y_P} \cdot \frac{E(y)}{E(x)} = \pi(y_P)\,\frac{E(y)}{E(x)} \tag{7.8}$$

Since E(x) and E(y) are constants, this shows that the left hand side of (7.8) is a monotone increasing function of y_P or P.

We may now note that the function $F(P) = F_2(P) - F_1(P)$ has zeroes at $P = 0$ and at $P = 1$. It must therefore have one or more extremums in (0,1). These should satisfy the condition that

$$F'(P) = F_2'(P) - F_1'(P) = 0 \tag{7.9}$$

Now if this holds for $P = P_0$, then

$$\frac{F_1'(P_0)}{F_2'(P_0)} = 1 \tag{7.10}$$

and (7.9) cannot hold for any other value of P since $F_1'(P)/F_2'(P)$ is nonotonic function of P. There is, thus, only one extremum of F(P) in the interval (0, 1), therefore,

$$F_1'(P) < F_2'(P) \qquad \text{for all } P < P_0 \tag{7.11}$$

$$F_1'(P) > F_2'(P) \qquad \text{for all } P > P_0 \tag{7.12}$$

The derivative of $F_1'(P)/F_2'(P)$ with respect to P is

$$\frac{F_2'(P)\, F_1''(P) - F_1'(P)\, F_2''(P)}{[F_2'(P)]^2} \tag{7.13}$$

and this must be positive for all P as the ratio $F_1'(P)/F_2'(P)$ is a monotone increasing function of P. We thus have

$$\frac{F_1''(P)}{F_2''(P)} > \frac{F_1'(P)}{F_2'(P)} \qquad \text{for all } P \tag{7.14}$$

In particular, for $P = P_0$, we have, in virtue of (7.10)

$$\frac{F_1''(P_0)}{F_2'(P_0)} > 1, \tag{7.15}$$

which means that $F''(P_0) = F_2''(P_0) - F_1''(P_0)$ is negative. Thus the single extremum at $P = P_0$ is a maximum of $F(P) = F_2(P) - F_1(P)$. As $F(P)$ is zero at $P = 0$ and $P = 1$, this also shows that $F(P)$ is positive throughout $(0, 1)$.

We may put these results into words as follows: The Lorenz curve for y is uniformly above that for x, the vertical distance between the curves reaching a maximum for that value of P for which

$$\pi(y_P) = \frac{E(x)}{E(y)} \tag{7.16}$$

Below this value of P, the slope of the curve for x is less than the slope for the curve for y the inequality being reversed for $P > P_0$. The slopes are equal at $P = P_0$.

As the Lorenz curve for y is uniformly above that for x the Lorenz measures of inequality will satisfy the inequality $L_x > L_y$. This follows from the geometrical interpretation (or definition) of the Lorenz ratio. The entire position will be reversed (e.g. L_y will be greater than L_x) if $\pi(y)$ is a monotone decreasing function of y.

7.3. The case of grouped distributions: Consider now the situation where the x-distribution is specified by $[\, p_i, \bar{x}_i [$ $(i = 1,2,\ldots, g)$ where p_i is the proportion of population in the i-th expenditure class, and $\bar{x}_i$ average expenditure of all persons in this class, there being in all g expenditure classes. This case deserves special mention because here (i) x need not be strictly continuous, and (ii) the y-distribution is obtained only approximately by using a single deflator for all x's within the i-th expenditure class $(i = 1,2,\ldots, g)$.

The broken Lorenz curve of x is obtained by joining the points (P_i, Q_i) $(i = 1,2,\ldots, g)$ where

$$P_i = \sum_{j=1}^{i} p_j$$

$$Q_i = (\sum_{j=1}^{i} p_j\bar{x}_j)/(\sum_{j=1}^{g} p_j\bar{x}_j) = (\sum_{j=1}^{i} a_j)/(\sum_{j=1}^{g} a_j), \quad \text{(say)} \qquad (7.17)$$

where $a_j = p_j\bar{x}_j$. The curve for y is obtained by plotting the points (P_i, Q_i') where

$$Q_i' = (\sum_{j=1}^{i} \frac{a_j}{\pi_j})/(\sum_{j=1}^{g} \frac{a_j}{\pi_j}) \qquad (7.18)$$

Multiplying the numerator of Q_i by the denominator of Q_i' and the denominator of Q_i by the numerator of Q_i' and comparing the coefficients of all the $a_i a_j$'s, it can be easily seen that

$$Q_i \leq Q_i' \quad \text{if} \quad \pi_1 \leq \pi_2 \leq \cdots \leq \pi_g \qquad (7.19)$$

and

$$Q_i \geq Q_i' \quad \text{if} \quad \pi_1 \geq \pi_2 \geq \cdots \geq \pi_g \qquad (7.20)$$

This means that the Lorenz curve for x is below that for y if the π's are monotone increasing, and above the curve for y if the π's are monotone decreasing. Conclusions regarding the two Lorenz ratios follow obviously from this.

The case where the distribution of x is specified in g fractile groups, with the group means $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_g$ (and the Lorenz curve based on such data,) is but a special case of the above. The proof of the above results is very similar but slightly simpler for this case.

The broken Lorenz curve defined by (7.17) leads to the following formula for the Lorenz ratio

$$L_x = 1 - \sum_{i=1}^{g} (P_i - P_{i-1})(Q_i + Q_{i-1}) \qquad (7.21)$$

which simplifies to

$$L_x = \frac{g-1}{g} - \frac{2}{g} \left(\frac{x_1 + x_2 + \dots + x_{g-1}}{x_g} \right) \qquad (7.22)$$

if the x-distribution is specified in g fractile groups, where $x_i = \bar{x}_1 + \dots + \bar{x}_i$. The formulae for L_y are analogous. The inequalities regarding the two Lorenz ratios can be directly proved from these formulae.

7.4. <u>The effect on concentration curves</u>. The concentration curve of any commodity shows the percentage Q of the total consumption of the commodity which is consumed by the poorest P % of the population. In most cases, however, the actual quantity consumed is not considered for such curves, but only the consumer expenditure incurred on the commodity, so that the curve shows the percentage Q of total consumer expenditure on the commodity incurred by the poorest P % of the population.

$$\frac{\frac{dQ}{dP}}{\frac{dQ'}{dP}} = \frac{E(y')}{E(y)} \cdot \frac{E(y \mid x)}{E(y' \mid x')} \qquad (7.25)$$

$$= c\, \pi\, (y \mid x), \text{ say, where}$$

$$c = \frac{E(y')}{E(y)} \qquad (7.27$$

The first factor on the right hand side is a constant, and the second is the consumer price index (for the item) for the current period expressed as a function of x. Equation (7.27) is completely analogous to equation (7.8) of Section 7.2, so that results similar to those proved in Section 7.2 can be proved for the two concentration curves.

7.5. Some illustrations: Table 7.1 presents the unweighted averages of the cost of living indices (CLI) for 23 towns and cities of West Bengal published by the State Statistical Bureau, West Bengal. The average CLI is seem to increase with the level of household expenditure in the years 1954 and 1955. This trend began to be reversed thereafter. In 1956 the CLI rose very little with the level of household expenditure, and from 1957 onwards, it actually fell with rising edpenditure levels. The indices with 1954 as base would show even greater variation in the CLI with the level of household expenditure. Actually, during the period covered in this table, the CLI was more sensitive at the lower levels of household expenditure, showing greater deviations from 100 in either direction.

Table 7.1: Average of cost of living indices for 23 towns and cities of West Bengal, by levels of monthly household expenditure.

(Base : November 1950 = 100)[5]

monthly household expenditure	average cost of living index 1954	1955	1956	1957	1958	1959	Autust 1960
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1 - 100	91.2	89.2	96.1	102.2	107.4	107.7	112.4
101 - 200	92.6	90.6	96.4	101.8	107.0	107.4	112.0
201 - 350	93.8	91.7	96.3	101.0	105.8	106.6	110.6
351 - 700	95.0	93.2	96.7	100.7	105.6	106.7	110.3
701 and above	96.2	94.3	96.9	100.5	105.2	106.6	109.6

Similar features were also observed in the corresponding indices for the food group which are published separately.

It is sometimes stated that in recent years in India, the CLI has increased more rapidly for the poorer classes of the population. Table 7.1 lends some support to this statement. If this were true in general, the stability over time of inequality measures of all-India expenditure distribution at current prices (observed by Bhattacharya and Iyengar [1] would really mean an increasing inequality in the corresponding distributions at current prices. The idea is clearly of considerable

5 Monthly Statistical Digest, West Bengal, November 1960.

importance ; but data in consumer price indices by levels of living are scanty and (or unrealiable), so that this line of thought could not be pursued successfully.

Recently in a study just completed, Iyengar, Chatterjee and Sarkar have constructed some new series of consumer price indices fractile groupwise for rural West Bengal entirely from the National Sample Survey materials. These indices have been calculated separately for food items, non-food items as well as for all items, for the year 1957-58 taking 1952-53 as the base year. Two sets of price deflators have been worked out in twenty fractile groups from two interpenetrating sub-samples of the National Sample Survey. Their main results are reproduced in the Appendix.

In the appendix, Table A.7.1 provides estimates of the distributions of total consumer expenditure as well as those of the relative distributions of food and non-food expenditures ; Table A.7.2 gives the corresponding differential price deflators.

In order to see the possible effects of ignoring the price differentials on measures of inequality some calculations were made on the above data. Lorenz ratios and specific concentration coefficients were calculated for the base year 1952-53; these calculations were also extended to undeflated and deflated distributions of 1957-58 using the formulae of Section 7.3. The main results are summarised in Table 7.2, in which the pooled estimates are shown in columns (4), (7), and (10); the pooled estimates were of course obtained by taking simple averages of sub-sample estimates.

Table 7.2: Concentration ratios for the distributions of Table A.7.1.

item	1952-53			1957-58					
				undeflated			deflated		
	s.s.1	s.s.2	pooled	s.s.1	s.s.2	pooled	s.s.1	s.s.2	pooled
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
food	0.2435	0.2306	0.2370	0.1997	0.1943	0.1970	0.2225	0.2073	0.2149
non-food	0.4730	0.3896	0.4313	0.4463	0.3477	0.3970	0.4687	0.3352	0.3020
all items	0.3333	0.2899	0.3116	0.2676	0.2259	0.2468	0.2842	0.2370	0.2606

From the fractile graphs constructed for the price index series of Table A.7.2, it appears that the overall consumer price index is a slightly decreasing function of the level of living. This is true, even to a larger extent, of the food index which in general is seen to be higher for the lower income groups than for the higher groups. The differentials in non-food prices suggest a small increasing trend which may not be statistically important. These graphs would indicate that in rural areas of West Bengal the price rise during 1951-52 to 1957-58 was higher for the poorer sections of the population, especially for foodgrains.

This would indirectly suggest that the coarser varieties of foodgrains generally consumed by them became more expensive.

From Table 7.2 it will be seen that the effect of price adjustment is not quite negligible. The adjustment tends to inflate the Lorenz ratio as well as specific concentration coefficients ; price adjusted specific concentration ratio for food, for example, is about 4 p.c. higher than the uncorrected. For total consumption, the adjusted Lorenz ratio exceeds the original by 6 p.c.

Now, if the distributions of 1952-53 are compared with those of 1957-58 the picture is one of reduction in inequality of food and non-food consumption by 9 p.c. and 7 p.c. respectively. The distribution of total consumption in rural West Bengal seems to have become appreciably more egalitarian (16 p.c. in the Lorenz ratio) during the five year period. The picture portrayed in this study should be extremely encouraging since, simultaneously with a reduction in disparity of levels of living, there is also a clear separation [4] between the 1952-53 fractile graphs and the deflated 1957-58 graphs constructed for average consumer expenditures.

The number of sample households in rural West Bengal during 1952-53 (fourth and fifth rounds of the National Sample Survey) was about 400 in each half sample, and only about 100 in each half sample in 1957-58 (thirteenth round of the National Sample Survey). Hence the precision of the estimates may not be very large so that our conclusions are only approximate.

The above conclusions may be true of other states as well. It would therefore be interesting to construct consumer price indices by levels of living, with rural-urban break-down, for all the Indian States, and with some break-downs by items of expenditure. This project has been taken up in the Indian Statistical Institute on the basis of National Sample Survey data.

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Table A.7.1. Average monthly per capita expenditure by fractile groups, rural West Bengal, 1952-53 and 1957-58.

percent of population	1952-53						1957-58					
	food		non-food		all items		food		non-food		all items	
	s.s.1	s.s.2	s.s.1	s.s.2	s.s.1	s.s.2	s.s.1	s.s.2	s.s.1	s.s.2	s.s.1	s.s.2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
bottom 5 p.c.	2.46	3.12	1.17	1.16	3.63	4.28	5.31	6.80	0.78	1.67	6.09	8.47
5 - 10 "	3.60	3.98	1.27	1.29	4.87	5.27	7.59	8.35	0.91	1.26	8.50	9.61
10 - 15 "	4.65	4.41	1.25	1.73	5.90	6.14	7.92	9.09	1.75	1.64	9.67	10.73
15 - 20 "	4.82	5.04	1.81	1.95	6.63	6.99	9.80	10.04	1.08	1.57	10.88	11.61
20 - 25 "	5.17	5.36	1.94	1.98	7.11	7.34	10.28	9.94	1.62	2.33	11.90	12.27
25 - 30 "	5.97	5.61	1.92	2.55	7.89	8.16	10.51	10.04	2.14	2.68	12.65	12.72
30 - 35 "	6.04	6.32	2.26	2.59	8.30	8.91	11.03	11.22	2.18	2.48	13.21	13.70
35 - 40 "	6.61	6.55	2.01	2.78	8.62	9.33	10.75	11.90	3.16	2.50	13.91	14.40
40 - 45 "	6.50	7.29	2.54	3.26	9.04	10.55	12.51	10.96	2.22	3.92	14.73	14.88
45 - 50 "	6.62	7.75	3.38	3.17	10.00	10.92	11.57	11.87	3.36	3.48	14.93	15.35
50 - 55 "	6.95	7.58	3.88	4.26	10.83	11.84	12.72	12.67	3.11	3.32	15.83	15.99
55 - 60 "	7.51	9.06	4.03	3.56	11.54	12.62	13.79	13.35	3.15	3.09	16.94	16.44
60 - 65 "	8.97	8.50	3.68	5.03	12.65	13.53	14.82	13.34	3.36	4.85	18.18	18.19
65 - 70 "	9.83	9.28	3.91	5.35	13.74	14.63	15.74	15.34	3.66	4.06	19.40	19.40
70 - 75 "	9.40	9.51	6.43	5.58	15.83	15.09	16.78	16.86	3.60	4.59	20.38	21.45
75 - 80 "	10.74	10.69	6.21	6.06	16.95	16.75	18.03	16.45	4.56	6.62	22.59	23.07
80 - 85 "	11.82	11.40	6.26	7.60	18.08	19.00	17.04	20.48	8.86	5.74	25.90	26.22
85 - 90 "	11.51	11.77	8.68	9.56	20.19	21.33	20.94	22.16	8.71	6.47	29.65	28.63
90 - 95 "	11.29	13.76	14.04	10.01	25.33	23.77	22.52	23.23	11.86	7.77	34.38	31.00
95 -100 "	19.28	17.27	26.06	17.62	45.34	34.89	23.63	23.47	20.92	13.93	44.55	37.40
0 - 100 p.c.	7.94	8.13	5.19	4.95	13.13	13.08	13.70	13.94	4.66	4.27	18.36	18.21

Table A.7.2: Consumer price index in 1957-58 with base: 1952-53 = 100 (Rural West Bengal)

percent of population	food		non-food		all times	
	s.s.1	s.s.2	s.s.1	s.s.2	s.s.1	s.s.2
(1)	(2)	(3)	(4)	(5)	(6)	(7)
bottom 5 p.c.	122.41	113.90	111.92	119.46	119.57	115.07
5 - 10 ''	121.01	123.55	117.61	94.43	120.32	118.23
10 - 15 ''	138.74	120.54	136.83	112.08	138.36	118.63
15 - 20 ''	138.21	115.54	120.73	119.32	134.76	116.37
20 - 25 ''	141.95	108.12	112.06	99.04	135.57	106.26
25 - 30 ''	121.13	107.00	114.36	112.71	119.82	108.47
30 - 35 ''	115.24	130.56	109.73	114.64	114.03	126.90
35 - 40 ''	114.74	119.31	120.87	100.48	116.09	115.02
40 - 45 ''	109.18	94.05	127.63	113.21	113.47	98.99
45 - 50 ''	114.59	116.71	124.18	102.46	117.32	113.22
50 - 55 ''	103.97	119.91	114.00	99.91	107.04	113.84
55 - 60 ''	111.43	88.75	117.96	99.52	113.41	91.33
60 - 65 ''	103.06	85.73	125.21	104.40	108.25	91.37
65 - 70 ''	101.72	87.74	104.64	110.14	102.51	95.10
70 - 75 ''	106.92	114.45	120.62	106.84	111.92	111.91
75 - 80 ''	94.57	119.03	122.14	118.34	103.32	118.82
80 - 85 ''	103.76	108.19	123.63	112.11	109.75	109.57
85 - 90 ''	101.87	102.75	125.87	115.66	111.42	108.07
90 - 95 ''	104.19	89.88	123.13	116.80	114.10	100.16
95 -100 ''	107.79	102.18	121.97	124.08	115.58	112.66
0 - 100 p.c.	110.49	102.68	117.90	115.35	113.11	106.85

Chapter 8

A STUDY OF DIFFERENTIAL PRICE MOVEMENTS : AN APPLICATION OF FRACTILE GRAPHICAL ANALYSIS

8.1 Recently several studies on intertemporal and interregional shifts in expenditure distribution in India have been reported; some of them are primarily concerned with inequality [1, 9, 10] while others [4, 5, 6] cover both aspects of the distribution, viz., inequality and average consumption levels. In the latter studies us has been made of fractile graphs to compare average consumption levels between two or more National Sample Survey (NSS) rounds. All the studies however employ concentration curves as a general tool to measure disparities in the levels of living[1]. The conclusions drawn from these studies do not seem to be valid in a strict sense because they are as a rule based on value comparisons[2]. This important limitation was noted in Chapter 6 and the effect of differentials in consumer price indices on inequality measures was theoretically examined by in the preceeding Chapter[3].

This Chapter represents a preliminary attempt to provide data on prices in a form which can be readily used in intertemporal analyses of consumer expenditure distributions, that is ,in the form of a series of price deflators, relating to different levels of living.

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1 The level of living is, of course, measured by monthly per capita total expenditure of households.

2 Quantitative comparisons have been made of consumption of cereals in rural and urban India between 8th and 13th rounds of the National Sample Survey by Mahalanobis [6] but not of total consumption.

As a starting point of a major project, one such index series was calculated for rural areas of West Bengal for some twenty fractile groups. Entirely based on the NSS household budget data, this series represents the movement of relative prices in 1957-58 taking the base as 1952-53. Our main results are given in Section 8.4. Section 8.2 gives a brief account of the basic material and method employed in the study. In Section 8.3 some of the special difficulties are discussed which were encountered in our work. A practical application involving the use of fractile graphical analysis is considered in Section 8.5. The effect of price differentials on Engel elasticities is examined in Section 8.6, and a few concluding remarks are made in the last Section.

8.2 The National Sample Survey Data : The National Sample Survey of India has been collecting detailed information on household budgets since its inception in 1951. The Survey has a continuous programme and is carried out in successive 'rounds' covering varying periods. In each round the sample households are drawn according to a probability design permitting valid estimation of population characteristics. Perhaps, the most important feature of the NSS sample design is that if provides for two or more independent and interpenetrating sub-samples from which a measure of uncertainty for the estimates may be approximately worked out.

In the construction of consumer price indices the basic ingredients are (i) weighting diagram, showing proportions of total outlay spent by an average household on specific commodities and services in some normal period called the 'reference' period, and (ii) price relatives, showing shifts in price levels in the given period called the 'current' period as compared with the reference period of each of the specific commodities and services. Both (i) and (ii) can be obtained from the NSS schedules relating to individual sample households. Information on consumption is available for a large number of consumer items both in terms of value and quantity. For each sample household interviewed in a given survey it is possible in principle to obtain the required price data, though implicitly. This possibility has been recently explored at the Indian Statistical Institute, and consumer price indices have been computed separately for three important categories of consumer items - bare essentials, other essentials and non-essentials [8]. The prcedure followed is formally different from the conventional methods which do not employ the same source of data [7] .

In the present study, which is restricted to rural areas of West Bengal, as mentioned earlier, use was made of the NSS household data relating to fourth, fifth and thirteenth rounds[3].

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3 The following are the periods covered by the NSS rounds :

4th .... April - September, 1952
5th .... December, 1952 - March 1953
13th .... September, 1957 - May 1958.

The fourth and fifth rounds were taken together to represent the reference period 1952-53 while thirteenth round was taken for the comparison period, 1957-58. Following Mahalanobis [5] , the sample households in each round were first classified into twenty fractile groups based on the distribution of their levels of living. Two sub-samples were used throughout the calculations. For purposes of convenience, the following major groups of consumer items were taken-food, non-food and miscellaneous, and all items[4]. For each of these groups, Laspeyres type of indices were constructed both fractile groupwise and sub-samplewise.

Appendix Table A.8.1 gives the distribution of sample households and persons in each fractile group for the fourth, fifth and thirteenth rounds of the NSS by two independent sub-samples. Whereas all the households interviewed during the fourth and fifth rounds were taken into account only a fraction of sample households surveyed in the thirteenth round was considered, viz., the households in the villages of West Bengal conforming to the self-weighting design which was partially adopted in the thirteenth round.

However, in working out the consumer price indices from the NSS data there are certain difficulties. We shall now turn to those problems in the following section.

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4 See footnotes to Table 8.1.

8.3 Method of Estimation. Let n_i be the number of sample households in the i th fractile group, and let for the j th household in this group M_{ij} be the corresponding probability weight, i.e., multiplier in the NSS terminology. Denoting by q_{ij}^t and v_{ij}^t the respective quantity (in standard units) and value of consumption of the t th item (t = 1,2, ... T say), we have the estimated quantity and value of the t th item in the fractile group :

$$q_i^t = \sum_j M_{ij} q_{ij}^t \qquad v_i^t = \sum_j M_{ij} v_{ij}^t \tag{8.1}$$

The total value of consumption of all items by the households in the i th fractile group is $v_i = \sum_t v_i^t$. The index weight to be associated with the t th item will then be

$$w_i^t = \frac{v_i^t}{v_i} \tag{8.2}$$

The implicit item prices were then derived by dividing the estimated value of consumption by the corresponding estimated quantity. That is,

$$p_i^t = \frac{v_i^t}{q_i^t} \tag{8.3}$$

Since the NSS fourth round was spread over six minths (April - September, 1952) and the fifth round over four months (December, 1952 - March 1953), the base period (1952-53) weights and prices, as obtained from (8.2) and (8.3), were worked out by combining the fourth found and fifth round weight and price data in the proportion 6:4. From the price data obtained for 1957-58 (thirteenth round), price relatives r_i^t were next calculated with reference to the prices of 1952-53. And finally, an index of consumer price was calculated as a weighted average of the price relatives, viz.,

$$I_i = \sum_t w_i^t \, r_i^t \qquad (8.4)$$

for each fractile group i (i = 1,2,3,........, 20)

The above procedure has certain practical limitations which are quite common to all index number problems. In the present study there are a few additional problems which arise from both statistical and conceptual considerations. On the statistical side the problem of small samples presents a serious difficulty, especially in the context of our attempt to construct index numbers for twenty fractile groups. On the conceptual side, it may be noted that for some items commonly appearing in the family budgets, the quantity of consumption cannot be obtained from the NSS schedules.

The items for which the price relatives could not be calculated from the household schedules are footwear, amusement, musical instruments, education, medicine, toilet, petty articles, conveyance (excluding railway), services, furniture, utensils, sundry equipments, ornaments, ceremonials, rents and taxes. For these items the price relatives were worked out from data gathered from a special study in a typical West Bengal village, Debalaya, about 35 miles away from Calcutta. The survey was carried out by competent field staff of the Indian Statistical Institute, and good deal of care was exercised in the selection of commodities which were qualitatively similar to the ones involved in the reference period. The price data collected through the survey were arbitrarily divided equally between two sub-samples called sub-sample 1 and sub-sample 2. As regards the item, railway journey, price relatives were formed by taking the ratio of third class ordinary passenger fare per mile during the two periods under consideration as given in the railway fare tables. Also, there are a few items for which prices were obtained from neither the NSS nor the special field study. For these items (which were small in number) prices were assumed to be constant. We should keep in view the various pracical limitations, partly enumerated above, in the evaluation and interpretation of the main results presented in the following sections.

8.4. Main Results. Table 8.1 summarises the main results of our study. The results are also displayed by means of fractile graphs in Figure 8.1.

Table 8.1. Consumer Price Indices in 1957-58, Rural West Bengal (Base : 1952-53 = 100)

fractile group (per cent)	all items including miscellaneous		all items excluding miscellaneous		food		non-food	
	S.S. 1	S.S. 2	S.S. 1	S.S. 2	S.S. 1	S.S. 2	S.S. 1	S.S. 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	116.94	111.76	117.82	112.24	121.60	113.32	107.18	107.58
2	118.53	111.89	119.61	112.02	120.48	123.57	113.00	75.72
3	138.36	113.67	140.43	113.85	135.99	121.00	147.17	94.95
4	128.11	114.87	130.50	115.20	137.42	115.28	103.29	113.82
5	133.74	104.01	136.06	102.60	141.93	107.89	111.88	93.53
6	118.38	107.25	118.92	106.65	120.97	106.59	110.32	108.70
7	112.43	123.77	112.94	125.61	114.48	129-60	106.94	109.56
8	116.09	114.32	116.30	114.72	113.92	119.73	123.23	101.54
9	111.58	99.00	111.45	96.30	108.01	93.81	120.23	110.60
10	117.46	117.85	117.77	118.93	111.45	116.25	129.23	121.74

Table 8.1 (contd.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
11	105.67	111.52	102.41	110.02	103.46	119.49	109.63	97.31
12	112.36	92.40	113.94	88.92	110.93	87.92	115.03	103.84
13	110.08	91.73	106.63	84.76	103.18	81.99	126.89	108.18
14	102.34	94.24	99.89	87.15	100.17	86.09	107.80	108.36
15	110.54	111.98	108.03	109.64	105.63	113.84	117.73	108.81
16	102.31	118.06	97.73	119.54	92.40	118.36	119.47	117.54
17	108.47	109.09	101.76	107.12	100.87	107.84	122.81	110.95
18	110.57	107.55	106.18	102.00	100.81	102.33	123.52	113.98
19	113.26	100.92	101.43	91.63	103.28	88.45	121.27	118.05
20	114.97	111.59	106.83	101.53	107.07	101.47	120.81	121.53
1 - 20	112.48	106.50	109.92	102.82	109.03	101.85	117.76	114.14

1957-58 is represented by NSS 13th round (September 1957 to May 1958). Base 1952-53 is represented by NSS 4th round (April to September 1952) and NSS 5th round (December 1952 to March 1953) taken together.

Food items include cereals, pulses, milk, milk and milk products, oil, vegetables, fruits, meat, fish, egg, sugar, gur etc., salt, spices, beverage and refreshment, pan supari.

Non-food items other than miscellaneous include fuel and light (coke and coal firewood, electricity and gas, keresene, matches etc.) clothing (mill-made and hosiery, handloom and khaddar, wool, silk bedding and upholstry), tobacco, drugs and intoxicants.

Miscellaneous items (also included under non-food) include amusement, education, medicine, toilet, petty articles, services, conveyance, ceremonials, rents, taxes, furniture, musical instruments, ornaments, domestic utensils, footwear, other equipment.

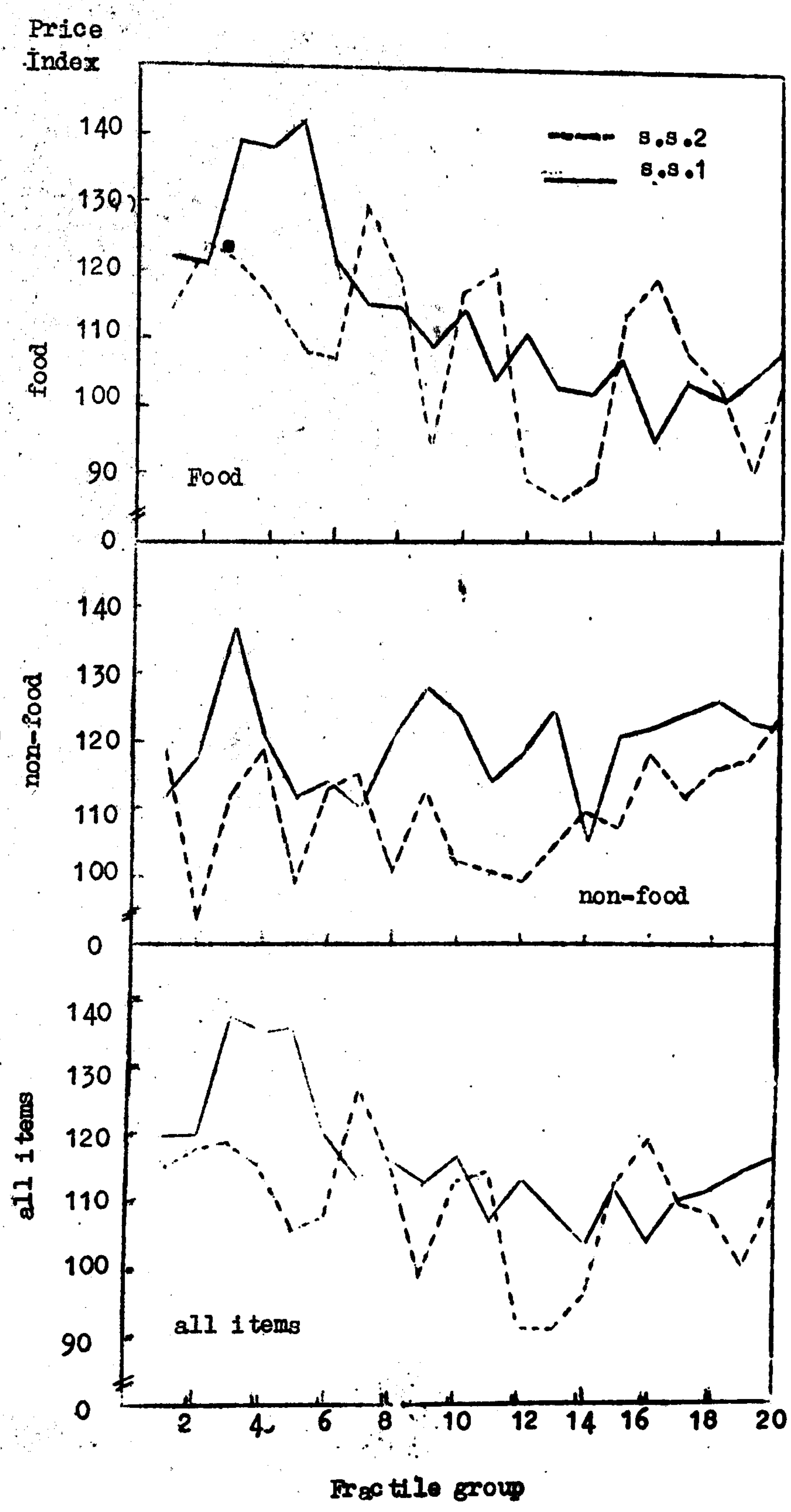
Price Index
s.s.2
s.s.1
140
130
120
110
100
90
0
food
Food
non-food
non-food
all items
all items
2
4
6
8
10
12
14
16
18
20
Fractile group

Figure 8.1

Notwithstanding appreciable divergence between the sub-sample estimates, certain broad conclusions can now be drawn from our calculations, In the first place it will be observed from the fractile graphs that the overall consumer price index is a slightly decreasing function of the level of living. This is true, perhaps to a larger extent, of the food index which in general is seen to be higher for the lower income groups than for the higher income groups.[9] The differentials in non-food prices suggest a mild increasing trend which may not be statistically important. More specifically, these graphs would indicate that in rural areas of West Bengal the price rise during 1952-53 to 1957-58 was generally positive but its distribution was not uniform; it was higher for the poorer sections of the population, especially with regard to foodgrains. In view of general preference for coarser varieties of foodgrains by the rural population [2] , this would suggest that the coarser varieties of foodgrains usually consumed by them tended to become relatively more expensive.

For all fractile groups combined, the overall index of consumer prices shows a rise of about 12 per cent and 6 per cent for sub-samples 1 and 2 respectively, If, however, we exclude from the budget the miscellaneous items for which price data are comparatively

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9 The fractile grouping according to total expenditure per capita is assumed to give approximately the same ordering on the corresponding income scale [5] .

inferior, the corresponding rises will be 10 per cent and 3 per cent. Since the miscellaneous group of items forms a small proportion of the total outlay, the overall results do not seem to be significantly affected by its inclusion or exclusion.

Though not comparable in any strict sense, it might be useful to point out that the State Statistical Bureau of West Bengal compiles unweighted averages of working class cost-of-living indices for 23 towns and cities of West Bengal. These indices also seem to confirm our hypothesis that the overall consumer price index is a generally decreasing function of the level of living.[10]

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10 This is evident from the following table which gives average cost of living indices for 23 towns and cities of West Bengal, by levels of monthly household expenditure, taken from the Monthly Statistical Digest, November 1960.

November 1950 = 100

monthly household expenditure (Rs.)	cost of living index 1957	cost of living index 1958
(1)	(2)	(3)
less than 100	102.2	107.4
101 to 200	101.8	107.0
201 to 350	101.0	105.8
351 to 700	100.7	105.6
701 and above	100.5	105.2

sented in Table 8.2 . The corresponding fractile graphs were also drawn, but the general conclusions do not seem to alter very much although the sub-sample errors are slightly reduced.

Table 8.2 : Consumer price index numbers in 1957-58 by fractile groups of population, excluding clothing, West Bengal, rural. (base 1952-53 = 100)

fractile group	all items including miscellaneous		all items excluding miccellaneous		food		non-food	
	s.s.1	s.s.2	s.s.1	s.s.2	s.s.1	s.s.2	s.s.1	s.s.2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	119.57	115.07	121.05	115.91	122.41	113.90	111.92	119.46
2	120.32	118.23	121.64	118.92	121.02	123.55	117.61	94.43
3	138.36	118.63	140.42	119.60	138.74	120.54	136.83	112.08
4	134.76	116.37	138.42	116.92	138.21	115.54	120.73	119.32
5	135.57	106.26	138.28	104.98	141.95	108.12	112.06	99.04
6	119.82	108.47	120.58	108.06	121.13	107.00	114.36	112.71
7	114.03	226.90	114.82	129.29	115.24	130.56	109.73	114.64
8	116.09	115.02	116.29	115.56	114.74	119.31	120.87	100.48
9	113.47	98.99	113.67	96.11	109.18	94.05	127.63	113.21
10	117.32	113.22	117.61	113.53	114.59	116.71	124.18	102.46
11	107.04	113.84	103.86	112.80	103.97	119.91	114.00	99.91
12	113.41	91.33	115.43	87.48	111.43	88.75	117.96	99.52
13	108.25	91.47	104.25	83.31	103.06	85.73	125.21	104.40
14	102.51	95.10	100.08	87.86	101.72	87.74	104.64	110.14
15	111.92	111.91	109.77	109.41	106.92	114.45	120.62	106.84
16	103.32	118.82	98.72	120.64	94.57	119.05	122.14	118.34
17	109.75	109.57	103.02	107.62	103.76	108.19	123.63	112.11
18	111.42	108.07	107.18	102.40	101.87	102.75	125.87	115.66
19	114.10	100.16	101.91	89.91	104.19	89.88	123.13	116.80
20	115.58	112.66	107.45	102.69	107.79	102.18	121.97	124.08
1-20	113.11	106.85	110.59	102.99	110.49	102.68	117.90	115.35

8.5 An application of Fractile Graphical Analysis. From the results of last Section we may draw some further conclusions which may be of great topical interest[11]. The appendix Tables A.8.2 to A.8.4 provide estimates of distributions of total consumer expenditure as well as those of relative distributions of food and non-food expenditure for years 1952-53 and 1957-58, at current prices. The distributions of 1957-58 are properly deflated to allow for differential price movements by using the series of consumer price indices given in Table 8.1.

Now, if the distributions of 1952-53 were compared with those of 1957-58 the picture would be one of reduction in inequality of food and non-food consumption by 9 per cent and 7 per cent respectively. The distribution of total consumption in rural West Bengal seems to have changed appreciably towards egalitarian distribution, with a decline of 16 per cent in the Lorenz ratio, during the period.

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11 The Mahalanobis Committee on Distribution of Income and Wealth is presently working on similar problems on all-India basis.

The same observation was made earlier in Chapter 7 in the context of our investigation into the effect of consumer price differentials on measures of inequality.

Although comparisons based on overall measures of inequality like the Lorenz ratio have an important place in empirical investigations, it might be useful in practice to consider changes in the relative shares of consumption enjoyed by different groups of population classified according to levels of living. We have therefore, calculated percentage shares in aggregative consumption accruing to bottom 10 and top 10 per cent of the population in rural West Bengal, and the results are presented in Table 8.3. These results provide additional empirical evidence supporting our conclusion.

Table 8.3. Percentage share in aggregative consumption accruing to bottom and top 10 percent of population (rural, West Bengal).

percentage population		percentage share in consumption								
		1952-53			1957-58			1957-58[a]		
		s.s.1	s.s.2	pooled	s.s.1	s.s.2	pooled	s.s.1	s.s.2	pooled
					all items					
bottom	10	3.24	3.65	3.44	4.04	5.00	4.52	3.79	4.62	4.20
top	10	26.92	22.45	24.68	21.95	13.92	20.38	21.45	19.14	20.30
					food items					
bottom	10	3.79	4.32	4.06	4.72	5.46	5.09	4.48	5.04	4.76
top	10	19.84	18.89	19.02	16.89	16.83	16.86	16.75	17.12	16.94
					non-food items					
bottom	10	2.38	2.52	2.45	1.82	3.62	2.72	1.76	3.23	2.50
top	10	39.03	28.46	33.74	35.32	26.83	31.08	35.49	25.81	30.65

a) price-adjusted

The picture portrayed in this study should be extremely encouraging because, simultaneously with a reduction in disparity of levels of living, there is a clear separation[12] between the 1952-53 graphs and the properly deflated 1957-58 graphs constructed for the average consumer expenditure. One may see this from the fractile graphs shown in Figure 8.3.

Certain analytical tests were also performed to examine whether the observed 'real' separation, $A_*^{(1,2)}$ between the 1952-53 distributions and those of 1957-58 is significant in some sense. For this purpose, sub-sample errors, to be denoted by $A_{1,2}^{(1)}$ and $A_{1,2}^{(2)}$ were computed by using the well-known formula for the area between the fractile graphs[13]. Separations were likewise calculated with the pooled estimates obtained by taking simple averages of the corresponding sub-sample estimates. Finally, these separations were divided by their respective total errors, $E_*^{(1,2)}$ defined as

$$E_*^{(1,2)} = \sqrt{[A_{1,2}^{(1)}]^2 + [A_{1,2}^{(2)}]^2} \quad (8.5)$$

The important stages of calculation are shown in Table 8.4.

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12 see Mahalanobis [5] . who treats separation as a kind of generalised distance between two size distributions.

13 see footnotes to Table 8.4.

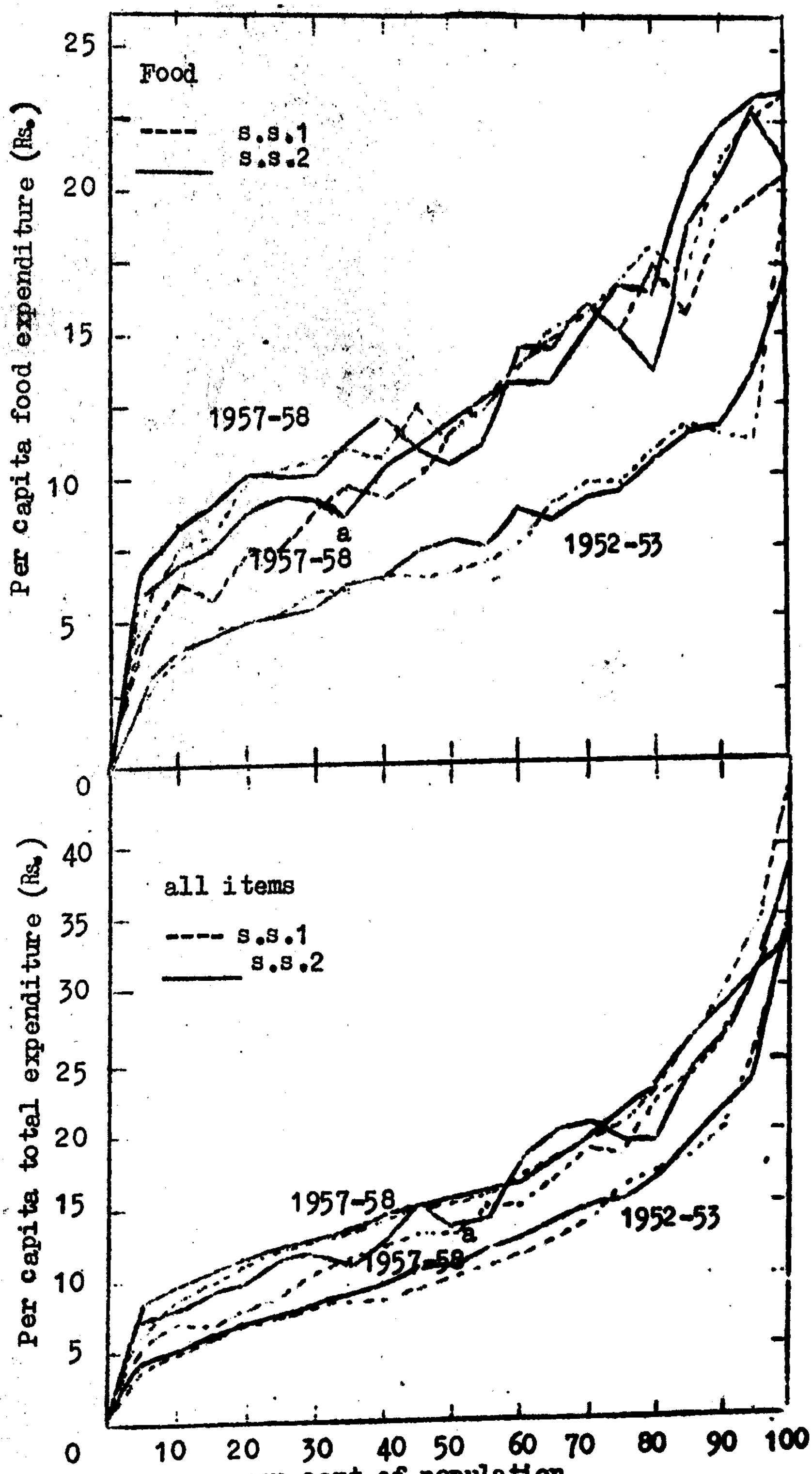
Food
s.s.1
s.s.2
1957-58
a
1957-58
1952-53
Per capita food expenditure (Rs.)
25
20
15
10
5
0
all items
s.s.1
s.s.2
1957-58
a
1957-58
1952-53
Per capita total expenditure (Rs.)
40
35
30
25
20
15
10
5
0
10
20
30
40
50
60
70
80
90
100
per cent of population

Figure 8.3

Table 8.4: Fractile Graphical Analysis.

item	$A_{1,2}^{(1)}$	$A_{1,2}^{(2)}$	$A_{1,2}^{(2^a)}$	$A_*^{(1,2)}$	$A_*^{(1,2^a)}$
(1)	(2)	(3)	(4)	(5)	(6)
food	0.4710	0.7277	1.2004	11.788	8.5388
non-food	0.8102	1.0912	1.0339	2.2805	1.7619
all items	0.9159	0.8705	1.3756	10.0565	6.9678

Table 8.4 (contd.

item	$E_*^{(1,2)}$	$E_*^{(1,2^a)}$	$A_*^{(1,2)}/E_*^{(1,2)}$	$A_*^{(1,2^a)}/E_*^{(1,2^a)}$
(1)	(7)	(8)	(9)	(10)
food	.8662	1.2964	12.7901 b)	6.5863
non-food	1.3591	1.3228	1.6779	1.3319
all items	1.2636	1.6526	7.9586 b)	4.2163 b)

a) price-adjusted b) significant

$A_{1,2}^{(1)}$: 'error' area between sub-sample graphs for the distributions in the period 1952-53 (=1)

$A_{1,2}^{(2)}$: 'error' area between sub-sample graphs for the distributions in the period 1957-58 (=2)

$A_*^{(1,2)}$: 'separation' area between the 'combined' graphs of periods 1 and 2.

$$A_{1,2}^{(1)} = \frac{1}{20} \sum_{i=1}^{20} \left[\frac{|w_i| + |w_{i+1}|}{2} - \delta(w_i, w_{i+1}) \frac{|w_i|\ |w_{i+1}|}{|w_i| + |w_{i+1}|}\right]$$

where $w_i = \bar{y}_{2i} - \bar{y}_{1i}$ (y represents the variable under study)

$\delta(w_i, w_{i+1}) = 1$ If the fractile graphs for the sub-samples (1) and (2) intersect between (1/20)th and (i+1)/20th coordinates on the fractile graph.

$= 0$ otherwise.

$E_*^{(1,2)}$: 'total error' of the combined fractile graph; it is defined as

$$\sqrt{[A_{1,2}^{(1)}]^2 + [A_{1,2}^{(2)}]^2}$$

The fractile graphical analysis and the associated graphs suggest that in real terms the distributions of total expenditure, as well as food expenditure, have indeed favourably changed in the villages of West Bengal during the five-year period 1952-53 to 1957-1958. The inequality of these distributions has appreciably declined simultaneously with a rise in the levels of real expenditure.

To see the implications of ignoring the price differentials in intertemporal studies of expenditure distributions we shall consider an important case that has some bearing on the previous Chapters, viz. when the distributions conform to the lognormal hypothesis. Empirical investigations made in this context indicate that the assumption of lognormality is highly plausible for the given distributions. The lognormal parameters θ and λ [14] were estimated for the total expenditure distributions of 1952-53 and 1957-58, with and without adjustment for price differentials, by using the methods of Chapter 2. The estimates given in Table 8.5 clearly reflect the effect of price adjustment on the lognormal parameters; the inequality parameter λ is inflated but θ is somewhat diminished.

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14 The notation is same as in earlier Chapters.

Table 8.5 : Estimates of lognormal parameters
(rural - West Bengal)

period	θ		λ	
	s.s. 1	s.s. 2	s.s. 1	s.s. 2
(1)	(2)	(3)	(4)	(5)
1952 - 53	2.3944	2.4305	.6008	.5302
1957 - 58 a/	2.7928	2.8191	.4845	.4070
1957 - 58	2.6478	2.7351	.5273	.4482

a/ Price-adjusted.

8.6 Effect of price differentials on Engel elasticity. One may also draw interesting conclusions from the price-adjusted size distribution data by examining, for instance, the differential price effects on Engel ratios or elasticities which are likely to be affected in the same manner as the inequality parameter λ. We shall illustrate below the effect of price differentials on the Engel elasticity. Assuming a double-log form for the Engel curve, the elastity was calculated for food items using, of course, the method of concentration curves. The elasticities are given in Table 8.6.

Table 8.6 : Engel elasticity of food items (rural, West Bengal)

period	constant elasticity		median elasticity b/		'mean' elasticity b/	
	s.s. 1	s.s. 2	s.s. 1	s.s. 2	s.s. 1	s.s. 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1952-53	.7413	.7882	.7164	.7682	.6344	.6933
1957-58	.7620	.8742	.7455	.8561	.6855	.7994
1957-58 a/	.8185	.8960	.7939	.8697	.7150	.7998

a/ Price-adjusted.

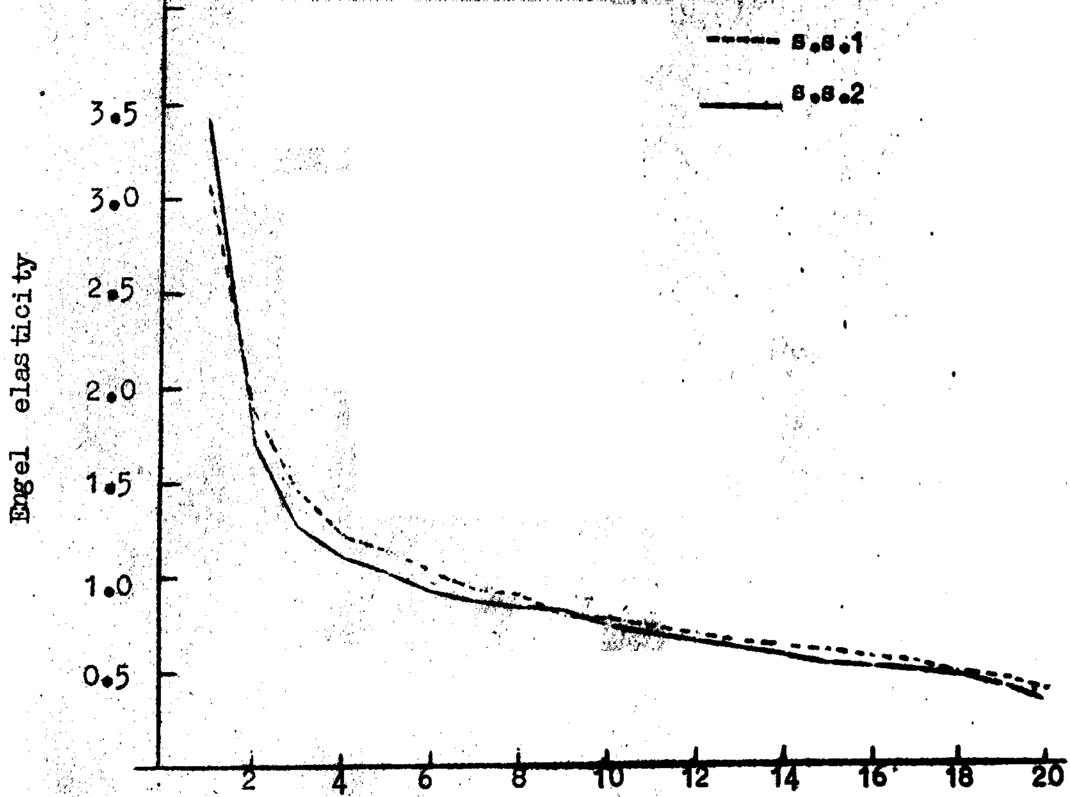
b/ The 'median' and 'mean' elasticities are based on semi-log Engel curve.

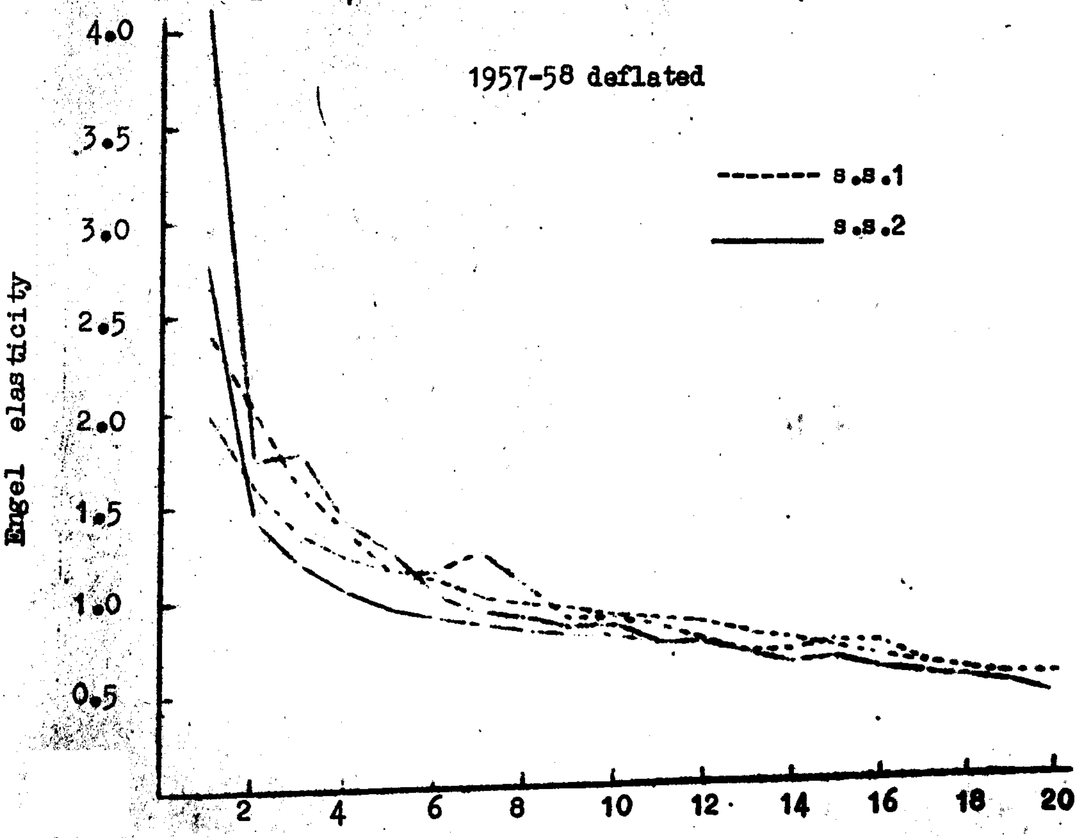
Ignoring for the moment the large divergence between sub-sample estimates, it seems clear that the elasticities computed from current-price distributions should not be compared in a strict sense if there are significant price differentials. As observed generally in Chapter 7, if price indices decline with rising levels of living as indeed is the case with food items, the elasticity calculated from the current-price distributions needs an upward revision before being compared with the elasticity obtained from the base-period distributions.

A similar upward adjustment is needed for the variable elasticity as well. The variable elasticity obtained from the semi-log Engel curve, perhaps illustrates the point. Following our methods of estimation, the semi-log Engel curves were completely derived for food expenditure for 1952-53 and 1957-58 (adjusted and un-adjusted). The variable elasticities were obtained for all fractile groups and were plotted. The resulting elasticity curves are given Table 8.7 as also shown in Figure 8.3. It will be noted that the curve based on price-adjusted data is generally above the curve based on un-adjusted 1957-58 distributions.

Table 8.7. Estimates of Engel elasticities of food expenditure by fractile groups (rural, West Bengal)

population (%)	1952-53		1957-58 unadjusted		1957-58 price-adjusted	
	S.S. 1	S.S. 2	S.S. 1	S.S. 2	S.S. 1	S.S. 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0 - 5	3.4405	3.0767	2.8155	2.0595	4.1846	2.4343
5 - 10	1.7110	1.8754	1.4523	1.6349	1.7663	1.9596
10 - 15	1.2877	1.4575	1.2231	1.3851	1.7977	1.6223
15 - 20	1.1197	1.2259	1.0690	1.2488	1.4286	1.3982
20 - 25	1.0383	1.1567	.9756	1.1683	1.2770	1.1609
25 - 30	.9372	1.0304	.9206	1.1209	1.0320	1.1405
30 - 35	.8947	.9448	.8852	1.0349	.9425	1.2591
35 - 40	.8654	.9054	.8466	.9840	.9132	1.0616
40 - 45	.8311	.8147	.8074	.9533	.8509	.8892
45 - 50	.7668	.7926	.7987	.9259	.8653	.9786
50 - 55	.7227	.7447	.7631	.8923	.7658	.9457
55 - 60	.6910	.7109	.7256	.8706	.7599	.7662
60 - 65	.6497	.6775	.6902	.8001	.6980	.7118
65 - 70	.6166	.6434	.6606	.7609	.6443	.6992
70 - 75	.5671	.6308	.6397	.7069	.6606	.7310
75 - 80	.5459	.5918	.6002	.6723	.5894	.7241
80 - 85	.5273	.5507	.5547	.6191	.5640	.6290
85 - 90	.4984	.5177	.5160	.5871	.5283	.5912
90 - 95	.4478	.4903	.4794	.5608	.4957	.5414
95 - 100	.3551	.4126	.4264	.5074	.4418	.5216
0 - 100	.6344	.6933	.6855	.7994	.7150	.7998





Fractile group

Figure 8.3

8.7 Some Concluding Observations. In conclusion ~~two~~ general observations may be worth recording : the desirability of using continuous cross-section data for purposes of index number calculation by some new and meaningful breakdowns, and the necessity of a suitable measure of reliability of the estimates of index numbers computed from sample survey data. Any realistic measure of reliability should take into account the sampling error, which is basically a problem of sample size and the probability design underlying the selection of sample households, and the non-sampling error which may dominate in a country like ours with a large illiterate population. The use of the technique of interpenetrating sub-samples in the NSS household surveys makes it possible to estimate in a practical sense the margin of both sampling and non-sampling errors.

The existing official (or non-official) consumer price indices are limited only to certain types of breakdowns; commoditywise, occupation or industrywise, state or reagionwise breakdowns are well known. But to provide index breakdown by household levels of living is relatively uncommon although such information can be highly useful in empirical investigations of income (or expenditure) distributions.

At the all India level, the NSS sample size is reasonably adequate for most analytical purposes, but this is perhaps not so at the State level. The position is even worse when considering the rural sample households only. A further sub-division of these households into twenty more or less equal groups on a fractile basis tends to diminish the efficiency of the final estimates to a considerable extent. And finally, the survey estimates of the weighting diagram appear more satisfactory than those of the price relatives. This may be easily verified from the sub-sample differences. To explain such situations further studies are needed, particularly on prices.

Our study is mainly of an exploratory nature confined as it is only to rural areas of West Bengal. It might be worthwhile to attempt large scale empirical work in this important area extending our method to other Indian States, covering both rural and urban areas. Special studies are also needed on the socio-economic factors that lead to concentration of income and consumption. Other related aspects are changes in the distribution of land-holdings and migration of rural population to urban areas, particularly in West Bengal. Probably, the migration of the poorest as well as the richest sections of the society from the villages to the fast developing industrial cities and towns has led to a new rural middle class, consisting of artisans and farmers with small land-holdings. Their position might have slightly improved apparently as a result of higher farm prices received by them, especially for foodgrains. This is purely a conjecture at this stage and requires empirical verification.

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Appendix

Table A. 8.1. Number of sample households used in the study (rural, West Bengal)

	per cent of population	1952-53* NSS 4th round ss1		NSS 4th round ss2		NSS 5th round ss1		NSS 5th round ss2		1957-58 NSS 13th round ss1		NSS 13th round ss2	
(0)	(1)	(2)		(3)		(4)		(5)		(6)		(7)	
1	Bottom 5%	13		12		6		8		5		6	
2	5 - 10%	15		9		5		7		4		5	
3	10 - 15%	15	68	14	56	7	32	6	36	4	22	5	25
4	15 - 20%	13		11		7		7		6		3	
5	20 - 25%	12		10		7		8		3		6	
6	25 - 30%	15		13		5		5		5		5	
7	30 - 35%	12		18		4		9		4		5	
8	35 - 40%	11	66	13	75	8	29	9	33	4	21	5	23
9	40 - 45%	13		18		5		5		4		4	
10	45 - 50%	15		13		7		5		4		4	
11	50 - 55%	15		11		7		6		7		5	
12	55 - 60%	15		17		6		8		5		4	
13	60 - 65%	14	71	6	59	7	37	8	35	7	25	6	26
14	65 - 70%	13		15		11		6		5		5	
15	70 - 75%	14		10		6		7		5		6	
16	75 - 80%	11		13		5		8		4		6	
17	80 - 85%	12		12		8		9		3		4	
18	85 - 90%	17	62	10	65	8	33	5	34	6	25	3	25
19	90 - 95%	7		14		6		6		5		4	
20	95 -100%	15		16		6		6		7		8	
all	0 -100%	267		255		131		138		97		99	

* approximately

Table A. 8.2. Average per capita expenditure on all items (rural, West Bengal)

	per cent of population	Average per capita total expenditure (Rs.)					
		1952-53		1957-58			
				undeflated		deflated	
		ss1	ss2	ss1	ss2	ss1	ss2
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	Bottom 5%	3.63	4.28	6.09	8.47	5.09	7.36
2	5 - 10%	4.87	5.27	8.50	9.61	7.06	8.13
3	10 - 15%	5.90	6.14	9.67	10.73	6.99	9.04
4	15 - 20%	6.63	6.99	10.88	11.61	8.07	9.98
5	20 - 25%	7.11	7.34	11.90	12.27	8.77	11.55
6	25 - 30%	7.89	8.16	12.65	12.72	10.56	11.73
7	30 - 35%	8.30	8.91	13.21	13.70	11.58	10.80
8	35 - 40%	8.62	9.33	13.91	14.40	11.98	12.52
9	40 - 45%	9.04	10.55	14.73	14.88	12.98	15.03
10	45 - 50%	10.00	10.92	14.93	15.35	12.73	13.56
11	50 - 55%	10.83	11.84	15.83	15.99	14.79	14.05
12	55 - 60%	11.54	12.62	16.94	16.44	14.94	18.00
13	60 - 65%	12.65	13.53	18.18	18.19	16.79	19.89
14	65 - 70%	13.74	14.63	19.40	19.40	18.92	20.40
15	70 - 75%	15.83	15.09	20.38	21.45	18.21	19.17
16	75 - 80%	16.95	16.75	22.59	23.07	21.86	19.42
17	80 - 85%	18.08	19.00	25.90	26.22	23.60	23.93
18	85 - 90%	20.19	21.33	29.65	28.63	26.61	26.49
19	90 - 95%	25.33	23.77	34.38	31.00	30.13	30.95
20	95 -100%	45.34	34.89	44.55	37.40	38.54	33.20
all	0 -100%	13.13	13.08	18.36	18.21	16.23	17.04

Table A . 8.3. Average per capita expenditure on food items (rural, West Bengal)

	per cent of population	Average per capita food expenditure (Rs.)					
		1952-53		1957-58			
				undeflated		deflated	
		ss1	ss2	ss1	ss2	ss1	ss2
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	Bottom 5%	2.46	3.12	5.31	6.80	4.44	5.91
2	5 - 10%	3.60	3.98	7.59	8.35	6.31	7.06
3	10 - 15%	4.65	4.41	7.92	9.09	5.72	7.66
4	15 - 20%	4.82	5.04	9.80	10.04	7.27	8.63
5	20 - 25%	5.17	5.36	10.28	9.94	7.58	9.35
6	25 - 30%	5.97	5.61	10.51	10.04	8.77	9.26
7	30 - 35%	6.04	6.32	11.03	11.22	9.67	8.84
8	35 - 40%	6.61	6.55	10.75	11.90	9.26	10.35
9	40 - 45%	6.50	7.29	12.51	10.96	11.02	11.07
10	45 - 50%	6.62	7.75	11.57	11.87	9.86	10.48
11	50 - 55%	6.95	7.58	12.72	12.67	11.88	11.13
12	55 - 60%	7.51	9.06	13.79	13.35	12.16	14.62
13	60 - 65%	8.97	8.50	14.82	13.34	13.69	14.58
14	65 - 70%	9.83	9.28	15.74	15.34	15.35	16.13
15	70 - 75%	9.40	9.51	16.78	16.86	14.99	15.07
16	75 - 80%	10.74	10.69	18.03	16.45	17.45	13.84
17	80 - 85%	11.82	11.40	17.04	20.48	15.53	18.69
18	85 - 90%	11.51	11.77	20.94	22.16	18.79	20.51
19	90 - 95%	11.29	13.76	22.52	23.23	19.74	23.19
20	95 -100%	19.28	17.27	23.63	23.47	20.44	20.83
all	0 -100%	7.94	8.13	13.70	13.94	12.11	13.05

Table A. 8.4. Average per capita expenditure on non-food item (rural, West Bengal)

	per cent of population	Average per capita non-food expenditure (Rs.)					
		1952-53		1957-58			
				undeflated		deflated	
		ss1	ss2	ss1	ss2	ss1	ss2
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	Bottom 5%	1.17	1.16	0.78	1.67	0.65	1.45
2	5 - 10%	1.27	1.29	0.91	1.26	0.76	1.07
3	10 - 15%	1.25	1.73	1.75	1.64	1.26	1.38
4	15 - 20%	1.81	1.95	1.08	1.57	0.80	1.35
5	20 - 25%	1.94	1.98	1.62	2.33	1.19	2.19
6	25 - 30%	1.92	2.55	2.14	2.68	1.79	2.47
7	30 - 35%	2.26	2.59	2.18	2.48	1.91	1.95
8	35 - 40%	2.01	2.78	3.16	2.50	2.72	2.17
9	40 - 45%	2.54	3.26	2.22	3.92	1.96	3.96
10	45 - 50%	3.38	3.17	3.36	3.48	2.86	3.07
11	50 - 55%	3.88	4.26	3.11	3.32	2.91	2.92
12	55 - 60%	4.03	3.56	3.15	3.09	2.78	3.38
13	60 - 65%	3.68	5.03	3.36	4.85	3.10	5.30
14	65 - 70%	3.91	5.35	3.66	4.06	3.57	4.27
15	70 - 75%	6.43	5.58	3.60	4.59	3.22	4.10
16	75 - 80%	6.21	6.06	4.56	6.62	4.41	5.57
17	80 - 85%	6.26	7.60	8.86	5.74	8.07	5.24
18	85 - 90%	8.68	9.56	8.71	6.47	7.82	5.99
19	90 - 95%	14.04	10.01	11.86	7.77	10.39	7.76
20	95 -100%	26.06	17.62	20.92	13.93	18.10	12.36
all	0 -100%	5.19	4.95	4.66	4.27	4.12	4.00