INDIAN STATISTICAL INSTITUTE
B. Stat. II Year (2003-2004), Analysis - III
Mid-Semestral Examination

Time: 3 hrs: Max. Marks 100: Date: 15-09-2003.

Note: You may answer all the questions. But the maximum you can score is 100.

1. Show that the union of two open balls $B(X_0, r_1)$ and $B(Y_0, r_2)$ in $I\!R^n$ is connected if, and only if $\|X_0 - Y_0\| < r_1 + r_2$. What will be the condition for the connectedness of the union if one or both the balls is closed? [15]

2. Let $L : I\!R^n \to I\!R^n$ be a linear function. Show from first principles that the function $f : I\!R^n \to I\!R$ defined by $f(X) = X \cdot L(X)$ is differentiable and find the derivative of f at $X_0 \in I\!R^n$. [10]

3. Let $u(x, y)$ and $v(x, y)$ be functions from $I\!R^2$ to $I\!R$. Let $f : C\!\!\!\!I \to C\!\!\!\!I$ ($C\!\!\!\!I$ = Complex plane), be defined by $f(x+iy) = u(x, y)+iv(x, y)$. Show that f is differentiable at x_0+iy_0 (as a function from $C\!\!\!\!I$ to $C\!\!\!\!I$) if, and only if the functions $u(x, y)$ and $v(x, y)$ are differentiable at (x_0, y_0) and satisfy the Cauchy-Riemann equations $\frac{\partial u}{\partial x}(x_0, y_0) = \frac{\partial v}{\partial y}(x_0, y_0)$ and $\frac{\partial u}{\partial y}(x_0, y_0) = -\frac{\partial v}{\partial x}(x_0, y_0)$. [25]

4. Assume that $f(x, y)$ and $\frac{\partial f}{\partial y}(x, y)$ are continuous on a rectangle $[a, b] \times [c, d]$. Let $p(y)$ and $q(y)$ be differentiable functions on $[c, d]$ taking values in $[a, b]$. For $y \in [c, d]$ let

$$F(y) = \int_{p(y)}^{q(y)} f(x, y)\ dx$$

Show that F is differentiable on (c, d), and find $F'(y)$.

[Hint: Consider the function $G(s, t, y) = \int_s^t f(x, y)dx$, $a \leq s, t \leq b, c \leq y \leq d$] [15]

P.T.O.

5. Let U be an open subset of $\mathbb{R}^3$ and let $f : U \to \mathbb{R}$ have continuous partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ on U. Let $(x_0, y_0, z_0) \in U$ and suppose $f(x_0, y_0, z_0) = 0$ and $\frac{\partial f}{\partial z}(x_0, y_0, z_0) \neq 0$. Show that there is $\epsilon > 0$ and a unique real-valued function $g(x, y)$ defined on $V = (x_0 - \epsilon, x_0 + \epsilon) \times (y_0 - \epsilon, y_0 + \epsilon)$ such that

   (a) $g(x_0, y_0) = z_0$

   (b) $f(x, y, g(x, y)) = 0$ for $(x, y) \in V$

   (c) g is continuously differentiable on V.

[20]

6. Find the rectangle of greatest perimeter inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [10]

INDIAN STATISTICAL INSTITUTE
Midsemester Examination : 2003-2004
B.Stat. (Hons.) II Year
Statistical Methods III

Date : 17.09.2003 **Maximum Marks : 100** **Duration : 3 Hours**

Answer all the questions . Marks allotted to each question are given within the parentheses . Standard notations and symbols are used .

1.(a) Find the MLE of θ for a random sample of size n from a distribution having the p.d.f.

$$f_\theta(x) = \begin{cases} \frac{1}{\theta}\exp(-x/\theta) & \text{if } 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

where $0 < \theta < \infty$.

(b) Obtain the Cramer-Rao lower bound to the variance of an unbiased estimator of θ and check whether the MLE of θ attains the Cramer-Rao lower bound .

(12 + 13) = [25]

2.(a) If Y has the binomial distribution b(n,p) where 0<p<1 and n is known , of what quantity is Y^2 an unbiased estimator ? Hence obtain unbiased estimators of p^2 and $\text{var}_p(Y)$.

(b) Let $X_{(1)}$, $X_{(2)}$ and $X_{(3)}$ be the order statistics of a random sample of size 3 from a distribution having the p.d.f.

$$f_\theta(x) = \begin{cases} \frac{1}{\theta} & \text{if } 0 < x < \theta \\ 0 & \text{elsewhere} \end{cases}$$

where $0 < \theta < \infty$. Show that $4X_{(1)}$, $2\,X_{(2)}$ and $\frac{4}{3}X_{(3)}$ are all unbiased estimators for θ . Find the variance and hence the efficiency of each .

(12 + 13) = [25]

3.(a) If X has the exponential distribution with p.d.f.

$$f(x) = \theta \exp(-\theta x) \quad , \; 0 < x < \infty ,$$

where $\theta > 0$, then what is the distribution of $\sum_{i=1}^{n} x_i$,

P.T.O.

x_i (i = 1,2,…, n) being random and independent observations from this distribution ?

(b) If x_1 , x_2 ,… , x_{2n} are random and independent observations from N(μ,σ^2) obtain the sampling distribution of

(i) $(x_1 - x_2) + (x_3 - x_4) + \ldots + (x_{2n-1} - x_{2n})$

(ii) $\sum_{i=1}^{n}(x_i - x_{n+i})^2$.

(12 + 13) = [25]

4. (a) A manufacturer of fluorescent tubes claims that no more than 6% of his products are defective . A sample of 20 tubes is found to contain 4 defective tubes . Does the manufacturer's claim seem justified in the light of these data ?

(b) It is required to compare two methods of treating a type of allergy . Method I was used on 15 patients and Method II on 14 . The results are shown below :

	Method I	Method II
Cured	6	11
Not cured	9	3

Is Method II better than Method I ?

(12 + 13) = [25]

INDIAN STATISTICAL INSTITUTE
B. Stat. II year : 2003–2004
C & Data Structures
Mid Semester Examination

Date : 19. 09. 2003 | Marks : 60 | Time : 3 Hours

Answer any part of any question. The question is of 70 marks. The maximum marks you can get is 60. Please write all the part answers of a question at the same place.

1. (a) Write a function in C which finds out GCD of two integers and show how the function executes when the two integers are 1155 and 969.

   (b) Write a C program for the following task. Input the last two digits of your roll number, say integer r. Input a prime p in the range 30 to 50. Create a data set of 11 integers $x_0, \ldots, x_{10}$, where $x_i = (r + 20) * (i + 1) \bmod p$.

   5+5 = 10

2. (a) Clearly explain the parameter passing strategy to functions in C programming language with discussion on 'printf' and 'scanf' functions.

   (b) Implement the factorial function using C in both recursive and iterative format. Explain which one is more efficient.

   $9 + 6 = 15$

3. (a) Clearly write down the program for heap sort in C.

   (b) Execute your program (explain with proper figures of binary tree) on the data set $x_1, \ldots, x_{10}$ available in the answer of 1b.

   (c) Write down the program of bubble sort and execute it on the data set $x_1, \ldots, x_{10}$ available in the answer of 1b. Compare the result with the result of heap sort in terms of 'number of swaps between two integers'.

   $10 + 5 + 10 = 25$

4. (a) Write a function in C to calculate a^b, where a, b are both integers. Do not use the 'pow' function available in C.

   (b) Describe the data structure of a binary search tree and write down the C program for insertion of a new data.

   (c) Execute your insertion function (explain with proper figures of binary tree) on the data set $x_1, \ldots, x_{10}$ available in the answer of 1b.

   $5 + 7 + 3 = 15$

5. Explain what happens when the following codes are executed.

   (a) `char *p, *q; while (*p++ = *q++);`

   (b)
   ```
   int i, k = 1, n = 5;
   for (i = 0; k < n+1; i = k-i) { printf("%d\n", k); k = k+i; }
   ```

   $2 + 3 = 5$

INDIAN STATISTICAL INSTITUTE
MID- SEMESTER EXAMINATION: 2003 – 2004
COURSE NAME: B. STAT. II
SUBJECT NAME: ECONOMICS-I

Date: 23/9/2003 Maximum Marks: 80 Duration: 3 Hours

Answer **ALL** questions

1. Prove the following statements. [4X7 = 28]
   (a) An increase in the price of a good leads to an increase in the total amount spent on purchase of the good if the demand for the good is inelastic.

   (b) Suppose x^* and y^* are two utility-maximizing bundles of a consumer. Then any convex combination of these two bundles is also an equilibrium bundle.

   (c) For any value of prices and income it cannot be the case that for a consumer all goods are inferior; so at least one good must be normal.

   (d) The assumption of diminishing marginal utility is not sufficient for the assumption of diminishing marginal rate of substitution between two goods.

2. Derive (formally or informally) the demand function for the commodity X for each of the following utility functions, given that prices of two goods are $p_x = 2 \text{ and } p_y = 3$, and money income of the consumer is $m = 30$.

   (a) $u = x^2 y^3$, (b) $u = 2x + 3y$, (c) $u = \min\left(\frac{x}{3}, \frac{y}{2}\right)$, and (d) $u = x + 18 \ln y$ [8X4 =32]

3. Explain the following situations: [5X2 = 10]
   (a) It is observed that compared to the last year, this year the price of rice is higher and at the same time the quantity sold is larger.

   (b) The local government imposes a tax per unit on a product sold with a view to increase total tax collections, but its tax collection in fact gets reduced.

4. (a) What do you mean by stability of equilibrium in a market? Show that equilibrium can be stable in Walrasian sense, but it is unstable in Marshallian sense.

   (b) Explain the nature of time path of price given the model described by the following equations (all notations are usual).
   $D_t = 100 - 0.5p_t$, $S_t = 50 - 0.1p_t$, and $p_t = p_{t-1} + 6(D_t - S_t)$. [5+5 =10]

Indian Statistical Institute

Mid-Semester Examination : (2003-2004)

Biology-I, B.Stat. II (23.9.03)

Full marks: 40, Answer any Five, Duration: 2hr 30min

(Equal marks for each question)

1. "Oxidation of fatty acid and glucose meet to a point for energy production" – describe it mentioning numbers of ATP produced after oxidation processes.

2. Mention the differences between DNA and RNA. How many DNA sequences are possible with 20 nucleotides containing all four bases (A, T, G, and C). Mention the numbers of sequences starting with (a) ATG codon (b) ATG codon but ending with TAA codon and (c) ATG codon, ending with TAA codon but no TAA codon within the sequences.

3. Describe mitosis and meiosis processes of cell division and also state when these divisions are necessary in human. Why monozygotic twins are identical. How they differ from dizygotic twins. Why the chromosomes of a pair are not identical but similar?

4. How proteins are digested in our system. "Amino acid composition of a protein determines its quality"- explain. Draw carbon and nitrogen cycles and mention their importance for human living.

5. What are aerobic and anaerobic metabolisms of glucose and mention the conditions when they undergo. How many ATPs are generated from each process and calculate the efficiencies of energy trapped in these processes.

6. How DNA and RNA are related to protein synthesis? What is a "codon" and its function. How these codons are related with protein synthesis. How it is possible to predict the length of a protein from the length of a gene:- describe with an example.

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination : (2003-2004)

B.Stat. (Hons.) II Year

PHYSICS I (Electrodynamics)

Group - A

Date : 23.9.2003 Maximum Marks : 15 Duration : 45 Minutes

Note : Answer any two of the following questions :

1. (a) Two spherical cavities, of radii a and b, are hollowed from the interior of a (neutral) conducting sphere of radius R. At the centre of each cavity a point charge q_a and q_b is placed respectively.

   (i) Find the surface charges σ_a, σ_b and σ_R.

   (ii) What is the field outside the conductor?

   (iii) What is the force on q_a and q_b ?

   (iv) Which of these answers would change if a third charge q_c is brought near the conductor?

   (b) A ring of radius R has a non-uniform charge density λ. Find the work done in taking a point charge q from the centre of the ring to infinity.

2. Find the electric field at a distance z above the centre of a circular loop of radius r which carries a uniform charge density λ.

3. The electric potential of some configuration is given by

$$V(\mathbf{r}) = \frac{Ae^{-\lambda r}}{r}$$

where A and λ are constants.

   (i) Find the charge density $\rho(r)$.

   (ii) Evaluate the total charge Q.

P.T.O

PHYSICS I

Group B

Date : 23.9.2003 Maximum Marks : 15 Duration : 45 Minutes

Answer any three

1. Show that the total angular momentum $\vec{L}$ of a system of particles about the origin O is given by

$$\vec{L} = \vec{R} \times M\vec{v} + \sum_i \vec{r_i}\,' \times \vec{p_i}\,'$$

$\vec{R}$ is the radius vector from O to the centre of mass.
M is the total mass of the system of particles.
$\vec{v}$ is the velocity of the centre of mass relative to O.
$\vec{r_i}\,'$ is the radius vector from the centre of mass to the ith particle.
$\vec{p_i}\,'$ is the linear momentum corresponding to $\vec{r_i}\,'$. (5)

2. a) A body of mass 'm' is in equilibrium on a smooth inclined plane (of inclination θ to the horizontal) under the action of a force $\vec{F}$ making on angle ϕ with the inclined plane. Apply the principle of virtual work to show that $F\cos\phi = \text{mg}\ \sin\theta$.

   b) A particle is constrained to move in a circle in the plane xy. Apply D' Alembert's principle to show that for equilibrium

$$\ddot{x}y - \ddot{y}x - gx = 0 \qquad (2\tfrac{1}{2} + 2\tfrac{1}{2})$$

   where $(\cdot)$ denotes differentiation with respect to time.

3. Set up the Lagrange's equations of a particle moving on the surface of earth using spherical polar coordinates.

$$\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned} \qquad (5)$$

and taking $V(r,\theta,\phi)$ as the potential energy.

4. A particle of mass 'm' moves on a plane in the field of force given by (in polar coordinates)

$$\vec{F} = \hat{n}_\theta \frac{\lambda}{r^2}$$

where λ is a constant and $\hat{n}_\theta$ is a unit vector in the θ-direction. Obtain Lagrange's equations of motion. (5)

[Given Curl $\vec{A} = \dfrac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{e}_1 & r\hat{e}_2 & r\sin\theta\,\hat{e}_3 \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_1 & rA_2 & r\sin\theta\, A_3 \end{vmatrix}$

where $\vec{A} = A_1\hat{e}_1 + A_2\hat{e}_2 + A_3\hat{e}_3$ and $\hat{e}_1, \hat{e}_2, \hat{e}_3$ are unit vectors.]

Indian Statistical Institute
Semester 1 (2003-2004)
B. Stat 2nd Year
Mid-semestral Exam
Probability Theory 3

Friday 26.9.2003, 10:30-1:30 Total Points $5 \times 6 = 30$

Answers must be justified with clear and precise arguments.

1. Let F and G be distribution functions on R and

$$U(x,y) = F(x)G(y)[1 + \alpha(1 - F(x)(1 - G(y))],$$

where $|\alpha| \leq 1$. Show that U is a dsitribution function on R^2 with marginals F and G

2. Suppose $(X_1, \cdots, X_n)'$ has a full normal distribution with mean $(0, \cdots, 0)'$ and variance-covariance matrix Σ. Calculate

$$Ee^{i(t_1 X_1 + \cdots + t_n X_n)},$$

where $(t_1, \cdots, t_n)' \in R^n$.

3. Suppose X_1, X_2, X_3 are i.i.d. exp(1). Write $U_1 = X_{(1)}, U_2 = X_{(2)}, U_3 = X_{(3)}$.

(a) Find the joint density of (V_1, V_2, V_3) where $V_1 = U_1, V_2 = U_2 - U_1, V_3 = U_3 - U_2$.

(b) Find the expectation of the sample range for the above sample of size 3.

4. Show that for the uniform distribution over $(0, 1)$ the correlation coefficient between the two order statistics $X_{(r)}$ and $X_{(s)}$ is

$$[(r(n - s + 1)/s(n - r + 1)]^{1/2}.$$

5. (a) If $\phi(t)$ is a characteristic function then show that $\psi(t) = e^{\phi(t)-1}$ is also a characteristic function.

(b) Show that for any real characteristic function the following inequality holds:

$$1 + \phi(2t) \geq 2\{\phi(t)\}^2.$$

(c) Suppose $\phi(t)$ is the characteristic function of X and $\alpha > 0$ is an *irrational* number such that

$$|\phi(t_0)| = 1 \text{ and } |\phi(\alpha t_0)| = 1,$$

for some $t_0 > 0$. Show that X is degenerate.

Indian Statistical Institute
Semester 1 Exam (2003-2004)
B. Stat 2nd Year
Probability Theory 3

Date and Time: 28.11.03 10:30-1:30 Total Points 70

Answers must be justified with clear and precise arguments. Each question must be answered on a separate page and more than one answers to a question will not be accepted. If there are more than one answers to a question, only the first answer will be examined.

1. Suppose X_1, X_2, X_3 are i.i.d. $N(0,1)$. Identify the joint distribution of

$$Y_1 = \frac{X_1 - X_2}{\sqrt{2}}, Y_2 = \frac{X_1 + X_2 - 2X_3}{\sqrt{6}}, Y_3 = \frac{X_1 + X_2 + X_3}{\sqrt{3}}.$$

10 pts.

2. Suppose X_1, X_2, X_3 are i.i.d. $N(0,1)$. Show that the expected sample range is $3\sqrt{\pi}$. 10 pts

3. In a sequence of Bernoulli trials let A_n be the event that a run of n consecutive successes occurs between the 2^nth and 2^{n+1}st trial (the second one included in the counting but not the first one). Show that if $p = 1/2$ then with probability 1 infinitely many A_n occur. 10 pts

4. Suppose X_n are independent random variables such that $P(X_n = \pm n^{\alpha}) = 1/2$. Show that the SLLN applies iff $\alpha < 1/2$. 10 pts.

5 (a) Suppose $X_n \Rightarrow X$ and $Y_n \to c$ in probability where c is a constant. Then show that $X_n Y_n \Rightarrow cX$.

(b) Identify the following as a probability to show that

$$e^{-n} \sum_{k=0}^{n} \frac{n^k}{k!} \to 1/2.$$

$5 + 5 = 10$ pts

6 Show that Lindeberg's condition is satisfied if for some $\delta > 0$

$$\frac{1}{s_n^{2+\delta}} \sum_1^n E(|X_k|^{2+\delta}) \to 0,$$

where the notation has its usual significance. 10 pts

7. X_n are independent exponential random variables with means $\frac{1}{\lambda_n}$ respectively. Show that $\sum_1^n X_i$ converges with probability 1 iff

$$\sum_1^{\infty} \frac{1}{\lambda_i} < \infty.$$

(For the only if part you may consider $Ee^{-\sum_1^n X_i}$ as $n \to \infty$.) 10 pts

INDIAN STATISTICAL INSTITUTE

1st Semestral Examination

B. Stat. II year : 2003–2004

C & Data Structures

Date : 01. 12. 2003 | Marks : 100 | Time : 3 Hours

1. (a) Write a function in C using iteration that can evaluate $f(n) = f(n-1) + f(n-2) - f(n-3)$ with initial conditions $f(0) = 0, f(1) = 1, f(2) = 3$.

   (b) Write a variable argument function in C that takes the number of arguments n as the first parameter and then calculates the standard deviation of n data supplied to the function as following n arguments.

   (c) Write a function $func1$ in C to search a substring in a string. Use the function $func1$ to write a function $func2$ in C that finds a substring in a circular string. As an example, the substring "abc" is absent in the string "cxyzsdfgab", but exists when the string is considered in a circular manner.

   (d) Write a C program that takes the preorder traversal data of a binary search tree as input and outputs the tree itself.

5+5+6+9 = 25

2. (a) Explain the meaning of hashing.

   (b) Briefly explain three strategies for collision resolution.

   (c) Select a specific hashing strategy and implement a function in C programming language that can manage search and insertion in a hash table.

   (d) Provide an approximate mathematical analysis for average search/insertion time complexity for double hashing.

5+9+6+5 = 25

3. (a) Define a height balanced tree.

   (b) Find the maximum possible height of a height balanced tree that contains n nodes?

   (c) Explain the insertion algorithm with suitable examples in a balanced binary search tree.

2+8+15 = 25

4. (a) Define a B-tree of order m.

   (b) Why a B-tree of order 3 is called a 2-3 tree?

   (c) What could be the maximum possible height of a B-tree of order m having N many keys?

   (d) Write the algorithms for search and insertion in a B-tree of order m.

   (e) Construct a B-tree of order 3 using your insertion algorithm with the keys 29, 21, 10, 15, 26, 39, 36, 38, 46, 50, 37, 5, 18, and 12.

2+3+5+10+5 = 25

INDIAN STATISTICAL INSTITUTE
B. Stat. II Year (2003-2004), Analysis - III
Semestral Examination

Time: 3 hrs: Max. Marks 60: Date: 05-12-2003.

Note: Answer only four questions. Each question carries 15 marks.

1. (a) Let S be a convex open subset of $I\!R^2$. Show that a continuously differentiable function $F(x,y) = (f_1(x,y), f_2(x,y))$ from S to $I\!R^2$ is the gradient of a real-valued function $\phi(x,y)$ if, and only if, $\frac{\partial f_1}{\partial y} = \frac{\partial f_2}{\partial x}$ on S.

   (b) Show that the function

$$F(x,y) = (\sin y - y\sin x + x, \cos x + x\cos y + y)$$

   is a gradient. Find a potential function of F.

   (c) Show that the function $F(x,y,z) = (y,x,x)$ is not the gradient of a real-valued function. Find a closed curve Γ such that $\int_\Gamma F \cdot d\alpha \neq 0$.

[7+4+4=15]

2. (a) Suppose $u(x,y)$ and $v(x,y)$ are continuously differentiable real-valued functions defined on an open set containing// $D = \{(x,y) : x^2 + y^2 \leq 1\}$. Let $F(x,y) = (v(x,y), u(x,y))$ and $G(x,y) = (\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x} - \frac{\partial v}{\partial y})$. Find

$$\int\int_D F \cdot G dxdy$$

   if it is given that $u = 1$ and $v = y$ on ∂D.

   (b) Let $\mathbf{a} \in I\!R^3$. For $t > 0$ let $V(t) = \{\mathbf{x} \in I\!R^3 : ||\mathbf{x} - \mathbf{a}|| \leq t\}$ and $S(t) = \{\mathbf{x} \in I\!R^3 : ||\mathbf{x} - \mathbf{a}|| = t\}$. Let $|V(t)|$ denote the volume of $V(t)$. If $F : I\!R^3 \to I\!R^3$ is continuously differentiable on an open set around $\mathbf{a}$, show that

$$\text{div } F(\mathbf{a}) = \lim_{t \to 0} \frac{1}{|V(t)|} \int\int_{S(t)} F \cdot \mathbf{n} \, dS.$$

   where $\mathbf{n}$ denotes the outward normal to $S(t)$.

[7+8=15]

P.T.O.

3. (a) If $a > 0$ evaluate

$$\int_0^a \left(\int_0^x \sqrt{x^2+y^2}\,dy\right)dx$$

by transforming into polar coordinates.

(b) Consider the mapping $\Phi(u,v) = (u+v, v-u^2)$. Find the Jacobian of Φ at (u,v). Let T be the triangle with vertices at $(0,0),(2,0),(0,2)$. Describe by means of a diagram the image $S = \Phi(T)$. Evaluate the area of S by transforming it to a double integral over T.

[7+8=15]

4. (a) Show that the area of the surface of revolution

$$x = u\cos v, y = u\sin v, z = f(u), 0 \le v \le 2\pi, a \le u \le b$$

(where $f : [a,b] \to \mathbb{R}$ is a differentiable function) is

$$2\pi \int_a^b \sqrt{1+(f'(u))^2}\ du.$$

(b) Use (a) to find the area of the sphere $x^2+y^2+z^2 = r^2$.

(c) Use Stokes' theorem to evaluate the line integral

$$\int_\Gamma y\,dx + (2x-z)\,dy + (z-x)\,dz$$

if Γ is the curve of intersection of $x^2+y^2+z^2 = 4$ and $z = 1$.

[5+4+6=15]

5. Let $\mathbb{C} = \{z = x+iy : x,y \in \mathbb{R}\}$ be the complex plane. Suppose $u(x,y)$ and $v(x,y)$ are real-valued functions which are continuously differentiable on $\mathbb{R}^2$ and such that the function $f : \mathbb{C} \to \mathbb{C}$ defined by $f(z) = f(x+iy) = u(x,y) + iv(x,y)$ is differentiable. Show that for any piecewise smooth Jordan curve Γ in $\mathbb{R}^2$, we have

$$\int_\Gamma u\ dx - v\ dy = 0 \text{ and } \int_\Gamma v\ dx + u\ dy = 0.$$

[15]

INDIAN STATISTICAL INSTITUTE
First Semestral Examination : 2003-2004
B.Stat. (Hons.) II Year
Statistical Methods III

Date : 08.12.2003 **Maximum Marks : 100** **Duration : 3 Hours**

Answer ANY FOUR questions . Marks allotted to each question are given within the parentheses . Standard notations and symbols are used .

1.(a) Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from the Pearsonian Type III population having the p.d.f.

$$f(x) = y_0 e^{-\gamma x}\left(1+\frac{x}{a}\right)^{\gamma a}, \quad -a \le x < \infty .$$

Obtain the MLE of γ.

(b) Obtain the Cramer - Rao lower bound to the variance of an unbiased estimator of γ in (a) above and check whether the MLE of γ attains this lower bound .

(10 + 15) = [25]

2. (a) If x_1 , x_2 , x_3 obey the joint distribution

$$\frac{1}{\pi\sqrt{\pi}}(x_1 x_2 x_3)^{-\frac{1}{2}} f(x_1 + x_2 + x_3)dx_1 dx_2 dx_3$$

in the range $(0, \infty)$, obtain the sampling distribution of (i) $u = x_1 + x_2 + x_3$,

(ii) $v = \frac{x_2}{x_1}$ and (iii) $w = \frac{x_3}{x_1 + x_2}$.

(b) Show that the sampling distribution of the standard deviations of samples of two observations from the rectangular population

$$dF = dx \quad , 0 \le x \le 1$$

is given by $f(s) = 4 (1-2s) , 0 \le s \le \frac{1}{2}$.

(15 + 10) = [25]

3. (a) Given a random sample of n pairs from a bivariate normal population with parameters $m_1, m_2, \sigma_1, \sigma_2, \rho$, discuss how you would test the null hypothesis $H_0 : m_1 = m_2$ against the alternative hypothesis $H_1 : m_1 \ne m_2$.

(b) Derive the sampling distribution of the test statistic proposed in (a) above under the null hypothesis . Indicate how you modify the

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distribution when the null hypothesis is not true.

(10 + 15) = [25]

4. (a) For 20 pairs of fathers and sons, the regression equation of height of son (y) on height of father (x), both measured in inches, was found to be

$$Y = 3.66 + 0.932\,x.$$

For the given data, $\bar{x} = 66.21, \sum (x_i - \bar{x})^2 = 120.56$ and $\sum (y_i - \bar{y})^2 = 145.61$. Obtain 95% confidence limits to the height of a son whose father's height is 5'4".

(b) A 5-foot specimen of a new type of fibre is found to have 13 defects, while the manufacturer claims that there are no more than 150 defects per 100 feet. Do the above data support this claim?

(15 + 10) = [25]

5. (a) A six-faced die was thrown 300 times, and the number of points obtained at each throw was recorded. In this way the following frequency distribution was obtained. Use these data to test whether the die is unbiased.

Number of points per throw	1	2	3	4	5	6
Frequency	31	52	46	40	54	77

(b) The breakdowns occurring during a year for each of 4 machines in a factory were classified as follows according to shift, there being 3 shifts daily. Judge whether the difference among the four distributions may be attributed to sampling fluctuations alone.

	Machine			
Shift	1	2	3	4
1	15	9	18	20
2	16	18	29	31
3	19	15	19	27

(10 + 15) = [25]

6. (a) The following table gives the data on goods carried by Indian Railways during 1948-1956. Obtain the trend values for the following series by fitting a second degree polynomial.

Year	Goods carried
1948-49	82,667
1949-50	91,581
1950-51	92,340
1951-52	97,871
1952-53	98,372
1953-54	99,361
1954-55	106,979
1955-56	115,283

(b) Trend equation for yearly sales (in '000 Rs.) for a commodity with 1991 as origin and unit 1 year is $T_t = 81.6 + 28.8$ t . Determine the trend equation to give the trend values for monthly sales with January 1992 as origin and hence calculate the trend value for March , 1992 .

(15 + 10) = [25]

Indian Statistical Institute

First Semestral Examination: 2003-2004

Economics 1

B. Stat. II: 2003-2004

Date: 12/12/03 Duration: 3 hours Maximum marks: 100

Answer any **Five** questions

1. (a) How do you derive the labor supply curve of a household? Explain the backward bend of labor supply curve in terms of income effect and substitution effect.
   (b) Consider a person whose present state of employment is the following. His salary per month is fixed at Rs. 2000/-, and the company bears 50% of his house rent. The market rent per sq. m. is Rs. 10/-, and the person is staying in a 100 sq. m house. The employer now gives him an option. The company will pay Rs. 2500/- per month as salary, but no house rent separately. Will the person switch to this alternative? Explain in terms of the indifference curve analysis. [(6+6)+8=20]

2. Consider a monopoly market for a product. Initially, there was a tax per unit of goods sold by the monopolist. Now, the government is thinking to switch to a value tax (i.e., a tax imposed as a percent of sales value) from the present unit tax, subject to the restriction that consumers' welfare should not be lowered. Will such a value tax lead to a higher tax collection of the government? {20]

3. (a) Consider a firm who is the only buyer in the labor market. He is also the only seller of his products in the product market. What will be the initial wage rate?
   (b) Now suppose that the government fixes a minimum wage. What could be its possible effects on employment, disemployment and unemployment?
   (c) Compare your results in (b) with those when both labor and product markets are competitive. [4+10+6=20]

4. Consider a two-consumer-two-commodity exchange economy. The two goods are X and Y, and the two individuals are 1 and 2. The endowments of individuals 1 and 2 are $(\bar{x}_1, \bar{y}_1) = (1, 0)$ and $(\bar{x}_2, \bar{y}_2) = (0, 1)$, respectively. The utility functions are $U_1(x_1, y_1) = x_1^{1/3} y_1^{2/3}$ and $U_2(x_2, y_2) = x_2^{1/2} y_2^{1/2}$, respectively.
   (a) Solve for the set of Pareto efficient allocations.
   (b) Solve for the competitive equilibrium (i.e., prices and consumption vectors). [10+10=20]

5. (a) State the Weak Axiom of the Revealed Preference (WARP). Portray diagrammatically the situations when the WARP is satisfied and when it is not.
   (b) Using the WARP prove that Hicks Substitution effect is negative.
   (c) Suppose you observe a consumer to buy the bundle $(x_1, x_2) = (1, 2)$ at prices $(p_1, p_2) = (1, 2)$, and the bundle $(y_1, y_2) = (2, 1)$ at prices $(q_1, q_2) = (2, 1)$. Examine whether the consumption behavior of this consumer is consistent with the WARP. [6+6+8=20]

6. Prove the following statements.
   (a) Marshallian demand function is homogeneous of degree zero in prices and money income.
   (b) If the production function is homogeneous of degree 1, then the expansion path is a straight line passing through the origin.
   (c) Price effect is decomposed into substitution effect and income effect. [4+6+10=20]

INDIAN STATISTICAL INSTITUTE

Semestral - I Examination : (2003-2004)

B.Stat. (Hons.) II Year

PHYSICS I

Group A (Electrodynamics)

Date : 12.12.2003 Maximum Marks : 30 Duration : 1.5 hours

Note : Answer any six questions.

1. Consider a pure dipole **p** sitting at the origin, pointing in the z direction.

   (a) What is the force on a point charge q at $(2b, 0, 0)$ (cartesian coordinates)?

   (b) What is the force on q at $(0, 0, 2b)$?

   (c) What is the work done on q in moving it from $(2b, 0, 0)$ to $(0, 0, 2b)$?

   (d) Sketch the electric field lines of a pure dipole. (1, 1, 2, 1)

2. A square loop of wire (side s) lies on a table at a distance l from a very long straight wire carrying current I.

   (a) What emf is generated if one pulls the loop directly away from the wire at speed v? In what direction (clockwise or counterclockwise) does the induced current flow?

   (b) What total charge passes a given point in the loop if we let $l = s$, and:

   (i) the current is switched off abruptly?

   (ii) the current is turned down gradually. $I(t) = (1 - at)I$ for $0 \leq t \leq 1/a$ and zero otherwise. (3, 1, 1)

3. Show that the fields

$$\mathbf{E}(\mathbf{r}, t) = -\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \theta(vt - r)\hat{\mathbf{r}}, \quad \mathbf{B}(\mathbf{r}, t) = 0$$

(where the step function $\theta(x) = 1$ if $x > 0$ and zero otherwise), satisfy all the Maxwell's equations. Determine the corresponding ρ and **J**. Describe the physical situation that gives rise to these fields. (5)

P.T.O

4. A long cable carries current in one direction uniformly distributed over its circular cross section. The current returns along the surface (there is a very thin insulating sheath separating the currents). Find the self inductance per unit length. (5)

5. A small loop of wire (radius a) lies a distance z above a large loop (radius b). The planes of the two loops are parallel to each other, and perpendicular to a common axis.

   (a) Suppose current I flows in the big loop. Find the flux through the little loop. (The little loop is so small that you may consider the field of the big loop to be essentially constant).

   (b) Suppose current I flows in the little loop. Find the flux through the big loop. (The little loop is so small that you may treat it as a magnetic dipole).

   (c) Find the mutual inductances of the loops, and confirm that $M_{12} = M_{21}$. (1, 2, 2)

6. Find the vector potential of an infinite solenoid with n turns per unit length, radius R, and current I. (5)

7. A sphere of radius R carries a volume charge density distributed as $\rho = k(R - r)$. On the surface it carries a surface charge density that is uniformly distributed. The total volume charge is Q and the total surface charge is $-Q$.

   (a) What is k in terms of Q and R?

   (b) What is the electric field inside and outside the sphere?

   (c) How much energy is stored in the electric field? (1, 2, 2)

8. Consider a parallel plate capacitor with the top plate having surface charge density $-\sigma$ and the bottom one $+\sigma$. A rectangular loop of wire is placed vertically, with one end in between the plates. The other end is way outside. The height of the loop is h and its resistance is R.

   (a) Evaluate the electric field in between the plates and also in the region outside.

   (b) What current flows in the loop? Explain. (2, 3)

INDIAN STATISTICAL INSTITUTE

First Semestral Examination : (2003-2004)

B.Stat. (Hons.) II Year

PHYSICS I

Group - B (Classical Mechanics)

Date : 12.12.2003 Maximum Marks : 30 Duration : 90 Minutes

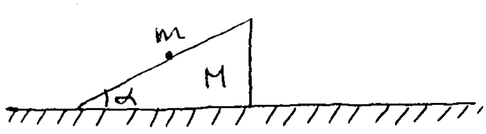
Answer any six of the following questions

1) A spaceship is moving away from the earth at a speed of v_1c metre/second (v_1 being a real constant such that $0 < v_1 < 1$, c the velocity of light) when it fires a missile parallel to the direction of motion of the ship.The missile moves at a speed of v_2c metre/second (v_2 being a real constant such that $0 < v_2 < 1$, c the velocity of light) relative to the ship. What would be the speed of the missile as measured by an observer on the earth? (5)

2) In inertial frame S, a red light and a blue light are separated by a distance d metres, with the red light at the larger value of x. The blue light flashes and t_1 seconds later the red light flashes. Frame S' is moving in the direction of increasing x with a speed of uc metres per second (u being a real constant such that $0 < u < 1$, c the velocity of light). What is the distance between the two flashes and the time between them as measured in S'? (2.5 + 2.5)

3) Consider the path of a particle of unit mass in a uniform gravitational field. Its displacement in time t is given by $x = ut + (1/2)gt^2$ where g is the acceleration due to gravity.Write the Lagrangian. Show that the action I for the actual path given above is smaller than on the varied path given by $x = ut + (1/2)gt^2 + \eta(t)$ where $\eta(t)$ (with $\dot{\eta}(t) \neq 0$) is a function satisfying certain conditions.What are the conditions? (1+3+1)

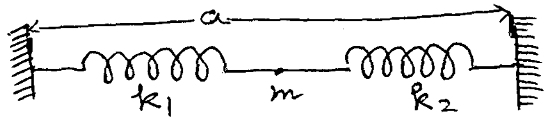
4) A wedge of mass M is free to move smoothly on a horizontal plane. The wedge has a perfectly smooth surface inclined at an angle α to the horizontal. A particle of mass m is free to move in the xy plane on the inclined surface as shown in the figure.



Set up the Lagrangian of the system. (5)

P. T. O

5) A particle of mass m can move in one dimension under the influence of two springs connected to fixed points a distance a apart as shown in the figure.

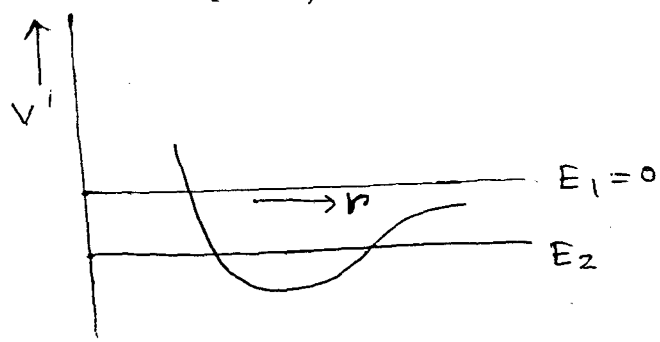


The springs obey Hooke's law and have zero unstretched lengths and force constants k_1 and k_2 respectively. Using the position of the particle from one fixed point as the generalized coordinate, find the Hamiltonian of the system and Hamilton's equations of motion. (3+2)

6) Suppose that a particle of mass m falls a given distance z_0 in time $t_0 = \sqrt{\frac{2z_0}{g}}$ but that the time of fall for distances other than z_0 is not known. Also the distance travelled in time t is always given by the relation $z = at + bt^2$ where the constants a and b are adjusted in such a way that the time to fall z_0 is correctly given by t_0. Show that the integral $\int_0^{t_0} L dt$ is an extremum for real values of the coefficients only when $a = 0$ and $b = g/2$. (5)

7) Consider a conservative system in which the potential energy V is not a function of velocities and that the kinetic energy is independent of generalized coordinates. If q_j represents a rotation coordinate then show that the generalized force is the component of the total applied torque about the axis of rotation. Hence show that if the coordinate q_j is cyclic then the conservation of its conjugate momentum corresponds to conservation angular momentum along the axis of rotation. (Relevant figure must be given). (5)

8 a) Discuss the motion of a particle having the energy $E_1 = 0$ and E_2 for a particular value of angular momentum under the action of the effective potential $V'(r) = V(r) + \frac{l^2}{2mr^2}$ as shown in the figure below (l being the angular momentum and m the mass of the particle).



b) A particle moves in a conservative field of force produced by a homogeneous mass distribution. The force generated by a volume element is derived from a potential that is proportional to the mass of the volume element and is a function only of the scalar distance from the volume element.State the conserved quantities in the motion of the particle if the mass is uniformly distributed in the half plane $z = 0, y > 0$. (2.5 + 2.5)

Indian Statistical Institute
Semester 1 (2003-2004)
B. Stat 2nd Year
Backpaper
Probability Theory 3

Date and Time: 27.1.04 (3 hrs.) Total Points 105

The maximum you can score is 100. Answers must be justified with clear and precise arguments. Each question must be answered on a separate page and more than one answers to a question will not be accepted. If there are more than one answers to a question, only the first answer will be examined.

1. Suppose (X, Y) follows bivariate normal with zero means, unit variances and correlation coefficient $\rho, \rho^2 < 1$. Find $E(Y|X)$.

2. Let Y denote the median of a random sample of size $2k+1$ from $N(\mu, \sigma^2)$. Show that the pdf of Y is symmetrical about μ and find EY.

3. Show that for any characteristic function ϕ the following inequality holds

$$1 - |\phi(t)|^2 \leq 4\{1 - |\phi(t)|^2\}.$$

4. Suppose X_n are independent random variables with

$$P(X_n = 0) = 1 - \frac{1}{n}, P(X_n = 1) = \frac{1}{n}.$$

Decide whether the sequence $\{X_n\}$ converges with probability 1, or does not converge with probability 1.

5. Suppose for each k, X_k is a χ_k^2 random variable. Show that

$$\frac{X_k - k}{\sqrt{2k}}$$

converges in distribution and identify the limit.

6. X_k are independent random variables with

$$P(X_k = \pm k) = \frac{1}{2\sqrt{k}}, P(X_k = 0) = 1 - \frac{1}{\sqrt{k}}.$$

Decide whether the WLLN holds for this sequence.

7. Let $X_1, X_2, \cdots, X_n$ be i.i.d. $U(0, \theta)$. Find the limiting distribution of $nX_{(n)}$.

(Back Paper)

Indian Statistical Institute

First Semestral Examination: 2003-2004

Economics 1

B. Stat. II: 2003-2004

Date: 30.1.04 Duration: 3 hours Maximum marks: 100

Answer **All** questions. All questions carry equal marks.

1. (a) Consider a Robinson Crusoe economy insulated from all trade and exchange. So there is only one person in the economy who has endowment $(\bar{x}, \bar{y})$ of two goods X and Y. No exchange is available to him, but production of these goods is possible using the initial endowments. Given his utility function, find his optimal position.

   (b) If production is not al all possible but exchange (or trade) of goods is possible at a given price (p_x, p_y), what will be the equilibrium of the consumer?

2. Consider a long run equilibrium of a competitive industry. If there is a shift of demand, how will the industry adjust before it reaches a new equilibrium?

3. The aggregate demand and supply functions in a competitive market are respectively, $D = 40 - P$ and $S = 20 + 2P$. Then estimate the dead weight loss if there is a tax of 5 per unit.

4. Consider a monopoly market for a product. The local government imposes a unit tax with a view to collect tax revenue. How do you solve for the optimal tax rate that will maximize government tax collections?

5. In an oligopoly there are n identical firms. The cost function of the i-th firm is: $C(q_i) = bq_i$ where q_i is the output of firm i. The market demand function for the product is: $P = a - \sum_i q_i$ $a > b > 0$. Find the symmetric oligopoly equilibrium. If now all firms collude to form a Cartel, what will be each firm's output under Cartel?

INDIAN STATISTICAL INSTITUTE
B. Stat. II Year (2003-2004), Analysis - III
Semestral Backpaper Examination

Time: 3 hrs: Max. Marks 100: Date: 30.1.2004.

Note: Answer all the questions.

1. Find the equation of the tangent plane to the surface $x^2+y^2-z=2$ at $(3,-1,8)$.

[10]

2. Find the direction in which the directional derivative of

$$f(x,y,z) = \sin(xy) - \cos(xz)$$

at $(\pi, \frac{1}{2}, 1)$ is maximised.

[10]

3. Find the point on the line of intersection of the two planes

$$a_1x + a_2y + a_3z + a_0 = 0$$

$$b_1x + b_2y + b_3z + b_0 = 0$$

which is nearest to the origin.

[10]

4. If $\Phi(r,\theta,\phi) = (r\cos\theta\sin\phi, r\sin\theta\sin\phi, r\cos\phi)$, find the Jacobian of Φ.

[10]

5. Find a scalar-valued function $\phi(x,y)$ such that the vector-valued function $F(x,y) = (2xe^y + y, x^2e^y + x - 2y)$ is equal to $\nabla\phi(x,y)$.

[10]

6. Show that the vector-valued function $F(x,y,z) = (xy, x^2+1, z^2)$ is not the gradient of a scalar-valued function. Also find a closed curve Γ such that $\int_\Gamma F \cdot d\alpha \neq 0$.

[10]

7. Use Green's theorem to evaluate

$$\int_\Gamma y^2dx + xdy$$

if Γ is the square with vertices $(\pm1, \pm1)$.

[10]

P.T.O

8. By transforming to polar co-ordinates, evaluate the integral

$$\int_0^{a\sin\beta}\left(\int_{y\cot\beta}^{\sqrt{a^2-y^2}}\log(x^2+y^2)dx\right)dy$$

where $a > 0$ and $0 < \beta < \frac{\pi}{2}$.

[10]

9. Use a suitable linear transformation to evaluate the double integral

$$\int\int_S (x-y)^2 \sin^2(x+y)dxdy$$

where S is the parallelogram with vertices $(\pi, 0), (2\pi, \pi), (\pi, 2\pi), (0, \pi)$.

[10]

10. Use Stokes' theorem to evaluate the line integral

$$\int_\Gamma y^2dx + xydy + xzdz$$

where Γ is the curve of intersection of the sphere $x^2 + y^2 + z^2 = 1$ and the plane $z = 0$.

[10]

INDIAN STATISTICAL INSTITUTE
Backpaper Examination : 2003- 2004
B.Stat. (Hons.) II Year
Statistical Methods III

Date : 28.1.04 **Maximum Marks : 100** **Duration : 3 Hours**

Answer ANY FOUR questions . Marks allotted to each question are given within the parentheses . Standard notations and symbols are used .

1. (a) Find the maximum- likelihood estimator of $\frac{1}{\theta}$ for an observation x from the discrete distribution

$$f(x) = (1-\theta)^{x-1}\theta \quad \text{for } x = 1,2,\ldots \; .$$

Show that the estimator is unbiased . What is the variance of this estimator ?

(b) A random sample of size n has been taken (without replacement) from a population of size N : N is unknown , but the number of individuals in the population with the character A is known to be N_1 . Let x among the n members of the sample have the character A . Find the maximum likelihood estimate of N .

Discuss how this method may be used in estimating the number of fish in a pond or the number of birds in an aviary .

(10 + 15) = [25]

2. (a) If x_i (i = 1,2,…, n) are a random sample from N (0 , 1) show that

$$y_1 = \frac{x_1}{\sqrt{\sum_{i=2}^{n} x_i^2}}, \quad y_2 = \frac{x_2}{\sqrt{\sum_{i=3}^{n} x_i^2}}, \quad \ldots, \quad y_{n-1} = \frac{x_{n-1}}{|x_n|}, \quad y_n = \sum_{i=1}^{n} x_i^2$$

are independently distributed , and determine the distribution of each y_i .

(b) If x_i (i = 1,2,…, n) are a random sample from N (μ,σ^2) show that

$$\frac{\sqrt{\frac{n}{n-1}}(x_1-\bar{x})}{\sqrt{\frac{\left\{(n-1)s^2 - \frac{n}{n-1}(x_1-\bar{x})^2\right\}}{n-2}}}$$

follows the t- distribution with (n-1) d.f.

(15 + 10) = [25] **P.T.O**

3.(a) Discuss how you would test for the equality of two population means based on two independent random samples drawn from two independent normal populations. State your assumptions clearly.

(b) Derive the sampling distribution of your test statistic proposed in (a) above under the null hypothesis. How do you modify the distribution when the null hypothesis is not tenable?

(10 + 15) = [25]

4.(a) The weights of ten boys before they are subjected to a change of diet and after a lapse of six months are recorded below.

Serial No.	Weight (in lb.) Before	After
1	109	115
2	112	120
3	98	99
4	114	117
5	102	105
6	97	98
7	88	91
8	101	99
9	89	93
10	91	89

Test whether there has been any significant gain in weight as a result of the change of diet.

(b) For the above data, judge whether the standard deviations of the two series are significantly different.

(10 + 15) = [25]

5. (a) An investigation of the performance of two machines, in a factory manufacturing large number of bobbins, gives the following results:

	No. of bobbins examined	No. of bobbins found defective
Machine 1	375	17
Machine 2	450	12

Test whether there is any significant difference in the performance of the two machines

(b) Obtain 99% confidence interval for the true difference between the proportions of defective bobbins manufactured by Machine 1 and Machine 2 .

(15 + 10) =[25]

6.(a) The seasonal indices of the sales of garments of a particular type in a certain shop are given below :

Quarter	Seasonal Index
Jan - Mar	97
Apr- Jun	85
Jul- Sep	83
Oct- Dec	135

If the total sales in the first quarter of a year be worth Rs.15,000/- , determine how much worth of garments of this type should be kept in stock by the shop-owner to meet the demand for each of the other three quarters of the year .

(b) From the following table showing the monthly receipts (Rs. Crores) of Government of India , obtain measures of seasonal variation .

	Month											
Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1952	23	39	82	17	18	16	20	17	12	22	20	18
1953	25	26	105	20	22	20	26	18	23	29	15	16
1954	32	36	93	21	21	22	29	21	15	27	27	21
1955	32	42	99	24	24	23	29	24	21	32	28	21

(10 + 15) = [25]

INDIAN STATISTICAL INSTITUTE

Second Mid Semestral Examination: 2003-2004
B.Stat. (Hons.). 2nd Year
Statistical Methods IV

Date: February 16, 2004 **Maximum Marks: 100** **Duration: 3 hours**

- You must state clearly any result stated and proved in class you might need in order to answer a particular question. Keep the answers brief, to the point and well structured.

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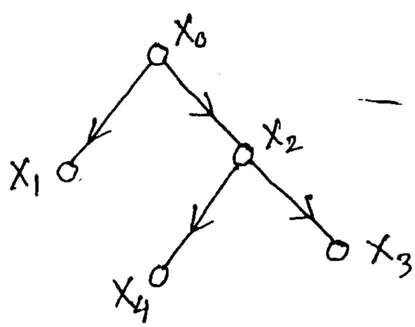
1. Let $f(x,y)$ be a bivariate probability mass function with $f(x|y)$ and $f(y|x)$ as respective conditionals. Suppose that X_0 is sampled from a proposal p.m.f g. Next, Y_0 is sampled from $f(y|x_0)$, X_1 is sampled from $f(x|y_0)$ and so on. Find a distribution g so that the distribution of X_1 is also g. Further, what is the joint distribution of (X_0, Y_0) with that g as the proposal p.m.f ? [20]

2. Consider the following graphical model where each of the variables $X_0, X_1, \ldots, X_4$ are binary. Assume that X_0 is the only unobserved node with $\Pr(X_0 = 1) = 0.5$. The conditional at X_1 is described by $\Pr(X_1 = 1|X_0 = t) = \frac{e^t}{(1+e^t)}$; the conditional at X_2 is described by $\Pr(X_2 = 1|X_0 = t) = \frac{e^{-t}}{(1+e^{-t})}$; the conditionals at X_3 and X_4 are identical and described by $\Pr(X_3 = 1|X_2 = t) = \frac{e^{2t}}{(1+e^{2t})}$ respectively. Obtain the conditional distribution of X_0 given X_1, X_2, X_3 and X_4. [20]



[P.T.O] ...

3. Suppose that $X_1, X_2, \ldots, X_n$ are independent with X_i having a $\mathrm{N}(\mu, w_i \sigma^2)$ distribution where $w_1, w_2, \ldots, w_n$ are known positive constants. Further let μ have a $\mathrm{N}(0, \tau^2)$ distribution and $\frac{1}{\sigma^2}$ a *Gamma* $(n, 1)$ distribution respectively (they are also independent). Find the conditional distribution of (μ, σ^2) given $X_1, X_2, \ldots, X_n$. [30]

4. Suppose the joint distribution of a pair of observations (X_1, X_2) is described by (i) the marginal distribution of X_1 is $\mathrm{Bin}\,(n, \theta + \beta)$ and (ii) the conditional distribution of X_2 given X_1 is $\mathrm{Bin}\left(n - X_1, \frac{2\beta}{(1-\theta-\beta)}\right)$ $(\theta, \beta > 0,\ \theta + 3\beta < 1)$. Find a suitable (multinomial) augmented data and obatin the basic updating equation of the EM algorithm applied to the observed data for obtaining the MLE of (θ, β). [30]

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination : (2003 – 2004)

B . STAT . II Year

PHYSICS - II

Date :- 18.2.04. Maximum Marks :- **30** Duration :- **3** *Hrs.*

Answer all questions . Each question carries 6 marks .

QUESTIONS

1) Give the general definition of a ***thermodynamic system*** . What is meant by its ***states*** and ***properties*** ?
Distinguish clearly between ***thermal equilibrium*** and ***thermodynamic equilibrium*** of a system. (3 + 3)

2) State and explain the **Zeroeth Law of Thermodynamics**.Using it, bring out the notion of ***empirical temperature*** of a ***thermodynamic system*** which is in the state of ***thermal equilibrium***. (2 + 4)

3) Deduce the **Virial equation** of ***Clausius*** for a gaseous system. Use it to obtain the most general form of the equation of state for a gas. (3 + 3)

4) **One gm.-mole** of a perfect gas , confined within an enclosure is in the state of ***equilibrium*** having a ***steady temperature*** of **T K** . Following ***Maxwell*** , obtain the expression for the number of gas molecules having their speeds lying within the range **c to c+dc** . Also , find out the speed possessed by maximum number of gas molecules . (5 + 1)

5) State and explain the **First Law of Thermodynamics**. What do we physically mean by ***internal energy*** of a thermodynamic system?
Apply the **First Law** to obtain an expression for the ***efficiency*** of a heat engine working in a ***reversible*** (***Carnot***'s) cycle. (2 + 1 + 3)

INDIAN STATISTICALINSTITUTE

Mid-semester Examination: 2003-2004

B.Stat. (Hons) II year

Biology II

Date : 18.02.2004 Maximum Marks : 30 Duration : 2 Hours

(Number of copies of the question paper required : 4)

1. Critically comments on the followings (any five) $5 \times 2 = 10$

a. Pedology
b. Puddling
c. Humic acid
d. Stoke's law
e. Micronutrients
f. Phosphetic fertilizer
g. Soil tilth

2. Answer the following questions (any five) $5 \times 2 = 10$

a. A soil having bulk density 1.4 $\mu g/m^3$ and particle density 2.65 $\mu g/m^3$. Find its porosity.
b. Establish the relationship between volumetric moisture content and gravimetric moisture content of soil.
c. Differentiate between manure and fertilizers.
d. Write short notes on soil texture and soil pH.
e. Biological nitrogen fixation.
f. Differentiate between consumptive use of water and conjunctive use of water.
g. Write the numerical formulae of water use efficiency and water distribution efficiency.

3. Answer any five of the following questions: $5 \times 2 = 10$

a. What is the definition of plant breeding? Write two major objectives of plant breeding.
b. What are the sexual groups for plant breeding? Define them.
c. What are the practical methods of plant breeding? Define them.
d. What are the two major similarities between plant and animal development?
e. What is the modular construction in plant? Describe with a figure.
f. What is Arabidopsis thaliana? Why it is called a model organism for developmental studies?
g. What is Agrobacterium tumifaciens? Give schematic description of how they transfer DNA into plant cells.

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination : (2003-2004)

B.Stat II Year

Economics II

18.2.2004 Maximum Marks: 40 Duration: 150 minutes.

Answer all questions. Maximum marks that can be scored is 40.

1. In a given period only two firms existed in the economy. Data regarding their activities and those of the government in the given period are given below:

All figures are in crores of rupees

Firm 1

Receipts from		Expenditure on	
(i) Sales to		(i) Purchases from	
Households	1000	Labour from Households	500
		Labour from foreigners	20
Government	100	Intermediate inputs from abroad	500
Foreigners	200	Machinery from abroad	1000
		Firm 2	50
Firm 2	50	(ii) Depreciation	50

(contd.)

Firm 2

Receipts from		Expenditure on	
(i) Sales to		(i) Purchases from	
Households	2000	Labour from Households	1500
Government	100	Intermediate inputs from abroad	500
Foreigners	200	Machinery from abroad	2000
		Firm 1	50
Firm 1	50	(ii) Depreciation	100
(ii) Subsidy	200	(iii) Indirect taxes	100
		(iv) Corporate Profit Tax	20
		(v) Business transfer	50

No inventory investment took place in either firm. Neither firm produced houses. Neither firm purchased capital goods from domestic sources.

Government Administration

Expenditure		Receipts	
Government Purchase from firms	200	Indirect taxes	100
Government wages	20	Corporate Profit Tax	20
Transfer	10	Personal taxes	10
Subsidy	200		

Find out

(contd.)

(a) NDP using all the three methods specifying the value of each component of final expenditure, each component of factor income and value added of every production unit

(b) NNP and NI

(c) Aggregate National Saving. Does the identity involving aggregate national saving , investment etc. hold here? (20)

2. State whether the following statements are true or false with reasons:

(i) In the identity $C + I + G + X - M \equiv \text{GDP}$, M does not include imported intermediate inputs.

(ii) In one case a firm was using up completely all the inputs it bought from other firms in production. In another case it was using up completely only half of the inputs it bought from other firms. In the latter case both GDP and aggregate national saving will be higher.

(iii) To calculate the profit of a firm the value of durable inputs it used must be subtracted.

(iv) An increase in interest payment by the government means an increase in aggregate national saving. (16)

3. The following information are given for an economy in a certain period. GNP is 1,200, disposable income is I,000, government budget deficit is 70, consumption is 850, capital account deficit is 20, depreciation is 20 and net foreign transfer from abroad is zero.(All figures are in millions of rupees).

(i) How large is private saving S ?

(ii) What is the size of investment (net), investment (gross) ?

(iii) How large is government spending ?

(10)

INDIAN STATISTICAL INSTITUTE
MID-TERM EXAMINATION
Second Semester : 2003 – 2004

B.STAT (Hons.) II
Subject : DEMOGRAPHY
Marks = 100

Date : 20.02.2004 **Time : 3 Hours**

Symbols have their usual meaning

(Attempt any five questions)

1. a) Define Demography. How does it differ from population studies ? What are the basic source books for demographic data ?

   b) Describe and discuss various systems of census.

   c) Derive a formula for evaluating errors in reporting ages between (10-99) years from census or survey data.

   6+6+8=20

2. a) Define vital statistics.

   b) Describe Dual Recording System of Registration of births and its merits and demerits.

   c) Derive the formula $P_t = P_0 e^{rt}$, where symbols have their usual meanings.

   d) Estimate the tripling time of a population with an annual growth rate 'r'

   2+8+6+4=20

3. a) Describe Logistic Curve of growth of population and its different phases.

   b) Describe the method of fitting the Logistic curve by Hoetelling method. What assumptions you have to make in fitting the Logistic curve by Hoetelling method.

   8+12=20

4. a) What do you mean by standardisation ?

   b) Briefly describe and derive the formula for direct standardisation to compare mortality of two population.

   c) Suggest a method in detail for comparison of mortality between two population when CDR is known for both population but ASDR is known only for one population, however, population by age is known for both population.

   4+8+8=20

P.T.O

5. a) Define Net Reproduction Rate and its drawbacks as a measure of growth.

b) Suggest a better method to measure replacement of population than NRR and describe the method. Why do you think about the superiority of this method ?

c) Do you think NRR=1 indicates population growth rate = 0 ? State reason behind your decision.

8+8+4=20

6. a) State assumptions in the construction of a life table.

b) A male child was born to a mother at age 20 and father at age 30. Find the probability that the child will be orphan at age 10. Given :

$l_{10}^{m} = 98544$ $l_{12}^{m} = 98509$ $l_{20}^{m} = 98184$ $l_{30}^{m} = 97551$ $l_{40}^{m} = 95570$ $l_{55}^{m} = 91760$

$l_{10}^{f} = 98921$ $l_{12}^{f} = 98806$ $l_{20}^{f} = 98246$ $l_{30}^{f} = 97824$ $l_{40}^{f} = 95720$ $l_{55}^{f} = 92012$

c) For a certain life table, $l_x = 20900 - 80x - x^2$

i) What is the ultimate age in the life table ?

ii) Find q_x

d) The force of mortality for a particular population follows Makenham's Law and $\mu_x = A + BC^x$.

Prove that : $l_x = ks^x g^{c^x}$, where k, s and g are constants.

4+4+8+4 =20

7. Write short notes (any five)

a) Defacto Method

b) Neonatal mortality rate

c) Comparative mortality index

d) Sprague's Multiplier

e) Census Measures of Fertility

f) UN age sex adjusted birth rate

4 x 5=20

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2003-2004
B. Stat. (Honas.) II Year
Elements of Algebraic Structures

Date: 23.02.04 Maximum Marks: 100 Duration: 3 Hours

All questions carry equal scores. Total score is 100.

I.(a) Let G be a group and a be an element of G such that $0(a) = m$. If $a^n = e$, show that m divides n.

(b) Let a,b be elements of a group G such that $ab = ba$. If $o(a)$ and $o(b)$ are relatively prime integers, show that $o(ab) = o(a).o(b)$.

II. Let A,B be subgroups of a group G. Show that

(a) the sets of the form

$$AxB = \{axb \mid a \in A, b \in B\}, x \in G$$

induce a partition of G.

(b) If A,B are two finite subgroups of a group G then show that AxB contains $\frac{|A|.|B|}{|A \cap (xBx^{-1})|}$ elements.

III. Prove or disprove

(a) A finite semigroup with both cancellation laws is a group.

(b) The set of nonzero elements of $\mathbb{Z}_n$ is a group under multiplication for any positive integer $n > 1$.

(c) For a subgroup H of a group G, let N be the intersection of all sets of the form

$$g^{-1}Hg \quad ; \quad g \in G$$

then N is a normal subgroup of G.

(d) If R,S are symmetric relations on a set A, then $S \circ R$ is a symmetric relation on A.

IV.(a) Let G be a group of order 36 with a subgroup H of order 9. Show that H is normal in G.

(b) Let H be a subgroup of a group G and K be a normal subgroup of G. Show that HK is a subgroup of G.

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INDIAN STATISTICAL INSTITUTE

B. Stat II: 2003 – 2004

Mid-semester Examination

Economic Statistics and Official Statistics

uration: 3 hours Maximum marks: 100 Date: 25 February 2004

[Answer Part I and Part II in separate answer scripts. Answer question no. 6 from Part II and any four from Part I.]

Part I: Economic Statistics

1. (a) State Pareto law. Give your comments on the universality of Pareto law stating the evidences for and against this law. How can you graphically test whether a given set of data is coming from a Pareto distribution? State and prove some properties of Pareto distribution. [20]
2. Suppose $y_1, y_2, \ldots, y_n$ are incomes of n persons in a community. Describe how you will find Lorenz ratio (LR) (i) graphically and (ii) numerically. Prove that LR found by numerical method is equivalent to the LR found by the formula using Gini's Mean Difference (GMD). [20]
3. (a) Suppose monthly per capita expenditures (x) of households in urban India follow Pareto law with inequality parameter $\nu = 2.0$ and the threshold parameter c = Rs. 10/-. Find
   (i) the average monthly per capita expenditure of all households,
   (ii) the average monthly per capita expenditure of households spending less than Rs 25/- per capita per month.

   (b) If households spending Rs. x per capita per month spend $x^{0.8}$ rupees per capita per month on a certain commodity then find the average monthly per capita expenditures of bottom 25% households on the commodity. [3+7+10=20]
4. Write down the important steps in deriving Atkinson's measures of inequality based on the Social Welfare Function Approach. How can one interpret the unknown parameter in the measure? [17+3]
5. Write short notes on any two of the following:
   (i) Properties of Log-normal distribution.
   (ii) Properties of Lorenz curve.
   (iii) Positive measures of inequality

Part II: Official Statistics

6. (i) What are the major functions of CSO?
   (ii) What is national income? Which office/division is responsible for estimating national income of India?
   (iii) Write the name of Statistical Offices under MOS&PI of Union Govt.?
   (iv) Write the broad steps of estimating the contribution of manufacturing sector in national income.
   (v) "Sample Survey is necessary to understand the economic parameters of the country." indicate three important points in support of the statement.
   (vi) What is the difference between house and household?
   (vii) What is SNA? [3+3+3+4+3+2+2=20]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: 2003-04
B. STAT II YEAR
Demography and SQC & OR

Time: 23.4.2004 Maximum Marks: 100 Duration: 3 Hours

GROUP A

Subject : DEMOGRAPHY

MAXIMUM MARKS : 50

Answer Question no. 4 and any two from the rest
(Symbols and notations have their usual meanings)

1. a) What do you understand by a 'Census'? Describe various systems of census mentioning their merits and demerits. What system of census was followed in the Indian Census of 2001?

   b) Describe Dual Recording System for the registration of births for any country. Critically evaluate the methodology of the dual recording system.

(2+6+2)+(6+4) = (20)

2. a) Describe various methods for obtaining inter-censal and post-censal estimates of population total.

   b) Starting from suitable assumptions regarding the relative growth rate of a population, derive the logistic equation and interpret its parameters.

(10+10) = (20)

3. a) What are the assumptions involved in the construction of a life table?

   b) If between age x and (x + t), ${}_tp_x$ is the probability of surviving and μ_{x+t} is the force of mortality, then prove that $\int_0^{\omega} {}_tp_x\, \mu_{x+t}\, dt = 1$. where ω has its usual meaning

   c) Given $\mu_x = \dfrac{1}{(a_0 + a_1x)(b_0 + b_1x)}$

   Find an expression for l_x.

[P.T.O.]

d) In a certain life table,

$\mu_x = 0.15 - 0.10x$ for $0 \le x \le \frac{1}{2},$ and $\mu_x = (0.01)^x$ for $\frac{1}{2} \le x \le 1$

Find l_1 assuming that $l_0 = 100{,}000$.

(5+5+5+5) = (20)

4. Answer any two

a) Define 'infant mortality rate'. How is it different from 'infant death rate'.

b) Establish mathematically the relationship between TFR and CBR.

c) Estimate ${}_nq_x$ and ${}_nL_x$ by Chiang's Method for the construction of a life table.

d) Describe Zelnik's Method.

(5+5)=(10)

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Group B

Subject : SQC & OR

(1) This paper carries 60 marks. You may answer as much as you can, but the maximum you can score is 50.
(2) Begin this group on a new answerscript.

1. State whether the following statements are True (T) or False (F), giving reasons for you answer.

(a) The principal function of a control chart is as a tool for process adjustment.

(b) The chance of getting a false out-of-control signal above the 3σ limit is the same for both an $\overline{X}$ and an R-chart.

(c) Sampling plans with the same per cent samples give the same quality protection.

(d) Quality characteristic falling within specification limits implies that the process is in statistical control.

(e) Fraction defective chart requires only the recording of number of defectives and therefore are more economic than $\overline{X}$ -charts.

(f) In an LP problem if the solution space is unbounded, the objective value always will be unbounded.

(g) In real-life problems, the variables of an LP model are always necessarily non-negative.

[14]

2.(a) A process is being controlled with a fraction non-conforming control chart. The process average has been found to be 0.2. Three sigma control limits are used and the procedure calls for taking 50 items daily.

(i) If the process is in control with $p = 0.2$, how often will a false-alarm (out of control signal) will be generated?

(ii) If the process shifts to $p = 0.3$, after how many samples on an average, will this shift be detected with a point outside any of the control limits?

(b) $\overline{X} - R$ charts, based on a subgroup size of 5 rings, are used to monitor rings produced by a forging process. The parameters of the $\overline{X} - R$ charts are the following :-

Parameter	$\overline{X}$ *- chart*	*R - chart*
UCL	74.014	0.049
Central Line	74.001	0.023
LCL	73.988	0.000

These charts are used at the shop floor. Since the process exhibited good control, the concerned supervisior wants to reduce the subgroup size to 3 rings. Find the control limits for the $\overline{X} - R$ charts.

[5+5+5=15]

[P.T.O.]

3.(a) Consider a double sampling acceptance rejection plan with the following parameters :-

$$n_1 = 50 \quad c_1 = 1 \qquad n_2 = 100 \qquad c_2 = 3$$

Find the probability of accepting a lot that has fraction defective $p = 0.05$, in the second sample.

(b) Suppose that a single sampling acceptance rectification plan $n = 150,\ c = 1$ is being used for receiving inspection where the vendor ships the product in lots of size $N = 3000$. Find the AOQL for this plan.

[6+10=16]

4.(a) A bank is in the process of formulating its loan policy for the next quarter. A total of Rs.12 crores is allocated for that purpose. The following table provides the types of loans, the interest rate charged by the bank, and the possibility of bad debt as estimated from past experience:

Type of Loan	Interest Rate	Probability of Bad Debt
Personal	0.140	0.10
Bar	0.130	0.07
Housing	0.120	0.03
Farm	0.125	0.05
Commercial	0.100	0.02

Bad debts are assumed unrecoverable and hence produce no interest revenue.
Competition with other financial institutions in the area requires that the bank allocate at least 40% of the total funds to farm and commercial loans; home loans must equal at least 50% of the personal, car and home loans. The bank also has a stated policy specifying that the overall ratio for bad debts on all loans must not exceed 0.04.

Formulate the bank's objective of maximizing its net return as can LPP.

(b) Maximize

$$x_0 = 2x_1 + x_2$$

subject to

$$x_1 + x_2 \le 5$$
$$-x_1 + x_2 \le 0$$
$$6x_1 + 2x_2 \le 21$$
$$x_1, x_2 \ge 0$$

[9+6=15]

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INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: 2003-04
B. Stat II Year
Economics II

Date: 26.04.04 | Maximum Marks: 60 | Duration: 3 Hours

Answer any three questions.
Marks allotted to each question is given at the bottom right hand Corner of the question.

1.(a) Consider a simple Keyensian Model and derive the condition for the equilibrium to be stable. Express the stability condition in terms of characteristics of the saving and investment schedules. Show the stable and unstable equilibrium situations in diagrams using saving and investment schedules as well as Keyensian cross. (You may consider a closed economy)

(b) Following set of information is given about a closed economy. Marginal propensity to consume with respect to disposable income is 0.8 and the marginal propensity to invest with respect to NDP(y) is 0.3. Tax function is given by

$$T = t.y \quad ; \quad t = 0 \quad \forall\ 0 < y < 1000$$
$$t = 0.5\ \forall\ y \geq 1000$$

Autonomous expenditure (which includes autonomous net investment) is 100.

(i) Specify and draw the aggregate demand function (expenditure function) against y.

(ii) Find out the minimum amount of autonomous expenditure that is needed to ensure existence of stable equilibrium in the model.

(20)

2. Consider an $IS-LM$ model where marginal propensity to spend with respect to output/income (y) lies between 0 and 1.

(a) How do the equilibrium y and the rate of interest (r) change, following an exogenous shift in the demand for commodity? Trace the path of the variables (y, r) from one equilibrium to another, assuming that the money market adjusts instantenously to correct any disequilibrium while commodity market takes longer to do so. Also rigorously derive the multiplier $\mu = \frac{dy}{dG}$ and explain it. (Use the diagrams you have drawn, to corroborate your explanation).

(b) Suppose, following an exogenous income in money supply by 2 units, the LM schedule shifts vertically by $(-.2)$ units and horizontally by 10 units. Also assume $p = 1$. (i) Derive the interest sensitivity and income sensitivity of demand for real balance, (ii) Derive the equation of the LM schedule under the assumption that $\overline{M} = 100$ and money demand is a linear function of

[P.T.O.]

y and r, (iii) if other things remaining same, the vertical shift of the LM schedule were $(-.1)$ instead of $(-.2)$, how would it have affected the effectiveness of fiscal policy in the model? Explain your answer in terms of crowding out effect.

(20)

3. Define broad money and narrow money. Discuss the process of money creation in a system where there is excess demand for credit and where there exists fractional reserve banking.
Hence derive a relationship between narrow money and high powered money.
Show that in a small open economy with capital mobility, money supply is endogenous under fixed exchange rate regime.

(20)

4. Discuss the effectiveness of monetary and fiscal policy in a small open economy with capital mobility under flexible exchange rate regime, using the Mandell-Fleming Model.

Hence argue, why the domestic rate of interest in this system, which cannot deviate from the world interest rate under perfect capital mobility, can well do so, if the capital mobility is imperfect.

(20)

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INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: (2003 – 2004)
B. Stat II Year
Biology II

Date 26.4.04 Maximum Marks 70 Duration Three hours .
(Number of copies of the question paper required Three)

1. Define Moisture Availability Index? Draw a suitable rice calendar with the following data 5+10

Week No.	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*
Rainfall (mm) at 0.5 Prob.	*0*	*8*	*5*	*0*	*12*	*27*	*38*	*25*	*48*	*60*	*68*	*72*	*90*	*80*	*25*	*12*	*5*	*0*	*0*
PET (mm)	*35*	*32*	*33*	*27*	*23*	*22*	*20*	*24*	*20*	*19*	*18*	*18*	*16*	*20*	*22*	*27*	*29*	*30*	*34*

2. Briefly explain the criteria for determining the essentiality of nutrients. Differentiate between micro and macro-nutrients. Calculate the quantity of DAP, Urea and Muriate of potash required for 600 sq. m. of rice crop to meet the nutrient requirement of 100kg N, 50 kg P_2O_5 and 50 kg K_2O per hectare. 2+3+10

3. Classify rice and rice culture depending on eco-geographical situation. Write down the suitable agrotechniques for optimization of rice production. Critically highlight the variation in yield of winter (Aman) rice and summer (Boro) rice. 2+10+3

4. Write short notes on any five of the following: 3 x 5
   a) Acidic fertilizers
   b) Soil humus
   c) Vegetative lag phase in rice
   d) Indices of assessing biological advantages/disadvantages of intercropping
   e) Biological nitrogen fixation
   f) Hybrid rice
   g) Intercropping and Mixed cropping
   h) Plant protection chemicals

5. Write in brief about any two of the following: 2X5
   a) The procedure for making a transgenic plant.
   b) Three broad classes of signaling events that take place during mammalian development.
   c) A morphogen gradient.
   d) The major practical methods of plant breeding.

OR

6. Write short notes on any four of the following: 2.5X4
   a) Growth rate follows a law of compound interest.
   b) Temperature effect of plant growth.
   c) Florigen synthesis initiates by P_{730}.
   d) Auxin usually presents in the colioptile part of a plant.
   e) ABA may be considered as anti-auxin.

INDIAN STATISTICAL INSTITUTE

Second semestral Examination : (2003 – 2004)

B . STAT . II Year

PHYSICS - II

Date :- 26.4.04 Maximum Marks :- **70** Duration :- **3** *Hrs.*

Answer all questions **.**

QUESTIONS

A *perfect gas* enclosed within a volume *V* and *completely isolated* has n molecules per unit volume . If the molecules are assumed to *move at random in all possible directions with equal probability having uniform speed* ***u*** , **obtain expressions for the mean molecular collision rate** and the **molecular mean free path** . **(12 + 4)**

Obtain an expression for the entropy **(*S*)** of a perfect gas in terms of its volume (*V*) and absolute temperature (*T*) .
In the framework of ***Kinetic Theory of Gases***, using ideas of ***molecular transport*** , **obtain an expression for the thermal conductivity** of a perfect gas . **(8 + 10)**

In connection with the ***black body radiation*** , **state and deduce** -
(a) **Kirchoff's Law of radiation** (b) **Wien's Displacement Law** .
(8 + 10)

Describe the phenomenon of the ***Photoelectric Effect*** . *Following* **Einstein** , **deduce** the **photoelectric equation** .
Describe with theory, **Millikan's Oil Drop experiment** to determine the ***electronic charge*** . **(8 + 10)**

INDIAN STATISTICAL INSTITUTE

B. Stat II: 2003 – 2004

Second Semester Examination

Economic Statistics and Official Statistics

Duration: 3 hours | Maximum marks: 100 | Date: 8. 5. 2004

[Answer Group A and Group B in separate answer scripts. Answer question no. 1 and any two from the rest of the questions of Group A and all questions of Group B. Allotted marks are given in brackets[] at the end of each question.]

Group A: Economic Statistics

1. Suppose the urban population is just one-third of the rural population in a country. The average monthly income of the bottom 10% population of rural and urban sectors are Rs. 35 and Rs. 45, respectively. The respective Lorenz Ratios are 0.28 and 0.40. Assume that income follows lognormal distribution separately for rural and urban sectors. Calculate the percentage of people with income more than Rs.100/- for the country as a whole. [40]
2. Describe in detail the statistical criteria for choosing an Engel Curve. [20]
3. Derive Sen's measure of Poverty. Show why this measure is superior to Head Count Ratio and Income Gap Ratio. Show that Sen's measure of Poverty does not satisfy strong transfer axiom. [20]
4. Suppose $y_1, y_2, \ldots, y_n$ are incomes of n persons in a community. Describe how you will find Lorenz ratio (LR) (i) graphically and (ii) numerically. Prove that LR found by numerical method is equivalent to the LR found by the formula using Gini's Mean Difference (GMD). [20]
5. State and prove the properties of a Cobb Douglas production function with two inputs. [20]
6. Write short notes on any two of the following:

   (i) Measures of Concentration in Business and Industry
   (ii) Fixed base indices vs. Chain base indices.
   (iii)Estimation of two-parameter lognormal distribution.
   (iv)Linear Expenditure System. [10+10=20]

Group B: Official Statistics

1. Write the 9-fold classification of land-use required in Agricultural Statistics. [5]
2. Write the names of price indices that are available in India as part of price statistics. Which of these indices is used to measure inflation? [5]
3. Write the names of major sectors contributing to GDP in descending order of their contribution. [3]
4. Which one of the following indicators is more acceptable to judge the overall development of a country – GDP, Green GDP or HDI? Give brief reasons. [5]

   or

   Write an outline of the sampling procedure for selecting households of a socio-economic survey. [5]
5. What are the sampling frames for selection of urban block and rural village in National Sample Survey? [2]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: 2003-2004
B.Stat. (Hons.). 2nd Year
Statistical Methods IV

).4.2004 **Maximum Marks: 100** **Duration: 3 hours**

ıe question paper carries 120 points. You must clearly state any result ated and proved in the class you might need in order to answer a rticular question. Keep the answers brief, to the point ıd well structured.

Define the family of multivariate normal distributions.

Show that for a standard normal random variable Z the joint moment generating ın of (Z, Z^2) is given by

$$M_{Z,Z^2}(t,s) = E(e^{tZ+sZ^2}) = \frac{1}{\sqrt{1-2s}}\, e^{\frac{t^2}{2(1-2s)}},$$

y real number t and $s < 1/2$. Hence show that for $Z_1, Z_2, \ldots, Z_n$ iid standard normal $a_2Z_2 + \ldots + a_nZ_n$ and $a'_1Z_1^2 + a'_2Z_2^2 + \ldots + a'_nZ_n^2$ are independent if and only if $a_i\, a'_i = 0$ i, where a_i's and a'_i's are real numbers. [5+(15+5)=25]

Let B be a $r \times n$ matrix. Prove that $\text{Col}(B) = \text{Col}(BB^t) = S_1 \oplus S_2 \oplus \cdots \oplus S_q$, where $\ldots, S_q$ are eigenspaces corresponding to distinct non-zero eigenvalues of BB^t. Hence that there is an orthonormal basis of $\text{Col}(B)$ consisting of eigenvectors of BB^t.

Let for a square matrix T, $\lambda(T) = \{\mu : Tx = \mu x \text{ for some } x \neq 0\}$ (spectrum of T). and B be $m \times n$ and $n \times m$ matrices respectively. Show that $\lambda(AB) - \{0\} = \lambda(BA) - \{0\}$.

[15+5 =20]

. Write a short note on Bayesian networks illustrating its usefulness in (i) describing te multivariate joint distributions and, (ii) learning causality among a set of variables, illustrative examples.

P.T.O ...

3 (b). There are three exponential populations with means 1, λ and 2λ respectively. In a two stage experiment the first stage consists of choosing the first population with probability w, the second with probability 0.5 and the third with probability $(0.5 - w)$ $(0 < w < 0.5)$ respectively. In the second stage a random sample is drawn from the population obtained in the first stage. On the basis of independent samples $X_1, X_2, \ldots, X_n$ from the above sampling experiment describe a computational algorithm for obtaining the MLE of (w, λ).

[10+15=25]

4 (a). Let $A_1, A_2, \ldots, A_q$ be $n \times n$ nonnegative definite matrices satisfying (i) $\sum_{i=1}^{n} A_i = B$ where $B^2 = B$ and (ii) $\sum_{i=1}^{n} \text{rank}(A_i) = \text{rank}(B)$. Show that (i) $A_i^2 = A_i$ for all i and (ii) $A_i A_j = 0$ for all $i \neq j$.

4 (b). Suppose that $Z_1, Z_2, \ldots, Z_{2n}$ are iid standard normal random variables. Define $U_e = \sum_{i=1}^{n} Z_{2i}$ and $U_o = \sum_{i=1}^{n} Z_{2i-1}$ respectively. Using 4(a) or otherwise show that the following random variables $(U_e + U_o)^2$, $(U_e - U_o)^2$ and $\sum_{i=1}^{n} \left(Z_{2i} - \frac{U_e}{n}\right)^2 + \sum_{i=1}^{n} \left(Z_{2i-1} - \frac{U_o}{n}\right)^2$ are independent.

[15+10=25]

5. For a mean zero multivariate normal vector X in $I\!R^n$ a Karhunen-Loéve expansion of X is defined by the following expression

$$X = \sum_{i=1}^{n} \sqrt{\lambda_i}\, z_i\, \phi_i,$$

where $\{z_1, z_2, \ldots, z_n\}$ are iid $N(0,1)$, $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ are nonnegative real numbers and $\{\phi_1, \phi_2, \ldots, \phi_n\}$ is an orthonormal basis of $I\!R^n$. Let X be multivariate normal with mean zero and covariance matrix Σ (positive definite). For a $r \times n$ matrix A of full rank, define $Y = AX$. Assuming the above Karhunen-Loéve expansion of X, obtain a Karhunen-Loéve expansion of Y solely in terms of λ_i's, z_i's and ϕ_i's. (*Hint.* Qn. 2.) [25]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: 2003-04
B. Stat. II Year
Elements of Algebraic Structures

Date: 5.5.04 Maximum Marks: 100 Duration: 3 Hours

You may answer all question-maximum score is 100.

I.(a) An element x in a finite group G has exactly two conjugates in G. Show that G has a normal subgroup different from (e) and G. [8]

(b) R is a ring and L a left ideal of R. Let $\lambda(L) = \{x \in R \mid xa = 0 \text{ for all } a \in L\}$. Show that $\lambda(L)$ is an ideal of R. [7]

II.(a) D is an integral domain and $a, b \in D$. Let m, n be positive integers which are relatively prime such that $a^m = b^m; a^n = b^n$.
Show that $a = b$. [10]

(b) Find all autopmorphisms of the ring of integers. [10]

III. Prove or disprove

(a) A subgroup of the multiplicative group of a field is cyclic. [10]

(b) If a group of order 28 has a normal subgroup of order 4, then it is abelian. [10]

(c) A group of order $p^2, (p \text{ prime})$ is abelian. [10]

IV.(a) R is a commutative ring and P is an ideal in R. P is called a prime ideal if $ab \in P$ implies $a \in P$ or $b \in P$ for any a, b in R.
Show that P is a prime ideal iff R/P is an integral domain. [10]

(b) Show that in a commutative ring with multiplicative identity every maximal ideal is a prime ideal. [10]

V.(a) Find the minimal polynomial of $\sqrt{2} - \sqrt{3}$ over Q. [10]

(b) Find all automorphisms of $Q(\sqrt[3]{2})$ fixing elements of Q. [10]

VI.(a) Test whether $x^4 + x + 1$ is irreducible over GF(2). [5]

(b) Write out the nonzero elements of a field of size 16 as powers of a single element. [10]

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INDIAN STATISTICAL INSTITUTE
Back-paper Examination
B.Stat. (Hons.) II Year, I Semester

Statistical Methods III

Date: 12.3.03 Maximum time: 3 hours Maximum marks: 10

This test is closed book, closed notes. Tables of statistical distributions may be used.

1. Let $X_1, X_2, \ldots, X_n$ be samples from the Poisson distribution with mean λ, and th parameter of interest is $p = e^{-\lambda}$.
   (a) Obtain an unbiased estimator of p based on the count of X_is which are positiv
   (b) Obtain the maximum lilkelihood estimator (MLE) of p based on the observe data. Is it unbiased?
   (c) Compare the mean squared errors of the two estimators. [3+(4+2)+6=1

2. Consider the mixture of exponential distributions

$$f(x) = p\lambda_1 e^{-\lambda_1 x} + (1-p)\lambda_2 e^{-\lambda_2 x}, \quad \lambda_1 > \lambda_2 > 0,\ 0 < p < 1.$$

   Describe a procedure to obtain the maximum likelihood estimates of the three pa rameters on the basis of n samples from the above distribution. Show clearly all th necessary steps. [1

3. Ordinary corn does not have as much of the amino acid lysine as animals need in thei feed. Plant scientists have developed varieties of corn that have increased amounts c lysine. In a test of the quality of the high-lysine corn as animal feed, an experiment group of 20 one-day-old chicks received a ration containing the new corn. A contr group of another 20 chicks received a ration that was identical except that it containe normal corn. Here are the weight gains (in grams) after 21 days.

Experimental				Control			
361	447	401	375	380	321	366	356
434	403	393	426	283	349	402	462
406	318	467	407	356	410	329	399
427	420	477	392	350	384	316	272
430	339	410	326	345	455	360	431

   (a) Is there good evidence that chicks fed high-lysine corn gain weight faster? Carr out a test and report your conclusions.
   (b) Give a 95% confidence interval for the mean extra weight gain in chicks fe high-lysine corn. [15+5=2

P. T. O

4. The measurements recorded by a digital communication receiver follows the 'signal plus noise' model

$$y_i = A\cos(2\pi f i + \phi) + \epsilon_i, \quad i = 1, 2, \ldots, n,$$

where f is a known frequency, A and ϕ are unspecified amplitude and phase and $\epsilon_1, \epsilon_2, \ldots, \epsilon_n$ are independent and normally distributed noise with zero mean and an unspecified variance. It has to be tested whether the signal is present. Formulate this problem as a test of hypothesis and derive a suitable test for it. [15]

5. Given samples $X_1, X_2, \ldots, X_n$ from an unknown distribution F, one has to check whether F is equal to the completely specified distribution F_0. Derive the chi-squared goodness-of-fit test and explain why the statistic should have the chi-square distribution (approximately) under the null hypothesis. How should the partition be chosen? [5+7+3=15]

6. For the above problem, describe the P-P plot and derive the pointwise 95% confidence limits for this plot. [2+8=10]

7. Define the autocorrelation function and spectral density of a stationary time series. Show that when the autocorrelation function is replaced by the usual (biased) estimator, the resulting estimator of the spectral density is nonnegative. What is this estimator called? [2+2+5+1=10]