INDIAN STATISTICAL INSTITUTE

First Semestral Examination : (1999-2000)
B.Stat. II & III years
Biology – I

Date: 10-11-99 Maximum marks: 60 Duration- 3hr

Answer any six questions

1. Mention the differences between DNA and RNA. How many DNA sequences are possible with 20 nucleotides but having only four different types of nucleotides? Mention the numbers of (a) sequences starting with only ATG and (b) starting with ATG but ending with TAA. "Of these sequences only few are biologically active" — comment on this statement.

2. How did Mendel arrive at the conclusion from his experiments that one of the alleles is dominant over the other. In the blood groups (A, B, O and AB), how many alleles are present and which is/are dominant, recessive and co-dominant

3. "Oxidation of fatty acid and glucose meet to a point for energy production"- describe this phenomenon mentioning numbers of ATP production after oxidation processes.

4. How protein is digested in our system. "Amino acid composition of a protein determines its quality"- explain. Draw carbon and nitrogen cycles and mention their importance for human living.

5. The biomolecules of living organisms are ordered into a hierarchy of increasing molecular complexity. Depict this with appropriate examples.

[P.T.O]

6. Although water molecule is electrically neutral, it is an electric dipole. What effect does this have on the following properties of water?
   a) Interaction between water molecules
   b) Polarity of the medium
   c) Ability to dissolve molecules with polar functional groups containing O or N , e.g. amine, alcohol, carboxylic acid
   d) Ability to dissolve non- polar molecules
   e) Ability to dissolve molecules with charged functional groups, e.g. ionic compounds.

7. Earth when formed some 4.6 billion years ago was a lifeless, inhospitable
place. A billion years later it was teeming with organisms resembling blue-green algae. How did they get there? How did life begin? Give a concise logical overview.

8. What are the major differences between plant and animal cells? Plant cells are able to withstand wider fluctuations of osmotic pressure of the surrounding medium than animal cells. Explain.

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INDIAN STATISTICAL INSTITUTE

Second Semestral Examination : (1999-2000)
Course Name : B.Stat. (Hons) II & III
Subject Name : Biology II
Date :20.4.2000 Maximum Marks : 60 Duration : 3 hrs.

1. Classify rice crop with names of two cultivars in each group. Write in brief the different cultural practices for transplanted rainfed rice. 10

OR

2. Write down the mathematical model on two species intercropping system and show that the inhibitory effect on the row-intercropping system plays an important role in the dynamics of the system. 10

3. What are the essential plant nutrients? If the recommended dose of nutrient for rice are 80 kg N + 50 kg P_2O_5 + 50 kg K_2O, find the requirement of FYM, Urea, Single super phosphate and KCl. 50% of the recommended Nitrogen should be given through FYM. 3+7

4. Write short notes on (any four)
a. Monsoon onset
b. Moisture availability index
c. Micro nutrients
d. Water holding capacity of soil
e. Soil texture
f. Yield attributing characters of rice
g. Reproductive phase of rice 10

4. How does a plant chemically protect itself from pathogens? Explain why Onion variety with red scale leaf is resistant to Smudge disease? How dose silica layer on epidermis in resistant rice variety help to protect the blast disease of rice? 2+4+2

6. What are enzymes? Write the salient features of an enzyme. What is meant by a competitive inhibitor? Write a brief account of the factors affecting the enzyme activity? 2+2+2+4

OR

7. What are the requisite natures of a chemotheraputant? What are the modes of action of a chematheraputant? What are the systemic fungicides and what are the types of this? 2+3+5

8. Write short notes on:
a. Totipotency
b. Photoperiodism
c. Typtophan path way of IAA synthesis
d. Michalis – Menten Constant
e. Prosthetic group
f. Quinone 6x2

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II and III Year (1999-2000) Semestral II Examination

Date of Exam. : 05 MAY 2000

ANTHROPOLOGY

Maximum Marks : 100 Duration : 3 hrs.

GROUP – A

Answer any five questions.

1. How do you define Anthropology ? What are the distinguishing features of Anthropology ? (2+8 = 10)

2. Why is man unique in the animal kindgom ? (10)

3. Describe the salient features of Darwin's theory of Evolution. Bring out the weakness of ~~this thing~~ this theory. (10)

4. Define Adaptation. What is the difference between Adaptation and Acclimatization. (2+8 = 10)

5. Write short notes on any two of the following :

   a) Marriage and Mating, b) Stable and Stationery Population, c) Race, d) Adaptive significance of skin colour, e) Homo erectus. (2 x 5 = 10)

6. Compare and Contrast the morphological features of Man and Chimpanzee. (10)

P. T. O

GROUP – B

Answer any five questions.

1. What is chromosome ? Describe the structure and function of chromosome. (2 + 8 = 10)

2. What is Mutation and how does it affect the Hardy-weinberg equilibrium ? (2 + 8 = 10)

3. Describe the various forms of chromosomal anomalies. (10)

4. Distinguish between the following (any two).

   a) Phenotype – Genotype
   b) Homozygote – Heterozygote
   c) DNA – RNA (2 x 5 = 10)

5. What are the laws of Mandal. Describe with suitable examples. (4 + 6 = 10)

6. Distinguish between Admixture and Inbreeding (10)

7. Write short notes of any two :

   a) Nucleus, b) Gene and ~~Alble~~ Allele, c) Twin, d) X- and Y- Chromosome, e) Polygenic Inheritance. (2 x 5 = 10)

INDIAN STATISTICAL INSTITUTE

SECOND SEMESTRAL EXAMINATION (1999-2000)
B.STAT (Hons.) - II & III yr.
ECONOMICS – II

DATE OF EXAMINATION : 20 APRIL 2000

Maximum Marks : 100 GROUP – A Duration : 3.5 hrs.

Answer any five questions, taking at least two from each group

Answer Goups A & B in separate answerscripts.

1. a) Consider a simple keyesian model (S.K.M.) for a closed economy (you may ingnore govt activities. Explain what is menat by unplanned inventory accumulation and derive the relationship among aggregate planned saving, aggregate planned investment and unplanned inventory accmmulation for any arbitrary level of Y.

   b) Consider a S.K.M. for a closed economy without govt. Suppose consumption (C) is a linear function of income (Y) and C=80 when Y=O, investment function is given by I=200 + .1Y; when Y=4800 the unintended inventory accumulation is 140. Derive the saving fucntion.

   [12 + 8 = 20]

2. a) In a S.K.M. for a closed economy with Govt., aggregate planned demand
   $E = C+I+G$

   $C + A+C.Y^d$, $a>0$, $0<c<1$

   $Y^d = Y-T$

   $T = T, I = \bar{I}, G = \bar{G}$

   Derive the relationship between private saving and public saving that exists in equilibrium. Now suppose $\bar{T}$ increases. Find out its effect on total saving (private + public), public saving and private saving in the new equilibrium.

   b) How do the results change if the investment function is replaced by

   $I = \bar{I} + i.Y$, $0<i<1$

   [14 + 6 = 20]

3. How would you extend the model of 2a) when planned investment is a function of rate of interest. Derive the Govt. expenditure multiplies for the extended model. Compare it with the corresponding multiplier of the model in (2a) above. Explain your answer.

   [20]

4. Consider an IS-LM model for a small open economy with perfect capital mobility under a perfectly flexible exchange rate regime. Show that the fiscal policy is totally ineffective in this model.

   [20]

P. T. O

GROUP – B

5. a) How will you present the accounting framework of an economy with n production sectors, a household sector, the government sector and the rest of the world Sector. [10+10]

b) Using the information given beow estimate gross domestic product at Market Prices (Rs. in crore)

		(Rs. in crore)
1.	Private Consumption Exp.	50,000/-
2.	Govt. Consumption Exp.	15,000/-
3.	Gross Fixed Capital Formation	10,000/-
4.	Increase in stocks	2,000/-
5.	Exports of Goods & Services	5,000/-
6.	Imports of Goods & Services	7,000/-
7.	Capital consumption allowances	6,500/-
8.	Net Indirect Taxes	5,000/-
9.	Net Foreign Income	5,000/

6. Discuss the dichotomisation of the pricing process in classical Macroeconomics. [20]

7. Consider the extended classical Macroeconomic system and work out the changes in the equilibrium values of Y, N, $\frac{W}{P}$, r, and P, resulting from the parameter shifts as follows. [20]

a) Saving schedule shifts to the right.

b) Money supply increases © supply of labour schedule shifts to the right. Take the standard meaning of the relations.

8. Write short notes on any two of the following. [10+10]

a) Wicksell's reformulation of the classical Macroeconomic System

b) Say's law of Markets (3) Effect of wage rigidity on the classical macrosystem.

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) III YEAR: 1999-2000
SEMESTRAL-I EXAMINATION
ECONOMICS-III

Date:10.11.99 Maximum Marks:100 Time: $3\frac{1}{2}$ Hours

Note: Answer any five questions. Each question carries equal marks.

1. How did the Nehru-Mahalanobis Programme of development generate inefficiency in the Indian economy? [20]

2. Explain, how the initial trade policy in India created a bias against exports? How does the new trade policy seek to remove this bias? [20]

3. Explain the rationale of the Narsimham committee's recommendation of doing away with the directed credit programme and administered interest policy. [20]

4. Does financial liberalisation lead to improvement in the efficiency of credit allocation when financial sector is not fully integrated as is the case in India? Explain. [20]

5. To what extent IMF-summers line of reasoning explain the currency crisis of the South East Asian countries? Discuss. [20]

6. How does Krugman's hypothesis of financial fragility explain the currency crisis of the South East Asian countries? Do you agree with this view? [20]

7. Explain the rationale of capital account convertibility. Can you recommend it for India in the light of the current recessionary experience of Japan? [20]

INDIAN STATISTICAL INSTITUTE
B.STAT.-III YEAR, 1999-2000
SEMESTRAL EXAMINATION - I
SAMPLE SURVEYS

Date : 12.11.99 Full Marks : 100 Duration : 3 Hours

1. a) Derive the expressions for π_i and π_{ij} for a probability proportional to size With Replacement design of n draws from N population units with a given size measure where π_i's and π_{ij}'s denote respectively the first and second order inclusion probabilities. Also verify whether $\pi_{ij} > \pi_i \pi_j$.

b) Mention six sources of non-sampling errors in a survey on expenditure of puja pandals in Calcutta city at the field operations stage.

c) A survey was conducted in a school consisting of 625 students by covering a sample of 50 students, selected using srswor scheme, to estimate the average study hours per week outside the school. The estimate was found to be 4.20 with a standard error of 0.47. Using this information, determine the sample size needed to estimate the same characteristic in a neighbouring school on the basis of a sample to be selected by srswor scheme such that the length of the confidence interval at 95% confidence level is 10% of the true value. State clearly the assumptions involved.

[4 + 3 + 10=17]

2. In each of the following practical situations, explain how you would deal with the problems(Mathematical justification should be provided) and make inference :

a) Allocation of sample size when Neyman's optimum allocation in one of the strata exceeds the stratum size.

b) In three strata of sizes 80, 40 and 30 the allocation computed turns out to be 7.9, 1.4 and 5.7 respectively.

c) A sampler has 6 strata and picks up one unit at random from each stratum and wishes to estimate the sampling error of the estimator of the population mean.

d) For estimating efficiently the population mean, a sample of size 10 was drawn at random from a population and the observed y-values on a study variate were found to be

24, 26, 78, 20, 29, 84, 21, 76, 20, 80.

e) A sampler wishes to use ratio method of estimation of population total together with stratification.

[4 + 4 + 6 + 6 + 6=26]

p.t.c

3. a) A population consists of N clusters of varying sizes M_i, i=1,2,...N. Suppose that k clusters are selected at random and with replacement.

Let Y_{ij} be the value taken by the variate on the jth unit of the ith. cluster, j=1,2,...M_i ; i=1,2,...,N. Let $\bar{Y}_i = \sum_{j=1}^{M_i} Y_{ij} / M_i$ be the ith. cluster mean.

i) Write down the conventional unbiased estimator for the population mean $\bar{Y}=\sum_i M_i\bar{Y}_i/k\bar{M}$ when M_i are known, ii) Suggest a method of estimating its sampling error, iii) When M_i's are unknown beforehand, construct a Murthy-Nanjamma type unbiased estimator for $\bar{Y}$, iv) Suggest a scheme for making $\hat{\bar{Y}}$ unbiased where $\hat{\bar{Y}}$ is defined as $\sum_1^k M_i\bar{Y}_i / \sum_1^k M_i$ based on the k sampled clusters.

b) For households consisting of four persons (husband, wife and two children) where the sex of the child is assumed to be binomially distributed, obtain the efficiency of selecting households by simple random sampling compared to selecting individuals by simple random sampling for estimating the proportion of males in a given area.

[2 + 4 + 10 + 4 + 6=26]

4. A sample survey was conducted in a district to estimate the total industrial output. A stratified two-stage sampling design was adopted with urban blocks as first stage units and factories within them as second stage units. From each stratum 4 blocks were selected with probability proportonal to size and with replacement and 4 factories were selected from each selected block with equal probability and without replacement. The data on output (in suitable units) for the sample factories together with information on selection probabilities are given belolw :

Stratum	sample block	Inverse of probability of selection	Total No. of factories	Output of sample factorie			
				1	2	3	4
I	1	440.21	28	104	182	148	87
	2	660.43	14	108	64	132	156
	3	31.50	240	100	115	50	172
	4	113.38	76	346	350	157	119
II	1	21.00	256	124	111	135	216
	2	16.80	288	123	177	106	138
	3	24.76	222	264	78	144	55
	4	49.99	69	300	114	68	111
III	1	67.68	189	110	281	120	114
	2	339.14	42	80	61	118	124
	3	100.00	134	121	212	174	106
	4	68.07	161	243	116	314	129

Using the above data

i) Obtain an unbiased estimate of the total industrial output in the district.

ii) Obtain an unbiased estimate of the variance of the estimator.
[12 + 19=31]

INDIAN STATISTICAL INSTITUTE

B-Stat III, Semester I, 1999-2000

STATISTICAL INFERENCE I

Date: 15.11.99 Semestral Examination Time: 3 hours

Answer as many as you can. Maximum you can score is 60.

Unless a proof has been specifically asked for, you can use any result proved in class by properly stating them.

1. Suppose that $X_1, \ldots, X_n$ form a random sample from a distribution modeled by the density $f_\theta(x)$, $\theta \in \Omega$. Show that the minimum variance unbiased estimator of θ must be a symmetric function of the observations. [6 points]

2. Consider the exponential model given by the density $f(x) = \frac{1}{\theta}\exp(-x/\theta)$, $x \geq 0$. Find the minimum variance unbiased estimator of $1/\theta$ on the basis of a random sample of size n from this distribution. [6 points]

3. Suppose that X has a Uniform $(\theta, \theta+1)$ distribution.

   (a) Consider a single observation from this distribution. Find the method of moments estimator of θ based on this observation. Is this estimator unbiased? Is this the minimum variance unbiased estimator of θ?

   (b) Consider now a random sample of three observations from this distribution. Find the method of moments estimator of θ based on these observations. Is this estimator unbiased? Is this the minimum variance unbiased estimator of θ? [4+4=8 points]

4. Prove that an estimator $aX + b$ $(0 \leq a \leq 1)$ of $E_\theta(X)$ is inadmissible (with squared error loss) under each of the following conditions.

   (i) if $E_\theta(X) \geq 0$ for all θ, and $b < 0$.

   (ii) if $E_\theta(X) \leq k$ for all θ, and $ak + b > k$. [4+4=8 points]

5. Let $f_\theta(x)$ be the probability mass function of a random variable X such that the set $\{x | f_\theta(x) > 0\}$ does not depend on θ. Suppose that for a random sample $\boldsymbol{X} = (X_1, \ldots X_n)$ of n observations from this distribution, there exists a function $T(\boldsymbol{X})$ such that for any two realisations $\boldsymbol{x} = (x_1, \ldots, x_n)$ and $\boldsymbol{y} = (y_1, \ldots, y_n)$ of $\boldsymbol{X}$ the likelihood ratio of the two realisations is constant as a function of θ if and only if $T(\boldsymbol{x}) = T(\boldsymbol{y})$. Show that $T(\boldsymbol{X})$ is a minimal sufficient statistic for θ. [6 points]

6. Suppose that X has density $f_X(x) = \frac{1}{\theta}\exp(-(x-\gamma)/\theta)$, $x > \gamma$. Let Y be any other random variable such that Y is independent of X, and has density $f_Y(y) = \frac{1}{\theta}g(y/\theta)$, g being any fixed known density. Consider testing $H_0 : \gamma = \gamma_0$, and let the rejection region be $\frac{X-\gamma_0}{Y} \leq a$, or $\frac{X-\gamma_0}{Y} \geq b$, such that the size of the test is α. Find the power of the test at any $\gamma < \gamma_0$, and show the power does not depend on the limits a, b, or the density g. [6 points]

7. Suppose that X and Y are two random variables with the joint density $f(x,y) = \lambda\mu\exp(-\lambda x - \mu y)$, $x, y \geq 0$. Find a UMP unbiased test of size $\alpha = 0.2$ for testing $H_0 : \lambda \leq \mu + 1$ against $H_1 : \lambda > \mu + 1$. [8 points]

8. Suppose that the distribution of the random observable X has the form $f_\theta(x) = c(\theta)h(x)\exp(\theta x)$. Let θ_1 and θ_2 be two elements of the parameter space, $\theta_1 < \theta_2$. Let the actions 'accept H_0' and 'reject H_0' be represented by a_0 and a_1 respectively. Consider the class of two sided tests given by

$$\phi(x) = \begin{cases} 1 & \text{if } x < x_1 \text{ or } x > x_2 \\ \delta_i & \text{if } x = x_i \quad x_i = 1,2 \\ 0 & \text{if } x_1 < x < x_2 \end{cases}$$

Suppose further that the loss function satisfies

$$\begin{aligned} L(\theta, a_1) - L(\theta, a_0) &\geq 0 \text{ if } \theta_1 < \theta < \theta_2 \\ L(\theta, a_1) - L(\theta, a_0) &\leq 0 \text{ if } \theta < \theta_1 \text{ or } \theta > \theta_2. \end{aligned}$$

If there exists $\theta_1' < \theta_2' < \theta_3'$ such that $L(\theta_1', a_1) - L(\theta_1', a_0) < 0$, $L(\theta_2', a_1) - L(\theta_2', a_0) > 0$, and $L(\theta_3', a_1) - L(\theta_3', a_0) < 0$, then every two sided test is admissible. [12 points]

9. Consider a random sample of size n from a Uniform $(0, \theta)$ distribution. Let $X_{(i)}$ be the i-th order statistic, and Y be the sample range.

(a) Show that if ξ is given by $\xi^{n-1}[n - (n-1)\xi] = \alpha$, then $\left(Y, \frac{Y}{\xi}\right)$ is a confidence interval for θ with confidence coefficient $(1 - \alpha)$.

(b) Show that $\left(X_{(n)}, \frac{X_{(n)}}{\alpha^{1/n}}\right)$ is also a confidence interval for θ with confidence coefficient $(1 - \alpha)$, but has shorter length than the confidence interval in (a). [4+4=8 points]

ii) Obtain an unbiased estimate of the variance of the estimator.
[12 + 19=31]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) III YEAR:1999-2000
SEMESTRAL-I EXAMINATION
DIFFERENTIAL EQUATIONS

Date:17.11.99 Maximum Marks:100 Time: 3 Hours

Note: Attempt all questions. The marks allotted to each question is indicated against each question.

1.(i) Find the curves that satisfy the following: the part of the tangent cut off by the axes is bisected by the point of tangency. (10)

(ii) An integral curve $y = u(x)$ of the differential equation

$$y'' - 4y' + 29y = 0$$

intersects an integral curve $y = v(x)$ of the differential equation

$$y'' + 4y' + 13y = 0$$

at the origin.

The two curves have equal slopes at the origin. Determine u and v if $u'(\pi/2) = 1$. (10)

2. Let $y_p(x)$ be a nontrivial solution of Bessel's equation

$$x^2y'' + xy' + (x^2 - p^2)y = 0.$$

Let d be the distance between two successive positive zeros of $y_p(x)$. Reduce the Bessel's equation in normal form and hence show that

(i) if $0 \leq p < \frac{1}{2}$ then $d \leq \pi$ and $d \to \pi$ as $x \to \infty$

(ii) if $p > \frac{1}{2}$ then $d > \pi$ and $d \to \pi$ as $x \to \infty$. (20)

3. The Legendre polynomial of degree n is given by Rodrigue's formula

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n$$

Show that $\{P_n(x) : n \geq 0\}$ is a sequence of orthogonal functions (using the above formula or otherwise) on $[-1, 1]$. Evaluate

$$\int_{-1}^{1} x P_n(x) P_{n-1}(x)\, dx.$$ (10+10)

p.t.o.

4. Show that the geodesics on a surface

$$G(x,y,z) = 0$$

satisfy the system

$$\frac{d/dt\ (\dot{x}/f)}{G_x} = \frac{d/dt\ (\dot{y}/f)}{G_y} = \frac{d/dt\ (\dot{z}/f)}{G_z}$$

where $f = [(\dot{x})^2+(\dot{y})^2+(\dot{z})^2]^{1/2}$

and $\dot{x} = \frac{dx}{dt}$; $\dot{y} = \frac{dy}{dt}$; $\dot{z} = \frac{dz}{dt}$.

Hence show that the geodesics on a sphere $x^2+y^2+z^2 = a^2$ are arcs of great circles. (20)

5. (i) Let A be a nxn matrix with real entries and e^{tA} denote the exponential of the matrix t A for any $t \in \mathbb{R}$. Show that A and e^{tA} commute and hence that e^{tA} is nonsingular for any $t \in \mathbb{R}$. (15)

(ii) Let F(p) be the Laplace transform of a function f(x). Then sh that the Laplace transform of $(-1)^n x^n f(x)$;

$$L[(-1)^n x^n f(x)] = F^{(n)}(p).$$

Hence evaluate $L[x \sin ax]$. (5)

Glass type	Phosphor Type A	B	C
1	280	300	270
	290	310	285
	285	295	290
2	230	260	220
	235	240	225
	240	235	230

.95 Quantiles of the studentized Range Distribution for parameters R, γ . (28)

γ	k = 2	3	4
7	3.34	4.16	4.68
8	3.26	4.04	4.53
9	3.20	3.95	4.42
10	3.15	3.88	4.33
11	3.11	3.82	4.26
12	3.08	3.77	4.20
13	3.06	3.73	4.15
14	3.03	3.70	4.11

INDIAN STATISTICAL INSTITUTE

SECOND SEMESTRAL EXAMINATION (1999-2000)
B.STAT (Hons.)– III yr.
Economics–IV

DATE OF EXAMINATION : 20 APRIL 2000

Maximum Marks : 100 Duration : 3 hrs.

Answer any four questions. All questions carry equal marks.

1. Consider the following regression in deviation form.

$$Y_t = \beta_1 X_{1t} + \beta_2 X_{2t} + U_t$$

with sample data

$n = 100, \Sigma Y^2 = \frac{493}{3}, \quad \Sigma X_1^2 = 30, \quad \Sigma X_2^2 = 3$

$\Sigma X_1 Y = 30, \Sigma X_2 y = 20, \Sigma X_1 X_2 = 0$

(i) Compute the least squares estimates of β_1 and β_2 and also calculate R^2.

(ii) Test the hypothesis $H_o : \beta_2 = 7$ against $H_1: \beta_2 \# 7$

(iii) Test the hypothesis $H_o : \beta 1 = \beta_2 = 0$ against $H_1 : \beta_1 \# 0$ or $\beta_2 \# 0$

(iv) Test the hypothesis $H_o: \beta 2 = 7\beta_1$ against $H_1 : \beta 2 \# 7\beta_1$

2.(a) Discuss the problem of prediction in the classical normal linear regression model.

(b) Suppose you are given an estimated linear regression

$$\hat{Y} = 1.36X_2 + o.11X_3$$

along with the following information

$n=9, \ \Sigma e^2 = 77.55, \ (X^1x)^{-1} = \begin{bmatrix} 0\cdot 0016 & 0\cdot 0003 \\ 0\cdot 0003 & 0 \ \ 0016 \end{bmatrix}$

Test whether a new observation

$Y = 2.7, X_2 = 17, X_3 = -16$

can be presented to be generated by same structure as the sample observations on the basis of which the relationship was estimated.

P.T.O

3. State the consequences of applying OLS to a linear regression model with auto correlated disturbances. Suggest alternative methods of estimating the parameters of the model when the disturbances follow a first order auto regressive process with an unknown auto correlation coefficient. Discuss the large sample and small sample properties of your estimators relative to the OLS estimators and also relative to each other.

4. Consider the following linear regression model

$Y_i = \beta x_i + u_i$ $(i = 1, 2, \ldots\ldots, n)$

Where the u_i's are serially uncorrelated disturbances each with zero mean and variance σ^2.

Suppose that the n observations are arranged in m groups, the i-th group contains n_i ($\Sigma n_i = n$) observation of y and x. The group means of y and x are denoted by $\overline{Y}_i$ and $\overline{X}_i$ $(i = 1, 2, \ldots.., n)$

Suppose that only the data on group means $(\overline{y}_i, \overline{x}_i)$, i = 1, 2, …n are available

(i) Derive the best linear unbiased estimator (BLUE) of β.
(ii) Derive the expression for the variance of your estimator in (i)
(iii) Find the variance of the OLS estimator of β and hence drive an expression for the efficiency of the OLS estimator relative to the BLUE.

Now suppose that the ungrouped data (y_i, x_i) i=1, 2, ….n, are available.

(iv) Find the BLUE of β in this case and verify that the variance of this estimator is less than or equal to the variance of your estimator in (i). Hence or otherwise, can you suggest any principle of grouping ?

5. Write short notes on any two.

(i) Consequences of applying OLS to a linear regression model with multicollinearity.
(ii) Distributed lag models and estimation of such models ,
(iii) Estimation of a omitted variable (OV) model.

Q.5. A set of data is given in the following table for a partially confounded 3^2 experiment in 6 blocks of 3 observations each. Identity the confounded effects. Prepare an analysis of variance table. Test any hypotheses that you think are of interest and state your conclusion about the two factors and their interaction. (Observations are shown in parenthesis)

Replication – I

Block I	00 (53)	12 (59)	21 (80)
Block II	01 (66)	10 (71)	22 (78)
Block III	02 (69)	11 (91)	20 (92)

Replication – II

Block I	00 (46)	11 (62)	22 (58)
Block II	01 (65)	12 (61)	20 (76)
Block III	02 (34)	10 (50)	21 (66)

[25]

INDIAN STATISTICAL INSTITUTE

SEMESTRAL - II

B.Stat. (Hons.) III Year (1999-2000)

Subject : Statistical Inference II

Maximum Marks : 100 (40 in Group A and 60 in Group B)

Time : 3 hours

Date:- 5.5.00

Use different answer-papers to answer questions in Group A and Group B respectively.

Group A

1. State and prove the Fundamental Identity and show with an example how you can calculate $P_\theta\{H_0 \text{ is rejected }\}$ for $\theta \neq \theta_0, \theta_1$ (Assume you have i.i.d. X_i's $\sim f_\theta$ and an SPRT ($B < 1 < A$) test $H_0 : \theta = \theta_0$ vs $H_1 : \theta = \theta_1$).

Or

2. Let X_i be a sequence of $r.v$'s and the joint density of $(X_1, \cdots, X_m)$ be $p_{jm}(x_1, \cdots, x_m)$ under hypothesis H_j. Let $B < 1 < A$ and the SPRT be the test that stops first time $\lambda_m = p_{1m}/p_{0m} \geq A$ or, $\leq B$. H_o is rejected if $\lambda_n \geq A$, accepted if $\lambda_n \leq B$ and no decision is made if $n = \infty$, where n is the stopping time of the test.

   Will the Wald inequalities remain true ? Will the Wald approximations remain valid ? 7+3 = 10

3. a) Assuming the dynamic programming theorem for finite horizons, show that the optimal rule in the secretary problem is to let m^* candidates be interviewed and then stop first time $Y_i = 1$ (You have to deduce the joint distribution of Y_i's, the form of X_n and the defining inequality for m^*)

   b) If the Secretary problem has a cost $c(> 0)$ per interview, how would the optimal stopping time change ? 15+5 = 20

P. T. O

Or

4. a) State the (exact) optimum property of an SPRT with $B < 1 < A$.

   b) Sketch the essential steps in the proof as done in class.

   c) Assuming the dynamic programming theorem for an infinite horizon, show that the Bayes stopping rule at stage m is as follows. Let $\prod_1 < \overline{\prod}_1$ be solutions of $\rho_o(\prod_1) = \rho_1(\prod_1)$. If the posterior probability of H_1 $\prod_{1m}$ given $Y_1, \cdots, Y_m$ is $\geq \overline{\prod}_1$ stop and reject H_o, if $\prod_{1m} \leq \underline{\prod}_1$, stop and accept H_0 and otherwise continue. 4+8+8 = 20

5. Assignment 10

Group B

Answer any three of the questions in this group.

1. Let $D_1, D_2, \cdots, D_n$ be observations from continuous symmetric, mutually independent random variables with the same median m. Let R_i = Rank of $|D_i|$ among $|D_1|, |D_2|, \cdots, |D_n|$ and T = Sum of R_i's, over all i for which $D_i > 0$.

   Show that T is the cardinality of the set given by

   $$\{(i, j) : D_i + D_j > 0, \; 1 \leq i \leq j \leq n\}$$

   Hence derive $100(1 - \alpha)\%$ confidence interval for m. 10 + 10 = 20

2. Derive the formula for asymptotic relative efficiency of the sign test relative to t-test for null and alternate hypotheses, where both hypotheses assume the same distribution except for a difference in location.

   Hence find ARE in the case of

   i) normal distribution

   ii) doubly exponential distribution 12+8 = 20

3. Let F be a continuous cumulative distribution function on R and F_n be the empirical distribution with underlying c.d.f. F.

   Show $\sup_x | F_n(x) - F(x) | \to 0$ as $n \to \infty$, almost surely.

   Show that $D_{mn} = \sup_x | F_n(x) - G_m(x) |$ is consistent estimator of $\sup_x | F(x) - G(x) |$, where G_m is the empirical distribution with underlying c.d.f. G.

   12+8 = 20

4. Describe the test based on total number of runs for randomness of a sample. Let n_1 and n_2 be the numbers of the two types of objects constituting the sample. Derive the exact null distribution of the total number of runs. State also its asymptotic distribution. 5+10+5 = 20

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons) IIIrd Yr.(1999-2000)
Semestral II Examination
Introduction to Stochastic Processes
Full Marks: 100

Date:08.05.2000 Time: $3\frac{1}{2}$ hours

Note: *The paper consists of 8 questions carrying a total of 120 marks. You can answer any part of any question. However the maximum you can score is 100.*

1. Consider a collection of $2N$ individuals, each one of whom is either in favour or against some issue. Let X_n denote the number of individuals in favour of the issue at time $n = 0, 1, 2, \cdots$. In the evolution, each one of the individuals will randomly re-decide his/her position under the influence of the current overall opinion as follows:
Let $\theta_n = X_n/2N$ denote the proportion in favour of the issue at time n. Then given $X_o, X_1, \cdots$
X_n each of the $2N$ individuals, independently of the choices of the others, elects to be in favour
with probability θ_n or against the issue with probability $1-\theta_n$.
Show that $\{X_n; n \geq 0\}$ is a Markov chain with state space $S = \{0, 1, \cdots, 2N\}$.
Find the one-step transition matrix $((p_{ij}))$. Classify the states. Find $E(X_{n+1}|X_n)$ and hence deduce that $E(X_{n+1}) = E(X_n)$ for $n \geq 0$.
[3+2+2+5=12]

2. (i) Show that a 2×2 stochastic matrix is the two-step transition matrix of a Markov chain if and only if the trace of the matrix is greater than or equal to 1.
(ii) Let a Markov chain contain r states. Prove the following :
(a) If a state k can be reached from j, then it can be reached in $(r-1)$ steps or less.
(b) If j is a recurrent state, there exists $\alpha (0 < \alpha < 1)$ such that for $n > r$ the probability that first return to state j occurs after n transitions is $\leq \alpha^n$.
(iii) Let $\{X_n; n \geq 0\}$ be a MC (Markov chain) with countable state space S and transition probabilities $((p_{ij}))$. For $i \in S$, let T_i denote the hitting time of the state i i.e.,
$T_i = min\{n > 0 : X_n = i\}$ if $X_n = i$ for some $n > 0$, otherwise $T_i = \infty$. Prove that
$$P(T_i < \infty | X_o = i) = p_{ii} + \sum_{j \neq i} p_{ij} P(T_i < \infty | X_o = j).$$
[4+4+8+6=22]

3. Consider a birth and death chain on the set $S = \{0, 1, 2, \cdots\}$ of non-negative integers. The one-step transition probabilities are
$$p_{ij} = \begin{cases} \mu_i, & j = i-1 \\ 1-\mu_i-\lambda_i, & j = i \\ \lambda_i, & j = i+1 \\ 0 & \text{otherwise} \end{cases}$$
where $\mu_0 = 0, 0 < \mu_i, \lambda_i$ for $i \geq 1$; $i, j \in S$.
For $i \in S$ let T_i denote the hitting time of the state i. Let $m < n$ be in S.
Set $u_i = P(T_m < T_n | X_o = i)$, $m < i < n$ and $u_m = 1$ and $u_n = 0$.

Please turn overleaf

(i) For $m < i < n$, express u_i in terms of u_{i-1}, u_i and u_{i+1}. Hence show that

$$u_i = \frac{\sum_{j=i}^{n-1} \rho_j}{\sum_{j=m}^{n-1} \rho_j} \quad \text{where } \rho_o = 1 \text{ and } \quad \rho_j = \frac{\mu_1\mu_2\cdots\mu_j}{\lambda_1\lambda_2\cdots\lambda_j} \quad \text{for } j \geq 1.$$

(ii) State sufficient condition which will ensure irreducibility of the birth and death chain. Assuming irreducibility show that the given birth and death chain is recurrent if and only if

$$\sum_{j=1}^{\infty} \rho_j = \sum_{j=1}^{\infty} \frac{\mu_1\cdots\mu_j}{\lambda_1\cdots\lambda_j} = \infty.$$

(iii) Consider the birth and death chain on the set of non-negative integers defined by

$$\mu_i = \frac{i}{2(i+1)}, \ \lambda_i = \frac{i+2}{2(i+1)}, \quad i \geq 0.$$

Determine whether the chain is recurrent or transient.

[8+6+4=18]

4.Consider a MC with state space $\{0, 1, 2\}$ and transition matrix

$$\mathbf{P} = \begin{pmatrix} 0 & 1 & 0 \\ 1-p & 0 & p \\ 0 & 1 & 0 \end{pmatrix}$$

(i)Find $\mathbf{P^2}, \mathbf{P^3}, \mathbf{P^4}$. Find a general formula for $\mathbf{P^n}$.

(ii) Find $f_{ii}^{(n)}$ for all i and $n \geq 1$. Also find $f_{02}^{(n)}$ for all $n \geq 1$.

(iii) Does $\lim_{n\to\infty} \mathbf{P}^n$ exist ? Where does $\frac{1}{n}\sum_{k=0}^{n-1} \mathbf{P}^k$ converge to ?

[4+6+4=14]

5.(i) If $\{N_t, t \geq 0\}$ is a Poisson process with parameter $\lambda > 0$, find $Cov(N_t, N_s)$ for $t, s > 0$.

(ii) Let $\{N_t^{(1)}, t \geq 0\}$, $\{N_t^{(2)}, t \geq 0\}$ be independent Poisson processes with parameters λ_1 and λ_2 respectively. Show that $\{N_t, t \geq 0\}$, defined as $N_t = N_t^{(1)} + N_t^{(2)}$, is a Poisson process. Also find $P(N_t^{(1)} = 0 | N_t = 2)$.

[4+6=10]

6.Call a MC with state space S an absorbing chain if every non-absorbing state leads to an absorbing state.
Show that for an absorbing chain, absorbing states are precisely the recurrent states.
Consider an absorbing chain with finite state space S,in which the set A of absorbing states are listed first, followed by the set T of transient states.

(i) Show that the transition matrix **P** is of the form $\begin{pmatrix} I & 0 \\ R & Q \end{pmatrix}$, where I is the identity matrix of order $k = \#(A)$, Q is a square matrix of order $l = \#(T)$, $\mathbf{0}$ is a matrix with all entries 0 and R is some matrix.

(ii) Show that $I_l - Q$ is invertible where I_l is the identity matrix of order l.

(iii) Let $M = (I_l - Q)^{-1}$.Show that $QM = MQ = M - I_l$.

[1+3+6+4=14]

7.(i) Consider an irreducible positive recurrent MC with transition matrix **P** and countable state space S. If $u = (u_i, i \in S)$ is a non-negative (row)vector satisfying $u\mathbf{P} \leq u$, prove that

$$\sum_{i \in S} u_i < \infty.$$

(ii) Let Π denote the class of all stationary distributions of a MC. Show that (a) Π is a convex set, (b) Π is precisely one of the three types: empty, singleton or uncountable.

(iii) Suppose that n states $a_1, a_2, \cdots a_n$ are arranged counter-clockwise in a circle.A particle jumps one unit in the clockwise direction with probability $p, 0 \leq p \leq 1$, or one unit in the counter-clockwise direction with probability $q = 1 - p$. Calculate the stationary distribution.

[8+3+2+5=18]

8.Let $\{X_t, t \geq 0\}$ be a stochastic process of real -valued random variables. If the distribution of the increments $X_{t+h} - X_t$ depends only on the length h of the interval and not on t, the process is said to have stationary increments.

(i) Suppose a process $\{X_t, 0 \leq t < \infty\}$ has stationary independent increments and has finite expectation. Let $f(t) = E(X_t) - E(X_o), t \geq 0$. Assuming that $f(t)$ is continuous in t, prove that $E(X_t) = m_o + m_1 t$ where $m_o = E(X_o)$ and $m_1 = E(X_1) - m_o$.

(ii) Consider a collection of particles which act independently in giving rise to successive generations of particles. Suppose that each particle,from the time it appears,waits a random length of time having an exponential distribution with parameter $\lambda > 0$ and then splits into two identical particles with probability $p, 0 < p < 1$,and disappears with probability $1 - p$. Let $X_t, 0 \leq t < \infty$,denote the number of particles present at time t. This branching process is a birth and death process on the non-negative integers. Find the birth and death rates.Write the backward and forward differential equations of the above process.

[6+6=12]

STOP