INDIAN STATISTICAL INSTIUTTE 2

Second Semestral Examination : (2000 – 2001)
BACK PAPER

B STAT [Hons.] II Year
Statistical Methods IV

Date : 24.7.01 Maximum Marks : 100 Duration : 3 Hours

NOTE : ATTEMPT Q4 AND ANY FOUR OF THE REST.
ALL QUESTIONS CARRY EQUAL MARKS.

Q1. For the Dirichlet distribution involving k [> 3] random variables ($X_1, X_2, \ldots, X_k$), work out the explicit form of the regression of X_1 on X_2 and X_3 and comment on the result. [Must show all the steps]

Q2. (X, Y) follow a joint distribution over R^2 with the joint pdf f (x, y) of the form:

$$\log f = - 6x^2 - 8y^2 + cxy - 10x + k, \text{ for a suitable constant } k.$$

Identify the distribution and work out suitable bounds for the unspecified parameter c. Taking c = 4, work out the value of the corr. coeff. between X and Y.

Q3. Show that in random samples from a BVN distribution, the sample means are distributed independently of the sample variances and the sample covariance.

Q4. The following computations are based on a random sample of n = 40 observations on (X, Y, Z) :

sample means : 30, 40, 50
sample variances : 16, 25, 12.25
sample corr. coeff.s : $r_{X,Y} = +0.35$, $r_{X,Z} = -0.45$, $r_{Y,Z} = -0.60$.

Work out the explicit form of the least squares linear regression equation of X on Y and Z and examine how far it explains the variation in X.

Q5. Derive the form of the joint distribution of the rth and sth order statistics in a random sample of size n from a population with a continuous density f. Hence, or otherwise, derive the distribution of the largest order statistic in a random sample from an exponential population with mean λ.

Q6. Work out large sample approximation to the variance of $\tanh^{-1} r$ where r refers to the corr. coeff. in a random sample of size n. Examine the usefulness of this result in testing homogeneity of a number of population corr. coeff.s.

INDIAN STATISTICAL INSTIUTTE

Second Semestral Examination : (2000 – 2001)

B STAT [Hons.] II Year

Statistical Methods IV

Date : ...2.5.01............. Maximum Marks : 100 Duration : 3 ½ Hours

NOTE : ATTEMPT ALL THE QUESTIONS.

Use of Calculators allowed.

Q1. A box contains a large number of balls having the following color combinations : Red – 35 %, White – 25 %, Yellow – 10 %, Blue – 15%, Green – 15 %. You are supposed to draw a total of 10 balls at random and with replacement. After you have drawn 4 balls, the sampling operation is stopped for a while and the color compositions in the box are altered to : 25 %, 10 %, 15 %, 15 % and 35 % in the order specified above. You are then allowed to continue to draw till the end.

Denote by R,W,Y,B and G the number of balls in the entire sample in the specified order of the colors.

(a) Work out $P[G = 1]$.
(b) Evaluate $P[R = 3 \mid G = 1]$.
(c) Evaluate $E[R \mid G = 1]$.
(d) Evaluate the form of the regression of R on G. [2 + 3 + 5 + 7 = 17]

Q2. Two random variables (X, Y) follow BVN distribution with unequal variances and corr. coeff. equal to + 0.50. It is further known that

$$P[X + Y > 50] = 0.50; \; P[X - Y < -10] = 0.50.$$

(a) Evaluate $P[X > 20, Y < 30]$.
(b) Evaluate $P[2X - Y > 10, X - 2Y < -40]$ given that $V(X) / V(Y) = 2.0$.
[5 + 8 = 13]

Q3. Three random variables (X, Y, Z) possess a non-singular joint distribution with the mean vector $(10, 20, 30)'$ and dispersion matrix given by

$$\Sigma = \begin{pmatrix} 9 & 3.6 & -3.0 \\ & 16 & 4.0 \\ & & 25 \end{pmatrix}$$

P.T.O

Q3. Continued

(a) Work out the value of the maximum absolute correlation between X and a a suitable linear function of Y and Z, to be specified by you.

(b) What is the value of the partial corr. coeff. between X and Y, eliminating the effect of Z from both ?

[10 + 5 = 15]

Q4. You are given n paired observations $[(x_i , y_i) \mid 1 \leq i \leq n]$ on (X,Y) which follow BVN distribution with all parameters unknown.

Define

$$SXX = \Sigma (x_i - \bar{x})^2 , SYY = \Sigma (y_i - \bar{y})^2 , \; SXY = \Sigma (x_i - \bar{x}) (y_i - \bar{y})$$
and $SYY.XX = SYY - SXY^2 / SXX$.

(a) Show that SXX and SYY.XX are independently distributed.
(b) Hence, or otherwise, show that $E [SXX.SYY - SXY^2] = k. \sigma_1^2 \sigma_2^2 (1 - \rho^2)$ [in usual notations] where k is a constant depending on n.

[10 + 10 = 20]

Q5. (a) Derive a formula for the expectation of the sample range in a random sample of size n from a simple exponential population with mean λ. Hence comment on the validity of the approx. formula :
E[Sample Range] = 6 Sigma !

(c) Show that E [r] = 0 in sampling from a BVN population with ρ = 0 where r (ρ) refers to the sample (population) corr. coeff. .

[10 + 8 = 18]

Q6. (a) What is meant by a variance – stabilizing transformation ? Derive its general form and the specific form for a binomial proportion.

(b) Illustrate how the Z-transformation of the correlation coefficient is useful in examining homogeneity of several population corr. coeff. in large samples.

[{ 2 + 3 + 3 } + 9 = 17]

INDIAN STATISTICAL INSTITUTE

B.STAT. (Hons.) – II YEAR (2000-2001)

II SEMESTRAL EXAMINATION

ECONOMICS - II

Answer any Five question taking at least two from each group. All questions carry equal marks.

GROUP - A

Date : 04.05.2001 Maximum Marks : 60 Duration : 3 hrs.

1. a) Consider the data given below :

	Rs. (in crores)
Indirect Taxes	10
Director Taxes	5
Transfers	2
Subsidy	1
Personal Saving	30
Consumption	100
Investment	40
Business Saving	5
Depreciation	1
Government's Budget Deficit	10

Find out GDP, NDP and Net Exports.

b) State with reasons whether the following statements are true or false ?

i) 'Value added' of a firm is obtained by subtracting from the value of output of a firm the value of all inputs purchased by the firm from other firms.

ii) A decline in indirect taxes together with an equal increase in corporate profit tax leaves personal income, personal saving and aggregate saving unaffected.

iii) Increase of wages and salaries in government administration and defence and increase of wages and salaries in public sector enterprises will have the same effect on the GDP measure.

iv) When farmers use less of their products for self-consumption and more for sale to the market, GDP falls.

(8 + 4)

2. a) Consider a simple Keynesian Model for a closed economy where investment is an increasing function of income. Derive the stability condition for equilibrium output. Illustrate the stable and unstable equilibrium situations interms of saving and investment schedules in a diagram.

b) In the same model the autonomous part of investment is 50 units. Saving function is given by $S(Y) = -80 + .2Y$. A shift in the saving schedule by 10 units causes Y to decline by 100 units. Derive the investment function.

(6 + 6)

3. a) Derive and explain the balanced budget multiplier in the simple Keynesian Model for an open economy.

b) Imagine an economy where the government spent all taxes but prevented (by balanced budget amendment) from spending any more,

Thus $G = t.Y$, where t is the tax rate.

i) Is the multiplier smaller or larger than the case in which Government spending is autonomous ? Explain.

ii) When t increases, does Y increase, decrease or remain the same ? Explain.

(6 + 6)

GROUP – B

1. a) Suppose, there is an increase in autonomous spending and assume a given marginal propensity to consume. Do you think that the incerase in income should always be equal to the simple multiplier times the change in autonomous spending. Use the IS-LM framework to explain your view.

   b) Explain why the slope of the IS curve may be a factor in determining the effectiveness of any monetary policy to be adopted by the Government.

(6+6=12)

2. What do you mean by 'Acceleration Principle'? Discuss the role of induced investment in the above context. Assume marginal prosperity to consume equal to .3 and accelerator equal to 2. Let Autonomous investment in each period be 100. Derive a numerical sequence to illustrate the income fluctuation process in a hypothetical economy.

(12)

3. Let, α = (Simple Multiplier) = 5
   b = (Interest Sensitivity of Investment) = 4
   h = (Interest sensitivity of demand for Money) = 4
   k = (Income sensitivity of Traveaction demand for Money) = .2

   a) What increase in Money Supply is needed to increase output by 100 ?

   b) What increase in Government spending is needed to have the same effect as in (a).

   c) Find the change in investment as a result of policy (a) and policy (b).

(4 + 4 + 4 = 12)

4. a) Indicate whether the following statements are True or False, giving reasons for your answer :

      i) the liquidity effect is large when investment is interest sensitive.

      ii) The output effect for a change in the money supply is large when investment is sensitive to the rate of interest and the demand for Money is interest insensitive.

      iii) An increase in Government spending always crowds-out investment spending.

   b) Obtain an expression for Money supply multiplier relating, 'currency deposit ratio', 'reserve deposit ratio', and 'High powered money'.

(4 + 8 = 12)

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination : 2000-2001
B.Stat II Year
SQC & OR

Date: 8.5.2001 Maximum Marks : 50 Time : 1 hr. 30 mts.

NOTE : Answer any <u>two</u> questions

1. Suppose for a given measurable quality characteristic (normally distributed) of a product

   U = Upper Specification Limit
   L = Lower Specification Limit
   σ = the standard deviation when the process is under statistical control

   a. What are the options available to the manufacturer when $6\sigma > U - L$
   b. Show that to achieve minimum proportion defective, the process average (μ) should be maintained at (U+L) / 2.
   c. A measurable quality characteristic is under statistical control with mean 2.2 cm and mean range in samples of size 5 of 0.025 cm. The specification limits are U = 2.02 cm. and L= 1.98 cm.
      (i) What percentage of items would fail to satisfy the specifications?
      (ii) If the process mean shifts to 2.30 cm., on an average how many samples will be collected before the shift is detected in the usual control chart for sample averages.

(5 + 10 + 10)

2. a. What do you understand by the Operating Characteristic (OC) curve of an acceptance sampling plan?
   b. How do the OC curves help in comparing the efficiency of sampling plans from the customers' viewpoint?
   c. Using Poisson approximation for Binomial probabilities show that for AQL = p_1, LTPD = p_2, producer's risk = α, consumer's risk = β for attribute sampling plan, the sample size n will satisfy the following inequalities

$$\chi^2_{\beta} / (2p_2 \leq n \leq \chi^2_{1-\alpha} / (2p_1)$$

   where Chi-square distribution has 2(c+1) degrees of freedom, c being the acceptance number. (5 + 5 + 15)

3 a. State the characteristic features of Linear Programming problems.
   b. A company makes two kinds of leather belts.Belt A is a high quality belt,and belt B is of lower quality. The respective profits are Rs 4 and Rs.3 per belt. Each belt of type A requires twice as much time as a belt of type B,and if all belts were of type B, the company could make 1000 per day.The supply of leather is sufficient for only 800 belts per day (for both A and B combined).Belt A requires a fancy buckle and only 400 per day are available.There are only 700 buckles a day available for belt B.Formulate the Linear Programming problem and solve by Simplex Method to determine the daily number of belts to be produced to maximise profit.

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(5+20)

INDIAN STATISTICAL INSTITUTE
B.STAT (HONS) II YEAR : 2000-2001

SEMESTER - II EXAMINATION

Date : 08 May 2001 Time: 1½ hours.

Group A

Demography

Maximum Marks 50

Note: Answer Question 5 and any THREE from the rest.

1. a) Define Vital Statistics
   b) Describe a suitable method to estimate the extent of coverage error in the sample registration of vital events in India.

   c) Derive a formula for evaluating errors in reporting ages over (10-99) years from census data.

   d) Describe the system of census that has been adopted for the Census of India, 2001.

2+5+5+3=15

2. a) Derive the logistic curve for a closed population at time t i.e.,

$$P_t = \frac{\omega}{1 + K \exp(-r\,t)}$$ where symbols have their usual meanings.

b) For a female population, the maximum and minimum age at first marriage are respectively 50 years and 15 years. The population of females remaining never married at age 50 increass from 5 per cent in 1980 to 15 percent in 2000. Estimate the changes in singulate mean age at marriage from 1980 to 2000.

c) Distinguish between infant mortality rate and infant death rate.

5+5+5=15

Turn Over

3. a) For a certain life table :
$l_x = 20900 - 80x - x^2$, estimate (i) the ultimate age in the life table, and (ii) ${}_{10}p_{20}$.

b) A maternity hospital delivers 10 new-born babies per week, 30% of them leave hospital within a week, 10% of the remaining one week old babies leave before they are two weeks old, 20% of the remainder leave before they are three weeks old, 40% of the remainder before they are four weeks old, 70% of the remainder before they are five weeks old and all the remainders before they are 6 weeks old. How many cots are needed t run the hospital ?

c)

Age group	Country A		Country B	
	Population	Deaths	Population	Deaths
0-4	306643	1202	39539	-
5-24	1001785	648	96453	-
25-64	1145408	6111	61312	-
65+	223093	15282	3855	-
Total	**2,676,929**	**23243**	**201159**	**1291**

From the above table, comment on relative mortality between two countries through some suitable technique.

4+6+5=15

4. a) Define total fertility rate an derive the mathematical relationship between crude birth rate and total fertility rate.
b) For a country if TFR= 3.6 and Sex Ratio at birth is 1.04, then estimate GRR.
c) Show that TFR is approximately equal to 35 GFR.
d) Find out the inter-relationship between total fertility and total fecundity rate (TF) through Bongaart's Model.

6+2+2+5=15

5. Write notes on any two of the following --- **2½ x2=5**
a) Maternal Mortality Ratio,
b) Replacement Index,
c) Comparative Morality Figure,
d) Sex Age Adjusted Birth Rate.

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B.STAT. (Hons.) – II YEAR (2000-2001)

II SEMESTRAL EXAMINATION

ECONOMIC STATISTICS AND OFFICIAL STATISTICS

Date : 11th MAY 2001 Duration : 3 hrs.

Maximum Marks : 100

Answer any ~~five~~ four questions from Group A and any one from Group B. All questions carry equal marks.

GROUP – A

1. Suppose income follows a two parameter lognormal distribution with Lorenz Ratio = 0.4 and median = e (2.71828). If the population is divided into two groups – one group with income $\leq$ e and the other group with income > e, find the mean value for each group. Also find the variance of income of the first group.

(15 + 5 = 20)

2. State the principle of Diminishing Transfer of income. Examine CV, LR and RMD in the light of the above principle.

(2 + 18 = 20)

3. Describe with derivations how one can estimate Engle elasticity from Specific concentration curve.

(20)

4. (i) State the Law of Proportionate Effect proposed by Kapteyn. Also state its modification due to Kalecki leading to lognormal distribution.

   (ii) Describe the graphical test and methods of estimating three-parameter lognormal distribution.

(10 + 10 = 20)

P.T.O

5. Write short notes on any two of the following :

(i) Linear Expenditure System.
(ii) Properties of Pareto Distribution.
(iii) Demand Projections Through Engel Curve.

(10 + 10 = 20)

6. (i) Define Laspeyres, Paasche's and Fisher's Ideal Index number formulae Examine these indices on the basis of Time Reversal, Factor Reversal an Circular Tests.

(ii) Compare chain-base-index-number formula over fixed-base-index numbe formula.

(10 + 10 = 20)

GROUP – B

1. Mention the major items on which data are collected in Indian population censuse What do you know about the quality of these data in terms of coverage errors an content errors ?

(8 + 12 = 20)

2. Briefly describe the methodology adopted in NSS enquiries on household budg data – mentioning concepts and definitions used, reference period used for dat collection etc. How are these data used for measuring the incidence of absolut poverty in the country ?

(12 + 8 = 20)

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination : (2000-2001)
B. Stat. (Hons.) – II Year
Physics – II

Date : 24.4.01 Maximum Marks : 60 Duration : 3.00 Hrs.

Note : There are two groups A & B, each consisting of three questions. Students are required to answer 4 questions in all – selecting two from each group.

QUESTIONS

Group A (Classical Thermodynamics)

Answer any two questions.

1. (a) Develop carefully the concept of "entropy" in the context of classical thermodynamics. Starting with the definition of entropy, how can one have an estimate of it for a gas in an arbitrary state ? Derive an expression for entropy of a perfect gas. Comment on the absolute character of entropy. (10 marks)

(b) Using the first law of thermodynamics, obtain the adiabatic equation of state for a perfect gas. (5 marks)

2. (a) In the context of classical thermodynamics, discuss "reversibility" and "irreversibility". Explain with necessary diagrams, working of a simple model heat engine. Apply the first law of thermodynamics to obtain an expression for the efficiency of a heat engine working in a reversible (Carnot's) cycle. (10 marks)

(b) Substantiate with necessary details, the statement – " No heat engine can have an efficiency greater than a reversible engine". In practice, how can we design such an optimally efficient engine ? Justify your answer. (5 marks)

3. (a) Following Clausius, obtain the "Virial equation" of state for a real gas. How does it essentially differ from the equation of state for a perfect gas ? In the context of the first law of thermodynamics, discuss internal energy (U) and change in internal energy (ΔU) of a gas (real as well as perfect) from molecular standpoint. (10 marks)

(b) In the framework of the "Kinetic Theory" of gases, state the Maxwellian law of distribution of molecular velocities in the case of an ideal gas. How can it be verified experimentally ? (5 marks)

P.T.O

Group B (Introduction to Quantum Theory and Quantum Mechanics)

Answer any two questions.

1. (a) State the basic idea involving exchange of radiation as introduced by Max Planck. (2 marks)

   (b) Derive Planck's distribution law for black body radiation, given that the total number of independent modes in the frequency range ν, $\nu + d\nu$ is $(8\pi\, \nu^2/c^3)\, d\nu$. (7 marks)

   (c) Show that, in the limiting cases, Planck's law reduces to Wien's law and Rayleigh-Jean's law. (6 marks)

2. (a) Describe "<u>Compton effect</u>" to account for the fact that radiation has a particle character involving both energy and momentum. (8 marks)

   (b) State and explain Bohr's atomic postulates. Deduce the energy spectrum for the electron in the Hydrogen atom. (7 marks)

3. (a) State de-Broglie's hypothesis about the wave nature of a particle. Write the expression for the Schrodinger equation of motion. (4 marks)

   (b) Solve the Schrodinger's equation for a particle in a one dimensional box and find the energy expression along with corresponding eigenfunctions. The box is represented by a potential function $V(x)$ defined as under-
   $V(x) = 0$ for $0 < x < a$; $V(x)$ infinite for all other x – values. (8 marks)

   (c) Consider a spin-1/2 particle associated with a 2-dimensional Hilbert space. Let the state of the particle be represented by the row vector $(\cos \theta/2 \;\; \sin \theta/2)$. On this state spin is measured along x-direction. What is the probability that the result of the measurement of spin will be $+ \frac{1}{2} h$? Given that the operator with spin measurement along x-direction is $\frac{1}{2} h\, \sigma_x$ where σ_x is the 2×2 matrix

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

(3 marks)

Indian Statistical Institute

Second Semestral Examination : 2000-2001

B.Stat.(Hons), II year

Elements of Algebraic Structures

30 April 2001 Max. marks: 100 Duration: 4 hrs.

Note: Answer as much as you can. The paper carries 112 marks but the maximum you can score is 100. Marks allotted to each question are shown in square brackets near the right margin. State clearly the theorems you use in solving problems.

1. Let G and H be groups.

   (a) If $G \times H$ is cyclic, show that G and H are cyclic. [4]

   (b) Let G be the infinite cyclic group. If $G \times H$ is cyclic, show that $o(H) = 1$. [9]

2. Let H be a subgroup of a group G. Let $\mathcal{L}$ (resp. $\mathcal{R}$) be the set of all left (resp. right) cosets of H.

   (a) Prove that $\mathcal{L}$ and $\mathcal{R}$ have the same cardinality by showing that $f : aH \mapsto Ha^{-1}$ is a bijection from $\mathcal{L}$ to $\mathcal{R}$. [9]

   (b) Find all the properties that are violated if f is changed to: $aH \mapsto Ha$. [6]

3. (a) State and prove Cayley's theorem on groups. [9]

   (b) Prove that every group of order n is isomorphic to a subgroup of GL_n, the multiplicative group of real $n \times n$ non-singular matrices.[9]

4. (a) Let R be a principal ideal domain (i.e., a commutative ring with at least two elements and without zero divisors and with the property that every ideal is aR for some $a \in R$).

      i. Show that R has an identity element. [5]

      ii. Show that an element of R is irreducible iff it is prime. [10]

   (b) Show that $\mathbb{Z}[x]$ is not a principal ideal domain. [6]

5. (a) Given an integral domain R, explain (without proof) how you would construct a field F such that R is a subdomain of F. [5]

P. T. O

(b) Show how one can get an infinite field with the property that $a + a = 0$ for all elements a. [9]

6. (a) State Eisenstein's criterion without proof. [4]

   (b) Let n be an arbitrary positive integer. Give an irreducible polynomial of degree n over $\mathbb{Q}$ and show that it is irreducible. (You may use (a).) [8]

   (c) Prove or disprove: there exists an irreducible polynomial of degree 2 over every field. [3]

7. (a) Let F be a field and $f(x)$ a polynomial over F such that the g.c.d. of $f(x)$ and its derivative $f'(x)$ is 1. Show that $f(x)$ does not have a multiple root in any extension of F. (You may assume elementary properties of derivative.) [6]

   (b) Show that for every prime number p and positive integer n, there exists a field of order p^n. (You may assume the existence of a splitting field for any polynomial over any field.) [10]

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B.STAT. SEMESTRAL – I EXAMINATION (2000-2001)

PHYSICS – I

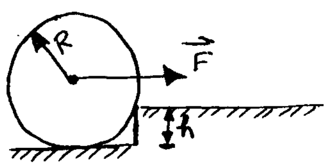
te : 06.12.2000 Duration : 2 hrs.

Maximum marks : 60

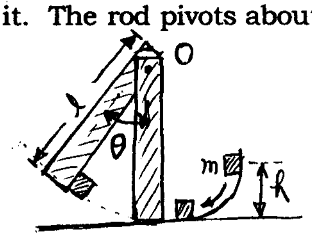
swer any five questions. All questions carry equal marks. (Only class notes allowed for this examination).

GROUP – I (Mechanics)

(a) What magnitude of force F applied horizontally at the axel of the wheel is necessary to raise the wheel over an obstacle of height h ? (Radius of the wheel is R, and its weight is W).



(b) A particle of mass 'm' slides down a frictionless surface and collides with a uniform vertical rod of mass 'M' and length 'l', sticks to it. The rod pivots about 0 through an angle θ.



Show that $\cos\theta = \left[1 - \frac{6m^2h}{(2m+M)(3m+M)l}\right]$

(6 + 6)

A Lagrangian for a physical system is given as

$$\mathcal{L} = \frac{m}{2}(a\dot{x}^2+2b\dot{x}\dot{y}+c\dot{y}^2) - \frac{k}{2}(ax^2+2bxy+cy^2),$$

where a,b, and c are arbitray constants. k and m are fixed for a particular system.

(a) Find the equations of motion, if $b^2-ac \neq 0$
(b) Discuss the equations of motion for a=c=1, and b=0
(c) What are equation of motion for b=0, c=-a.

(5 +4+3)

P. T. O

3. a) A particle is thrown up vertically with initial speed u_0, reaches a maximum height and falls back to the ground. Show that the coriolis deflection when it again reaches the ground is opposite in direction, and four times greater in magnitude, than the coriolis deflection when it is dropped at rest from the same maximum height.

b) A parallel beam of particles are scattered by a circular disc of radins 'a' and thickness 'b'. What are the maximum and the minimum values of total scattering cross-section. Assume that the incoming beam of particles covers an area much larger than that of the disc, which is completely inside the beam.

(8 + 4)

GROUP – II (Electrodynamics)

1. a) Calculate the electrostatic potential and field at points both interior and exterior to a spherical charge distribution of radius 'R'. The volume charge density is given by

$$\rho(\bar{r}) = c\,|\bar{r}|, \text{ for } r \leq R$$
$$= 0, \text{ for } r > R$$

where c is a constant and 'r' denotes the distance from the centre of the charge distribution. Use Coulomb's law of electrostatics.

b) Derive the differential form of Gauss law starting from the integral form.

c) Find the electric field for the above problem using Gauss law.

(5 + 3 +4 = 12)

2. a) Explain the fact that there is no electrostatic field inside a conductor. How will the situation change if the conductor has a cavity with a point charge present inside the cavity ?

b) Find the electric field at points outside a spherical conductor of radius 'R' having a charge 'q' present inside a cavity in the conductor. The cavity is of arbitrary shape and can be anywhere inside the conductor.

Find the electric field at points just outside a conductor having a surface charge density σ (r).

(4 + 5+ 3 = 12)

3. a) Find the magnetic field at a distance r from a straight wire of infinite extent, carrying a steady current I.

b) Show that two parallel wires carrying steady currents can attract or repel each other. Find the magnitude of this force per unit length of the wires.

c) Find the magnetic field inside as well as outside of an infinite solenoid carrying current. Sketch the magnetic fields for an infinite solenoid and a solenoid of finite extent.

(3 + 3 + 6 =12)

Indian Statistical Institute
First Semestral Examination:(2000-2001)
B.Stat.. - II yr.
Subject : Calculus - III

Date:11.12.2000 **Maximum Marks : 60** **Duration:3 hrs.**

Answer any six questions.

1. a) Let $S \subseteq \mathbb{R}^n$ be such that for every sequence $\{a_n\} \subseteq S$ there is a convergent subsequence $\{a_{n_k}\}$ converging to some point $a \in S$. Show that S is closed and bounded.

   b) Show that if $f: S \to \mathbb{R}^k$ is a continuos function where S is a closed and bounded subset of $\mathbb{R}^n$, then $f(S)$ is closed and bounded.

   [4+6 =10]

2. a) Let f be a real real valued function on an open subset of $\mathbb{R}^n$ such that f has continuous partial derivatives at a point a. Show that f is differentiable at a.

   b) Let ϕ be an infinitely differentiable function on $\mathbb{R}$ such that $\phi(t) = 1$ if $0 \le t \le 1$ and $\phi(t) = 0$ if $2 < t < \infty$. Show that the function $\phi(|x-a|)$ is infinitely differentiable on $\mathbb{R}^n$, where a is a fixed point in $\mathbb{R}^n$.

   [7+3=10]

3. a) Let $f = (f_1, f_2)$ be the inversion map of $\mathbb{R}^2 - \{(0.0)\}$, i.e., $f(x,y) = (x/(x^2+y^2), y/(x^2+y^2))$. Show f preserves the angle between any two intersecting curves γ and ϕ in $\mathbb{R}^2 - \{(0,0)\}$.

   b) Let g be a harmonic function on $\mathbb{R}^2$ and f be as in (a) above. Show that g of is harmonic on $\mathbb{R}^2 - \{(0,0)\}$

   [5+5 =10]

4. a) Let $k < n$ and let $f, \phi_1, \ldots\ldots \phi_k$ be continuously differentiable functions on $\mathbb{R}^n$. Suppose at $a \in \mathbb{R}^n$, $f(x)$ has an extreme value subject to the conditions $\phi_i(x) = \phi_i(a)$, $i = 1, , \ldots\ldots k$. Show that if ζ is tangent to the surface $S = \{x : \phi_i(x) = \phi_i(a), i = 1, \ldots\ldots k\}$ at the point a, then

   (i) $\zeta \perp \nabla f(a)$

   (ii) $\zeta \perp \nabla \phi_i$ $i = 1, 2 \ldots\ldots$

P.T.O

b) If all symbols denote positive quantities, then the stationary values of $lx+ny+nz$ subject to the condition $x^p + y^p + z^p = c^p$ is $c(l^q + m^q + n^q)^{1/q}$ where $q = p/p-1$.

[7+3 = 10]

a) Let $f = (f_1, f_2)$ be the mapping of $\mathbf{R}^5$ into $\mathbf{R}^2$ given by
$f_1(x_1,x_2,y_1,y_2,y_3) = 2e^x + x_2y_1 - 4y_2 + 3$
$f_2(x_1,x_2,y_1,y_2,y_3) = x_2\cos x_1 - 6x_1 + 2y_1 - y_3$.
If g is the unique function given by the implicit function theorem on a neighbourhood of (3,2,7) such that g (3,2,7) = (0,1) and f (g(y), y) = 0, find J_g (3,2,7).

b) Find the tangent hyperplane to the hypersurface $S = \{ f(x_1,x_2,y_1,y_2,y_3) = 0 \}$ at the point (0,1,3,2,7).

[7+3 = 10]

a) Let f_1 and f_2 be continuous functions on $\mathbf{R}^2$, such that for any rectangle R, $\int_{\delta R} f_1dx + f_2dy = 0$ if δR is the closed curve bounding R. Show that there exists g on $\mathbf{R}^2$ such that $\partial g/\partial x = f_1(x,y)$ and $\partial g/\partial y = f_2(x,y)$ for all $(x,y) \in R$.
Is it necessary that there exists h on $\mathbf{R}^2$ such that $\partial h/\partial x = f_2(x,y)$ and $\partial h/\partial y = f_1(x,y)$?

b) Calculate $\iint \sqrt{x^2 + y^2}\, d\sigma$ Where S is the surface given by $z = xy$, $0 \le x^2 + y^2 < 1$ and $d\sigma$ stands for integration with respect to surface area.

[6+4 = 10]

a) Find the equation to the tangent plane of the ellipsoid
$S : x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ at the point (x,y,z) .

b) Denote by p(x,y,z) the distance of the tangent plane at (x,y,z) from the centre (0,0,0) of the ellipsoid. Find an expression for p(x,y,z).

c) Compute $\iint p\, d\sigma$ where $d\sigma$ stands for integration with respect to surface area .

[2+3+5 = 10]

8. a) Let $T = \{ (x,y) : 0 \leq x/a + y/b \leq 1 \}$ Where $a > 0$, $b > 0$. Assume that a function f has a continuous second order partial derivative $D_{1,2} f$ on T . Show that there exists a point (x_0, y_0) on the line segment joining (a,0) and (0,b) such that

$$\int D_{1,2} f(x,y)\, dxdy = f(0,0) - f(a,0) + aD_1 f(x_0,y_0)$$

(Hint : Begin with picture of T.)

b) If D is a bounded solid with a boundary δD which is a parametric surface in patches, express the volume of D by a surface intregral on δD .

[8+2 = 10]

INDIAN STATISTICAL INSTITUTE
First Semestral Examination : 2000-2001
B.Stat.(Hons) - 2nd Year
Probability Theory and its Applications III
Maximum Score : 130 pts

15
te : ~~14~~.12.2000 Time : 4 Hours

te : This paper carries questions worth a **total of 150 points.** Answer as much as u can. The **maximum** you can score is **130 points.**

1. Let X, Y, Z be random variables with joint density function given by

$$f(x,y,z) = \begin{cases} 2(x+y)e^{-(x+y)z} & \text{if } x,y,z > 0 \text{ and } x+y<1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the conditional distribution of Z given (X,Y).

   (b) Find the distribution of the random variable $W = (X+Y)Z$.

(10+10)=[20]

2. (a) Write down the formula for the n-dimensional normal density function with mean vector $\underset{\sim}{\mu}$ and dispersion matrix Σ.

   (b) Let the random vector $\underset{\sim}{X} = (X_1, \cdots, X_n)'$ have n-dimensional normal density function as in (a) and let $\underset{\sim}{Y} = (X_1, \cdots, X_m)'$ and $\underset{\sim}{Z} = (X_{m+1}, \cdots, X_n)'$, where $1 < m < n$. Get a suitable matrix A such that $\underset{\sim}{Y} + A\underset{\sim}{Z}$ and $\underset{\sim}{Z}$ are independent. Hence find the conditional distribution of $\underset{\sim}{Y}$ given $\underset{\sim}{Z}$.

(5+10+5)=[20]

3. Suppose that four points are chosen at random from the unit interval $(0,1)$, independently of one another.

   (a) Find the expected value of Y, the distance between the farthest two among the chosen points.

   (b) Find the expected value of Z, the shortest distance between the points chosen. [Hint: For $0 < x < \frac{1}{3}$, calculate $P(Z > x)$.]

(10+15)=[25]

4. (a) Let X be a random variable taking only integer values and let ϕ be the characteristic function of X. Show that: (i) ϕ is periodic with period 2π, and (ii) for any integer k, $P(X=k) = \frac{1}{2\pi}\int_0^{2\pi} e^{-ikt}\phi(t)dt$.

   (b) If ϕ is a characteristic function which is non-negative and integrable, then show that the corresponding density function has a unique maximum at the point 0.

(10+10+10)=[30]

P.T.O

5. (a) State Chebyshev's inequality.
Prove that if $X_1, X_2, \cdots$ is a sequence of i.i.d. random variables with finite variance, then $\frac{2}{n(n+1)} \sum_{j=1}^{n} jX_j \xrightarrow{P} E(X_1)$ as $n \to \infty$.

(b) State the two Borel-Cantelli lemmas.
Prove that if $X_1, X_2, \cdots$ is a sequence of independent random variables with X_n having an exponential distribution with parameter $\alpha \log(n+1)$ for each n, then $\limsup_{n\to\infty} X_n = \frac{1}{\alpha}$ with probability 1.

(5+10+5+15)=[35]

6. (a) Let $\{X_n\}$ be a sequence of random variables converging in distribution to a non-degenerate random variable X. Show that if X_n has $N(0, \sigma_n^2)$ distribution for each n, then σ_n^2 converges to a number $\sigma^2 > 0$ and X has $N(0, \sigma^2)$ distribution.

(b) Let $X_1, X_2, \cdots$ be a sequence of i.i.d. random variables with $P(X_1 = 1) = P(X_1 = 2) = \frac{1}{2}$. I claim that, by the CLT, $P\left(\prod_{j=1}^{100} X_j \leq (1.5)^{10}\right)$ is approximately equal to $\Phi(\alpha)$ for an appropriate α. What is α? Give reason.

(10+10)=[20]

B-Stat (Hons.)
Second Year, First Semester
2000–2001
Statistical Methods III
Semestral Examination (December 18, 2000)

Total Marks : 100 *Time Allowed : 3 hours*

Answer All Questions.
Marks will be deducted if your answer is not clear and to the point.

(1). Suppose that an observation X follows negative binomial distribution with probability mass function

$$P(X = x) = \{\Gamma(\alpha)\Gamma(x+1)\}^{-1}\Gamma(\alpha + x)p^{\alpha}(1-p)^{x}$$

for $x = 0, 1, 2, \ldots$, where $0 < p < 1$ is an unknown parameter and $\alpha > 1$ is a known constant. Is the standard family of beta priors for p a conjugate family in this case ? Justify your answer. Obtain a simplified expression for the Bayes estimate of p with respect to the beta prior

$$\{\beta(m,n)\}^{-1}p^{m-1}(1-p)^{n-1},$$

where $0 < p < 1$ and $m, n > 0$. Also, obtain an unbiased estimate of p in this case with adequate justification.

[10 + 10 = 20]

(2). Consider the following data on the heights and the ages of 8 teen aged boys. The age (the first co-ordinate) is in years, and the height (the second co-ordinate) is in feet.

$$(13, 3.9), \quad (15, 4.8), \quad (17, 4.2), \quad (15, 3.8),$$
$$(18, 4.2), \quad (19, 4.6), \quad (17, 4.0), \quad (16.3.6).$$

Assume a simple linear regression model with normally distributed residuals for the relationship between the height (the dependent variable) and the age (the independent variable). Obtain 95% confidence intervals for the slope and the intercept parameters.

[10 + 10 = 20]

(3). Consider two hypotheses H_0 and H_A, where H_0 says that the observation X has a standard normal distribution, and H_A says that the observation

P.T.O

X has standard double exponential distribution with p.d.f $(1/2)\exp(-|x|)$ where $-\infty < x < \infty$. Suppose that the prior probability π_0 for H_0 is 2/3, and the penalty for incorrectly rejecting H_0 is 4 times the penalty for incorrectly rejecting H_A. Describe (with full justification) the set of those values of the observation X that will lead to the rejection of H_0 in the optimal Bayes solution of this testing problem.

[20]

(4). Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d continuous observations and θ is the unique median of the common distribution of the X_i's. The common distribution of the X_i's is unknown, and we want to test $H_0 : \theta = 2.5$ against $H_A : \theta \neq 2.5$ at 5% level when n is very large. Describe cleary and with sufficient justification how you would carry out the test. What will happen to the power of your test as $n \longrightarrow \infty$? Justify your answer.

[7 + 8 = 15]

(5). Suppose that we observe X following $Bin(n, p)$ distribution and estimate the logarithm of the odds ratio $\log_e\{p/(1-p)\}$ by the natural estimate $\log_e\{X/(n-X)\}$ (assume that $0 < p < 1$ and n is sufficiently large so that $P(X = 0)$ and $P(X = n)$ are negligible). Derive the large sample variance of this estimate by obtaining the limiting distribution of $n^{1/2}\log_e\{(X - pX)/(np - pX)\}$ as $n \longrightarrow \infty$.

[15]

(6). Assignments.

[10]

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: 2000 – 2001
B. Stat. – II Year
Economics I

Date: 20.12.2000 Maximum Marks: 60 Time: 2 Hours

Answer Group A and Group B in separate Books

Group A: Answer any three questions

1. (a) If the input prices are constant, and the production function is homogeneous of degree r (r > 0), show that the expansion path will be a straight line passing through the origin.
   (b) Derive the expression of the expansion path for the production function $Q = 2\log L + 4\log K$. [5+5 = 10]

2. Given the demand function $P = 20 - Q$ and total cost function $C = Q^2 + 8Q + 2$, answer the following questions.
   (a) Find the profit maximizing level of price (P) and output (Q).
   (b) If the objective of the firm is to maximize sales revenue, what will be the corresponding values of P and Q?
   (c) If the firm maximizes sales revenue subject to the profit constraint that $\pi \geq 8$, what will be the values of P and Q? [3+3+4 = 10]

3. A monopolist supplies his output into two isolated markets. The demand functions are $P_1 = 12 - Q_1$ and $P_2 = 20 - 3Q_2$. The total cost function of the monopolist is $C = 3 + 2(Q_1 + Q_2)$.
   (a) Determine the prices and sales in the two markets under a regime of price discrimination.
   (b) Find the corresponding values of the variables, if the monopolist cannot discriminate. [5+5 = 10]

4. The demand for a product in the Indian market is given by $P = 90 - Q$. The products are supplied by both Indian firms and Japanese firms. For simplicity, assume that there is a single representative firm in each country that behaves competitively. The cost function for the product is given by $C(Q) = \frac{Q^2}{2}$ in each country.
   (a) What is the equilibrium price and quantity sold?
   (b) Now if the domestic industry lobbies for protection and the local government agrees to put a tariff of Rs 3 per unit on foreign products, what is the new price for the product as paid by the consumers?
   (c) How many units of the good are supplied by foreign firms and by domestic firms? [3+5+2 = 10]

P.T.O

Group B: Answer any two questions

5. A consumer lives for two periods and earns income I_1 and I_2 in the two periods. He has given preferences defined over consumption in the two periods. He cannot borrow from the market, but can lend in the first period at a given rate of interest r.

(i) Draw his budget constraint.
(ii) What will happen to his level of lending if the rate of interest rises?
(iii) Draw his level of lending as a function of the market rate of interest.

(4+4+7=15)

6. Consider an economy producing two goods, 1 and 2, and using three factor of production. To produce good 1, labour has to be combined with capital. To produce good 2, labour has to be combined with land.

(i) Assuming constant returns to scale and diminishing returns in each sector and free mobility of labour across sectors, characterize the equilibrium.
(ii) What will happen to the factor returns, the relative price and the levels of output if there is a shift in demand in favour of good 1?

(9+6=15)

7. In a two person zero sum game, player 1 has a strategy set $((\alpha_1, \alpha_2)$ and player 2 has a strategy set (β_1, β_2)).The payoff matrix is given by

$$\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

Determine the equilibrium pair of mixed strategies and show that it maximizes each player's security level.

[15]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: (2000 -2001)

B.Stat.-II yr.

Biology – I

Date:22.12.20000 Maximum marks: 60 Duration- 3hr.

Answer any six questions. All questions carry equal marks .

1. How do photosynthetic cells differ from heterotrophic cells? Why is ATP so important to the living cells? Which class of biomolecules are responsible for the self-regulation of cellular reactions and how?
2. Give an account of the detailed structure of the eukaryotic nucleus. How does this differ from the prokaryotic nucleoid and what are the similarities between the two?
3. a) Carbon is the major element of all biological molecules. How did carbon reach the primitive earth?

   b) What is prebiotic soup? Design an experiment to synthesize primitive life molecule (s) from prebiotic soup.

   c) Give reasons why RNA and not DNA would be a better candidate for primitive "life – molecules".
4. What are the major differences between plant and animal cells with respect to structures and functions of various organelles? What is turgor pressure and what advantage do plant cells get out of it?
5. Describe digestion, metabolism and energetic of fat with an example .
6. What are gene and allele? Describe how replication, transcription and translation are related with cell division.
7. How many DNA sequences are possible with 30 nucleotides considering the presence of four different nucleotides. Calculate the number of sequences beginning with ATG and ending with TAA but without any ATG and TAA within the sequence.
8. (a) At the ABO blood-groups locus, describe the genotype-phenotype relationships and clearly indicate the dominance/co-dominance relationships among the alleles .

   (b) Discuss mitosis, meiosis and their importance in cell division and growth.