INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) I YEAR: 1996-97
SEMESTRAL-I EXAMINATION
CALCULUS-I

Date:26.11.96 Maximum Marks:100 Time: 3 Hours

Note:Each question carries ten marks. You may answer as many as you can. But the maximum you can score is 100.

1.(a) Define supremum of a set of real numbers. Show that supremum of a set is uniquely determined whenever if exists. [5]

(b) Determine the supremum for the following sets, whenever they exist. Justify your answer.

(i) $\left\{ \frac{1}{x} : 0<x\leq 1\right\}$, (ii) $\left\{ x\in\mathbb{Q} : x\geq 0,\ x^2<2 \right\}$, (iii) $\left\{ \frac{2^n-1}{2^n} : n\geq 1\right\}$. [5]

2.(a) Define accumulation point of a set of real numbers. Suppose $(a_n)_{n=1}^{\infty}$ is a sequence of reals and x is an accumulation point of the range $\left\{a_n : n\geq 1\right\}$. Show that there exists a subsequence of $(a_n)_{n=1}^{\infty}$ converging to x. [5]

(b) Find a convergent subsequence of the sequence $(a_n)_{n=1}^{\infty}$,

where $a_n=(-1)^n\ (1-\frac{1}{n})$. [5]

3. Discuss the convergence of the following series.
State explicitly the results that you use.

(a) $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ (ii) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$. [5]

(b) Use Taylor's theorem to prove

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$ [5]

4.(a) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be continuous at $a \in \mathbb{R}$. Show that for every sequence $(x_n)_{n=1}^{\infty}$ converging to a, the sequence $(f(x_n))_{n=1}^{\infty}$ converges to f(a). [5]

(b) Let $f,g : \mathbb{R} \longrightarrow \mathbb{R}$ be continuous functions. Show that the set $D = \left\{x\in\mathbb{R} : f(x)=g(x)\right\}$ is a closed set. [5]

5.(a) State intermediate value property of continuous functions of reals.

(b) Use this property to prove the following:
If n is a positive integer and a>0, then there exists exactl one positive real b such that $b^n=a$.

6. Define uniform continuity of a function $f:S\longrightarrow \mathbb{R}$, $S\subseteq \mathbb{R}$. Prove tl the function $f:(2.5,3)\longrightarrow\mathbb{R}$ defined by $f(x) = \frac{3}{x-2}$ is uniformly continuous.

7. Let $f:(a,b)\longrightarrow\mathbb{R}$ be an increasing function. Prove that the set $D=\{x\varepsilon(a,b): f \text{ is discontinuous at } x\}$ is countable.

8. Prove that every compact subset of $\mathbb{R}$ is bounded. Show that the set $S = (0,1]$ is not compact by producing an open cover which does not admit any finite subcover.

9. Let $f:\mathbb{R}\longrightarrow\mathbb{R}$ be a continuous function such that $f(x+y)=f(x)f(y$ for all $x,y\varepsilon\mathbb{R}$. Prove that either $f(x)=0$ for all $x\varepsilon\mathbb{R}$ or there exists $a>0$ such that $f(x)=a^x$ for all $x\varepsilon\mathbb{R}$.

10.(a) Let $f:\mathbb{R}\longrightarrow\mathbb{R}$ be a function and $x\varepsilon\mathbb{R}$. Assume that there exists a constant A and a real valued function E defined on a neighbourhood of 0, E(0)=0 and E is continuous at 0 such that for every h in that neighbourhood we have $f(x+h)=f(x)+hA+hE(h)$. Prove that f is differentiable at x and find $f'(x)$. [

(b) Discuss the differentiability of the following function:

$$f(x) = x^2\sin\frac{1}{x} \qquad x\neq 0$$
$$= 0 \qquad x=0$$

Is it a C'-function? Justify your answer. [

11.(a) State Rolle's Theorem.
Let $f:[a,b]\longrightarrow\mathbb{R}$ be continuous, f' exist on (a,b) and $f(a) = f(b) = 0$. Show that for every real λ there exists $c\,\varepsilon\,(a,b)$ such that $f'(c) = \lambda f(c)$. [

(b) Prove that if $f:[a,b]\longrightarrow\mathbb{R}$ is continuous and $f'(x)$ exists and $f'(x)\neq 0$ for all $x\varepsilon(a,b)$, then f is one to one. [

12. (a

12.(a) Prove that $|\sin x - \sin y| \leq |x-y|$. [5]

(b) Consider the function $f(x)=[x]$. Does there exist a function g such that $g'(x)=f(x)$ for all x. Justify your answer. [5]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) I Year : 1996-97
SEMESTRAL - I EXAMINATION
Probability Theory and its Applications I

Date: 28.11.1996 Maximum Marks: 100 Time: 3 hours

Note: Answer FIVE questions.

1.(a) It is known that on the average, one person out of 10,000 has a certain rare disease. A test for this disease is known to make a correct diagnosis with probability 95%. What is the probability that someone who has been tested positive by this test, has the disease ?

(b) Suppose every packet of detergent TIDE contains a coupon bearing one of the letters of the word TIDE. A customer who has all the letters get a free packet. Each letter has the same chance of appearing in a packet. Find the probability that if you buy 8 packets, you will get at least one free packet.

(10+10) = [20]

2.(a) Consider Polya's urn scheme with the specifications: Initially the urn has r red balls and b black balls; after every draw, the ball is replaced and c balls of the colour drawn are added to the urn. Let $X_n = 1$ if the ball drawn at the nth trial is red and 0 otherwise.

Find the joint distribution of (X_2, X_4).

(b) Let X have the distribution $\begin{pmatrix} a_1 & a_2 & a_3 & \cdots \\ p_1 & p_2 & p_3 & \cdots \end{pmatrix}$

and Y the distribution $\begin{pmatrix} -a_1 & -a_2 & -a_3 & \cdots \\ p_1 & p_2 & p_3 & \cdots \end{pmatrix}$

and suppose X and Y are independent. Show that

$$P(X+Y = t) = P(X+Y = -t) \quad \text{for all } t.$$

(12+8) = [20]

3.(a) Let X, Y be independent random variables each having the distribution

$$\begin{pmatrix} 0 & 1 & 2 & \cdots\cdots \\ p & qp & q^2p & \cdots\cdots \end{pmatrix}$$

Find the distribution of $X \wedge Y = \min(X, Y)$.

contd..... 2/-

(b) Find a function f in the problem (a) such that f(X) has the same distribution as $X \wedge Y$.

(c) Does (b) mean that $X \quad Y = f(X)$? Give reasons for your answer.

(10+6+4) = [20]

4.(a) Let X be a nonnegative integer valued random variable whose probability generating function $\varphi_X(t)$ is finite for all t and let $\lambda > 0$. Prove that

$$P(X \le \lambda) \le \frac{\varphi_X(t)}{t^{\lambda}}$$

for $0 \le t \le 1$.

(b) Let $X_1, \ldots\ldots, X_k$ be non-negative independent identically distributed random variables. Find

$$E\left(\frac{X_1}{X_1 + \cdots + X_k}\right).$$

(12+8) = [20]

5. A random sample of size n is drawn from N tickets marked 1, ... N. Let X be the sum of the numbers sampled. Find E(X) and Var(X) when

(i) sampling is done with replacement
(ii) sampling is done without replacement.

[20]

6.(a) Let X and Y be two random variables such that for some real number a

$$P(X = t) \le P(Y = t) \quad \text{if } t < a$$
$$P(X = t) \ge P(Y = t) \quad \text{if } t \ge a.$$

Assuming that E(X) and E(Y) exist, prove that

$$E(X) \ge E(Y).$$

(b) Let $X_1, \ldots X_n$ be independent non-negative random variables. Prove that

$$P(X_1 + \cdots + X_n > a) \le 1 - \prod_{i=1}^{n} P\left(X_i < \frac{a}{n}\right).$$

(12+8) = [20]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) I Year : 1996 - 97
SEMESTRAL - I EXAMINATION
Statistical Methods I

Date: 29.11.1996 Maximum Marks: 100 Time: 3 hours

Note: Attempt ALL questions. Scores allocated to each question are given in parantheses [ ].

1. The table below represents the frequency distribution of salaries (per month) of 'computer specialists' in the present day job market:

Class Intervals ('00 s)	Number of specialists
55 - 64	3
65 - 74	21
75 - 84	78
85 - 94	182
95 - 104	305
105 - 114	209
115 - 124	81
125 - 134	21
135 - 144	5

(a) Compute the median for the data by direct computation and also from the ogive (less than type) and compare the results. Also obtain the Inter Quartile Range from the ogive. Explain whether you prefer mean to median as a measure of central tendency in this case and give reasons.

(b) The Computer Society claims that the coefficient of variation for this type of data cannot exceed 5%. Is the claim correct ? Verify.

[(6+6+4+2) + (10)] = [28]

2.(a) For classification of data by attributes, explain the terms (i) ultimate class frequencies and (ii) positive class frequencies. Explain with an illustration why it is never necessary to enumerate more than the ultimate frequencies.

contd..... 2/-

INDIAN STATISTICAL INSTITUTE
B.STAT. (HONS.) I YEAR: 1996-97
SEMESTRAL-I EXAMINATION
VECTOR AND MATRICES-I

Date: 2.12.96 Maximum Marks: 100 Time: $3\frac{1}{2}$ Hours

Note: There are 9 questions carrying 119 marks. The maximum you can score is 100. Marks allotted to each question are shown in brackets.

1. Let $\mathcal{V}$ be the real vector space of all real-valued functions defined on the real line $\mathbb{R}$. Determine whether each of the following subsets of $\mathcal{V}$ is dependent or independent. Compute the dimension of the subspace spanned by each set:
(i) $\{\cos x, \sin x\}$ (ii) $\{1, \cos 2x, \sin^2 x\}$. [4+4=8]

2. If $f \in (\mathcal{M}_n(\mathbb{C}))'$ = Dual of $\mathcal{M}_n(\mathbb{C})$, the set of all nxn complex matrices, such that $f(AB)=f(BA)$ for all $A, B \in \mathcal{M}_n(\mathbb{C})$, show that $f(A)=c\,tr(A)$ for some $c \in \mathbb{C}$ where $tr(A)$=trace of A. [12]

3. Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation whose matrix relative to the standard bases $\{e_1, e_2\}$ and $\{e_1, e_2, e_3\}$ is

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ -3 & -1 \end{pmatrix}$$

Let new bases be defined by $\{\alpha_1 = e_1 - 2e_2,\ \alpha_2 = e_1 + e_2\}$ and $\{\beta_1 = e_1 + e_2,\ \beta_2 = e_2 + e_3,\ \beta_3 = e_1 + e_3\}$ in $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively. Compute the matrix which represents T relative to the latter bases. [10]

4. Show that

$$\begin{vmatrix} 1 & a_1 & a_1^2 \cdots\cdots a_1^{n-2} & a_1^n \\ 1 & a_2 & a_2^2 \cdots\cdots a_2^{n-2} & a_2^n \\ \vdots & \vdots & \vdots & \vdots \\ 1 & a_n & a_n^2 \cdots\cdots a_n^{n-2} & a_n^n \end{vmatrix} = (a_1 + a_2 + \ldots + a_n) \prod_{1 \le i < j \le n} (a_j - a_i)$$

[14]

5. Let A be an mxn real matrix. Prove that rank $(A'A)$=rank (A). Give an example of a matrix A with complex entries such that rank $(A'A)$<rank (A). What can you say if A' is replaced by $(\bar{A})'$ where $\bar{A}$ is the complex conjugate of A and why? [6+8+6=20]

Indian Statistical Institute

BStat–I : 1996-97

Computational Techniques & Programming – I

Semestral–I Examination

Date 4 December 1996 — Time 3 hrs.

Answer as many questions as you can.
Maxium you can score is 100

1. Write short notes on
   (i) Subprogram (ii) Debugger
   (iii) File Organizations and Access Methods

   [5 + 5 + 10 = 20]

2. (a) Describe, briefly, different parts of a digital computer.

   (b) Calculate the number of 80-column input records that can be stored on a magnetic tape of 3600 feet length assuming :

   i. one record per block, 1600 characters per inch density, and $\frac{1}{4}$ inch inter-record gaps.

   ii. Twenty-five records per block, 3200 character per inch and $\frac{1}{4}$ inch interblock gaps.

   [6 + 6 = 12]

3. (a) Write VAX DCL commands to create a text module library named TEXTLIB and add to it two modules named MODULE1 and MODULE2, which are given in the files 'TEST1.FOR' and 'TEST2.PAS' respectively. Then replace MODULE1 by the content of the file 'TEST2.FOR'

   (b) Explain clearly what you have to do to access the modules in an object module library called 'OBJECT.OLB' from your VAX FORTRAN program.

   [8 + 5 = 13]

4. Convert the following decimal numbers to their binary forms, using binary arirhmetic. Describe the algorithm you are using.

   (i) −99.625 (ii) 128.0 (iii) −107 (iv) 0.1

   Show how these numbers are stored internally in VAX/VMS system. Assume that the integer numbers are stored in two bytes and real numbers are stored in four bytes.

   [10 + 5 + 10 = 25]

5. Given an integer number N, write a FORTRAN Function subprogram to find the sum of its digits.

   [15]

provided EX exists.

(7+7+6) → [20]

6. Consider the following VAX FORTRAN statements

```
      DIMENSION A(10), B(3,3), C(2,3,2)
      EQUIVALENCE (A(10), B(3,3)), (A(10), C(2,3,2))
      N = 3
      OPEN (UNIT=1,FILE='TESTBI.DAT',STATUS='OLD')
      OPEN (UNIT=2,FILE='TESTBI.OUT',STATUS='NEW')
      READ (1,111) C
      WRITE (2,222) ((B(I,J), J=1,3), I=1,3)
111   FORMAT (< N + 1 > F2.0)
222   FORMAT (1X, < N > F6.1)
      END
```

The file 'TESTBI.DAT' contains the following records (from first column onwards)

Record 1 : 1 ƀ 2 ƀ 3 ƀ 4 ƀ 5 ƀ 6 ƀ 7 ƀ 8 ƀ 9 ƀ 10 ƀ

Record 2 : 11 ƀ 12 ƀ 13 ƀ 14 ƀ 15 ƀ 16 ƀ 17 ƀ

Record 3 : 2 ƀ 2 ƀ 3 ƀ 3 ƀ 4 ƀ 4 ƀ 5 ƀ 5 ƀ

Record 4 : 21 ƀ 22 ƀ 23 ƀ 24 ƀ 25 ƀ 26 ƀ 27 ƀ

Show the allignment of the three arrays. What will be the content (indicate the columns) of the file 'TESTBI.OUT' when the above program is run on VAX machine ?

[5 + 10 = 15]

7. The following information is available on each employee of an organisation.

   (a) Employee ID
   (b) Name and address
   (c) Department/Unit to which the employee is currently attached
   (d) Date of
       (i) birth (ii) joining (iii) increment (iv) last promotion
   (e) Salary consisting of
       (i) Basic Pay (ii) City Compensetory Allowance
       (iii) Dearness Allowance (iv) House Rent Allowance

   Administration of the organisation would like know the following information from time to time 3

   - Information about an employee
   - List of all employees in a department/unit
   - Designation wise list of employees

   Describe a suitable data structure to store these information. How would you create a file so that employer would be able to access the information efficiently and effectively ?

   [6 + 9 = 15]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) I Year : 1996-97
BACKPAPER SEMESTRAL - I EXAMINATION
Probability Theory and its Applications I

Date: 30.12.96 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL questions.

1.(a) A car is parked among N cars in a row, not at either end. On his return the owner finds that exactly r of the N places are still occupied. What is the probability that both the neighbouring places are empty ?

(b) In the random placement of r balls in n cells find the probability that all the cells are occupied.

(10+10) = [20]

2.(a) A cointossing experiment continues until one run of r heads is obtained. Let p be the probability for the coin to fall heads and q = 1 - p. Show that, if p_n is the probability for the experiment to conclude in n or fewer tosses, then

$$p_{n+1} = p_n + (1 - p_{n-r})\, p^r q$$

for all $n \geq r$.

(b) Give an example to show that the joint distribution of two random variables X and Y is not determined by the distribution of X and the distribution of Y.

(12+8) = [20]

3.(a) Prove that for a random variable X, EX^3 exists if $E(X^4)$ exist

(b) Show that if X and Y are random variables such that Var X and Var Y exist, then

$$|\operatorname{cov}(X,Y)| \leq \sqrt{\operatorname{Var} X \,.\, \operatorname{Var} Y}.$$

(c) Show that if X is a non-negative integer valued random variable, then

$$EX = \sum_{n=1}^{\infty} P(X \geq n)$$

provided EX exists.

(7+7+6) = [20]

4.(a) Show that if X and Y have Poisson distribution with param λ and μ respectively and are independent, then the r.v.) has a Poisson distribution.

(b) Show that the result in (a) may not hold if X and Y are n independent.

(c) Prove that if X has Poisson distribution with parameter λ then

$$P(X > 2\lambda) \leq \frac{1}{\lambda}.$$

(8+4+8) = [

5.(a) Define the conditional expectation E(X|Y). Prove that

$$E(E(X|Y)) = EX.$$

(b) If X and Y are independent having the common geometric distribution,

0	1	2	3	4	.....
p	qp	q^2p	q^3p		

find the conditional distribution of X given X+Y = n.

(8+12) = [

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) I Year : 1996 - 97
BACK-PAPER SEMESTRAL - I EXAMINATION
Statistical Methods I

Date: 31.12.96 Maximum Marks: 100 Time: 3 hours

Note: Attempt ALL questions. Scores allocated to each question are given in parentheses [ ].

1. The table given below represents the frequency distribution of intelligence quotients of school children from 5 to 14 years of age:

Class-limits	Number of children
55 - 64	3
- 74	21
- 84	78
- 94	182
- 104	305
- 114	209
- 124	81
- 134	21
- 144	5

(a) Find the median by direct computation and check this value by drawing the ogive (less than type) and the Ogive (more than type for the (reletive) frequency distribution. Also obtain the inter-quartile range from the Ogive (less than).

(b) A psychometrician claims that the coefficient of variation for this type of data cannot exceed 5%. Is this claim correct ? Explain.

[(4+6+4) + (10)] = [24

2.(a) Let (X) denote the number of individuals possessing attribute X. In a population of N individuals, if $\frac{(X)}{N}$, $\frac{(Y)}{N}$ and $\frac{(Z)}{N}$ are α, 2α and 3α respectively while $\frac{(XY)}{N}$, $\frac{(YZ)}{N}$ and $\frac{(XZ)}{N}$ are all equal to β, show that the value of neither α nor β can exceed 1/4.

(b) A newspaper reported that "of the 998 cases dealt with by high court, 313 involved foreign exchange, 526 related to illegal property while bribery in contracts was the main ground in 471 cases. Only 43 cases involved both foreign exchange and illegal property while 148 cases related to foreign exchange as well as bribery. In 87 cases all the three causes were cited". Examine if the report is corr

(10+10) = [

3.(a) What do you understand by "Kurtosis" and "Skewness".

(b) With the usual notation, prove that $\beta_2 \geq \beta_1 + 1$. When do the equality occur ?

(c) A student has calculated the following quantities for N = observations:

$\Sigma x_i = -114$, $\Sigma x_i^2 = 1030$, $\Sigma x_i^3 = -1590$, $\Sigma x_i^4 = 1424$.

Explain with reasons if these are possible.

[6 + (10+2) + 8] = [

4.(a) A class teacher wishes to consider the "best two" class t scores out of three held during a semester, for computing the final grade. He decides to take those two test score for which ρ_{ij}, the correlation coefficient between the ith and jth test scores is the largest. Explain with an illustration if this is a good procedure.

(b) The railways planned to minimize the 'discomfort' for passengers of chair car. They defined 'discomfort' as the average deviation of knee heights and height of the chair In a survey of 420 adult passengers, it war found that the average knee height was 45 cm. and the railways then order chairs to be made with a height of 45 cm. for the next bat of chair cars. Comment on the decision of the railways wi relevant theory.

(c) An institute plans to conduct a survey in South Calcutta t find out the average household expenditure, during the 3 da of puja. If you are hired as a statistical consultant, explain the various steps you would take in conducting thi survey, briefly outlining the schedule you would use for data collection.

(10+10+10) = [3

:bcc:

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) I YEAR : 1996-97
SEMESTRAL-I BACKPAPER EXAMINATION
CALCULUS-I

:e: 1.1.97 Maximum Marks:100 Time: 3 Hours

Note: Each question carries 10 marks. You may answer as many as you can. But the maximum you can score is 100.

(a) State the least upper bound axiom for the set of real numbers. Deduce that every non-empty set of reals that is bounded below has a greatest lower bound. [5]

(b) Determine the greatest lower bound and the least upper bound for the following sets, whenever they exist. Justify your answ

(i) $\{x \in Q : \sqrt{2} < x\} \cap \{x \in Q : x < 2\}$

(ii) $S = (-\infty, 0] \cup \{\frac{1}{2^m} + \frac{1}{3^n} : m, n \geq 1\}$. [5]

(a) Define accumulation point of a set of real numbers. Suppose the sequence $(a_n)_{n=1}^{\infty}$ converges to l and the range $\{a_n : n \geq 1\}$ of the sequence is an infinite set. Show that l is an accumulati point of $\{a_n : n \geq 1\}$. [5]

(b) Prove that if the sequence $(a_n)_{n=1}^{\infty}$ converges to l, then the sequence $(|a_n|)_{n=1}^{\infty}$ converges to $|l|$. Is the converse true? Justify your conclusion. [5]

(a) Let ΣA_n be an absolutely convergent series and $(b_n)_{n=1}^{\infty}$ be a bounded sequence. Prove that the series $\Sigma a_n b_n$ is absolutely convergent. [5]

(b) Prove that the series $\sum_{n} (-1)^{n+1} \frac{n+1}{n^5}$ is convergent. [5

(a) Let $f: \mathbb{R} \to \mathbb{R}$ be continuous at $a \in \mathbb{R}$. Show that for every sequence $(x_n)_{n=1}^{\infty}$ converging to a, the sequence $(f(x_n))_{n=1}^{\infty}$ converges to $f(a)$. [5]

(b) Let $f(x) = \begin{cases} \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$

Use part (a) to show that f is not continuous at $x = 0$. [5]

p.t.o.

5. State intermediate value property of continuous functions of . Prove that if $f:[0,1] \to [0,1]$ is continuous then there exists $c \in [0,1]$ such that $f(c)=c$.

6. Let $f: S \to \mathbb{R}$ be continuous and $S \subseteq \mathbb{R}$ be compact. Prove that 3.1.1997 uniformly continuous on S.

7. Prove that the function $f(x) = x - [x]$, $x \in \mathbb{R}$ is periodic an determine its period. Determine the points where f is discont and the nature of discontinuity of f at those points.

8. (a) Determine which of the following sets are compact. Justify answer.

(i) Q (the set of rationals) (ii) $\{1, 2, 3, \ldots\ldots\}$

(iii) $\{\pi : 0 < x \leq 1\}$, (iv) $\{\frac{1}{n} ; n \geq 1\} \cup \{0\}$.

(b) Let $f: S \to \mathbb{R}$ be continuous and $S \leq \mathbb{R}$ be compact. Prove that there exists reals A,B such that $A \leq f(x) \leq B$ for all $x \in S$.

9. (a) Suppose $f, g: \mathbb{R} \to \mathbb{R}$ are continuous functions. Show that th functions max (f,g) and min (f,g) are also continuous.

(b) Prove that the function $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ -x & \text{if } x \text{ is irrational} \end{cases}$

is continuous only at $x=0$.

10. (a) Let $f(x) = \begin{cases} \frac{1}{|x|} & \text{if } |x| > c \\ a + bx^2 & \text{if } |x| \leq c. \end{cases}$

Determine the values of a and b (in terms of c) such that $f'(c)$ exists.

(b) If $0 < a < b$ and $x > 1$, then prove that

$$\frac{x^a - 1}{a} < \frac{x^b - 1}{b}.$$

11. (a) State 'Mean Value Theorem.' Prove that if $f:[a,b] \to \mathbb{R}$ is con and $f'(x)$ exists and $f'(x) > 0$ for all $x \in (a,b)$, then f is strictly increasing function.

(b) Use 'Mean Value Theorem' to deduce the following inequality

$$ny^{n-1}(x-y) \leq x^n - y^n \leq nx^{n-1}(n-y) \text{ if } 0 < y \leq x, n \geq 1.$$

12. Determine the intervals of convexity, concavity and the point inflection for the function $f(x) = e^{-x^2}$. Hence give a rough ske of the graph of the function.

INDIAN STATISTICAL INSTITUTE
BSTAT—I : 1996-97
COMPUTATIONAL TECHNIQUES & PROGRAMMING – I
SEMESTRAL–I EXAMINATON
(BACK PAPER)

3.1.1997 Maximum Marks 100 Time 3 hrs.

Answer as many questions as you can.

(a) How would you classify computers ?

(b) Describe, in brief, two external storage (mass storage) devices. Describe the different types of files, which can be stored on them.

[8 + 6 + 6 = 20]

What are the differences between

(a) a compiler and an interpreter

(b) a function subprogram and a subroutine subprogram

(c) a function subprogram and a statement function, and

(d) flow chart and algorithm

[12]

3. (a) Find a binary number which approximate π to within 10^{-3}.

(b) Convert the following octal numbers to their decimal equivalent

(i) $(.614)_8$ (ii) $(7.776)_8$ (iii) $(777)_8$

Show how these numbers are stored internally in VAX/VMS system. Assume that integer numbers are stored in 2 bytes and real numbers are stored in 4 bytes.

[6 + 9 = 15]

4. Consider the following VAX FORTRAN statements

```
      CHARACTER * 6 WORD
      READ (1,111) WORD
111   FORMAT( A4 )
      WRITE (2,222) WORD
222   FORMAT( 5X, A7)
```

What will the be content of WORD ? Describe exactly, indicating column numbers, the content of disk file FOR002.DAT when the following statements are executed with each of the data given below. Each data indicated is from 1st column onward, columns after the last character shown are blanks.

(i) ƀ ISI (ii) John ƀ F. ƀ Kennedy
(iii) HOLD (iv) DIGITAL

If the FORMAT with number 111 is changed to A8, what will be the content of WORD and the corresponding output ?

[16]

P.T.O.

5. Given two positive integers N and M, write a FORTRAN Function subprogam to find their Highest Common Factor.

[15]

6. A book in a library is identified by its catalogue number and its accession number. Accession number is unique for each book that is if there are more than one copy of a book in the library, catalogue number in each copy of the same book will be same but the accession number will be different. The library wants the information computerised so that it can

(a) list all books by a an author,

(b) list all books added to the library during a given period,

(c) list all books on a subject,

(d) list all books on special topic

Describe the necessary information you need to solve the above problem. Justify your requirements. Describe how would you store the above information so that it can be used efficiently and effectively. Also, define a data structure to represent the information you need.

[10 + 5 + 5 + 5 = 25]

—x—

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) I YEAR: 1996-97
SEMESTRAL-II EXAMINATION
CALCULUS-II

Date: 28.4.97 Maximum Marks:100 Time: 3 Hours

Note: The paper carries of total 110 marks. You can answer as many as you can, but the maximum you can score is 100.

1.(a) Define a convex function.

(b) Suppose $f: [a,b] \to \mathbb{R}$ is an increasing function and $\int_a^x f(t)\,dt$ exists for all $x \in [a,b]$. Prove that the function F defined by $F(x) = \int_a^x f(t)\,dt$, $x \in [a,b]$ is convex. [3+7=1

2.(a) Let $g : [a,b] \to \mathbb{R}$ be continuous and $f: [a,b] \to \mathbb{R}$ be continuously differentiable function and $f'(x) \geq 0$ for all $x \in [a,b]$. Then show that

$$\int_a^b f(x)g(x)dx = f(a)\int_a^c g(x)dx + f(b)\int_c^b g(x)\,dx \text{ holds}$$

for some $c \in [a,b]$.

(b) Use part a) to deduce: If φ'' is continuous and non-zero on $[a,b]$ and if there exists $m > 0$ such that $\varphi''(t) > m$ for all t in $[a,b]$, then

$$\left| \int_a^b \sin \varphi(t)\,dt \right| \leq \frac{4}{m}.$$

[10+5=15]

3.(a) Define $f : [0,1] \to \mathbb{R}$ as follows:

$$f(x) = \begin{cases} \sqrt{1-x^2} & \text{if } x \text{ is rational} \\ 1-x & \text{if } x \text{ is irrational} \end{cases}$$

Determine whether f is Riemann-integrable or not.

(b) Let A be a dense subset of $[0,1]$ and $f:[0,1] \to \mathbb{R}$ be an integrable function in the Riemann sense such that $f(x)=0$ for all $x \in A$. Prove that $\int_0^1 f(x)dx=0$. [10+5=15]

4.(a) Determine the range of p for which $\int_1^\infty \frac{x^{p-1}}{1+x}dx$ is convergent.

(b) Evaluate $\int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2}$ [10+5=15]

5.(a) Prove that $\Gamma(a) = \int_0^\infty x^{a-1}e^{-x}dx$ converges if and only if $a > 0$

(b) Evaluate $\int_0^\infty e^{-t^2}\,dt$. [10+5=15]

p.t.o.

6.(a) Investigate the existence of the two iterated limits and double limit of the double seequence

$$f(p,q) = (-1)^{p+q}\left(\frac{1}{p} + \frac{1}{q}\right).$$

(b) Show that the double series $\sum_{m,n} \frac{1}{m^4+n^4}$ is convergent. [2+2+2+

7.(a) Let $(r_n)_{n=1}^\infty$ be a sequence of rational numbers in $[0,1]$ containing each rational number in $[0,1]$ exactly once. D for each positive integer n,

$$f_n(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 0 & \text{if } x=r_i,\ i > n \\ 1 & \text{if } x=r_i,\ i \leq n. \end{cases}$$

Show that (f_n) converges pointwise on $[0,1]$ to a function that is not Riemann-integrable.

(b) Show by an example that there may exist a sequence (f_n) of differentiable functions having a pointwise limit func but the sequence (f_n') may not have a pointwise limit [

8. Let $0<\delta<\pi$, prove that the series $\sum_{n=1}^\infty \frac{e^{inx}}{n}$ converges unifor on $[\delta, 2\pi-\delta]$.

9.(a) Find the radius of convergence of the power series

$$\frac{1}{3} - x + \frac{x^2}{3^2} - x^3 + \frac{x^4}{3^4} - x^5 + \ldots\ldots$$

(b) Suppose $f: [0,1] \to \mathbb{R}$ is continuous and $\int_0^1 f(x)x^n\,dx = 0$ for all $n = 0,1,2,\ldots\ldots$. Prove that f is zero identically on $[0,1]$. [

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) I YEAR: 1996-97
SEMESTRAL-II EXAMINATION
CALCULUS-II

Date: 28.4.97 Maximum Marks:100 Time: 3 Hours

Note: The paper carries of total 110 marks. You can answer as many as you can, but the maximum you can score is 100.

1. (a) Define a convex function.

(b) Suppose $f: [a,b] \longrightarrow \mathbb{R}$ is an increasing function and $\int_a^x f(t)\,dt$ exists for all $x \in [a,b]$. Prove that the function F defined by $F(x) = \int_a^x f(t)\,dt$, $x \in [a,b]$ is convex. [3+7=

2. (a) Let $g : [a,b] \longrightarrow \mathbb{R}$ be continuous and $f: [a,b] \longrightarrow \mathbb{R}$ be continuously differentiable function and $f'(x) \geq 0$ for all $x \in [a,b]$. Then show that

$$\int_a^b f(x)g(x)dx = f(a)\int_a^c g(x)dx + f(b)\int_c^b g(x)\,dx \text{ holds}$$

for some $c \in [a,b]$.

(b) Use part a) to deduce: If ϕ'' is continuous and non-zero on $[a,b]$ and if there exists $m > 0$ such that $\phi''(t) > m$ for all t in $[a,b]$, then

$$\left|\int_a^b \sin\phi(t)\,dt\right| \leq \frac{4}{m}.$$

[10+5=1

3. (a) Define $f : [0,1] \longrightarrow \mathbb{R}$ as follows:

$$f(x) = \begin{cases} \sqrt{1-x^2} & \text{if } x \text{ is rational} \\ 1-x & \text{if } x \text{ is irrational} \end{cases}$$

Determine whether f is Riemann-integrable or not.

(b) Let A be a dense subset of $[0,1]$ and $f:[0,1] \longrightarrow \mathbb{R}$ be an integrable function in the Riemann sense such that $f(x)=0$ for all $x \in A$. Prove that $\int_0^1 f(x)dx=0$. [10+5=15]

4. (a) Determine the range of p for which $\int_1^\infty \frac{x^{p-1}}{1+x}dx$ is convergent.

(b) Evaluate $\int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2}$ [10+5=15]

5. (a) Prove that $\Gamma(a) = \int_0^\infty x^{a-1}e^{-x}dx$ converges if and only if $a > C$

(b) Evaluate $\int_0^\infty e^{-t^2}dt$. [10+5=15]

p.t.o.

6. (a) Investigate the existence of the two iterated limits and double limit of the double seequence

$$f(p,q) = (-1)^{p+q}\left(\frac{1}{p} + \frac{1}{q}\right).$$

(b) Show that the double series $\sum_{m,n} \frac{1}{m^4+n^4}$ is convergent. [2+2+2+4

7. (a) Let $(r_n)_{n=1}^\infty$ be a sequence of rational numbers in $[0,1]$ containing each rational number in $[0,1]$ exactly once. De for each positive integer n,

$$f_n(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 0 & \text{if } x=r_i,\ i > n \\ 1 & \text{if } x=r_i,\ i \leq n. \end{cases}$$

Show that (f_n) converges pointwise on $[0,1]$ to a function that is not Riemann-integrable.

(b) Show by an example that there may exist a sequence (f_n) differentiable functions having a pointwise limit func but the sequence (f_n') may not have a pointwise limit [

8. Let $0<\delta<\pi$, prove that the series $\sum_{n=1}^\infty \frac{e^{inx}}{n}$ converges unifo on $[\delta, 2\pi-\delta]$.

9. (a) Find the radius of convergence of the power series

$$\frac{1}{3} - x + \frac{x^2}{3^2} - x^3 + \frac{x^4}{3^4} - x^5 + \dots\dots$$

(b) Suppose $f: [0,1] \longrightarrow \mathbb{R}$ is continuous and $\int_0^1 f(x)x^n\,dx = 0$ for all $n = 0,1,2,\dots\dots$.

Prove that f is zero identically on $[0,1]$.

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) I Year: 1996 - 97
SEMESTRAL-II EXAMINATION
Statistical Methods II

Date: 30.4.1997 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL the questions.

1.(a) What do you understand by the term "Intra Class Correlation Coefficient" ρ_c. Obtain the bounds for ρ_c.

(b) A random sample of n units is divided into k categories such that the ith category has n_i units. Assuming the multinomial probability model, derive $E(n_i)$, $V(n_i)$ and $Corr.(n_i, n_j)$ $i \neq j$. How do you compare the observed and expected frequencies ?

(c) In order to see the Hale - Bopp comet, a company planned to manufacture the following standard types of binoculars:

type	A	B	C	D	E
distance between the eye-pieces in mm.	54	58	62	66	70

From extensive optimetric data, it was found that the distribution of inter-pupillary distance (i.p.d.) of adults was Normally distributed with mean 64.90 mm. and s.d. 3.81 mm. Assuming that persons having i.p.d. differing by not more than 2 mm. from the distance between the eye-pieces have no difficulty in using the binoculars, find how may of each typ of these should be manufactured by the company in a lot of 1000 and why ?

[(5+3) + (3+3+2+2) + (12)] = [30]

2.(a) Derive the expression

$$\rho_{03.12} = \frac{\rho_{03.1} - \rho_{02.1}\,\rho_{32.1}}{\sqrt{(1-\rho^2_{02.1})(1-\rho^2_{32.1})}}$$

where the symbols have their usual meaning.

(b) After a state government has taken over a milk dairy, there has been a controversy that the per-capita consumption of

contd..... 2/-

milk (Y) has fallen down while the government claimed th the retail price of milk X_1 depended on the cost of proc ssing of milk (X_2) and cost of agricultural products (X_3 The statistician-in-charge has collected data on all the variables (measured from their means) for a period of 40 months. The following table gives sums of squares ar products of the deviations of these variables from thei

	Y	X_1	X_2	X_3
Y	1369.54	-352.55	-536.48	938.86
X_1		1581.49	850.33	1235.76
X_2			2534.80	730.78
X_3				2626.99

(i) Calculate the multiple correlation coefficient of Y the other variables and

(ii) the partial correlation coefficient between the variables X_3 and X_2 eliminating the effect of Y and X_1.

(iii) Comment on the calculations.

[(12)+(11+17+2)] = [4

3.(a) Draw a simple random sample of size 5 from the Cauchy distribution with density

$$f(x) = \frac{1}{\pi}\frac{1}{1+x^2}, \qquad -\infty < x < \infty.$$

(b) The following definition was given by a student: "A rando variable Y = log X is said to have a lognormal distributi if X has a Normal distribution".

Correct the definition pointing out the fallacies and wri down the density function. By writing the moment generat function or otherwise obtain the mean and variance of thi distribution.

Point out a situation where the lognormal distribution is

(c) Suppose that X has a Uniform distribution in the in Derive the distribution of Y = -2 log X. Based on a r sample of size n, say $X_1, X_2, \ldots, X_n$, write dow of the distribution of $Z = \sum_1^n Y_i$ explaining clearly all results you have used.

[(5)+(2+3+4+5+1)+(8)] =

:bcc:

INDIAN STATISTICAL INSTITUTE
B.STAT. (HONS.) I YEAR: 1996-97
SEMESTRAL-II EXAMINATION
COMPUTATIONAL TECHNIQUES AND PROGRAMMING II

:2.5.97 Maximum Marks:100 Time: 3 Hours

Note: Attempt all the questions.

fine spline function of degree K. Without using formula, constr cubic spline interpolating to the following date. Use clamped line end conditions, given $f'(0)=0$, $f'(3)=15$.

x	0	1	3
f(x)	0	1	19

[3+17=20]

efine degree of precision of a quadrature formula. Show that the of precision of the Lagrangian m point quadrature formula t exceed 2m-1. Derive a quadrature formula (determine A,B,C,α f the form

$$\int_{-1}^{1} f(x)dx \approx A\ f(-\alpha) + B\ f(0) + C\ f(\alpha)$$

which is exact for all polynomials of as high degree as possible. What is the degree of precision of the formula derived? [2+3+13+2=2

Define order of convergence of an iterative scheme. Show that the iteration formula $x_{n+1}=g(x_n)$, where $g(x)= x(x^2+3a)/(3x^2+a)$, converges cubically to a, Provided that x_0 is sufficiently close to $\sqrt{a}$. What is the asymptotic error constant for the above iteration? Calculate $\sqrt{17}$ using the above iteration. Give result correct upto four places of decimals. [3+7+10=20]

Derive Euler-Maclaurin summation formula.

Evaluate $S = \frac{1}{100^2} + \frac{1}{101^2} + \frac{1}{102^2} + \ldots\ldots \infty$,

upto 10 places of decimals.

[$B_0 = 1$, $B_1 = -\frac{1}{2}$, $B_2 = \frac{1}{6}$, $B_4 = -\frac{1}{30}$] [12+8=20]

Show that Gauss-Seidel iteration for solving AX = b, converges if A is strictly diagonally dominant.
Determine whether Gauss-Seidel iteration will converge for the following system. If the method converges, find the solution correct to three places of decimals.

$$\begin{aligned} 2x_1 - x_2 + x_3 &= 2 \\ -x_1 + x_2 - x_3 &= -1 \\ -x_1 - x_2 + 2x_3 &= 0 \end{aligned}$$

[10+10=20]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) I Year: 1996 - 97
SEMESTRAL-II EXAMINATION

Probability Theory II

Date: 5.5.1997 Maximum Marks: 60 Time: 3 hours

Note: Answer any FIVE questions.

1.(a) Let X be a non-negative random variable on a probability space. Show that there exists an increasing sequence of non-negative simple random variables s_n converging to X.

(b) Let X be a random variable on the probability space (Ω, α, P). Show that $X^{-1}(B) \in \alpha$ for all Borel subsets B of $\mathbb{R}$. (6+6) = [12]

2.(a) Show that for a non-negative random variable X,

$$EX = \int_0^\infty (1 - P(X \leq a))\, da .$$

(b) Let m(t) be the moment generating function of a non-negative random variable X. Prove that m is a continuous function on $[0, \infty)$. (6+6) = [12]

3.(a) Let X and Y be independent random variables having the common exponential distribution with parameter 1. Prove that X + Y and X - Y are not independent.

(b) Let X and Y be as in part (a) and U = max (X , Y) and V = min (X , Y). Prove that V and U-V are independent. (6+6) = [12]

4.(a) Let X have the normal distribution having mean μ and variance 1. Find the expectation of $Y = \frac{1 - \Phi(X)}{\varphi(X)}$ where Φ and φ are, respectively, the distribution function and the density function of X.

(b) Let X_1, X_2, X_3, be independent having the common distribution U[0,1]. Find $E(X_{(2)} - X_{(1)})$. (6+6) = [12]

p.t.o.

5.(a) Let $Y_1, Y_2, \ldots$ be independent and identically distributed random variables each having the exponential distribution with parameter 1. Let Z be a random variable, independent of the $\{Y_i\}$ and distributed uniformly on $[T, T+1]$, $T > 0$. Let N be the random variable defined by

$$Y_1 + \ldots + Y_N \leq Z < Y_1 + \ldots + Y_{N+1}.$$

Find the distribution of N.

(b) Let X have the uniform distribution on [0,1]. Find a function f on [0,1] such that Y = f(X) has the distribution function

$$F(t) = \begin{cases} \frac{1}{2} e^t & \text{if } t \leq 0 \\ \frac{1}{2}(1+t) & \text{if } 0 \leq t \leq 1 \\ 1 & \text{if } 1 \leq t. \end{cases}$$

(7+5) = [12]

6.(a) Let X and Y be independent random variables having the Cauchy density $\frac{1}{\pi}\frac{1}{1+x^2}$. Find the distribution of XY.

(b) Prove that, if $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$, then

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q} \quad \text{for all } a, b > 0.$$

(7+5) = [12]

:bcc:

Date: 7.5.199
1.(a) Show th
are ort
(b) Show th
inner p
(c) Prove t
space a
for al
2. Show th
nal ma
3. Find t
symmet
4. An n x
if the
Prove
tic ma
Prove
$|\lambda| \leq 1$

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) I Year: 1996 - 97

SEMESTRAL-II EXAMINATION

Vectors and Matrices II

Date: 7.5.1997 Maximum Marks: 60 Time: 3 hours

Note: There are 8 questions carrying a total of 75 marks. Maximum one can score is 60.

1.(a) Show that two vectors x and y in a <u>real</u> innerproduct space are orthogonal if and only if $\|x+y\|^2 = \|x\|^2 + \|y\|^2$.

(b) Show that the above statement becomes false on a <u>complex</u> inner product space.

(c) Prove that two vectors x and y in a complex inner product space are orthogonal if and only if $\|\alpha x + \beta y\|^2 = \|\alpha x\|^2 + \|\beta y\|^2$ for all pairs of scalars α and β.

(2+4+6) = [12]

2. Show that the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ is not similar to a diagonal matrix.

[6]

3. Find the eigen values and a diagonalizing matrix P for the symmetric matrix

$$A = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{pmatrix}.$$

[15]

4. An n x n real matrix $A = ((a_{ij}))$ is called a <u>stochastic matrix</u> if the following conditions are satisfied:

$$0 \leqq a_{ij} \leqq 1, \quad \sum_{j=1}^{n} a_{ij} = 1 \quad \text{for each } i = 1,2,\ldots,n.$$

Prove that the product of two stochastic matrices is a stochastic matrix. Do such matrices form a group under multiplication? Prove that every eigen value of a stochastic matrix satisfies $|\lambda| \leqq 1$.

(2+2+4) = [8]

p.t.o.

5. Represent the following quadratic form by a real symmetric matrix, and by reducing it to the diagonal form determine the rank and the signature of the quadratic form

$$q(x_1, x_2, x_3) = 16x_1x_2 - x_3^2.$$

(2+8+2) = [12]

6. Suppose A is a normal operator on a finite-dimensional complex inner product space (i.e., $AA^* = A^*A$).
Prove that a necessary and sufficient condition that x be an eigen vector of A is that it be an eigen vector of A^* (i.e., if $Ax = \lambda x$, then $A^* x = \bar{\lambda} x$). (Hint: Show that $\|(A-\lambda I)x\| = \|(A-\lambda I)^* x\|$.

[6]

7. (i) Without expanding a 4x4 determinant show that a-b, a-c and a-d are eigen values of the matrix

$$A = \begin{pmatrix} a & b & c & d \\ b & a & c & d \\ b & c & a & d \\ b & c & d & a \end{pmatrix}$$

(ii) What is the fourth eigen value ?

(iii) Find general conditions on the numbers a, b, c, d which are sufficient for A to be diagonable.

(5+3+2) = [10]

8. Let $B = ((b_{ij}))$ be any nxn real, symmetric, positive definite matrix. Show that (i) $b_{jj} > 0$ for each j = 1, 2, ... n

(ii) $\det(B) \leq \left(\frac{tr(B)}{n}\right)^n$.

(2+4) = [6]

:bcc:

Date:27.6.97

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) I YEAR:1996-97
SEMESTRAL-II. BACKPAPER EXAMINATION
STATISTICAL METHODS-II

Maximum Marks:100 Time: 3 Hours

Note: Answer all questions

1. (a) Suppose that Y is the number of trials required to get r successes in a situation. Writedown the distribution of Y and obtain its moment generating function. Hence or otherwise derive the mean and variance of Y.

(b) In an examination, a student fails and is allowed to take back paper examinations if he scores less than 400, gets an ordinary pass if the score is between 400 and 500 and gets honours if he scores 500 or more. In one year, the percentage of failures, ordinary pass and pass with honours were respectively 23, 62 and 15 percent. Under a suitable model for the scores find the mean and the variance of the scores of (i) all the candidates and (ii) candidates passing with honours. (3+6+2+4)+(7+11)=33

2. (a) Derive the expression

$$\rho_{03.12} = \frac{\rho_{03.1}-\rho_{02.1}\,\rho_{32.1}}{\sqrt{(1-\rho^2_{02.1})\,(1.\rho^2_{32.1})}}$$

where the symbols have their usual meaning.

(b) An anthropologist has predicted the index Y by the relation $Y=\beta_1X_1+\beta_2X_2+\beta_3X_3$ where X_1, X_2, X_3 are three measurements on 86 subjects (The measurements are taken around the means.). The corrected sums of squares and products matrix is calculated as:

Y	X_1	X_2	X_3
0.12692	0.03030	0.04410	0.03629
	0.01875	0.00848	0.00684
		0.02904	0.00878
			0.02886

He is now interested in the (i) multiple correlation coefficient of Y on the other variables and (ii) the partial correlation coefficient between X_3 and X_2, eliminating the effect of Y and X_1. Obtain these values. (12)+(13+17) = (42)

p.t.o.

3.(a) (i) Find the (cumulative) distribution function for the Caucby distribution with density

$$f(x) = \frac{1}{\pi} \frac{1}{1+x^2}, \quad -\infty < x < \infty.$$

(ii) Draw a random sample of size 5 from the above.

(b) State clearly what you understand by the 'memory-less property' of the exponential distribution and establish the result giving the practical interpretation.

(c) Let X have a U (0,1) distribution. Based on a random sample of size 2, consider $\frac{1}{2} Y = -(\log X_1 + \text{by } X_2)$. Derive the distribution of Y, explaining clearly <u>all</u> the results you have used. (4+2)+(3+6+1)+(9)=(25)
