Bilateral Trade and Partnership with Loss Averse Agents

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To my parents, sister and teachers

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Chapter 1

Introduction

This thesis consists of three essays on bilateral trading and partnership with loss averse agents. All the three essays consider the same model where two loss averse agents (Tversky and Kahneman (1991), Köszegi and Rabin (2006)) hold some shares of an asset and now wants to dissolve the partnership.

The prospect theory (Kahneman and Tversky (1979)) laid the foundational stone for the behavioral economics in which the shortcomings of the expected utility theory were addressed. According to the prospect theory, gains and losses are evaluated differently by the economic agents. Thaler (1980) described the experimental findings through endowment effect in which the agents who owns the object (which is an endowment) values it more as compared to those who do not have the object. Meanwhile, new behavioral theories have also started to come up. To name a few are Gilboa and Schmeidler (1989), Klibanoff et al. (2005) and Borah and Kops (2016). We consider a particular type of preferences known as loss averse preferences in which the outcomes are evaluated in an absolute way as well as relative to a reference point and the sensitivity associated with the losses (relative to reference point) is higher as compared to the gains.

The first chapter shows the impossibility result for loss averse agents i.e. there does not exist a feasible, incentive compatible and individually rational mechanism which can implement an efficient outcome. The second chapter talks about the dissolution mechanism intermediated by a broker which is optimum in a sense different than ex-post efficiency. Set of all optimal mechanisms are characterised for the bilateral trade as well as partnership model. The third chapter shows non-existence of a dissolution mechanism which is incentive-compatible, interim individually rational, budget-balanced and efficient for equal share partnership when the degree of loss aversion exceeds a cut off, which is in contrast to Cramton et al. (1987) result.

A brief description of each chapter is provided below.

1.1 Dissolving a Bilateral Partnership

In this chapter, we are looking for a dominant strategy incentive compatible, individually rational, feasible and efficient mechanism to dissolve a partnership between two loss averse agents. We consider a model where partnership between two loss averse agents, each holding some share (endowment) of the asset, has to be dissolved. The framework is similar to Cramton et al. (1987) except that we only look at the two agent model (bilateral partnership), consider dominant strategy incentive compatible mechanisms instead of Bayesian incentive compatible mechanisms and our agents have loss averse preferences as modeled in Köszegi and Rabin (2006, 2007) and Tversky and Kahneman (1991). Apart from the standard utility from the ownership of the good and money, called as "material utility", "gain-loss utility" with respect to ownership of the good is introduced. The reference point, relative to which agents evaluate an outcome, is the initial share of the asset. Cramton et al. (1987) is a generalised version of the bilateral trading problem considered by Myerson and Satterthwaite (1983) where one agent (seller) has the full ownership of the asset. Myerson and Satterthwaite (1983) showed that in the bilateral trading problem, there exists no mechanism which is Bayesian incentive compatible, interim individually rational, efficient and budget balanced. Cramton et al. (1987), on the other hand, finds that efficient trade is possible with a Bayesian incentive compatible, interim individually rational and a budget balanced mechanism if partners have equal shares. The model specified by Cramton et al. (1987) can be used to study the problems of economic resources. For example, Chaturvedi (2020) modeled the problem of ownership of land being shared by various landholders and a buyer who wants the entire land as a multilateral trading problem. The question that we ask is if there exists an efficient, dominant strategy incentive compatible, and budget-balanced mechanism when agents are loss averse. Green and Laffont (1979) showed that there is no efficient, dominant strategy incentive compatible, and budget-balanced mechanism, if preferences are quasi-linear. Lahkar and Mukherjee (2020) considers a model of public goods with large population and constructs a mechanism which is strictly dominant strategy incentive compatible, satisfies individual rationality and strong budget balance condition. We are interested to see whether the impossibility proved by Green and Laffont (1979) continues to hold in our model with loss averse preferences.

Recent papers on mechanism design with loss averse agents have considered the case where reference points are formed endogenously using rational expectations as mentioned in Köszegi and Rabin (2006, 2007). Lange and Ratan (2010), Eisenhuth and Grunewald (2018), Eisenhuth (2018) focus on auction setting with the loss averse traders in two-dimensional model where gain-loss utility is separable across good dimension and money dimension (known as narrowly bracketing) and one-dimensional model where gain-loss utility is defined over entire risk neutral payoff (known as widely bracketing). Benkert (2023) builds on the model of Eisenhuth (2018) in the bilateral trade setting. Our paper is the first to study the partnership model with loss averse agents with exogenous reference point and study the impact of endowment effect on the trade.

We first characterise the incentive compatible mechanisms for loss averse agents. The characterization of incentive compatible mechanisms is a generalization of Cramton et al. (1987) and Myerson and Satterthwaite (1983) without loss aversion. Because of the gain-loss utility terms, the application of envelope theorem gives a piecewise utility function which depends on whether the agent is losing his share or gaining additional share.

Next, we look into the problem of efficiency. Benkert (2023) also studies the efficiency problem but it defines an efficient allocation rule as the one which maximizes "material valuations" of the traders. In our paper, efficient allocation is the one which maximizes the valuations of the partners (including the gain-loss utilities in ownership). In the absence of loss aversion, the allocation in which the agent with highest valuation gets the full ownership of the object is efficient. But with loss averse preferences, the object may not be allocated to an agent even if he has the highest valuation due to higher loss sensitivity associated with an allocation less than the endowment. As a result, loss averse preferences inhibit the trade as the set of valuations where trade takes place is reduced.

We find that, with loss averse agents, the bilateral partnership cannot be dissolved by a dominant strategy incentive compatible, individually rational, efficient and feasible mechanism, irrespective of the initial shares or endowments. The impossibility could be due to the way efficiency is defined in our model. The set of values at which trade should take place shrinks because of the higher sensitivity of loss associated with losing the share after the trade. The other reason is different payment functions of the agents depending on whether the final allocation is more or less than initial share, leading to violation of budget balance condition. When a partnership is dissolved, one agent receives the full ownership of the company and pay the other agent who has lost his share. The partners evaluate the gain (loss) terms of share and with loss aversion, losses loom larger than gains, inhibiting the trade. The partner who has lost his share suffers more loss in utility than the gain experienced by the partner who gets the full ownership. To compensate the loss, the agent losing his share has to be paid much more than the amount paid by the the agent gaining the ownership if he is to participate in the trade. But then differences in the transfer (payments) received (paid) by the agents violate the condition of transfer rule to be budget balanced.

1.2 Optimal Dissolution Mechanisms

Myerson and Satterthwaite (1983) showed that, when entire ownership of the object belongs to one person, trade between two agents (who have incomplete information about each other types') cannot be efficient. Since then, a long line of literature explores the limit of this impossibility. Because one agent owns the entire object, the worst off type of both the agents is known ex-ante.

In this paper, we try to design a mechanism in order to dissolve partnership between two agents who have some share of the object and have per unit valuation for the object which is private information. The agents are loss averse with respect to the initial share (endowment) (Köszegi and Rabin (2006, 2007); Tversky and Kahneman (1991)). The agent's role as a seller or a buyer depends on the the realized valuation of the object which cannot be determined prior to dissolution.

This paper focuses on the mechanisms that are optimal, where optimality is defined as maximising a weighted average of expected gains from dissolution and expected revenue generated from dissolution. We give a characterisation result for the ex-ante efficient mechanisms and the revenue-maximizing mechanisms. In this case, the efficient mechanism is the one that maximizes the material utility as well as the gain-loss utility associated with endowment. In the standard bilateral trade model discussed by Myerson and Satterthwaite (1983), the minimum utility in an incentive-compatible and individually rational mechanism is always achieved by the lowest valuation for the buyer and the highest valuation for the seller. So, the agent's virtual valuation functions are known ex-ante and, therefore, are independent of the mechanism. Individual rationality constraints are reduced to the worst-off valuation, which is the highest and lowest valuation, depending on the role of the agent, and is binding at the optimum. Assuming that the distribution of valuations satisfy regularity, i.e., hazard rates are increasing, the monotonicity of the allocations is satisfied, and optimal allocations are given by point-wise solutions. We follow this methodology for bilateral trade with loss-averse agents. The only difference is that optimal allocations compare the effective virtual valuation (i.e., inclusive of additional gain-loss effects with respect to the endowment). However, when both agents have some ownership in the object, it is no longer clear which agent is playing the role of seller and which agent is playing the role of buyer. We adapt the multilateral trade setting of Lu and Robert (2001) and Loertscher and Wasser (2019) in the partnership setting for loss-averse agents. When both agents have some shares in the object, it is no longer clear who will sell his shares (acting as a seller) and who will get the additional shares, making him the owner of the object (acting as a buyer), before the revelation of valuations. So, the agent with a high valuation expects to get full ownership of the object, and the low type expects to lose his shares. The minimum utility valuations for the agents whose role is ex-ante unidentified are in the middle, where on average he neither wants to be a buyer nor a seller and usually depends on the dissolution mechanism. Also, despite the regularity of the distribution of valuation, the virtual valuation function does not satisfy monotonicity. As a result, optimal dissolution mechanisms are characterised by ironed virtual valuations in which the object is transferred to the highest effective ironed virtual valuation (which takes into account the loss aversion parameters).

Because of ironing, there is bunching phenomena due to which ties occur with positive probability. So, the optimal allocation rule consists of a randomizing rule to break the ties in Loertscher and Wasser (2019) and Lu and Robert (2001). Ties cannot be broken arbitrarily because bunching is not because of the irregularity of the distributions. The tie breaking rule has to be such that the agents who have the valuations in the middle expects to be neither a buyer nor a seller. However in this paper, whenever the virtual valuations are same, according to the optimal allocation rule, the agents will keep their shares. Despite the positive probability of ties within the common bunching range, there is no requirement of tie breaking rule because whenever virtual valuations tie is in the common bunching range, optimality requires the agents to keep their initial shares i.e. no trade in the bunching range.

1.3 Efficient Dissolution Mechanisms

The classical work of Myerson and Satterthwaite (1983) laid the foundation for the bargaining models and showed that under extreme ownership shares where one agent has all the shares of the object (seller) and the other agent has none (buyer), efficient outcome can be implemented by an incentive compatible and individually rational mechanism if and only if an outside party provides a subsidy. Cramton et al. (1987) showed that efficient outcome is possible if the ownership structure is symmetric. Since then, literature has explored the possibility (impossibility) of reallocation of object to achieve ex-post efficiency by analyzing the conditions on the initial ownership shares (Makowski and Mezzetti (1993), McAfee (1991)).

In chapters 1 and 2, we have explored the literature on loss-averse preferences, and the evidence shows that the possibility of trade or dissolution is reduced due to the higher loss sensitivity associated with losing the share compared to the gains. We try to answer the following question: We try to answer the following question: Does the Cramton et al. (1987) possibility result hold with loss-averse agents? i.e., with an equal-share partnership, is it still possible to dissolve the partnership with an efficient, Bayesian incentive compatible and interim individually rational mechanism, or does it depend on the values of the loss aversion parameters? We find that with loss-averse agents, it is not always possible to dissolve an equal-share partnership efficiently. There exists a cut-off point for the loss aversion parameters such that the partnership cannot be dissolved by a Bayesian incentive compatible, individually rational, and efficient mechanism, even when the agents have equal shares, for any distribution. However, we cannot say that for parameter values less than the cutoff point, an equal share partnership is dissolvable by a Bayesian incentive compatible, interim individually rational, budget balanced and efficient mechanism. The particular values of loss aversion parameters such that the equal share partnership can be dissolved efficiently depend on the specific distribution. Therefore, without knowledge of the loss aversion parameters and distribution functions, it is not possible to decide whether efficient trade can take place. This is in contrast to Cramton et al. (1987) and our result generalises the result of Cramton et al. (1987) (when there is no loss aversion). The reason the possibility result breaks down is the following: As sensitivity to losses increases, the set of values at which trade could be implemented is reduced. This leads to a shrinking of the set of values at which dissolution takes place. A higher sensitivity to losses means it is less efficient to dissolve the partnership. Using the example of a unit interval uniform distribution, we provide a range for the loss aversion parameters at which the dissolution of a partnership is efficient.

We also consider a one owner partnership model (bilateral trade) and show that the impossibility result of Myerson and Satterthwaite (1983) still persists. The result is quite intuitive since the agents are less willing to participate in the trade due to the loss aversion with respect to losing the initial share. However, we find that the minimal subsidy required to implement the efficient outcome decreases as the loss sensitivity of agents increases. The possible reason for this is could be the following: As sensitivity to the losses increase, the set of values at which trade could be conducted is reduced. With less possibility of trade, the requirement for minimal subsidy also reduces. There could occur a possibility that the loss aversion parameters are so high that the agents do not participate in the trade and therefore, the minimal subsidy would be 0.

There are few papers in the literature that talk about the departure from quasi-linear preferences in the bilateral trade setting. Chatterjee and Samuelson (1983) showed that as agents become infinitely risk-averse, double auctions are efficient asymptotically. Garratt and Pycia (2023) relaxed the assumption that agents have quasilinear preferences in Myerson and Satterthwaite (1983) model. They showed that if the agents are risk-averse or the utility of the agents from the object is dependent on wealth, then there is a possibility that the trade among the agents is ex-post efficient. Under risk aversion or wealth effects, they give conditions to realize all gains from trade. Their results show that the impossibility of bilateral trade is due to the assumption of quasilinear preferences. Under quasilinear preferences, the reason for the impossibility is that the gains from trade are not sufficient to cover the information rents (due to private information) of the agents. On the other hand, additional efficiency gains are generated from risk aversion. Wolitzky (2016) examines efficiency within a bilateral trade model where both the buyer and seller know the expected valuation of each other. He demonstrates that efficient trade is feasible under certain parameter conditions and gives an exact characterization of that.

Benkert (2023) introduced loss aversion in the bilateral trade setting of Myerson and

Satterthwaite (1983). He applies expectation-based loss averse preferences (Köszegi and Rabin (2006, 2007)) by adapting the narrow bracketing model of Eisenhuth (2018). Eisenhuth (2018) considers the problem of designing optimal auction for loss averse agents and he used two forms of utility functions: (a), gains and losses are evaluated in the good dimension and in the money dimension separately known as narrow bracketing and (b), gains and losses are evaluated over the entire risk neutral utility, known as wide bracketing. Benkert (2023) discusses that loss aversion decreases the buyer's information rent due to which there is a possibility that the gains from trade (which are also decreased due to reduction in agent's expected utility) can cover the information rent, depending on the parameters of loss aversion. Note that Benkert (2023) talks about implementing the materially efficient outcome. Our paper is different because of two reasons: 1) We consider a fixed reference point which is the initial share/endowment. 2) Benkert (2023) considers loss aversion with respect to transfers as well. Benkert (2023) also showed that a lower subsidy would be required to implement the efficient outcome.

Literature on the partnership dissolution focuses on the ownership structure that will implement efficient outcome. Fieseler et al. (2003) with positive interdependent valuation, showed that it may not be possible to achieve ex-post efficiency even with equal ownership. It is impossible to decide whether ex-post efficient reallocation can take place or not without the knowledge of distribution of private values. Schweizer (2006) showed that the possibility result holds for all prior distributions if the ex-post efficient surplus is sufficient to cover ex-post information rents and the value of outside option at the critical valuation for all type profiles. The impossibility result is true if the ex-post efficient surplus is lower than the ex-post information rents and the value of outside option at the critical valuation for all type profiles, irrespective of prior distribution. For the rest of the cases, the possibility or impossibility result depends on the prior distribution. In a partnership setting where agent's type is private information and types are drawn from different distributions, Figueroa and Skreta (2012) try to find the ownership structure to dissolve the partnership efficiently. They showed that if the agents' critical types (types at which the gains from trade are lowest) are equal, partnership can be dissolved efficiently. When types are drawn from symmetric distribution, equal property rights guarantee equal critical valuations for agents. In the case of asymmetric distributions, equal critical types hold for extremely unequal property rights. They also show that the agents with highest valuation must have a larger share of the object in the partnership. We could not find any paper in the literature that talks about departure from quasi-linear preferences in the partnership dissolution and hence ours is the first paper to study dissolution of partnership for the efficient outcome when preferences are non-standard.

Chapter 2

Dissolving a Bilateral Partnership

2.1 Introduction

A partnership is defined as ownership of an indivisible asset by at least two individuals. There are scenarios where the partners do not wish to share the ownership of the asset together any longer due to various reasons like disputes, completion of the partnership contract or bankruptcy. As a result, the partnership is dissolved i.e., reallocation of the asset takes place through buying and selling of the shares of the asset within the partners. There are many economic problems that fit into this framework such as termination of joint-ventures, inheritance, divorce, privatizations. In these cases, the partners/agents start with some share of the asset which can be treated as an endowment and in the process of dissolution of partnership, the agents either lose their endowment or gain shares over and above endowment.

In standard preferences theory, individual's current endowment does not affect the preferences over different commodity bundles. But evidence suggests that preferences depend on endowment, which act as a reference point. (Knetsch (1992); Tversky and Kahneman (1991)) observe that disutility from losing commodities is more than the utility from gaining them. This started the study of reference dependent preferences. The fundamental intuition behind reference dependent preferences is that outcomes are not evaluated on an absolute scale, but rather evaluated relative to some point of reference and losses relative to the reference point have more weightage than commensurate gains. The most well defined general theory of this kind is explained by reference-dependence model of Tversky and Kahneman (1991), which builds on Prospect Theory of Kahneman and Tversky (1979). The "loss aversion" (people dislike losses more than they like gains compared to a reference point) assertion was shown to provide an elegant explanation for a wide variety of behavioral phenomena.

We consider a model where partnership between two loss averse agents, each holding some share (endowment) of the asset, has to be dissolved. The framework is similar to Cramton et al. (1987) except that we only look at the two agent model (bilateral partnership), consider dominant strategy incentive compatible mechanisms instead of Bayesian incentive compatible mechanisms and our agents have loss averse preferences as modeled in Köszegi and Rabin (2006, 2007) and Tversky and Kahneman (1991). Apart from the standard utility from the ownership of the good and money, called as "material utility", "gain-loss utility" with respect to ownership of the good is introduced. The reference point, relative to which agents evaluate an outcome, is the initial share of the asset. Cramton et al. (1987) is a generalised version of the bilateral trading problem considered by Myerson and Satterthwaite (1983) where one agent (seller) has the full ownership of the asset. Myerson and Satterthwaite (1983) showed that in the bilateral trading problem, there exists no mechanism which is Bayesian incentive compatible, interim individually rational, efficient and budget balanced. Cramton et al. (1987), on the other hand, finds that efficient trade is possible with a Bayesian incentive compatible, interim individually rational and a budget balanced mechanism if partners have equal shares. The model specified by Cramton et al. (1987) can be used to study the problems of economic resources. For example, Chaturvedi (2020) modeled the problem of ownership of land being shared by various landholders and a buyer who wants the entire land as a multilateral trading problem. The question that we ask is if there exists an efficient, dominant strategy incentive compatible, and budget-balanced mechanism when agents are loss averse. Green and Laffont (1979) showed that there is no efficient, dominant strategy incentive compatible, and budget-balanced mechanism, if preferences are quasi-linear. Lahkar and Mukherjee (2020) considers a model of public goods with large population and constructs a mechanism which is strictly dominant strategy incentive compatible, satisfies individual rationality and strong budget balance condition. We are interested to see whether the impossibility proved by Green and Laffont (1979) continues to hold in our model with loss averse preferences.

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(1987) and Myerson and Satterthwaite (1983) without loss aversion. Because of the gain-loss utility terms, the application of envelope theorem gives a piecewise utility function which depends on whether the agent is losing his share or gaining additional share.

Next, we look into the problem of efficiency. Benkert (2023) also studies the efficiency problem but it defines an efficient allocation rule as the one which maximizes "material valuations" of the traders. In our paper, efficient allocation is the one which maximizes the valuations of the partners (including the gain-loss utilities in ownership). In the absence of loss aversion, the allocation in which the agent with highest valuation gets the full ownership of the object is efficient. But with loss averse preferences, the object may not be allocated to an agent even if he has the highest valuation due to higher loss sensitivity associated with an allocation less than the endowment. As a result, loss averse preferences inhibit the trade as the set of valuations where trade takes place is reduced.

We find that, with loss averse agents, the bilateral partnership cannot be dissolved by a dominant strategy incentive compatible, individually rational, efficient and feasible mechanism, irrespective of the initial shares or endowments. The impossibility could be due to the way efficiency is defined in our model. The set of values at which trade should take place shrinks because of the higher sensitivity of loss associated with losing the share after the trade. The other reason is different payment functions of the agents depending on whether the final allocation is more or less than initial share, leading to violation of budget balance condition. When a partnership is dissolved, one agent receives the full ownership of the company and pay the other agent who has lost his share. The partners evaluate the gain (loss) terms of share and with loss aversion, losses loom larger than gains, inhibiting the trade. The partner who has lost his share suffers more loss in utility than the gain experienced by the partner who gets the full ownership. To compensate the loss, the agent losing his share has to be paid much more than the amount paid by the the agent gaining the ownership if he is to participate in the trade. But then differences in the transfer (payments) received (paid) by the agents violate the condition of transfer rule to be budget balanced.

The remainder of the paper is structured as follows. Section 2.2 explains the formal framework in detail. Section 2.3 states the main result. Section 2.4 presents the literature review while section 2.5 concludes. All proofs are relegated to an appendix 2.6 at the end.

2.2 Model

2.2.1 Types, Information and Preferences

Two agents, denoted by $i \in \{1, 2\}$, hold the shares of an indivisible asset. Agent 1 owns a share r_1 of the asset and agent 2 owns r_2 where $r_1 + r_2 = 1$. Valuation for the entire asset by agent i is v_i which is private information, where $v_i \in [\underline{v}, \overline{v}]$. We assume that $\underline{v} > 0$. Agents have loss averse preferences with respect to the share r_i which is acting as an endowment in this case.

Following Köszegi and Rabin (2006), preferences of the loss averse agent are represented using the following utility function.

$$\hat{u}_i(s_i, t_i | v_i, r_i) = \underbrace{s_i v_i + t_i}_{\text{material utility}} + \underbrace{\eta \mu_i \Big(s_i v_i - r_i v_i \Big)}_{\text{gain-loss utility in ownership}}$$

where

$$\mu_i = \begin{cases} 1 & \text{if } s_i \geq r_i, \\ \lambda > 1 & \text{if } s_i < r_i \end{cases}.$$

$s_i \in [0, 1]$ is the actual consumption, $r_i \in [0, 1]$ is the reference level of endowment and $t_i \in \mathbb{R}$ is transfer (payment). Per unit valuation of the asset v_i is the private information of the agent, $s_i v_i$ is the intrinsic utility of the object and $r_i v_i$ is the reference utility. The term $s_i v_i + t_i$ is the material utility that incorporates the transfers. Gain-loss utilities are considered with respect to the loss in the reference utility. The loss aversion parameters are η and λ where $\eta > 0$ captures the importance of gain-loss utility relative to intrinsic utility, and $\lambda > 1$ captures the degree of loss aversion. We assume that both the agents have identical loss aversion parameters.

Note that μ_i is an indicator function: If $s_i \geq r_i$, then there is gain of endowment and if $s_i < r_i$, then there is loss. Greater weight to loss is reflected by the fact that $\lambda > 1$.

Value of μ_i can vary across the agents as it depends on the difference between allocation received and initial endowment, even though λ is same for all agents.

2.2.2 Efficiency

An allocation rule is ex-post efficient if it maximises the sum of the valuations of the traders. In the absence of loss aversion, an allocation is efficient if the agent with the highest valuation gets the entire object. With loss averse preferences, we define the efficient allocation in the following way

For a given (v_1, v_2), an allocation $(s_1, s_2) \in [0, 1]^2$ is efficient if for every $(\hat{s}_1, \hat{s}_2) \in [0, 1]^2$

$$\sum_{i \in \{1,2\}} \Big(s_i(v_i, v_j) + \eta \mu_i (s_i(v_i, v_j) - r_i) \Big) v_i \geq \sum_{i \in \{1,2\}} \Big(\hat{s}_i(v_i, v_j) + \eta \mu_i (\hat{s}_i(v_i, v_j) - r_i) \Big) v_i$$

Benkert (2023) also studies the efficiency problem but he defines an ex-post efficient allocation rule as the one which maximizes "material valuations" of the traders i.e.

$\sum_{i\in\{1,2\}} s_i(v_i, v_j)v_i$. We incorporate gain-loss utility terms in the material utility in our definition of efficiency.

For a given $\eta > 0$, $\lambda = 1$ implies that agents are not loss averse and the allocation in which the agent with highest valuation gets the full ownership of the object is efficient i.e. $s^e(v) \in \arg\max_s \sum_{i\in\{1,2\}} \big(s_i(v_i, v_j) + \eta(s_i(v_i, v_j) - r_i)\big)v_i$. But with loss averse preferences, the object may not be allocated to an agent even if he has the highest valuation. We describe the intuition behind this below.

Maximising $\sum_{i\in\{1,2\}} \Bigg(s_i(v_i, v_j) + \eta\mu_i(s_i(v_i, v_j) - r_i)\Bigg)v_i$ is equivalent to maximizing $\sum_{i\in\{1,2\}} \Bigg((1 + \eta\mu_i)s_i(v_i, v_j) - r_i\Bigg)v_i$ because r_i is a constant and just have a scalar effect. Suppose the object is transferred to agent 1 after the trade. His valuation for the object is $(1 + \eta)r_2v_1$. Agent 2 has lost his share r_2 after the trade, generating a negative valuation of $(1 + \eta\lambda)r_2v_2$. This allocation generates pareto efficient outcome if the positive valuation generated from the object to agent 1 dominates the negative valuation of agent 2. Similarly, agent 1 is not allocated the object (or loses his share) if the negative valuation associated with the loss of his share $(1 + \eta\lambda)r_1v_1$ is less than the valuation of agent 2 after gaining full ownership of the object $(1 + \eta)r_1v_2$. If, due to transfer of full ownership, the valuation associated with gaining additional share is not sufficient enough to make up for the negative valuation of the other agent associated with losing own share, then each agent will keep his/her respective share i.e. there will be no re-allocation of the shares. With loss aversion, we are comparing the effective valuation associated with the object/shares.

PROPOSITION **2.1** *The ex-post efficient allocation rule s^e, for all i, for all v_i, for all v_j is*

$$s^e_i(v_i, v_j) = \begin{cases} 1 & \text{if } v_i \geq \left(\dfrac{1+\eta\lambda}{1+\eta}\right)v_j, \\ r_i & \text{if } \left(\dfrac{1+\eta}{1+\eta\lambda}\right)v_j < v_i < \left(\dfrac{1+\eta\lambda}{1+\eta}\right)v_j, \\ 0 & \text{if } v_i \leq \left(\dfrac{1+\eta}{1+\eta\lambda}\right)v_j. \end{cases}$$

Note that as $\eta \to 0$, the loss aversion parameters disappear and we have the standard definition of efficiency i.e. the agent with the highest material valuation will be allocated the object.

To compare the efficient allocation in the case of loss averse preferences and standard preferences, consider the following example.

EXAMPLE **2.1** *There are two agents with $v_1 = 1$, $v_2 = 0.8$ and have equal shares of the object i.e. $r_1 = r_2 = 0.5$. Suppose the agents are loss averse with parameters given as $\eta = 0.5$,*

$\lambda = 3$. *The sum of valuations of agents if* $(s_1, s_2) = (1, 0)$ *is*

$$(1 + 0.5(1 - 0.5)) + (0 + 0.5 \cdot 3(0 - 0.5))0.8 = 0.65$$

The sum of valuations of agents if $(s_1, s_2) = (0.5, 0.5)$ *is*

$$(0.5 + 0.5(0.5 - 0.5)) + (0.5 + 0.5(0.5 - 0.5))0.8 = 0.9$$

In $(s_1, s_2) = (1, 0)$, agent 1 gets the entire object implying that $\mu_1 = 1$ because he is gaining more than his endowment whereas $\mu_2 = \lambda = 3$ as agent 2 is losing his initial share. On the other hand, $(s_1, s_2) = (0.5, 0.5)$ means that trade does not take place and $\mu_1 = \mu_2 = 1$ as both agents will continue to hold their initial shares. This shows that no trade is better than in terms of efficiency.

Example **2.2** *The same environment described in Example 2.1 but the loss aversion parameters are* $\eta = 0.2$ *and* $\lambda = 2$. *The sum of valuations of agents if* $(s_1, s_2) = (1, 0)$ *is*

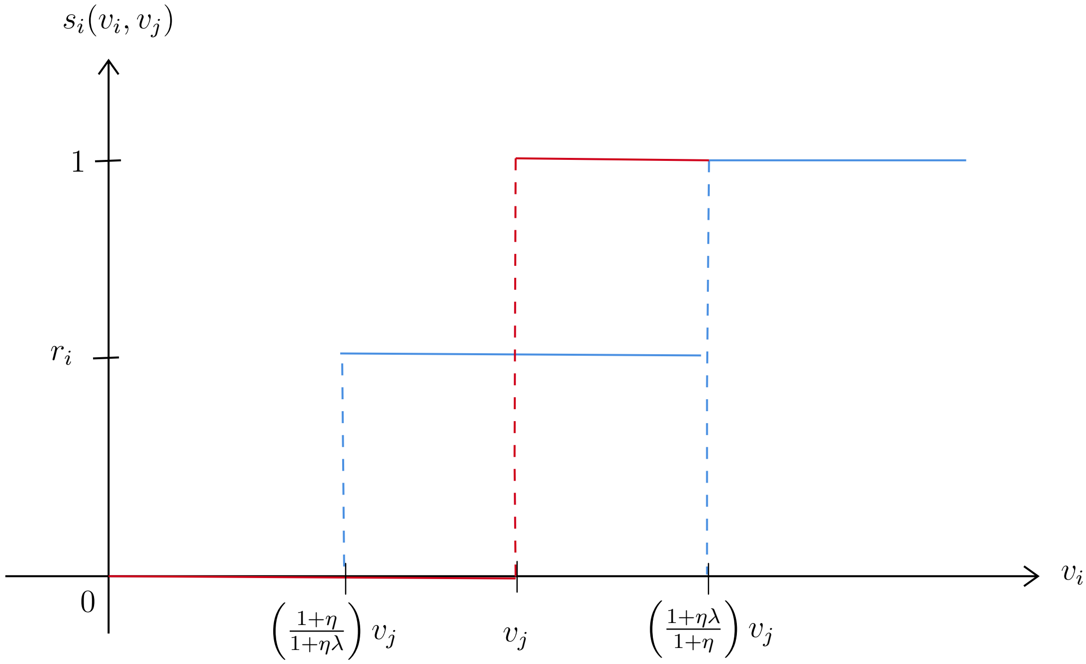
$$(1 + 0.2(1 - 0.5)) + (0 + 0.2 \cdot 2(0 - 0.5))0.8 = 0.94$$

The sum of valuations of agents if $(s_1, s_2) = (0.5, 0.5)$ *is*

$$(0.5 + 0.2(0, 5 - 0.5)) + (0.5 + 0.2(0.5 - 0.5))0.8 = 0.9$$

In this case, trading between the agents is an efficient outcome.

Had there been no loss aversion, agent 1 would have got the object as he has the highest valuation. But due to loss aversion, the trade does not take place in the first example (agents keep their shares r_1 and r_2) despite the fact that agent 1 values the object more than agent 2. The second example shows that the changing the values of the parameters of loss aversion while keeping the valuations and shares of the agents same as given in Example 2.1, changes the outcome from no trade to trade where agent 1 gets the entire object.



The graph depicts the efficient allocation for agent i as a function of v_i for a given value of v_j. The efficient allocation in case of loss averse agents is shown in blue. For comparison, the efficient allocation in case of standard preferences is also shown (depicted in red). One can see from the graph that the region of the trade is reduced. For $v_i \in \left[\left(\frac{1+\eta}{1+\eta\lambda}\right)v_j, \left(\frac{1+\eta\lambda}{1+\eta}\right)v_j\right]$, there is no trade as agents keep their respective shares. Therefore, trade with loss averse agents is restricted in comparison to the standard case.

The conditions for the efficient allocation is similar to the one mentioned in Tversky and Kahneman (1991). If ν_1 and ν_2 denote the intrinsic valuation for the two goods, namely good 1 and good 2, then people endowed with good 1 will trade for good 2 if $\nu_2 > \left(\frac{1+\eta\lambda}{1+\eta}\right)\nu_1$. On the other people endowed with good 2 (no endowment for good 1) will trade for good 1 if $\nu_2 > \left(\frac{1+\eta}{1+\eta\lambda}\right)\nu_1$. Loss aversion implies that the agent, who has to give up his share, has the highest cutoff and therefore, is least likely to trade. On the other hand, the agent who will gain the additional share has the lowest cutoff and is most likely to trade.

2.2.3 Mechanism

A direct revelation mechanism consists of four maps (s_1, s_2, t_1, t_2) where, $s_i : [\underline{v}, \overline{v}]^2 \longrightarrow [0, 1]$ with $s_1(v_1, v_2) + s_2(v_1, v_2) \leq 1, \quad \forall v \in [\underline{v}, \overline{v}]^2$ and $t_i : [\underline{v}, \overline{v}]^2 \longrightarrow \mathbb{R}$. The term $s_i(v_i, v_j)$ is the allocation share of agent i and t_i is the transfer (payment) received (paid) by agent i. The agents directly report their valuations for the object, and then receive the share of the ownership $s_i(v_i, v_j)$ and the transfers $t_i(v_i, v_j)$.

Utility function of agent i from the mechanism when other agent reports v_j is

$$\hat{u}_i(v_i, v_j) = s_i v_i + t_i + \eta\mu_i\Bigg(s_i(v_i, v_j)v_i - r_i v_i\Bigg) \quad \forall v_i \in [\underline{v}, \overline{v}]$$

Define $s_i^{ref}(v_i, v_j)$ as the modified allocation where

$$s_i^{ref}(v_i, v_j) = s_i(v_i, v_j) + \eta\mu_i\Bigg(s_i(v_i, v_j) - r_i\Bigg)$$

This allows us to compactly write $\hat{u}_i(v_i, v_j) = s_i^{ref}(v_i, v_j)v_i + t_i(v_i, v_j)$. Net payoff from the mechanism, denoted as $u_i(v_i, v_j)$, is defined as the difference between the utility from the trade $\hat{u}_i(v_i, v_j)$ and the reference utility $r_i v_i$. This net utility is useful for the analysis.

$$u_i(v_i, v_j) = \Bigg(s_i^{ref}(v_i, v_j) - r_i\Bigg)v_i + t_i(v_i, v_j)$$

Following are the required properties of the mechanism.

DEFINITION **2.1** *The mechanism (s_1, s_2, t_1, t_2) is* ***dominant strategy incentive compatible (DSIC)*** *if for all i and for every $v_i, \hat{v}_i \in [\underline{v}, \overline{v}]$,*

$$u_i(v_i, v_j) \geq \Bigg(s_i^{ref}(\hat{v}_i, v_j) - r_i\Bigg)v_i + t_i(\hat{v}_i, v_j) \quad \forall v_j \in [\underline{v}, \overline{v}]$$

This condition ensures that the agents report their valuations for the asset truthfully.

DEFINITION **2.2** *The mechanism (s_1, s_2, t_1, t_2) is* ***ex-post individually rational*** *if for all i and for all $v_i \in [\underline{v}, \overline{v}]$,*

$$u_i(v_i, v_j) \geq 0 \quad \forall v_j \in [\underline{v}, \overline{v}]$$

DEFINITION **2.3** *A payment rule (t_1, t_2) is* ***feasible*** *if*

$$t_1(v_1, v_2) + t_j(v_1, v_2) \leq 0 \quad \forall v_1, v_2 \in [\underline{v}, \overline{v}]$$

.

The following lemma develops a necessary and sufficient condition for a mechanism to be DSIC.

LEMMA **2.1** *A mechanism (s_1, s_2, t_1, t_2) is DSIC if and only if for each i*

1. *$(1+\eta)(s_i(v_i, v_j) - r_i)$ is non-decreasing in v_i,*

2. *$(1+\eta\lambda)(r_i - s_i(v_i, v_j))$ is non-increasing in v_i and*

3. for i and j, for all $v_j \in [\underline{v}, \overline{v}]$, and for all $v_i \in [\underline{v}, \overline{v}]$

$$u_i(v_i, v_j) = \begin{cases} u_i(v_i^*, v_j) + \int_{v_i^*}^{v_i} (1+\eta)(s_i(x_i, v_j) - r_i)dx_i & \text{if } v_i \in [v_i^*, \overline{v}], \\ u_i(v_i^*, v_j) + \int_{v_i^*}^{v_i} (1+\eta\lambda)(s_i(x_i, v_j) - r_i)dx_i & \text{if } v_i \in [\underline{v}, v_i^*]. \end{cases} \tag{2.1}$$

where v_i^* solves $s_i^{ref}(v_i^*, v_j) - r_i = 0$

For any allocation (s_1, s_2) such that $s_i^{ref}(v_i, v_j)$ is non-decreasing for all i, v_i^* is called the worst-off types of agent i. Equation (2.1) implies that net utility $u_i(v_i, v_j)$ is continuous and convex in v_i and is minimized at v_i^*. At v_i^*, $s_i(v_i, v_j) - r_i = 0$ which means that v_i^* is the valuation at which the agent i will not trade and will keep his own share of the asset. A worst-off type is neither a buyer nor a seller of the good. For a given $\eta > 0$, $\lambda = 1$ indicates there is no loss aversion and $s_i^{ref}(v_i, v_j) - r_i = (1+\eta)(s_i(v_i, v_j) - r_i)$. Application of envelope theorem (Milgrom and Segal (2002)) for $\lambda = 1$ results in the characterization similar to Cramton et al. (1987) with derivative of the utility function equal to $s_i^{ref}(v_i, v_j) - r_i = (1+\eta)(s_i(v_i, v_j) - r_i)$ almost everywhere. If $r_1 = 0$ and $r_2 = 1$, then the model is the same as standard bilateral trade setting considered by Myerson and Satterthwaite (1983) where agent 1 is buyer with $v_1^* = \underline{v}$ and agent 2 is seller with $v_2^* = \overline{v}$. When $\lambda > 1$, derivative of the utility function depends on the worst off type. For $v_i > v_i^*$, slope of utility function of agent i will be $(1+\eta)(s_i(v_i, v_j) - r_i)$ whereas for $v_i < v_i^*$, slope will be $(1+\eta\lambda)(s_i(v_i, v_j) - r_i)$. Applying envelope theorem gives a piecewise-defined functional form for utility. Note that Cramton et al. (1987) characterize the set of Bayesian incentive compatible mechanisms. Eisenhuth (2018) and Benkert (2023) consider endogenous reference points formed using rational expectations. The characterization of the incentive compatible mechanisms is similar to Myerson and Satterthwaite (1983) except that the expected allocation in these papers incorporate gain-loss sensitivity terms. Note that as $\eta \to 0$, $s_i^{ref}(v_i, v_j)$ approaches $s_i(v_i, v_j)$ and the characterization result defined in lemma 2.1 approaches the characterization result of Cramton et al. (1987).

Using Lemma 2.1, a transfer function that implements the efficient allocation is as follows:

$$t_i(v_i, v_j) = \begin{cases} -r_j(1+\eta\lambda)v_j & \text{if } i \text{ wins the object,} \\ 0 & \text{if } i \text{ gets his original share } r_i, \\ r_i(1+\eta)v_j & \text{if agents } i \text{ gets nothing.} \end{cases}$$

for all i, for all v_i and for all v_j.

This is a generalized transfer function which incorporates the case of standard preferences. For $\eta > 0$ if the agents are not loss averse ($\lambda = 1$), efficiency requires that the person who gets the object needs to pay the other agent his reservation utility. Specifically, if agent 1 wins the object, he needs to pay agent 2 his utility he was getting before the trade i.e.

$r_2(1+\eta)v_2$ to compensate for the loss. If one agent has the entire ownership, for example, $r_1 = 0$ and $r_2 = 1$, then then we have the case of standard bilateral trade problem where, if the object is transferred to agent 1, he needs to pay agent 2 his reservation utility which is $(1+\eta)v_j$, which is equivalent to payment rule of the second-price auction.

We are now prepared to answer the central question of this paper: What partnerships can be dissolved with an DSIC, individually rational and feasible mechanism?

2.3 Impossibility Result

The main negative result of this paper is as follows: If agents are loss averse, then partnership cannot be dissolved by a DSIC, ex-post efficient, ex-post individually rational and feasible mechanism.

Theorem **2.1** *There exists no mechanism (irrespective of initial shares) satisfying, DSIC, efficiency, individual rationality and feasibility.*

The impossibility result is true for any shares (endowments) of the agents. This contrasts with the Cramton et al. (1987) result that the equal shares ($r_1 = r_2 = 0.5$) partnership can be dissolved by a budget-balanced, Bayesian incentive compatible and interim individually rational mechanism. Note that for η small and λ equal to 1, the ex-post efficient allocation in our model is the same as defined in Cramton et al. (1987) and we are close to the possibility result of Cramton et al. (1987).

The impossibility is due to the way efficiency is defined in our model. We are trying to implement an allocation which maximizes the overall valuations of the agents (including gain-loss terms). This reduces the set of values at which trade should take place. The presence of loss aversion parameters restrict the trade because of the higher sensitivity of loss associated with losing the share after the trade.

The other reason is different payment functions of the agents depending on whether the final allocation is more or less than initial share, leading to violation of feasibility. When a partnership is dissolved, one agent receives the full ownership of the company and pay the other agent who has lost his share. The partners evaluate the gain (loss) terms of share and with loss aversion, losses loom larger than gains, inhibiting the trade. The partner who has lost his share suffers more loss in utility than the gain experienced by the partner who gets the full ownership. To compensate the loss, the agent losing his share has to be paid much more than the amount paid by the the agent gaining the ownership if he is to participate in the trade. But then differences in the transfer (payments) received (paid) by the agents violate the condition of payment rule to be budget balanced. As the loss sensitivity increases

(λ increases), the disparity in the transfers between the agents keep on increasing, thereby restricting the trade.

2.4 Literature Review

2.4.1 Literature on loss averse preferences

There has been a growing literature on how loss aversion makes economic agents behave differently. Endowment effect was one of the early examples of loss aversion, frequently observed in both experimental and survey research. Thaler (1980) came up with the phenomena of endowment effect, based on the observation that buyers are willing to pay less to buy goods compared to the price at which the owners are willing to sell. The reason for the disparity is explained by loss aversion. Since owners are losing the object, the disutility due to loss in the endowment is more than the gain buyers experience. As a result, the sellers needs to be compensated by an amount which is larger than what the buyers are willing to pay. Experimental evidence of endowment effect were also shown in Knetsch (1989), Kahneman et al. (1990) and Tversky and Kahneman (1992). Other candidates for the reference point has been mentioned by the researchers e.g. expectations. Köszegi and Rabin (2006, 2007) assumed that the reference point is determined endogenously using rational expectations

The phenomena of loss aversion has been used to explain real-world phenomenon. Camerer et al. (1997) observed negative wage elasticity in the study of New York cab drivers. The explanation is that the cab drivers are "loss-averse" around an income target. The well known "equity premium puzzle" (the huge disparity between equity and bond return) can be potentially explained by reference dependent loss aversion (Benartzi and Thaler (1995)). Benartzi and Thaler (1995) propose that individuals experience utility from financial-portfolio statement compared to the prior statement. If the intensity of losses is high compared to the gains, then individuals will display increased risk-aversion in portfolio choice. Odean (1998) studied a sample of retail investors and observed that sellers tend to sell assets whose valuation has increased and keep the assets whose valuation has decreased (disposition effect). As a result, there has been a growing interest in the application of loss averse preferences in the mechanism design.

2.4.2 Literature on Mechanism Design with loss averse preferences

There is a small literature on incorporation of loss averse preferences in the study of mechanism design. Lange and Ratan (2010) compares the first price auction with the second price auction when agents are loss averse and finds that the expected revenue was higher under the first price auction as compared to the second price auction. Shunda (2009) considered a different notion of reference dependence and showed that expected revenue can be increased using buy-now price. Eisenhuth (2018) considers the problem of designing optimal auction for loss averse agents and he used two forms of utility functions: (a), gains and losses are evaluated in the good dimension and in the money dimension separately and (b), gains and losses are evaluated over the entire risk neutral utility. In the first case, an all pay auction with a minimum bid is an optimal auction and in the second one, first price auction with minimum bid is an optimal auction.

Our paper also contributes on the literature on bilateral trade with loss averse agents. A long line of literature has focused on the ex-post efficiency of the mechanism in the bilateral trade and partnership setting. While Myerson and Satterthwaite (1983) showed the impossibility to implement an efficient outcome through a Bayesian incentive compatible and interim individually rational mechanism without a deficit. Cramton et al. (1987), on the other hand, showed the possibility result if the agents have symmetric property rights. A shortcoming of the approach used in these papers is the assumption that all agents have priors over the possible valuation of the object and the priors are a common knowledge. Hagerty and Rogerson (1987) characterises the class of mechanisms considered in Myerson and Satterthwaite (1983) which are DSIC, individually rational and budget balance and they find that the posted price mechanism is the only mechanism which satisfies all the above mentioned properties. Loss aversion in the bilateral trade model is introduced by Benkert (2023) by adapting the framework of Eisenhuth (2018) in Myerson and Satterthwaite (1983) setting and studies the impact of endowment effect and attachment effect (the agent who does not have the object can get attached to it which increases his valuation for the asset) on the agents' information rents. The impossibility result still persists although impossibility is less severe due to buyer's loss aversion.

2.5 Conclusion

This paper tries to find the existence of an incentive compatible and feasible mechanism to dissolve a partnership between loss averse agents in an efficient way. First, we characterize the set of all dominant strategy incentive compatible dissolution mechanisms for a partnership model. We also showed how the parameters of loss aversion affect the efficient allocation. Because of the loss averse preferences, there is a disparity in the transfer to be received by selling his share and the transfer to be paid while getting the full ownership of the asset, thereby inhibiting the trade. Hence, the impossibility result shown by Green and Laffont (1979) continues to hold for loss averse agents.

Cramton et al. (1987) showed the existence of a dissolution mechanism that is "Bayesian" incentive compatible and budget balanced to implement an efficient allocation, if the partnership is centred around equal shares. It will be interesting to see if this result will hold even in the case of loss averse preferences. Therefore, one possible extension of this paper is to relax the condition of dominant strategy incentive compatible and look for the possibility of trade in the class of Bayesian incentive compatible mechanisms.

2.6 Appendix

Proof of Lemma 2.1

Proof: Necessity: Suppose that the mechanism (s_1, s_2, t_1, t_2) is DSIC. Then,

$$u_i(v_i, v_j) \geq u_i(\hat{v}_i, v_j) + (s_i^{ref}(\hat{v}_i, v_j) - r_i)(v_i - \hat{v}_i) \tag{2.2}$$

which gives

$$u_i(v_i, v_j) - u_i(\hat{v}_i, v_j) \geq (s_i^{ref}(\hat{v}_i, v_j) - r_i)(v_i - \hat{v}_i)$$

Exchanging the roles of v_i and $\hat{v}_i$

$$u_i(\hat{v}_i, v_j) \geq u_i(v_i, v_j) + (s_i^{ref}(v_i, v_j) - r_i)(\hat{v}_i - v_i)$$

This implies

$$u_i(v_i, v_j) - u_i(\hat{v}_i, v_j) \leq (s_i^{ref}(v_i, v_j) - r_i)(v_i - \hat{v}_i) \tag{2.3}$$

(2.2) and (2.3) together imply that

$$(s_i^{ref}(v_i, v_j) - r_i)(v_i - \hat{v}_i) \geq u_i(v_i, v_j) - u_i(\hat{v}_i, v_j) \geq (s_i^{ref}(\hat{v}_i, v_j) - r_i)(v_i - \hat{v}_i) \tag{2.4}$$

This shows that if $v_i > \hat{v}_i$, $s_i^{ref}(v_i, v_j) - r_i \geq s_i^{ref}(\hat{v}_i, v_j) - r_i$. Therefore, $s_i^{ref}(\cdot, v_j) - r_i$ is non-decreasing.

Claim **2.1** *$u_i(\cdot, v_j)$ is Lipschitz continuous.*

Proof: To prove that $u_i(\cdot, v_j)$ is Lipschitz continuous, we need to show that there exists $M > 0$, such that

$$|u_i(v_i, v_j) - u_i(\hat{v}_i, v_j)| \leq M|v_i - \hat{v}_i|$$

If $v_i > \hat{v}_i$,

$$u_i(v_i, v_j) - u_i(\hat{v}_i, v_j) \leq (s_i^{ref}(v_i, v_j) - r_i)(v_i - \hat{v}_i) \leq (1 + \eta)(1 - r_i)(v_i - \hat{v}_i)$$

If $v_i < \hat{v}_i$,

$$u_i(v_i, v_j) - u_i(\hat{v}_i, v_j) \geq (s_i^{ref}(\hat{v}_i) - r_i)(v_i - \hat{v}_i)$$

which can also be written as

$$\begin{aligned} -(u_i(v_i, v_j) - u_i(\hat{v}_i, v_j)) &\leq -(s_i^{ref}(\hat{v}_i, v_j) - r_i)(v_i - \hat{v}_i) \\ &\leq -(s_i^{ref}(v_i, v_j) - r_i)(v_i - \hat{v}_i) \end{aligned}$$

Therefore,

$$\begin{aligned} |u_i(v_i, v_j) - u_i(\hat{v}_i, v_j)| &\leq (s_i^{ref}(v_i, v_j) - r_i)|(v_i - \hat{v}_i)| \\ &\leq (1 + \eta)(1 - r_i)|(v_i - \hat{v}_i)| \end{aligned}$$

For $M = (1 + \eta)(1 - r_i)$, we have proved that $|u_i(v_i, v_j) - u_i(\hat{v}_i, v_j)| \leq M|v_i - \hat{v}_i|$. Hence, $u_i(\cdot, v_j)$ is Lipschitz continuous. ■

This means that $u_i(\cdot, v_j)$ is differentiable almost everywhere. From equation (2.4), we have

$$s_i^{ref}(v_i, v_j) - r_i \geq \frac{u_i(v_i, v_j) - u_i(\hat{v}_i, v_j)}{v_i - \hat{v}_i} \geq s_i^{ref}(\hat{v}_i, v_j) - r_i$$

This implies

$$\frac{du_i(v_i, v_j)}{dv_i} = s_i^{ref}(v_i, v_j) - r_i$$

and

$$u_i(v_i, v_j) = u_i(v_i^*, v_j) + \int_{v_i^*}^{v_i} (s_i^{ref}(x_i, v_j) - r_i)dx_i, \tag{2.5}$$

$\forall v_i, v_i^* \in [\underline{v}, \overline{v}]$, where v_i^* satisfies $s_i^{ref}(v_i^*, v_j) - r_i = 0$.
Substituting the expression for $u_i(v_i, v_j)$ in the above equation gives

$$(s_i^{ref}(v_i, v_j) - r_i)v_i + t_i(v_i, v_j) = u_i(v_i^*, v_j) + \int_{v_i^*}^{v_i} (s_i^{ref}(x_i, v_j) - r_i)dx_i$$

which can be rewritten as

$$t_i(v_i, v_j) = u_i(v_i^*, v_j) - (s_i^{ref}(\cdot, v_j) - r_i)v_i + \int_{v_i^*}^{v_i} (s_i^{ref}(x_i, v_j) - r_i)dx_i$$

If (s_1, s_2, t_1, t_2) is DSIC, then, according to equation (2.5), i's net utility function $u_i(v_i, v_j)$ is increasing on $v_i \in [v_i^*, \overline{v}]$ and decreasing on $v_i \in [\underline{v}, v_i^*]$. This means that $u_i(v_i, v_j)$ is minimized at v_i^*. Because the worst off type of an agent is revealed once the true valuations of all agents are known, the worst off type v_i^* depends on v_j i.e. valuation of the other agent. As v_j changes, the worst type of agent i changes. This means v_i^* is a function of v_j i.e.

$v_i^* = g(v_j)$ where $g : [\underline{v}, \overline{v}] \longrightarrow [\underline{v}, \overline{v}]$ is a function which defines the worst off type of agent i on v_j.

Given the mechanism (s_1, s_2, t_1, t_2) is DSIC, the net utility of agent i is minimum if he does not participate in the trade i.e. v_i^* satisfy $s_i(v_i^*, v_j) - r_i = 0$. This implies that he is neither a buyer nor a seller. Also, $s_i(v_i^*, v_j) - r_i = 0 \implies s_i^{ref}(v_i^*, v_j) - r_i = 0$.

This shows $s_i^{ref}(v_i, v_j) - r_i = (1+\eta)(s_i(v_i, v_j) - r_i) \geq 0$ for $v_i \geq v_i^*$ and $s_i^{ref}(v_i, v_j) - r_i = (1+\eta\lambda)(s_i(v_i, v_j) - r_i) \leq 0$ for $v_i \leq v_i^*$. Therefore, The set of worst off types is $V_i^*(v_j) = \{g(v_j) | s_i^{ref}(u, v_j) - r_i \leq 0 \quad \forall u \leq g(v_j); s_i^{ref}(u, v_j) - r_i \geq 0 \quad \forall u \geq g(v_j)\}$.

We showed that $s_i^{ref}(\cdot, v_j) - r_i$ is non-decreasing. For $v_i \geq v_i^*$, $s_i^{ref}(v_i, v_j) - r_i = (1+\eta)(s_i(v_i, v_j) - r_i) \geq 0$ which is non-decreasing in v_i. For $v_i \leq v_i^*$, $s_i^{ref}(v_i, v_j) - r_i = (1+\eta\lambda)(s_i(v_i, v_j) - r_i) \leq 0$ which is non-increasing in v_i.

Equation (2.5) can be re-written as

$$u_i(v_i, v_j) = \begin{cases} u_i(v_i^*, v_j) + \int_{v_i^*}^{v_i} (1+\eta)(s_i(x_i, v_j) - r_i) dx_i & \forall v_i \in [v_i^*, \overline{v}], \\ u_i(v_i^*, v_j) + \int_{v_i}^{v_i^*} (1+\eta\lambda)(r_i - s_i(x_i, v_j)) dx_i & \forall v_i \in [\underline{v}, v_i^*]. \end{cases}$$

Sufficiency: Suppose that the mechanism (s_1, s_2, t_1, t_2) is such that $(1+\eta)(s_i(v_i, v_j) - r_i)$ is non-decreasing, $(1+\eta\lambda)(r_i - s_i(v_i, v_j))$ in non-increasing and $u_i(v_i, v_j)$ satisfies (2.1).

Now, $(1+\eta)(s_i(v_i, v_j) - r_i)$ is $s_i^{ref}(v_i, v_j) - r_i$ for $v_i \in [v_i^*, \overline{v}]$ and $(1+\eta\lambda)(r_i - s_i(v_i, v_j))$ is $s_i^{ref}(v_i, v_j) - r_i$ for $v_i \in [\underline{v}, v_i^*]$. It is equivalent to say that $s_i^{ref}(v_i, v_j) - r_i$ is non-decreasing and (2.1) can be written as

$$u_i(v_i, v_j) = u_i(v_i^*, v_j) + \int_{v_i^*}^{v_i} (s_i^{ref}(x_i, v_j) - r_i) dx_i,$$

$\forall v_i, v_i^* \in [\underline{v}, \overline{v}]$, where v_i^* satisfies $s_i^{ref}(v_i^*, v_j) - r_i = 0$.

Note that,

$$\begin{aligned} u_i(v_i, v_j) - u_i(v_i^*, v_j) = \int_{v_i^*}^{v_i} (s_i^{ref}(x_i, v_j) - r_i) dx_i &\geq \int_{v_i^*}^{v_i} (s_i^{ref}(v_i^*, v_j) - r_i) dx_i \\ &= (s_i^{ref}(v_i^*, v_j) - r_i)(v_i - v_i^*) \end{aligned}$$

Therefore,

$$u_i(v_i, v_j) \geq u_i(v_i^*, v_j) + (s_i^{ref}(v_i^*, v_j) - r_i)(v_i - v_i^*)$$

Substituting the expression for $u_i(v_i^*, v_j)$ in the above equation gives

$$u_i(v_i, v_j) \geq (s_i^{ref}(v_i^*, v_j) - r_i) v_i + t_i(v_i^*, v_j)$$

Hence, (s_1, s_2, t_1, t_2) is DSIC. ■

Proof of Proposition 2.1

Proof: The efficient allocation rule s^e is the one which maximizes the sum of valuations for the object by the agents. In particular, for every $v \in [\underline{v}, \overline{v}]^2$

$$s^e(v) \in \arg\max \sum_{i \in \{1,2\}} (s_i(v_i, v_j) + \eta\mu_i(s_i(v_i, v_j) - r_i))v_i$$

Consider three allocation rules $\overline{s} = (\overline{s}_1, \overline{s}_2)$, $s^* = (s^*_1, s^*_2)$ and $\tilde{s} = (\tilde{s}_1, \tilde{s}_2)$ where

- $\overline{s}_1 > r_1$ in $\overline{s}$
- $\tilde{s_1} < r_1$ in $\tilde{s}$
- $s^*_1 = r_1$ in s^*

We look at each case and find out the sufficient condition for each case to be efficient. Sum of the valuations is denoted by z.

$$\begin{aligned} z &= (s_1 + \eta\mu_1(s_1 - r_1))v_1 + (s_2 + \eta\mu_2(s_2 - r_2))v_2 \\ &= s_1 v_1(1 + \eta\mu_1) + s_2 v_2(1 + \eta\mu_2) - \eta\mu_1 r_1 v_1 - \eta\mu_2 r_2 v_2 \end{aligned}$$

- If $\overline{s}_1 > r_1$, then $s_2 < r_2$ since $r_1 + r_2 = 1$. So, $\mu_1 = 1$ and $\mu_2 = \lambda$. Hence,

  $$\overline{z} = s_1 v_1(1 + \eta) + s_2 v_2(1 + \eta\lambda) - \eta r_1 v_1 - \eta\lambda r_2 v_2$$

  If $v_1(1 + \eta) > v_2(1 + \eta\lambda)$ or $v_1 > \left(\frac{1 + \eta\lambda}{1 + \eta}\right) v_2$, then by point-wise maximization, set the maximum value to s_1 i.e. $s_1 = 1$. If $s_1 = 1$, then $s_2 = 0$ because $s_1 + s_2$ cannot be greater than 1.
  Then,

  $$\overline{z} = v_1(1 + \eta) - \eta r_1 v_1 - \eta\lambda r_2 v_2$$

- If $\tilde{s_1} < r_1$, then $s_2 > r_2$. So, $\mu_1 = \lambda$ and $\mu_2 = 1$. Hence,

  $$\tilde{z} = s_1 v_1(1 + \eta\lambda) + s_2 v_2(1 + \eta) - \eta\lambda r_1 v_1 - \eta r_2 v_2$$

  If $v_1(1 + \eta\lambda) < v_2(1 + \eta)$ or $v_1 < \left(\frac{1 + \eta}{1 + \eta\lambda}\right) v_2$, then by point-wise maximization, set the maximum value to s_2 i.e. $s_2 = 1$. If $s_2 = 1$, then $s_1 = 0$ because $s_1 + s_2$ cannot be greater than 1.
  Therefore,

  $$\tilde{z} = v_2(1 + \eta) - \eta\lambda r_1 v_1 - \eta r_2 v_2$$

- If $s_1^* = r_1$, then $s_2 = r_2$ since $r_1 + r_2 = 1$. So, $\mu_1 = 1$ and $\mu_2 = 1$. Hence,

$$\begin{aligned} z^* &= s_1 v_1(1+\eta) + s_2 v_2(1+\eta) - \eta r_1 v_1 - \eta r_2 v_2 \\ &= r_1 v_1 + r_2 v_2 \end{aligned}$$

Now, compare $\overline{z}$, $\tilde{z}$ and z^* with one another to find the sufficiency condition for each case to be efficient.

- **Finding the case when $s_1 > r_1$ or $s_1 = 1$ is efficient**
  Compare $\overline{z}$ and $\tilde{z}$

$$\overline{z} - \tilde{z} = v_1(1+\eta) - \eta r_1 v_1 - \eta\lambda r_2 v_2 - v_2(1+\eta) + \eta\lambda r_1 v_1 + \eta r_2 v_2$$

  Adding and subtracting $v_2(1+\eta\lambda)$

$$\begin{aligned} \overline{z} - \tilde{z} &= v_1(1+\eta) - v_2(1+\eta\lambda) + v_2(1+\eta\lambda) - \eta r_1 v_1 - \eta\lambda r_2 v_2 - v_2(1+\eta) + \eta\lambda r_1 v_1 + \eta r_2 v_2 \\ &= v_1(1+\eta) - v_2(1+\eta\lambda) + v_2(1+\eta\lambda) - v_2(1+\eta) - \eta r_1 v_1 + \eta\lambda r_1 v_1 + \eta r_2 v_2 - \eta\lambda r_2 v_2 \\ &= v_1(1+\eta) - v_2(1+\eta\lambda) + v_2\eta(\lambda-1) + \eta r_1 v_1(\lambda-1) - \eta r_2 v_2(\lambda-1) \end{aligned}$$

  Use $r_1 + r_2 = 1$

$$\overline{z} - \tilde{z} = v_1(1+\eta) - v_2(1+\eta\lambda) + v_2 r_1 \eta(\lambda-1) + \eta r_1 v_1(\lambda-1)$$

  $\overline{z} > \tilde{z}$ if $v_1(1+\eta) > v_2(1+\eta\lambda)$ or $v_1 > \left(\dfrac{1+\eta\lambda}{1+\eta}\right) v_2$.
  Compare $\overline{z}$ and z^*

$$\begin{aligned} \overline{z} - z^* &= v_1(1+\eta) - \eta r_1 v_1 - \eta\lambda r_2 v_2 - r_1 v_1 - r_2 v_2 \\ &= v_1(1+\eta) - r_1 v_1(1+\eta) - r_2 v_2(1+\eta\lambda) \\ &= (1-r_1)v_1(1+\eta) - r_2 v_2(1+\eta\lambda) \\ &= r_2 v_1(1+\eta) - r_2 v_2(1+\eta\lambda) \\ &= r_2(v_1(1+\eta) - v_2(1+\eta\lambda)) \end{aligned}$$

  $\overline{z} > z^*$ if $v_1(1+\eta) > v_2(1+\eta\lambda)$ or $v_1 > \left(\dfrac{1+\eta\lambda}{1+\eta}\right) v_2$.
  Hence, $s_1 = 1$ and $s_2 = 0$ is efficient if $v_1 > \left(\dfrac{1+\eta\lambda}{1+\eta}\right) v_2$.

- **Finding the case when $\tilde{s}_1 < r_1$ or $s_1 = 0$ maximizes the sum of the valuations.**
  Compare $\tilde{z}$ and $\overline{z}$

$$\tilde{z} - \overline{z} = v_2(1+\eta) - \eta\lambda r_1 v_1 - \eta r_2 v_2 - v_1(1+\eta) + \eta r_1 v_1 + \eta\lambda r_2 v_2$$

Adding and subtracting $v_1(1+\eta\lambda)$

$$\begin{aligned}
\tilde{z}-\overline{z} &= v_2(1+\eta)-v_1(1+\eta\lambda)+v_1(1+\eta\lambda)-\eta\lambda r_1v_1-\eta r_2v_2-v_1(1+\eta)+\eta r_1v_1+\eta\lambda r_2v_2\\
&= v_2(1+\eta)-v_1(1+\eta\lambda)+v_1\eta(\lambda-1)-r_1v_1\eta(\lambda-1)+\eta r_2v_2(\lambda-1)\\
&= v_2(1+\eta)-v_1(1+\eta\lambda)+(1-r_1)\eta(\lambda-1)v_1+\eta r_2v_2(\lambda-1)\\
&= v_2(1+\eta)-v_1(1+\eta\lambda)+r_2\eta(\lambda-1)v_1+\eta r_2v_2(\lambda-1)
\end{aligned}$$

$\tilde{z}>\overline{z}$ if $v_1(1+\eta\lambda)<v_2(1+\eta)$ or $v_1<\left(\dfrac{1+\eta}{1+\eta\lambda}\right)v_2$.

Compare $\tilde{z}$ and z^*

$$\begin{aligned}
\tilde{z}-z^* &= v_2(1+\eta)-\eta\lambda r_1v_1-\eta r_2v_2-r_1v_1-r_2v_2\\
&= v_2(1+\eta)-r_1v_1(1+\eta\lambda)-r_2v_2(1+\eta\lambda)\\
&= (1-r_1)v_2(1+\eta)-r_2v_2(1+\eta\lambda)\\
&= r_2(v_2(1+\eta)-v_2(1+\eta\lambda))
\end{aligned}$$

$\tilde{z}>z^*$ if $v_1(1+\eta\lambda)<v_2(1+\eta)$ or $v_1<\left(\dfrac{1+\eta}{1+\eta\lambda}\right)v_2$.

Therefore, $s_1=0$ and $s_2=1$ if $v_1<\left(\dfrac{1+\eta}{1+\eta\lambda}\right)v_2$.

- **Finding the case when $s^*=r_1$ is efficient.**

  Compare z^* and $\overline{z}$

$$\overline{z}-z^*=r_2(v_1(1+\eta)-v_2(1+\eta\lambda))$$

$\overline{z}<z^*$ if $v_1(1+\eta)<v_2(1+\eta\lambda)$ or $v_1<\left(\dfrac{1+\eta\lambda}{1+\eta}\right)v_2$.

Compare z^* and $\tilde{z}$

$$\tilde{z}-z^*=r_2(v_2(1+\eta)-v_2(1+\eta\lambda))$$

$\tilde{z}<z^*$ if $v_1(1+\eta\lambda)>v_2(1+\eta)$ or $v_1>\left(\dfrac{1+\eta}{1+\eta\lambda}\right)v_2$.

Hence, $s_1=r_1$ and $s_2=r_2$ if $\left(\dfrac{1+\eta\lambda}{1+\eta}\right)v_2>v_1>\left(\dfrac{1+\eta}{1+\eta\lambda}\right)v_2$

■

Proof of Theorem 2.1

Proof: We will prove this by contradiction. Let $M=(s,t)$ be any DSIC, feasible and efficient mechanism. Lemma 2.1 implies that for any type profile (v_1,v_2) with $v_1>\left(\dfrac{1+\eta\lambda}{1+\eta}\right)v_2$, we have

$$t_1(v_1, v_2) = u_1(v_1^*, v_2) - (1+\eta)(1-r_1)v_1 + \int_{v_1^*}^{v_i} (1+\eta)(1-r_1)dx_i$$
$$= u_1(v_1^*, v_2) - (1+\eta)(1-r_1)v_1 + (1+\eta)(1-r_1)\left(v_1 - \left(\frac{1+\eta\lambda}{1+\eta}\right)v_2\right)$$
$$= u_1(v_1^*, v_2) - r_2(1+\eta\lambda)v_2 \quad \because r_1 + r_2 = 1$$

$u_1(v_1^*, v_2) = 0$ for all values of v_2 because agent 1 is at worst off type and hence, he will not take part in the trade. Similarly,

$$t_2(v_1, v_2) = u_2(v_1, v_2^*) - (1+\eta\lambda)(0-r_2)v_2 + \int_{v_2}^{v_2^*} (1+\eta\lambda)(r_2-0)dx_i$$
$$= u_2(v_1, v_2^*) + (1+\eta\lambda)r_2v_2 + (1+\eta\lambda)r_2\left(\left(\frac{1+\eta}{1+\eta\lambda}\right)v_1 - v_2\right)$$
$$= u_2(v_1, v_2^*) + r_2(1+\eta)v_1$$

and $u_2(v_1, v_2^*) = 0$. Hence,

$$u_1(v_1^*, v_2) + u_2(v_1, v_2^*) = 0 \tag{2.6}$$

Since the mechanism is feasible, $t_1(v_1, v_2) + t_1(v_1, v_2) \leq 0$ This implies,

$$u_1(v_1^*, v_2) + u_2(v_1, v_2^*) - r_2(1+\eta\lambda)v_2 + r_2(1+\eta)v_1 \leq 0$$

which means

$$u_1(v_1^*, v_2) + u_2(v_1, v_2^*) + r_2(1+\eta)\left(v_1 - \left(\frac{1+\eta\lambda}{1+\eta}\right)v_2\right) \leq 0$$

Rewriting the above equation

$$u_1(v_1^*, v_2) + u_2(v_1, v_2^*) \leq -r_2(1+\eta)\left(v_1 - \left(\frac{1+\eta\lambda}{1+\eta}\right)v_2\right)$$

$r_2(1+\eta)\left(v_1 - \left(\frac{1+\eta\lambda}{1+\eta}\right)v_2\right) > 0$ because $v_1 > \left(\frac{1+\eta\lambda}{1+\eta}\right)v_2$ which means

$$u_1(v_1^*, v_2) + u_2(v_1, v_2^*) < 0$$

violating equation (2.6) ■

Chapter 3

Optimal Dissolution Mechanisms

3.1 Introduction

Myerson and Satterthwaite (1983) showed that, when entire ownership of the object belongs to one person, trade between two agents (who have incomplete information about each other types') cannot be efficient. Since then, a long line of literature explores the limit of this impossibility. Because one agent owns the entire object, the worst off type of both the agents is known ex-ante.

In this paper, we try to design a mechanism in order to dissolve partnership between two agents who have some share of the object and have per unit valuation for the object which is private information. The agents are loss averse with respect to the initial share (endowment) (Köszegi and Rabin (2006, 2007); Tversky and Kahneman (1991)). The agent's role as a seller or a buyer depends on the the realized valuation of the object which cannot be determined prior to dissolution.

This paper focuses on the mechanisms that are optimal, where optimality is defined as maximising a weighted average of expected gains from dissolution and expected revenue generated from dissolution. We give a characterisation result for the ex-ante efficient mechanisms and the revenue-maximizing mechanisms. In this case, the efficient mechanism is the one that maximizes the material utility as well as the gain-loss utility associated with endowment. In the standard bilateral trade model discussed by Myerson and Satterthwaite (1983), the minimum utility in an incentive-compatible and individually rational mechanism is always achieved by the lowest valuation for the buyer and the highest valuation for the seller. So, the agent's virtual valuation functions are known ex-ante and, therefore, are independent of the mechanism. Individual rationality constraints are reduced to the worst-off valuation, which is the highest and lowest valuation, depending on the role of the agent, and is binding at the optimum. Assuming that the distribution of valuations satisfy regularity, i.e., hazard rates are increasing, the monotonicity of the allocations is satisfied, and optimal allocations are given by point-wise solutions. We follow this methodology for bilateral trade with loss-averse agents. The only difference is that optimal allocations compare the effective virtual valuation (i.e., inclusive of additional gain-loss effects with respect to the endowment). However, when both agents have some ownership in the object, it is no longer clear which agent is playing the role of seller and which agent is playing the role of buyer. We adapt the multilateral trade setting of Lu and Robert (2001) and Loertscher and Wasser (2019) in the partnership setting for loss-averse agents. When both agents have some shares in the object, it is no longer clear who will sell his shares (acting as a seller) and who will get the additional shares, making him the owner of the object (acting as a buyer), before the revelation of valuations. So, the agent with a high valuation expects to get full ownership of the object, and the low type expects to lose his shares. The minimum utility valuations for the agents whose role is ex-ante unidentified are in the middle, where on average he neither wants to be a buyer nor a seller and usually depends on the dissolution mechanism. Also, despite the regularity of the distribution of valuation, the virtual valuation function does not satisfy monotonicity. As a result, optimal dissolution mechanisms are characterised by ironed virtual valuations in which the object is transferred to the highest effective ironed virtual valuation (which takes into account the loss aversion parameters).

Because of ironing, there is bunching phenomena due to which ties occur with positive probability. So, the optimal allocation rule consists of a randomizing rule to break the ties in Loertscher and Wasser (2019) and Lu and Robert (2001). Ties cannot be broken arbitrarily because bunching is not because of the irregularity of the distributions. The tie breaking rule has to be such that the agents who have the valuations in the middle expects to be neither a buyer nor a seller. However in this paper, whenever the virtual valuations are same, according to the optimal allocation rule, the agents will keep their shares. Despite the positive probability of ties within the common bunching range, there is no requirement of tie breaking rule because whenever virtual valuations tie is in the common bunching range, optimality requires the agents to keep their initial shares i.e. no trade in the bunching range.

The rest of the paper is structured as follows. Section 3.2 explains the formal framework in detail. We characterize the set of all Bayesian incentive compatible and interim individually rational mechanisms and construct mechanisms that maximizes the weighted average of expected gains from dissolution and expected revenue to the broker in section 3.3. Section 3.4 presents the literature review and section 3.5 concludes. All the proofs are relegated to the appendix 3.6 at the end.

3.2 Model

3.2.1 Types, Information and preferences

Two agents, denoted by $i \in \{1, 2\}$, hold the shares of an asset. Agent 1 owns a share r_1 of the asset and agent 2 owns r_2 where $r_1 + r_2 = 1$. Valuation for the entire asset by agent i is v_i which is a private information and $v_i \sim F[\underline{v}, \overline{v}]$ with positive continuous density f where $\underline{v} > 0$. Agents have loss averse preferences with respect to the share r_i, which acts as an endowment in this case.

Following Köszegi and Rabin (2006), preferences of the loss averse agent are represented using the following utility function.

$$\hat{u}_i(s_i, t_i | v_i, r_i) = \underbrace{s_i v_i + t_i}_{\text{material utility}} + \underbrace{\eta \mu_i \bigg(s_i v_i - r_i v_i \bigg)}_{\text{gain-loss utility in ownership}}$$

where

$$\mu_i(s_i) = \begin{cases} 1 & \text{if } s_i \geq r_i, \\ \lambda > 1 & \text{if } s_i < r_i \end{cases}.$$

$s_i \in [0, 1]$ is the actual consumption, $r_i \in [0, 1]$ is the reference level of endowment and $t_i \in \mathbb{R}$ is transfer (payment). Per unit valuation of the asset v_i is the private information of the agent, $s_i v_i$ is the intrinsic utility of the object and $r_i v_i$ is the reference utility. The term $s_i v_i + t_i$ is the material utility that incorporates the transfers. Gain-loss utilities are considered with respect to the loss in the reference utility. The loss aversion parameters are η and λ where $\eta > 0$ captures the importance of gain-loss utility relative to intrinsic utility, and $\lambda > 1$ captures the degree of loss aversion. We assume that both the agents have identical loss aversion parameters.

Note that μ_i is an indicator function: If $s_i \geq r_i$, then there is gain of endowment and if $s_i < r_i$, then there is loss. Greater weight to loss is reflected by the fact that $\lambda > 1$.

Value of μ can vary across the agents as it depends on the difference between allocation received and initial endowment, even though λ is same for all agents. Throughout the analysis, we assume that $k\underline{v} < \frac{\overline{v}}{k}$ where $k = \frac{1 + \eta\lambda}{1 + \eta}$.

3.2.2 Mechanism

A direct revelation mechanism is $(s, t) \equiv (s_1, s_2, t_1, t_2)$ where, $s_i : [\underline{v}, \overline{v}]^2 \longrightarrow [0, 1]$ with $s_1(v_1, v_2) + s_2(v_1, v_2) \leq 1, \quad \forall v \in [\underline{v}, \overline{v}]^2$ and $t_i : [\underline{v}, \overline{v}]^2 \longrightarrow \mathbb{R}$. The agents directly report their valuations for the object, and then receive the share of the ownership $s_i(v_i, v_j)$ and the transfers $t_i(v_i, v_j)$.

Utility function of agent i from the mechanism when the other agent reports v_j is

$$\hat{u}_i(v_i, v_j) = s_i v_i + t_i + \eta\mu_i\bigg(s_i(v_i, v_j)v_i - r_i v_i\bigg) \quad \forall v_i \in [\underline{v}, \overline{v}]$$

Define $s_i^{ref}(v_i, v_j)$ as the modified allocation where

$$s_i^{ref}(v_i, v_j) = s_i(v_i, v_j) + \eta\mu_i\bigg(s_i(v_i, v_j) - r_i\bigg)$$

This allows us to compactly write $\hat{u}_i(v_i, v_j) = s_i^{ref}(v_i, v_j)v_i + t_i(v_i, v_j)$. Net payoff from the mechanism, denoted as $u_i(v_i, v_j)$, is defined as the difference between the utility from the trade $\hat{u}_i(v_i, v_j)$ and the reference utility i.e.

$$u_i(v_i, v_j) = \bigg(s_i^{ref}(v_i, v_j) - r_i\bigg)v_i + t_i(v_i, v_j)$$

The expected modified share and expected money transfer for player i when he announces v_i are $S_i^{ref}(v_i)$ and $T_i(v_i)$ where

$$\begin{aligned}
S_i^{ref}(v_i) - r_i &= \int_{v_j} (s_i(v_i, v_j) + \eta\mu_i(s_i(v_i, v_j) - r_i) - r_i) f(v_j) dv_j \\
&= \int_{v_j : s_i \geq r_i} (s_i(v_i, v_j) - r_i + \eta(s_i(v_i, v_j) - r_i)) f(v_j) dv_j \\
&+ \int_{v_j : s_i < r_i} (s_i(v_i, v_j) - r_i + \eta\lambda(s_i(v_i, v_j) - r_i)) f(v_j) dv_j \\
&= \int_{v_j} (1 + \eta\mu_i)(s_i(v_i, v_j) - r_i) f(v_j) dv_j
\end{aligned}$$

and

$$T_i(v_i) = \int_{v_j} t_i(v_i, v_j) f(v_j) dv_j$$

So the agent's expected net payoff is

$$U_i(v_i) = (S_i^{ref}(v_i) - r_i)v_i + T_i(v_i) \tag{3.1}$$

This model generalizes the bilateral trade setting considered by Myerson and Satterthwaite (1983). If agent 1 does not have any share of the object ($r_1 = 0$) and agent 2 has the entire ownership of the good ($r_2 = 1$), then the utility function of agent 1 is $(1 + \eta)s_1(v_1, v_2)v_1 + t_1(v_1, v_2)$ where $(1 + \eta)s_1(v_1, v_2)$ is the gain utility in the share of the object and the utility function of agent 2 is $(1 + \eta\lambda)(s_2(v_1, v_2) - 1)v_2 + t_1(v_1, v_2)$ where $(1 + \eta\lambda)(s_2(v_1, v_2) - 1)$ is loss in the share of the object due to loss aversion.

We define incentive compatibility and individual rationality of the mechanism.

DEFINITION **3.1** *The mechanism* (s_1, s_2, t_1, t_2) *is* ***Bayesian incentive compatible*** *if for all* i *and for every* $v_i, \hat{v}_i \in [\underline{v}, \overline{v}]$,

$$U_i(v_i) \geq (S_i^{ref}(\hat{v}_i) - r_i)v_i + T_i(\hat{v}_i)$$

DEFINITION **3.2** *The mechanism* (s_1, s_2, t_1, t_2) *is* ***interim individually rational*** *if for all* i *and for all* $v_i \in [\underline{v}, \overline{v}]$,

$$(S_i^{ref}(v_i) - r_i)v_i + T_i(v_i) \geq 0$$

We now give a necessary and sufficient condition for a mechanism to be incentive incompatible.

LEMMA **3.1** *The mechanism* (s_1, s_2, t_1, t_2) *is incentive compatible if and only if for agent* i *and* j, $S_i^{ref}(v_i)$ *is non-decreasing and*

$$U_i(v_i) = U_i(v_i^*) + \int_{v_i^*}^{v_i} (S_i^{ref}(x_i) - r_i)dx_i \tag{3.2}$$

For a given monotone allocation rule, payoff equivalence pins down interim expected payoffs U_i and payments T_i up to a constant. This characterization result is similar to Myerson and Satterthwaite (1983) and Cramton et al. (1987) Equation (3.2) implies that expected net utility $U_i(v_i)$ is continuous and convex in v_i. The continuity of $U_i(v_i)$ implies it has a minimum at some $v_i^* \in [\underline{v}, \overline{v}]$ where v_i^* is defined in the following lemma.

LEMMA **3.2** *Given a Bayesian incentive-compatible mechanism* (s, t), *agent* i*'s net utility is minimized at*

$$v_i^* \in \Omega(S_i^{ref}) = \{v_i : S_i^{ref}(z) - r_i \leq 0 \ \forall z < v_i; S_i^{ref}(z) - r_i \geq 0 \ \forall z > v_i\} \tag{3.3}$$

Note that $\Omega(S_i^{ref})$ is non-empty because $S_i^{ref}(v_i)$ is non-decreasing (Lemma 3.1)

LEMMA **3.3** *An incentive-compatible mechanism* (s, t) *is interim individually rational if and only if for all* $i \in \{1, 2\}$

$$U_i(v_i^*) \geq 0 \tag{3.4}$$

For any allocation (s_1, s_2) such that $S_i^{ref}(v_i)$ is non-decreasing for all $i \in \{1, 2\}$, $\Omega_i(S^{ref})$ is well-defined in (3.3) and is called the set of worst off valuations. Equations (3.2) and (3.3) implies that expected net utility $U_i(v_i)$ is continuous and convex in v_i and is minimized at $v_i^* \in \Omega(S_i^{ref})$. The modified expected share $S_i^{ref}(v_i)$ is a continuous function with r_i in its range. The worst off valuation v_i^* satisfies $S_i^{ref}(v_i^*) - r_i = 0$ i.e. the agent with worst-off valuation expects to receive a share equal to his initial ownership share r_i. As in Cramton et al. (1987) and Lu and Robert (2001), this means, on an average, the agents with worst-off valuation expects to be neither a buyer nor a seller of the asset. Therefore, he has no incentive to overstate or understate his valuation. Hence, he does not need to be compensated in order to induce him to report his valuation truthfully. It is no longer clear who is selling and who is buying prior to revelation of types, but on an average agent i is a buyer if his type $v_i \geq \max_{v_i^* \in \Omega(S_i^{ref})} v_i^*$ and a seller if his type $v_i \leq \min_{v_i^* \in \Omega(S_i^{ref})} v_i^*$. The next lemma (along with lemma 3.1) characterizes implementable allocations.

Lemma **3.4** *For any allocation s such that $S_i^{ref}(v_i)$ is non-decreasing for all $i \in \{1, 2\}$, there exists a payment function t such that (s, t) is Bayesian incentive compatible and interim individually rational.*

3.3 Optimal Dissolution Mechanisms

Consider a situation where dissolution is intermediated by a broker, to whom the agents simultaneously reports their valuations. The broker, then, determines who is allocated the asset and what will be the payment or transfer. The broker himself cannot own the object. He can either subsidize or exploit the agents. We seek a mechanism that maximizes weighted average of expected gains from dissolution and expected revenues to the broker subject to the incentive compatibility and individual rationality constraints for traders. For that, we first define, for any $\alpha \geq 0$ and $v_i \in [\underline{v}, \overline{v}]$,

$$\omega_i^B(v_i|\alpha) = v_i - \alpha\frac{(1 - F(v_i))}{f(v_i)} \quad \text{and} \quad \omega_i^S(v_i|\alpha) = v_i + \alpha\frac{F(v_i)}{f(v_i)} \tag{3.5}$$

where $\omega_i^B(v_i|\alpha)$ and $\omega_i^S(v_i|\alpha)$ are referred to as the α-virtual valuation of agents buyer-type and seller-type respectively (Lu and Robert (2001); Loertscher and Wasser (2019)). We will impose the regularity assumption that each agent's α-weighted valuation is strictly increasing, i.e.,

$$\frac{d}{dv}\omega_i^B(v|\alpha) \geq 0 \quad \text{and} \quad \frac{d}{dv}\omega_i^S(v|\alpha) \geq 0 \quad \forall v \in [\underline{v}, \overline{v}]$$

Before defining the objective function, we give functional form of the expected revenue from an incentive compatible mechanism.

Lemma **3.5** *The expected revenue from an incentive compatible mechanism with allocation s is*

$$R(s) = \sum_{i=1}^{2}\left(\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}} \omega_i(v_i|v_i^*, 1)\Big\{(1 + \eta\mu)(s_i(v_i, v_j) - r_i)\Big\}f(v_i)f(v_j)dv_i dv_j - U_i(v_i^*)\right) \tag{3.6}$$

where

$$\omega_i(v_i|v_i^*,1)=\begin{cases}\omega_i^B(v_i|1) & \text{if} \quad v_i>v_i^*,\\ v_i^* & \text{if} \quad v_i=v_i^*,\\ \omega_i^S(v_i|1) & \text{if} \quad v_i<v_i^*.\end{cases}$$

Now, we define the objective function. For any $\alpha \in [0,1]$, let

$$W_\alpha(s)=(1-\alpha)\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}}\sum_{i=\{1,2\}}\Big(s_i^{ref}(v_i,v_j)v_i\Big)f(v_i)f(v_j)dv_idv_j+\alpha R(s)$$

where $R(s)$ is the revenue generated from the mechanism.

We are looking for a mechanism that maximizes $W_\alpha(s)$ among all incentive compatible and individually rational mechanisms. Next, we characterise the optimal mechanisms.

Theorem **3.1** *For any incentive-compatible mechanism with broker, $S_i^{ref}(v_i)$ is non-decreasing in v_i for all i and*

$$W_\alpha(s)=\sum_{i=1}^{2}\Bigg(\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}}\omega_i(v_i|v_i^*,\alpha)\Big\{(1+\eta\mu_i)(s_i(v_i,v_j)-r_i)\Big\}f(v_i)f(v_j)dv_idv_j-\alpha U_i(v_i^*)\Bigg)$$

where

$$\omega_i(v_i|v_i^*,\alpha)=\begin{cases}\omega_i^B(v_i|\alpha) & \text{if} \quad v_i>v_i^*,\\ v_i^* & \text{if} \quad v_i=v_i^*,\\ \omega_i^S(v_i|\alpha) & \text{if} \quad v_i<v_i^*.\end{cases}$$

If $r_i=0$, then $\underline{v}\in\Omega(S^{ref})$ for all implementable allocations s. For $v_i^*=\underline{v}$, $\omega_i(v_i|\underline{v},\alpha)=v_i-\alpha\dfrac{(1-F(v_i))}{f(v_i)}=\omega_i^B(v_i|\alpha)\quad\forall v_i>\underline{v}$ implying agent i has buyer-type virtual valuations. If $r_i=1$, then $\overline{v}\in\Omega(S^{ref})$ for all implementable allocations s. For $v_i^*=\overline{v}$, $\omega_i(v_i|\overline{v},\alpha)=v_i+\alpha\dfrac{F(v_i)}{f(v_i)}=\omega_i^S(v_i|\alpha)\quad\forall v_i<\overline{v}$ implying agent i has seller-type virtual valuations. For $r_i\in(0,1)$, v_i^* is between $\underline{v}$ and $\overline{v}$ which means that agent i has virtual valuations of both, buyer-type and seller-type.

Note that, when $\alpha=1$, the above problem is equivalent to maximizing the expected revenue to the broker subject to the constraints. When $\alpha=0$, the above problem is equivalent to maximizing expected gains from dissolution subject to the constraints.

3.3.1 Optimal Mechanism Under Bilateral Trade

We first solve the maximization problem under bilateral trade setting i.e. one agent has an entire ownership of the object. With extreme shares of partnership, the worst off value is independent of the mechanism. The role of agents (buyer-type or seller-type) is identified prior to the revelation of type. The corollary below characterises the optimal mechanisms for $r_1 = 0$ and $r_2 = 1$.

Corollary **3.1** *If $r_1 = 0$ and $r_2 = 1$, then for any incentive-compatible mechanism with a broker, $S_1(v_1)$ is non-decreasing in v_1, $1 - S_2(v_2)$ is decreasing in v_2 and*

$$W_\alpha(s) = \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \Big((1+\eta)s_1(v_1,v_2)\omega_1^B(v_1|\alpha) + (1+\eta\lambda)(s_2(v_1,v_2)-1)\omega_2^S(v_2|\alpha) - \alpha U_1(\underline{v}) - \alpha U_2(\overline{v})\Big) f(v_1)f(v_2)dv_1dv_2$$

The optimal mechanism in this case is given below.

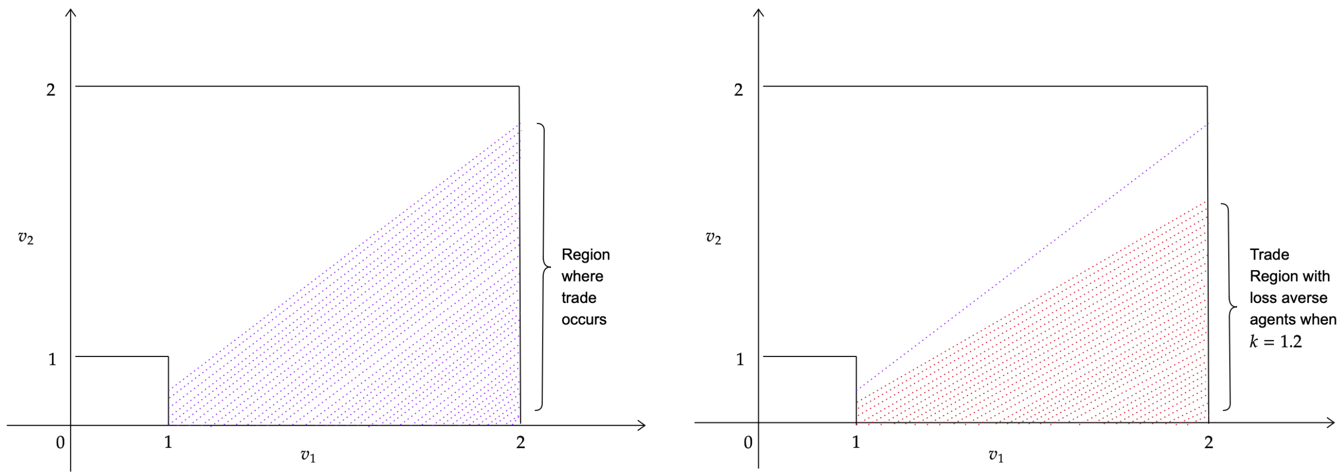
Theorem **3.2** *For $r_1 = 0$ and $r_2 = 1$, the objective function $W_\alpha(s)$ is maximized by an incentive-compatible and individually-rational mechanism in which the object is transferred to the agent 1 if and only if $\omega_1^B(v_1|\alpha) \geq k\omega_2^S(v_2|\alpha)$ where $k = \dfrac{1+\eta\lambda}{1+\eta}$, otherwise agent 2 keeps the object.*

Myerson and Satterthwaite (1983) showed that the agent who has the highest virtual value for the object is allocated the object. Specifically, agent 1 is allocated the object if $\omega_1^B(v_1|\alpha) > \omega_2^S(v_2|\alpha)$ otherwise, agent 2 keeps the object himself. With loss averse preferences, the allocation rule suggested by Myerson and Satterthwaite (1983) is not optimal. Since agent 1 is a buyer if he is allocated the object, it is a gain in the material valuation of the object (over and above his endowment, which is 0). So effective virtual valuation for agent 1 is $(1+\eta)\omega_1^B(v_1|\alpha)$. Similarly, if agent 2 loses ownership of the object, he becomes the seller. This results in additional losses (relative to his endowment), generating a negative effective virtual valuation for agent 2 as $(1+\eta\lambda)\omega_2^S(v_2|\alpha)$ (since virtual valuation is negative, we can refer to this as effective virtual cost). Allocating the object to the buyer is optimal if and only if the effective virtual valuation of the buyer is greater than the effective virtual cost of the seller. Otherwise, the seller keeps the object. Instead of comparing the virtual valuation and virtual cost, effective virtual valuation is compared with effective virtual cost, and this reduces the region of the trade compared to the case when there is no loss aversion.

The example below shows how loss aversion reduces the set of values at which trade takes place when the broker is maximizing the revenue.

Example **3.1** *Agent 1 is a buyer with v_1 as the valuation for the object and agent 2 is the seller with valuation v_2 where v_1 and v_2 are uniform random variables on $[1, 2]$. Both the agents are loss averse with identical parameters η and λ. The virtual valuations of the agents* *when* $\alpha = 1$ *are* $\omega_1(v_1|1) = v_1 - (1-(v_1-1)) = 2v_1 - 2$ *and* $\omega_2(v_2|1) = v_2 + v_2 - 1 = 2v_2 - 1$. *The broker's optimal mechanism transfers the object if and only if*

$$\omega_1(v_1|1) = 2v_1 - 2 \geq k(2v_2 - 1) = \omega_2(v_2|1) \quad or \quad v_1 \geq kv_2 + 1 - \frac{k}{2}$$



For $k = 1$, *there is no loss aversion, and we have Myerson and Satterthwaite (1983) result. For* $1 < k \leq 2$ *as* k *increases, the gap between* v_1 *and* v_2 *will keep on increasing in order to transfer the object, thereby restricting the trade.*

The next example shows the decrease in the expected revenue due to loss aversion which occurs because the set of values at which trade takes place is reduced.

Example **3.2** *Consider example 1 again where* v_1 *and* v_2 *follows* $U[1,2]$. *When* $k = 1$, *there is no loss aversion and the expected revenue from the mechanism is*

$$\begin{aligned} R &= \int_1^2 \int_1^2 ((2v_1 - 2)s_1(v_1, v_2)) - (2v_2 - 1)(1 - s_2(v_1, v_2)))dv_2 dv_1 \\ &= \int_{1.5}^2 \int_1^{v_1 - 0.5} ((2v_1 - 2) - (2v_2 - 1))dv_2 dv_1 \\ &= 0.04167 \end{aligned}$$

On the other hand with loss averse preferences, the expected revenue in the same envi-

ronment will be

$$\begin{aligned}
R &= \int_1^2 \int_1^2 ((1+\eta)(2v_1-2)s_1(v_1,v_2)) - (1+\eta\lambda)(2v_2-1)(1-s_2(v_1,v_2)))dv_2dv_1 \\
&= \int_{1+k/2}^2 \int_1^{0.5+(v_1-1)/k} ((1+\eta)(2v_1-2) - (1+\eta\lambda)(2v_2-1))dv_2dv_1 \\
&= (1+\eta) \int_{1+k/2}^2 \int_1^{0.5+(v_1-1)/k} ((2v_1-2) - (2v_2-1)k)dv_2dv_1 \\
&= (1+\eta) \int_{1+k/2}^2 (v_1^2 - v_1(2+k) + (0.5k+1)^2)dv_1 \\
&= -(1+\eta)\frac{(k-2)^3}{24}
\end{aligned}$$

In the above example, for a given $\eta > 0$, it is straightforward to check that as k increases, expected revenue decreases. Note that because of the restriction on k, $1 < k < \sqrt{2}$.

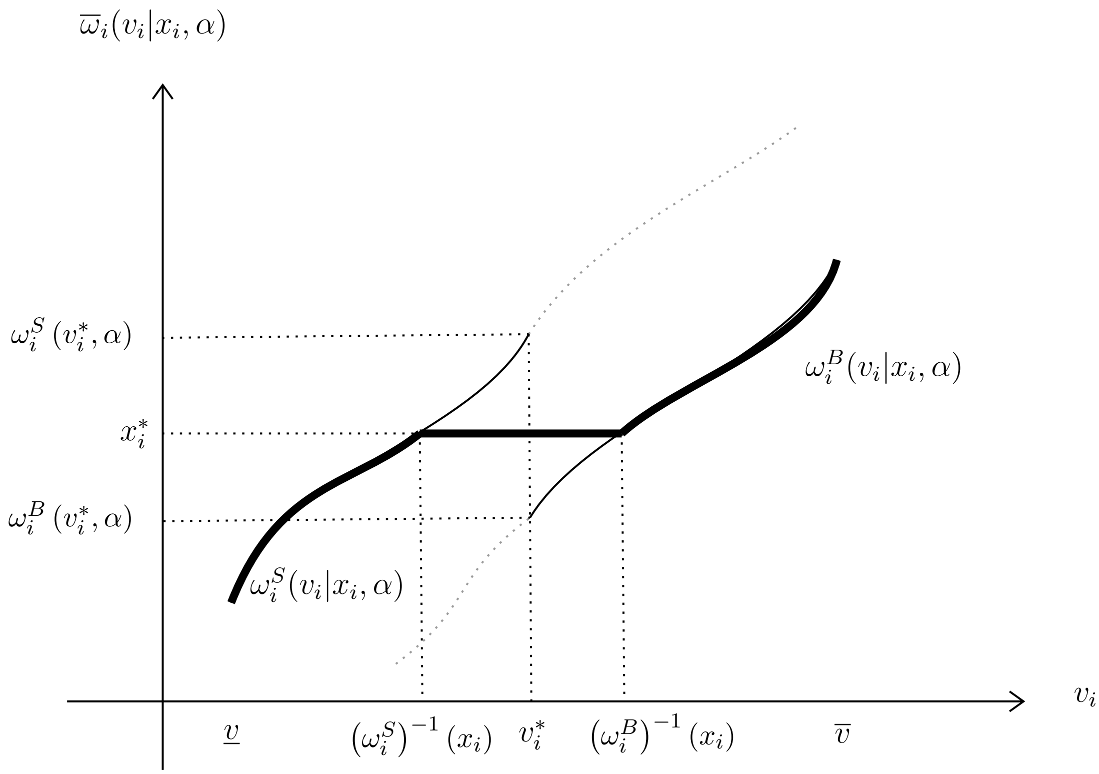
3.3.2 Optimal Mechanism Under Partnership

Problem arises when $0 < r_i < 1$ for $i \in \{1, 2\}$ because the worst off valuation for each agent is unknown. For $\alpha > 0$, agent's virtual valuation has upward distortion above his true valuation when he expects to be a seller. Similarly, agent's virtual valuations has downward distortion below his true valuation when he expects to be a buyer. Also, $\omega_i(v_i|v_i^*, \alpha)$ is discontinuous at $v_i = v_i^*$ since $\omega_i^B(v_i|\alpha) < v_i < \omega_i^S(v_i|\alpha)$ for $\alpha > 0$ and does not satisfy monotonicity in v_i over $[\underline{v}, \overline{v}]$ for all distributions of valuations.

We define an ironed virtual value function $\overline{\omega}_i(v_i|x_i, \alpha)$ that is monotonic and then find the optimal allocation based on ironed virtual value. Define the ironed virtual value as:

$$\overline{\omega}_i(v_i|x_i,\alpha) = \begin{cases} \omega_i^S(v_i|\alpha) & \text{if} \quad \omega_i^S(v_i|\alpha) < x_i, \\ x_i & \text{if} \quad \omega_i^B(v_i|\alpha) \le x_i \le \omega_i^S(v_i|\alpha), \\ \omega_i^B(v_i|\alpha) & \text{if} \quad \omega_i^B(v_i|\alpha) > x_i. \end{cases}$$

where $x_i \in [\omega_i^B(v_i^*|\alpha), \omega_i^S(v_i^*|\alpha)]$ is the ironing parameter. The figure below illustrates the ironed virtual valuation function. Agent i's ironed virtual valuation $\overline{\omega}_i(v_i|x_i, \alpha)$ is constant and equal to x_i for an interval of valuations that contains the worst off type v_i^*, and it is strictly increasing otherwise.



Consider the modified optimization problem below:

$$\begin{aligned} \max_s \quad & \overline{W}_\alpha(s) = \sum_{i=1}^{2} \left(\int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \overline{\omega}_i(v_i|x_i,\alpha) \Big\{ (1+\eta\mu)(s_i(v_i,v_j) - r_i) \Big\} f(v_i) f(v_j) dv_i dv_j \right) \\ \text{s.t.} \quad & s_1(v_1,v_2) + s_1(v_1,v_2) = 1 \end{aligned}$$

We first show that the allocation s^* as defined below solves the modified optimization problem.

$$s_i^*(v_i,v_j) = \begin{cases} 1 & \text{if} \quad \overline{\omega}_i(v_i|x_i^*,\alpha) > k\overline{\omega}_j(v_j|x_j^*,\alpha), \\ r_i & \text{if} \quad \dfrac{\overline{\omega}_j(v_j|x_j^*,\alpha)}{k} < \overline{\omega}_i(v_i|x_i^*,\alpha) < k\overline{\omega}_j(v_j|x_j^*,\alpha), \\ 0 & \text{if} \quad \overline{\omega}_i(v_i|x_i^*,\alpha) < \dfrac{\overline{\omega}_j(v_j|x_j^*,\alpha)}{k}. \end{cases} \tag{3.7}$$

PROPOSITION **3.1** *Among all incentive-compatible, individually-rational mechanisms, the expected ironed virtual value function is maximized by a mechanism with allocation s^* as defined in* (3.7)*.*

Then, we show the existence of x_i such that s^* as defined in (3.7) solves the original maximization problem if s^* satisfies $S_i^{ref}(v_i) - r_i = 0$.

Next, we prove that the allocation rule that maximizes the modified problem also solves the original problem.

THEOREM **3.3** *There exists an* $x^* = (x_1^*, x_2^*) \in [\omega_i^B(v_i^*|\alpha), \omega_i^S(v_i^*|\alpha)]$ *such that the allocation* $s^* = (s_1^*, s_2^*)$ *defined in* (3.7) *satisfies* $S_i^{ref}(v_i) - r_i = 0$ *for* $v_i \in \left[(\omega_i^S)^{-1}(x_i^*), (\omega_i^B)^{-1}(x_i^*)\right]$ $\forall i \in \{1, 2\}$. *Then the allocation* s^* *satisfies the original maximization problem.*

According to Loertscher and Wasser (2019) and Lu and Robert (2001), when agents have risk-neutral preferences, the optimal allocation rule allocates the object to the agent with the highest ironed virtual valuation. On the other hand, in the case of loss-averse preferences, the object is allocated to the agent with the highest effective ironed virtual valuation in the optimal allocation rule. Effective ironed virtual valuation includes gain-loss terms associated with the gain or loss of the object. Because of ironing, there is bunching phenomenon, due to which ties occur with positive probability. So, the optimal allocation rule consists of a randomizing rule to break the ties in Loertscher and Wasser (2019) and Lu and Robert (2001). Ties cannot be broken arbitrarily because of the irregularity of the distributions. The tie-breaking rule has to be such that the agent who has the valuation in the middle expects to be neither a buyer nor a seller, that is, $S_i^{ref}(v_i) - r_i = 0$. However, in this model, whenever the virtual valuations are same, according to the optimal allocation rule, the agents will keep their shares. Despite the positive probability of ties within the common bunching range, there is no requirement of a tie-breaking rule because whenever virtual valuations tie in the common bunching range, optimality requires the agents to keep their initial shares, i.e., no trade in the bunching range.

We will show the example of ironing for uniform distribution below.

EXAMPLE **3.3** *Consider the setup of example* 1 *where* v_1 *and* v_2 *follows* $U[1, 2]$. α*-virtual valuation of seller is* $\omega_i^S(v_i|\alpha) = (1 + \alpha)v_i - \alpha$ *and of buyer is* $\omega_i^B(v_i|\alpha) = (1 + \alpha)v_i - 2\alpha$.

$$(1 + \alpha)v_i - \alpha = \frac{x^*}{k}$$

This gives $(\omega_i^S)^{-1}\left(\frac{x^*}{k}\right) = \frac{x^* + k\alpha}{k(1 + \alpha)}$

$$(1 + \alpha)v_i - 2\alpha = kx^*$$

This gives $(\omega_i^B)^{-1}(kx^*) = \frac{kx^* + 2\alpha}{1 + \alpha}$

Using $S_i^{ref}(v_i) - r_i = 0$, *we get*

$$F\left(\frac{x^* + k\alpha}{k(1 + \alpha)}\right) + kF\left(\frac{kx^* + 2\alpha}{1 + \alpha}\right) = k$$

This implies

$$\frac{x^* + k\alpha}{k(1 + \alpha)} - 1 + k\left(\frac{kx^* + 2\alpha}{1 + \alpha} - 1\right) = k$$

Solving the above equation gives

$$x^* = \frac{2k^2 + k}{1 + k^3}$$

Observe that $k = k\underline{v} < x^* < \frac{\overline{v}}{k} = \frac{2}{k}$. *To calculate* $(\omega_i^S)^{-1}(x^*)$, *we use* $(1+\alpha)v_i - \alpha = x^*$. *This gives* $(\omega_i^S)^{-1}(x^*) = \frac{2k^2 + k + \alpha + \alpha k^3}{(1+k^3)(1+\alpha)}$. *Similarly, to calculate* $(\omega_i^B)^{-1}(x^*)$, *we use* $(1+\alpha)v_i - 2\alpha = x^*$. *This gives* $(\omega_i^B)^{-1}(x^*) = \frac{2k^2 + k + \alpha + \alpha k^3}{(1+k^3)(1+\alpha)}$.

3.4 Literature

Our paper contributes to the literature on mechanism designs with loss averse agents. In the literature, loss averse preferences in mechanism design problems are based on Köszegi and Rabin (2006, 2007). Lange and Ratan (2010) discussed how bidding behaviour is affected due to loss aversion in first-price and second-price auctions where reference point is endogenous. They showed that expected revenue is higher under first price auction as compared to second price auction. Eisenhuth and Grunewald (2018), on the other hand, derived the equilibrium bidding strategy in the case of first price auction and all pay auctions under two specifications: (a) gains and losses are evaluated in the good dimension and in the money dimension separately (narrow bracketing) and (b), gains and losses are evaluated over the entire risk neutral utility (wide bracketing). They showed that under narrow bracketing, expected revenue is higher in the case of all pay auction in comparison to first price auction and under wide bracketing, the revenue ranking is opposite. Eisenhuth and Grunewald (2018) also put their theoretical findings into test through laboratory experiments. Their results were consistent with the experimental results in the case of wide bracketing but inconsistent in the case of narrow bracketing.

Eisenhuth (2018) considers the problem of designing the revenue maximizing mechanisms where the seller of the object is risk neutral and the buyers are loss averse under wide bracketing and narrow bracketing of the pay off function. In case of narrow bracketing, all pay auction with minimum bid is an optimal auction and in case of wide bracketing, first price auction with minimum bid is an optimal auction. Agents with loss averse preferences as described by Köszegi and Rabin (2006, 2007) do not like variation in ex-post utility. Under narrow bracketing of payoff function, the amount that can be extracted by the seller from the agents is reduced due to uncertainty in the ex-post transfers, making all pay auction optimal. Similarly under wide bracketing, the variation in the ex-post utility can be reduced by first price auction.

Benkert (2023) introduced loss aversion in the preferences of the seller as well as the buyer in the bilateral trade setting of Myerson and Satterthwaite (1983) using the narrow bracketing framework of Eisenhuth (2018). Benkert (2023) also addressed the question of designing the revenue maximizing mechanisms, where the designer is an intermediator who provides a platform to trade. He showed that with loss averse agents, mechanisms that maximize revenue involves interim-deterministic transfers i.e. transfers of an agent are not dependent on other agent's reported valuation of the object. This provides full insurance to the agents, by reducing the fluctuation in ex-post utility. Results in our paper are different because revenue maximizing mechanisms in our setup allocates on the basis of effective ironed virtual valuation.

Our paper also contributes to the literature related to partnership dissolution. There is a long line of literature that focuses on ex-post efficiency and property rights which lead to efficient outcome, subject to individual rationality, incentive compatibility and budget balanced. Cramton et al. (1987) showed the existence of ex-post efficient outcome with equal partnership. On the other hand, Fieseler et al. (2003) showed that with positive interdependent valuation, it may not be possible to achieve ex-post efficiency even with equal ownership. It is impossible to decide whether ex-post efficient reallocation can take place or not without the knowledge of distribution of private values. Loertscher and Wasser (2019) addresses the question of identifying optimal ownership structure for independent private value and general distribution functions, where optimality is defined as a weighted average of revenue and gains from dissolution. In case of identical distribution, equal share ownership always lies in the set of optimal ownership structure, irrespective of the weight on the revenue. Also, increasing the weight on the revenue expands the set of optimal ownership structures. On the other hand if distributions are not identical, optimal ownership structures are asymmetric and depends on the weight put on the revenue.

3.5 Conclusion

We give a full characterization of the optimal dissolution mechanisms for both, bilateral model and general bilateral partnership model in the case of loss averse preferences. For bilateral trade model, the optimal mechanisms allocate on the basis of virtual valuations which is independent of the mechanism. For bilateral partnership, the optimal mechanisms allocate on the basis of ironed virtual valuations functions, which is a constant over a range of values for each agent and the corresponding seller (buyer)type virtual valuation for lower (higher) valuations. We are capturing the loss aversion with respect to fixed endowment, thereby contributing to the literature on mechanism design and loss aversion.

One possible extension is to introduce the expectations based loss aversion (Köszegi and Rabin (2006), Köszegi and Rabin (2007)) in the general partnership model and characterise the optimal dissolution mechanisms. With the expectations based loss aversion, the definition of efficiency might be different compared to the one used in this paper. It will be interesting to see how the results change with a different notion of reference point. Also, throughout the analysis we have assumed that the parameters of loss aversion are commonly known. Another take would be to drop the assumption that the loss aversion parameters are common knowledge. Assuming that the parameters have private information will lead to a multi-dimensional mechanism design problem.

3.6 Appendix

Proof of Lemma 3.1

Proof: Necessity: Suppose that the mechanism (s, t) is Bayesian incentive compatible. Then,

$$U_i(v_i) \geq U_i(\hat{v}_i) + (S_i^{ref}(\hat{v}_i) - r_i)(v_i - \hat{v}_i) \tag{3.8}$$

which gives

$$U_i(v_i) - U_i(\hat{v}_i) \geq (S_i^{ref}(\hat{v}_i) - r_i)(v_i - \hat{v}_i)$$

Exchanging the roles of v_i and $\hat{v}_i$

$$U_i(\hat{v}_i) \geq U_i(v_i) + (S_i^{ref}(v_i) - r_i)(\hat{v}_i - v_i)$$

This implies

$$U_i(v_i) - U_i(\hat{v}_i) \leq (S_i^{ref}(v_i) - r_i)(v_i - \hat{v}_i) \tag{3.9}$$

(3.8) and (3.9) together imply that

$$(S_i^{ref}(v_i) - r_i)(v_i - \hat{v}_i) \geq U_i(v_i) - U_i(\hat{v}_i) \geq (S_i^{ref}(\hat{v}_i) - r_i)(v_i - \hat{v}_i) \tag{3.10}$$

This shows that if $v_i > \hat{v}_i$, $S_i^{ref}(v_i) \geq S_i^{ref}(\hat{v}_i)$. Therefore, $S_i^{ref}(\cdot)$ is non-decreasing.

Claim **3.1** *$U_i(\cdot)$ is Lipschitz continuous.*

Proof: Show that there exists $M > 0$, such that

$$|U_i(v_i) - U_i(\hat{v}_i)| \leq M|v_i - \hat{v}_i|$$

If $v_i > \hat{v}_i$,

$$U_i(v_i) - U_i(\hat{v}_i) \leq (S_i^{ref}(v_i) - r_i)(v_i - \hat{v}_i) \leq (1 + \eta)(1 - r_i)(v_i - \hat{v}_i)$$

If $v_i < \hat{v}_i$,

$$U_i(v_i) - U_i(\hat{v}_i) \geq (S_i^{ref}(\hat{v}_i) - r_i)(v_i - \hat{v}_i)$$

which can also be written as

$$\begin{aligned}-(U_i(v_i) - U_i(\hat{v}_i)) &\leq -(S_i^{ref}(\hat{v}_i) - r_i)(v_i - \hat{v}_i) \\ &\leq -(S_i^{ref}(v_i) - r_i)(v_i - \hat{v}_i)\end{aligned}$$

Therefore,

$$\begin{aligned}|U_i(v_i) - U_i(\hat{v}_i)| &\leq (S_i^{ref}(v_i) - r_i)|(v_i - \hat{v}_i)| \\ &\leq (1+\eta)(1-r_i)|(v_i - \hat{v}_i)|\end{aligned}$$

For $M = (1+\eta)(1-r_i)$, we have proved that $|U_i(v_i) - U_i(\hat{v}_i)| \leq M|(v_i - \hat{v}_i)|$. Therefore, $U_i(\cdot)$ is Lipschitz continuous. ■

This means that $U_i(\cdot)$ is differentiable almost everywhere. From equation (3.10), we have

$$S_i^{ref}(v_i) - r_i \geq \frac{U_i(v_i) - U_i(\hat{v}_i)}{v_i - \hat{v}_i} \geq S_i^{ref}(\hat{v}_i) - r_i$$

This implies

$$\frac{dU_i(v_i)}{dv_i} = S_i^{ref}(v_i) - r_i$$

and

$$U_i(v_i) = U_i(v_i^*) + \int_{v_i^*}^{v_i} (S_i^{ref}(x_i) - r_i)dx_i \tag{3.11}$$

$\forall v_i, v_i^* \in [\underline{v}, \overline{v}]$.
Substituting the expression for $U_i(v_i)$ in the above equation gives

$$(S_i^{ref}(v_i) - r_i)v_i + T_i(v_i) = U_i(v_i^*) + \int_{v_i^*}^{v_i} (S_i^{ref}(x_i) - r_i)dx_i$$

which can be rewritten as

$$T_i(v_i) = U_i(v_i^*) - (S_i^{ref}(v_i) - r_i)v_i + + \int_{v_i^*}^{v_i} (S_i^{ref}(x_i) - r_i)dx_i$$

Sufficiency: Suppose that the mechanism (s,t) is such that $S_i^{ref}(v_i)$ is non-decreasing and $U_i(v_i)$ satisfies (3.2)

$$\begin{aligned}U_i(v_i) - U_i(v_i^*) = \int_{v_i^*}^{v_i} (S_i^{ref}(u) - r_i)du &\geq \int_{v_i^*}^{v_i} (S_i^{ref}(v_i^*) - r_i)du \\ &= (S_i^{ref}(v_i^*) - r_i)(v_i - v_i^*)\end{aligned}$$

Therefore,

$$U_i(v_i) \geq U_i(v_i^*) + (S_i^{ref}(v_i^*) - r_i)(v_i - v_i^*)$$

Substituting the expression for $U_i(v_i^*)$ in the above equation gives

$$U_i(v_i) \geq (S_i^{ref}(v_i^*) - r_i)v_i + T_i(v_i^*)$$

Hence, (s,t) is Bayesian incentive compatible.

■

Proof of Lemma 3.2

Proof: By Lemma 3.1, the net utility function of trader i with valuation v_i is continuous and convex in v_i. Hence, $U_i(v_i)$ is minimized at the point where the derivative of net utility function is 0. Derivative of $U_i(v_i)$ is $S_i^{ref}(v_i) - r_i$ almost everywhere with $S_i^{ref}(v_i)$ is increasing in v_i. If $S_i^{ref}(v_i) - r_i > 0 \quad \forall v_i \in [\underline{v}, \overline{v}]$, then $U_i(v_i)$ is minimized at $v_i^* = \underline{v}$. Similarly, if $S_i^{ref}(v_i) - r_i < 0 \quad \forall v_i \in [\underline{v}, \overline{v}]$, then $U_i(v_i)$ is minimized at $v_i^* = \overline{v}$. On the other hand, if there exists p and q such that $S_i^{ref}(p) - r_i \leq 0$ and $S_i^{ref}(q) - r_i \geq 0$, then $U_i(v_i)$ is minimized at v_i^* where $S_i^{ref}(v_i^*) - r_i = 0$. The set of valuations at which $S_i^{ref}(v_i) - r_i = 0$ is denoted by

$$\Omega_i(S_i^{ref}) = \{v_i : S_i^{ref}(z) - r_i \leq 0 \; \forall z < v_i; S_i^{ref}(z) - r_i \geq 0 \; \forall z > v_i\}.$$

■

Proof of Lemma 3.3

Proof: A mechanism is interim individually rational if

$$U_i(v_i) \geq 0 \quad \forall v_i \in [\underline{v}, \overline{v}]$$

Because of Lemma **??**, $U_i(\cdot)$ is increasing. Therefore, we need to check individual rationality at the valuation v_i^* only.

$$U_i(v_i^*) \geq 0$$

■

Proof of Lemma 3.4

Proof: We need to construct a transfer function $t(v_1, v_2)$ such that (s,t) is incentive compatible and individually rational. There are many such functions which could be used, we will consider the following function.

$$t_i(v_i, v_j) = \begin{cases} -(1+\eta)(s_i(v_i, v_j) - r_i)v_i + \int_{v_i^*}^{v_i}(1+\eta)(s_i(x_i, v_j) - r_i)dx_i & \text{if } s_i(v_i, v_j) \geq r_i, \\ -(1+\eta\lambda)(s_i(v_i, v_j) - r_i)v_i + \int_{v_i^*}^{v_i}(1+\eta\lambda)(s_i(x_i, v_j) - r_i)dx_i & \text{if } s_i(v_i, v_j) < r_i. \end{cases}$$

where $v_i^* \in \Omega_i(S_i^{ref})$. This transfer function gives

$$T_i(v_i) = -(S_i^{ref}(v_i) - r_i)v_i + \int_{v_i^*}^{v_i}(S_i^{ref}(x_i) - r_i)dx_i$$

or

$$U_i(v_i) = \int_{v_i^*}^{v_i}(S_i^{ref}(x_i) - r_i)dx_i$$

with $U_i(v_i^*) = 0$. From lemma 3.1 and 3.3, the mechanism (s, t) is incentive compatible and individually rational. ■

Proof of Lemma 3.5

Proof: First, we define the modified virtual value function:

$$\omega_i(v_i|v_i^*, 1) = \begin{cases} \omega_i^B(v_i|1) & \text{if} \quad v_i > v_i^*, \\ v_i^* & \text{if} \quad v_i = v_i^*, \\ \omega_i^S(v_i|1) & \text{if} \quad v_i < v_i^*. \end{cases}$$

The expected revenue of the mechanism is

$$\begin{aligned} R(s) &= -\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}}\big(t_1(v_1, v_2) + t_1(v_1, v_2)\big)f(v_2)f(v_1)dv_2dv_1 \\ &= -\Big(\int_{\underline{v}}^{\overline{v}}T_1(v_1)f(v_1)dv_1 + \int_{\underline{v}}^{\overline{v}}T_2(v_2)f(v_2)dv_2\Big) \end{aligned}$$

From Lemma 1, we know that

$$U_1(v_1) = U_1(v_i^*) + \int_{v_i^*}^{v_i}(S_i^{ref}(x_1) - r_i)dx_i$$

This gives,

$$T_i(v_i) = U_i(v_i^*) - (S_i^{ref}(v_i) - r_i)v_i + \int_{v_i^*}^{v_i}(S_i^{ref}(x_1) - r_i)dx_i$$

Taking expectation of $T_i(v_i)$ over v_i, we get

$$
\begin{aligned}
\int_{\overline{v}}^{\overline{v}} T_i(v_i)f(v_i)dv_i &= \int_{\underline{v}}^{\overline{v}} \left(U_i(v_i^*) - (S_i^{ref}(v_i) - r_i)v_i + \int_{v_i^*}^{v_i} (S_i^{ref}(x_i) - r_i)dx_i \right) f(v_i)dv_i \\
&= U_i(v_i^*) - \int_{\underline{v}}^{\overline{v}} (S_i^{ref}(v_i) - r_i)v_i f(v_i)dv_i + \int_{\underline{v}}^{\overline{v}} \int_{v_i^*}^{v_i} (S_i^{ref}(x_i) - r_i)dx_i \Big) f(v_i)dv_i \\
&= U_i(v_i^*) - \int_{\underline{v}}^{\overline{v}} (S_i^{ref}(v_i) - r_i)v_i f(v_i)dv_i + \int_{v_i^*}^{\overline{v}} [1 - F(x_i)](S_i^{ref}(x_i) - r_i)dx_i \\
&- \int_{\underline{v}}^{v_i^*} F(x_i)(S_i^{ref}(x_i) - r_i)dx_i \\
&= U_i(v_i^*) - \int_{\underline{v}}^{v_i^*} (S_i^{ref}(v_i) - r_i)v_i f(v_i)dv_i - \int_{\underline{v}}^{v_i^*} F(x_i)(S_i^{ref}(x_i) - r_i)dx_i \\
&- \int_{v_i^*}^{\overline{v}} (S_i^{ref}(v_i) - r_i)v_i f(v_i)dv_i + \int_{v_i^*}^{\overline{v}} [1 - F(x_i)](S_i^{ref}(x_i) - r_i)dx_i \\
&= U_i(v_i^*) - \int_{v_i^*}^{\overline{v}} (S_i^{ref}(v_i) - r_i) \left(v_i - \frac{(1 - F(v_i))}{f(v_i)} \right) f(v_i)dv_i \\
&- \int_{\underline{v}}^{v_i^*} (S_i^{ref}(v_i) - r_i) \left(v_i + \frac{F(v_i)}{f(v_i)} \right) f(v_i)dv_i \\
&= U_i(v_i^*) - \int_{\underline{v}}^{\overline{v}} \omega_i(v_i|v_i^*, 1)(S_i^{ref}(v_i) - r_i)f(v_i)dv_i \\
&= U_i(v_i^*) - \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \omega_i(v_i|v_i^*, 1) \Big\{ (1 + \eta\mu)(s_i(v_i, v_j) - r_i) \Big\} f(v_i)f(v_j)dv_i dv_j
\end{aligned}
$$

Substituting the expression of $\int_{\underline{v}}^{\overline{v}} T_i(v_i)f(v_i)dv_i$ in the revenue function gives

$$
R(s) = \sum_{i=1}^{2} \left(\int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \omega(v_i|v_i^*, 1) \Big\{ (1 + \eta\mu)(s_i(v_i, v_j) - r_i) \Big\} f(v_i)f(v_j)dv_i dv_j - U_i(v_i^*) \right)
$$

■

Proof of Theorem 3.1

Proof: First, we define the modified virtual value function:

$$
\omega_i(v_i|v_i^*, \alpha) = \begin{cases} \omega_i^B(v_i|\alpha) & \text{if} \quad v_i > v_i^*, \\ v_i^* & \text{if} \quad v_i = v_i^*, \\ \omega_i^S(v_i|\alpha) & \text{if} \quad v_i < v_i^*. \end{cases}
$$

The objective function is

$$W_\alpha(s) = (1-\alpha)\sum_{i=1}^{2}\left(\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}}\Big((1+\eta\mu_i)s_i(v_i.v_j)v_i\Big)f(v_i)f(v_j)dv_idv_j\right) + \alpha R(s)$$

Now, we substitute the revenue function from lemma 3.5 in the above equation.

$$\begin{aligned}
W_\alpha(s) &= (1-\alpha)\sum_{i=1}^{2}\left(\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}}\Big((1+\eta\mu_i)s_i(v_i.v_j)v_1\Big)f(v_i)f(v_j)dv_idv_j\right.\\
&\quad + \alpha\sum_{i=1}^{2}\left(\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}}\omega_i(v_i|v_i^*,1)\Big\{(1+\eta\mu_i)(s_i(v_i,v_j)-r_i)\Big\}f(v_i)f(v_j)dv_idv_j - U_i(v_i^*)\right)\\
&= \sum_{i=1}^{2}\left(\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}}\omega_i(v_i|v_i^*,\alpha)\Big\{(1+\eta\mu_i)(s_i(v_i,v_j)-r_i)\Big\}f(v_i)f(v_j)dv_idv_j - \alpha U_i(v_i^*)\right)
\end{aligned}$$

■

Proof of Theorem 3.2

Proof: The maximization problem can be written as

$$\begin{aligned}
\max_s \quad & W_\alpha(s) = \int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}}\Big((1+\eta)s_1(v_1,v_2)\omega_1^B(v_1|\alpha) + (1+\eta\lambda)(s_2(v_1,v_2)-1)\omega_2^S(v_2|\alpha)\\
& - \alpha U_1(\underline{v}) - \alpha U_2(\overline{v})\Big)f(v_1)f(v_2)dv_1dv_2\\
\text{s.t.} \quad & s_1(v_1,v_2)+s_1(v_1,v_2)=1\\
& S_1(v_1) \text{ is non-decreasing in } v_1\\
& S_2(v_2) \text{ is non-decreasing in } v_2\\
& U_1(\underline{v}) \geq 0\\
& U_2(\overline{v}) \geq 0
\end{aligned}$$

Individual rationality constraint is binding. So,

$$\begin{aligned}
\max_s \quad & W_\alpha(s) = \int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}}\Big((1+\eta)s_1(v_1.v_2)\omega_1^B(v|\alpha) + (1+\eta\lambda)(s_2(v_1,v_2)-1)\omega_2^S(v|\alpha)\Big)f(v_1)f(v_2)dv_1dv_2\\
\text{s.t.} \quad & s_1(v_1,v_2)+s_1(v_1,v_2)=1\\
& S_1(v_1) \text{ is non-decreasing in } v_1\\
& S_2(v_2) \text{ is non-decreasing in } v_2
\end{aligned}$$

Because of the regularity of the distribution, we can use the point-wise maximization to find the allocation (s_1, s_2) that solves the above maximization problem. Consider the following allocation $s = (s_1, s_2)$:

$$s_1(v_1, v_2) = \begin{cases} 1 & \text{if } \omega_1^B(v_1|\alpha) \geq k\omega_2^S(v_2|\alpha), \\ 0 & \text{if } \omega_1^B(v_1|\alpha) < k\omega_2^S(v_2|\alpha). \end{cases} \quad \text{and} \quad s_2(v_1, v_2) = \begin{cases} 1 & \text{if } \omega_2^S(v_2|\alpha) \geq \dfrac{\omega_1^B(v_1|\alpha)}{k}, \\ 0 & \text{if } \omega_2^S(v_2|\alpha) < \dfrac{\omega_1^B(v_1|\alpha)}{k}. \end{cases} \tag{3.12}$$

where $k = \dfrac{1 + \eta\lambda}{1 + \eta}$.

First, we show that the allocation s satisfies the constraint. When $\omega_1^B(v_1|\alpha) \geq k\omega_2^S(v_2|\alpha)$, then $s_1(v_1, v_2) = 1$ according to the mechanism. It is implicit from $\omega_1^B(v_1|\alpha) \geq k\omega_2^S(v_2|\alpha)$ that $\dfrac{\omega_1^B(v_1|\alpha)}{k} > \omega_2^S(v_2|\alpha)$ and according to the mechanism $s_2(v_1, v_2) = 0$. This gives $s_1(v_1, v_2) + s_2(v_1, v_2) = 1$. Similarly, $\omega_1^B(v_1|\alpha) < k\omega_2^S(v_2|\alpha)$ implies that $\omega_2^S(v_2|\alpha) > \dfrac{\omega_1^B(v_1|\alpha)}{k}$ and according to the mechanism $s_1(v_1, v_2) = 0$ and $s_2(v_1, v_2) = 1$ which gives $s_1(v_1, v_2) + s_2(v_1, v_2) = 1$.

Next, we show that this allocation rule maximizes $W_\alpha(s)$. Since the allocation satisfies $s_1(v_1, v_2) + s_1(v_1, v_2) = 1$,

$$\begin{aligned} W_\alpha(s) &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \Big((1 + \eta)s_1(v_1, v_2)\omega_1^B(v_1|\alpha) - (1 + \eta\lambda)s_1(v_1, v_2)\omega_2^S(v_2|\alpha) \Big) f(v_1)f(v_2)dv_1dv_2 \\ &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \Big((1 + \eta)\omega_1^B(v_1|\alpha) - (1 + \eta\lambda)\omega_2^S(v_2|\alpha) \Big) s_1(v_1, v_2)f(v_1)f(v_2)dv_1dv_2 \end{aligned}$$

The function is maximized at $s_1(v_1, v_2) = 1$ if $(1 + \eta)\omega_1^B(v_1|\alpha) > (1 + \eta\lambda)\omega_2^S(v_2|\alpha)$ or $\omega_1^B(v_1|\alpha) > k\omega_2^S(v_2|\alpha)$. That means, $s_2(v_1, v_2) = 0$ if $\omega_1^B(v_1|\alpha) > k\omega_2^S(v_2|\alpha)$ or $\omega_2^S(v_2|\alpha) < \dfrac{\omega_1^B(v_1|\alpha)}{k}$. On the other hand, if $(1+\eta)\omega_1^B(v_1|\alpha) < (1+\eta\lambda)\omega_2^S(v_2|\alpha)$ or $\omega_1^B(v_1|\alpha) < k\omega_2^S(v_2|\alpha)$, then $W_\alpha(s)$ attains its maximum value (which is 0) at $s_1(v_1, v_2) = 0$. That means, $s_2(v_1, v_2) = 1$ if $\omega_1^B(v_1|\alpha) < k\omega_2^S(v_2|\alpha)$ or $\omega_2^S(v_2|\alpha) > \dfrac{\omega_1^B(v_1|\alpha)}{k}$.

3.6.1 Proof of Proposition 3.1

The optimization problem under consideration is

$$\begin{aligned} \max_s \quad & \sum_{i=1}^{2} \left(\int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \overline{\omega}_i(v_i|x_i^*, \alpha) \Big\{ (1 + \eta\mu_i)(s_i(v_i, v_j) - r_i) \Big\} f(v_i)f(v_j)dv_idv_j \right) \\ \text{s.t.} \quad & s_1(v_1, v_2) + s_2(v_1, v_2) = 1 \end{aligned}$$

We prove that

$$s_i^*(v_i, v_j) = \begin{cases} 1 & \text{if} \quad \overline{\omega}_i(v_i|x_i^*, \alpha) > k\overline{\omega}_j(v_j|x_j^*, \alpha), \\ r_i & \text{if} \quad \dfrac{\overline{\omega}_j(v_j|x_j^*, \alpha)}{k} < \overline{\omega}_i(v_i|x_i^*, \alpha) < k\overline{\omega}_j(v_j|x_j^*, \alpha), \\ 0 & \text{if} \quad \overline{\omega}_i(v_i|x_i^*, \alpha) < \dfrac{\overline{\omega}_j(v_j|x_j^*, \alpha)}{k}. \end{cases}$$

solves the above maximization problem.

We first show that the allocation s^* satisfies the constraint. When $\overline{\omega}_i(v_i|x_i^*, \alpha) > k\overline{\omega}_j(v_j|x_j^*, \alpha)$, then $s_1^* = 1$ according to the mechanism. It is implicit from $\overline{\omega}_i(v_i|x_i^*, \alpha) > k\overline{\omega}_j(v_j|x_j^*, \alpha)$ that $\dfrac{\overline{\omega}_i(v_i|x_i^*, \alpha)}{k} > \overline{\omega}_j(v_j|x_j^*, \alpha)$ and according to the mechanism $s_j^* = 0$. This means $s_i^*(v_i, v_j) + s_j^*(v_i, v_j) = 1$. Similarly, $\overline{\omega}_i(v_i|x_i^*, \alpha) < \dfrac{\overline{\omega}_j(v_j|x_j^*, \alpha)}{k}$ implies that $k\overline{\omega}_i(v_i|x_i^*, \alpha) < \overline{\omega}_j(v_j|x_j^*, \alpha)$ and according to the mechanism $s_i^* = 0$ and $s_j^* = 1$ which gives $s_i^*(v_i, v_j) + s_j^*(v_i, v_j) = 1$. Whenever $\dfrac{\overline{\omega}_j(v_j|x_j^*, \alpha)}{k} < \overline{\omega}_i(v_i|x_i^*, \alpha) < k\overline{\omega}_j(v_j|x_j^*, \alpha)$, it is implicit that $\dfrac{\overline{\omega}_i(v_i|x_i^*, \alpha)}{k} < \overline{\omega}_j(v_j|x_j^*, \alpha) < k\overline{\omega}_i(v_i|x_i^*, \alpha)$ and according to s^*, $s_1^* = r_1$ and $s_2^* = r_2$ which gives $s_1^*(v_1, v_2) + s_2^*(v_1, v_2) = r_1 + r_2 = 1$.

$$\begin{aligned}
&\sum_{i=1}^{2} \left(\int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \overline{\omega}_i(v_i|x_i^*, \alpha) \Big\{ (1+\eta\mu_i)(s_i(v_i, v_j) - r_i) \Big\} f(v_i) f(v_j) dv_i dv_j \right) \\
&= \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \Bigg(\overline{\omega}_1(v_1|x_1^*, \alpha) \Big\{ (1+\eta\mu_1)(s_1(v_1, v_2) - r_1) \Big\} \\
&+ \overline{\omega}_2(v_2|x_2^*, \alpha) \Big\{ (1+\eta\mu_2)(s_2(v_1, v_2) - r_2) \Big\} \Bigg) f(v_1) f(v_2) dv_1 dv_2 \\
&= \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \Bigg(\overline{\omega}_1(v_1|x_1^*, \alpha) \Big\{ (1+\eta\mu_1)(s_1(v_1, v_2) - r_1) \Big\} \\
&+ \overline{\omega}_2(v_2|x_2^*, \alpha) \Big\{ (1+\eta\mu_2)(1 - s_1(v_1, v_2) - 1 + r_1) \Big\} \Bigg) f(v_1) f(v_2) dv_1 dv_2 \\
&= \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \Bigg(\overline{\omega}_1(v_1|x_1^*, \alpha) \Big\{ (1+\eta\mu_1)(s_1(v_1, v_2) - r_1) \Big\} \\
&- \overline{\omega}_2(v_2|x_2^*, \alpha) \Big\{ (1+\eta\mu_2)(s_1(v_1, v_2) - r_1) \Big\} \Bigg) f(v_1) f(v_2) dv_1 dv_2 \\
&= \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \Bigg(\Big\{ (1+\eta\mu_1)\overline{\omega}_1(v_1|x_1^*, \alpha) - (1+\eta\mu_2)\overline{\omega}_2(v_2|x_2^*, \alpha) \Big\} (s_1(v_1, v_2) - r_1) \Bigg) f(v_1) f(v_2) dv_1 dv_2
\end{aligned}$$

There are three possible cases:

- Case 1: $s_1(v_1, v_2) > r_1$

If $s_1(v_1, v_2) > r_1$, then $s_2(v_1, v_2) < r_2$ since $r_1 + r_2 = 1$. So, $\mu_1 = 1$ and $\mu_2 = \lambda$. Hence,

$$\overline{W}_\alpha(s) = \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \left(\left\{ (1+\eta)\overline{\omega}_1(v_1|x_1^*, \alpha) - (1+\eta\lambda)\overline{\omega}_2(v_2|x_2^*, \alpha) \right\} (s_1(v_1, v_2) - r_1) \right) f(v_1) f(v_2) dv_1 dv_2$$

If $(1+\eta)\overline{\omega}_1(v_1|x_1^*, \alpha) > (1+\eta\lambda)\overline{\omega}_2(v_2|x_2^*, \alpha)$ or $\overline{\omega}_1(v_1|x_1^*, \alpha) > k\overline{\omega}_2(v_2|x_2^*, \alpha)$, then by point-wise maximization, set the maximum value to $s_1(v_1, v_2)$ i.e. $s_1(v_1, v_2) = 1$. This implies $s_2(v_1, v_2) = 0$ and

$$\overline{W}_\alpha(1, 0) = \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \left(\left\{ (1+\eta)\overline{\omega}_1(v_1|x_1^*, \alpha) - (1+\eta\lambda)\overline{\omega}_2(v_2|x_2^*, \alpha) \right\} (1 - r_1) \right) f(v_1) f(v_2) dv_1 dv_2$$

- Case 2: $s_1(v_1, v_2) < r_1$
  If $s_1(v_1, v_2) < r_1$, then $s_2(v_1, v_2) > r_2$ since $r_1 + r_2 = 1$. So, $\mu_1 = \lambda$ and $\mu_2 = 1$. Hence,

$$\overline{W}_\alpha(s) = \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \left(\left\{ (1+\eta\lambda)\overline{\omega}_1(v_1|x_1^*, \alpha) - (1+\eta)\overline{\omega}_2(v_2|x_2^*, \alpha) \right\} (s_1(v_1, v_2) - r_1) \right) f(v_1) f(v_2) dv_1 dv_2$$

If $(1+\eta\lambda)\overline{\omega}_1(v_1|x_1^*, \alpha) < (1+\eta)\overline{\omega}_2(v_2|x_2^*, \alpha)$ or $\overline{\omega}_1(v_1|x_1^*, \alpha) < \dfrac{\overline{\omega}_2(v_2|x_2^*, \alpha)}{k}$, then by point-wise maximization, set the lowest value to $s_1(v_1, v_2)$ i.e. $s_1(v_1, v_2) = 0$.This implies $s_2(v_1, v_2) = 1$ and

$$\overline{W}_\alpha(0, 1) = \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \left(\left\{ (1+\eta)\overline{\omega}_2(v_2|x_2^*, \alpha) - (1+\eta\lambda)\overline{\omega}_1(v_1|x_1^*, \alpha) \right\} (r_1) \right) f(v_1) f(v_2) dv_1 dv_2$$

- Case 3: $s_1(v_1, v_2) = r_1$
  If $s_1(v_1, v_2) = r_1$, then $s_2(v_1, v_2) = r_2$ since $r_1 + r_2 = 1$. So, $\mu_1 = \mu_2 = 1$ and

$$\overline{W}_\alpha(r_1, r_2) = 0$$

Note that $\overline{W}_\alpha(r_1, r_2)$ is optimal if $\overline{W}_\alpha(r_1, r_2) > \overline{W}_\alpha(1, 0)$ and $\overline{W}_\alpha(r_1, r_2) > \overline{W}_\alpha(0, 1)$. It is straightforward to find that $\overline{W}_\alpha(r_1, r_2) > \overline{W}_\alpha(1, 0)$ gives $(1+\eta)\overline{\omega}_1(v_1|x_1^*, \alpha) < (1+\eta\lambda)\overline{\omega}_2(v_2|x_2^*, \alpha)$ or $\overline{\omega}_1(v_1|x_1^*, \alpha) < k\overline{\omega}_2(v_2|x_2^*, \alpha)$. Similarly, $\overline{W}_\alpha(r_1, r_2) > \overline{W}_\alpha(0, 1)$ gives $(1+\eta\lambda)\overline{\omega}_1(v_1|x_1^*, \alpha) > (1+\eta)\overline{\omega}_2(v_2|x_2^*, \alpha)$ or $\overline{\omega}_1(v_1|x_1^*, \alpha) > \dfrac{\overline{\omega}_2(v_2|x_2^*, \alpha)}{k}$. Therefore, (r_1, r_2) is optimal for $\dfrac{\overline{\omega}_2(v_2|x_2^*, \alpha)}{k} < \overline{\omega}_1(v_1|x_1^*, \alpha) < k\overline{\omega}_2(v_2|x_2^*, \alpha)$

■

Proof of Theorem 3.3

Proof: Assuming the existence part is true, we first prove that if s^* as defined in (3.7) satisfies, then s^* solves the original maximization problem. Assume that s^* satisfies $S_i^{ref}(v_i) - r_i = 0$ for some $x_i \in [\omega_i^B(v_i^*|\alpha), \omega_i^S(v_i^*|\alpha)]$ then s^* solves W_α.

From (3.7), $s^*(v)$ satisfies

$$s_i^*(v_i, v_j) = \begin{cases} 1 & \text{if} \quad \overline{\omega}_i(v_i|x_i^*, \alpha) > k\overline{\omega}_j(v_j|x_j^*, \alpha), \\ r_i & \text{if} \quad \dfrac{\overline{\omega}_j(v_j|x_j^*, \alpha)}{k} < \overline{\omega}_i(v_i|x_i^*, \alpha) < k\overline{\omega}_j(v_j|x_j^*, \alpha), \\ 0 & \text{if} \quad \overline{\omega}_i(v_i|x_i^*, \alpha) < \dfrac{\overline{\omega}_j(v_j|x_j^*, \alpha)}{k}. \end{cases}$$

We have assumed that s^* satisfies $S_i^{ref}(v_i) - r_i = 0$ for $v_i \in \Big[(\omega_i^S)^{-1}(x_i), (\omega_i^B)^{-1}(x_i)\Big]$. This implies that $\Omega_i(S^{ref}) = \Big[(\omega_i^S)^{-1}(x_i), (\omega_i^B)^{-1}(x_i)\Big]$ for all $i \in \{1, 2\}$. So s^* satisfies all constraints in the original problem. Now consider any alternative implementable allocation $\hat{s}$. For any $v_i^* \in \Omega_i(S_i^{ref})$ and $\hat{v}_i \in \Omega_i(\hat{S}_i^{ref})$, we have:

$$\sum_{i=1}^{2} \left(\int_{\underline{v}}^{\overline{v}} \omega(v_i|v_i^*, \alpha) \Big\{ S_i^{ref}(v_i, v_j) - r_i) \Big\} f(v_i) dv_i \right) \tag{3.13}$$

$$= \sum_{i=1}^{2} \left(\int_{\underline{v}}^{\overline{v}} \overline{\omega}(v_i|x_i^*, \alpha) \Big\{ S_i^{ref}(v_i, v_j) - r_i \Big\} f(v_i) dv_i \right) \tag{3.14}$$

This equality is because $\overline{\omega}(v_i|x_i^*, \alpha) = \omega(v_i|v_i^*, \alpha)$ for $v_i \notin [(\omega_i^S)^{-1}(x_i^*), (\omega_i^B)^{-1}(x_i^*)]$ and from second condition of s^*. Because s^* is the optimal allocation, we have

$$\sum_{i=1}^{2} \left(\int_{\underline{v}}^{\overline{v}} \overline{\omega}(v_i|x_i^*, \alpha) \Big\{ S_i^{ref}(v_i, v_j) - r_i \Big\} f(v_i) dv_i \right) \tag{3.15}$$

$$\geq \sum_{1=1}^{2} \left(\int_{\underline{v}}^{\overline{v}} \overline{\omega}(v_i|x_i^*, \alpha) \Big\{ \hat{S}_i^{ref}(v_i, v_j) - r_i \Big\} f(v_i) dv_i \right) \tag{3.16}$$

Combining equations (3.13) to (3.16) give

$$\sum_{i=1}^{2} \left(\int_{\underline{v}}^{\overline{v}} \omega(v_i|v_i^*, \alpha) \Big\{ S_i^{ref}(v_i, v_j) - r_i \Big\} f(v_i) dv_i \right)$$

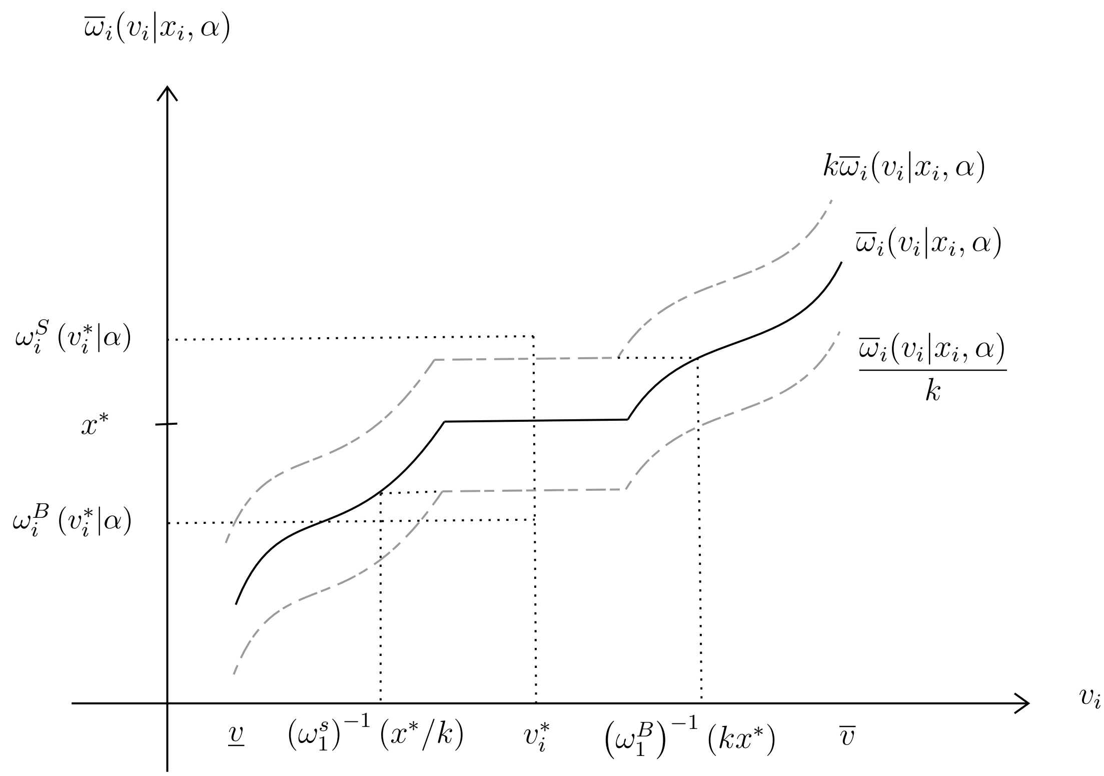
$$\geq \sum_{1=1}^{2} \left(\int_{\underline{v}}^{\overline{v}} \overline{\omega}(v_i|x_i^*, \alpha) \Big\{ \hat{S}_i^{ref}(v_i, v_j) - r_i \Big\} f(v_i) dv_i \right)$$

$$\geq \sum_{1=1}^{2} \left(\int_{\underline{v}}^{\overline{v}} \omega(v_i|\hat{v}_i, \alpha) \Big\{ \hat{S}_i^{ref}(v_i, v_j) - r_i \Big\} f(v_i) dv_i \right)$$

The last inequality is explained as follows: When $v_i < \hat{v}_i$, $\overline{\omega}(v_i|x_i^*, \alpha) \leq \omega(v_i|\hat{v}_i, \alpha)$ and $S_i^{ref}(\hat{v}_i) - r_i \leq 0$. If $v_i > \hat{v}_i$, $\overline{\omega}(v_i|x_i^*, \alpha) \geq \omega(v_i|\hat{v}_i, \alpha)$ and $S_i^{ref}(\hat{v}_i) - r_i \geq 0$. This implies, $\overline{\omega}(v_i|x_i^*, \alpha)(S_i^{ref}(\hat{v}_i) - r_i) \geq \omega(v_i|\hat{v}_i, \alpha)(S_i^{ref}(\hat{v}_i) - r_i)$. Hence s^* maximizes the original objective function.

Now, we prove the existence of x^* such that the allocation s^* satisfies (3.7) for that particular x^*. Suppose that agents share the same x^* and have the same ironed virtual valuation function i.e. $\overline{\omega}_1(v_1|x^*, \alpha) = \overline{\omega}_2(v_2|x^*, \alpha) = \overline{\omega}(v_i|x^*, \alpha), \quad \forall i \in \{1, 2\}$. Since $S_i^{ref}(v_i) - r_i = 0$ for $v_i \in \Big[(\omega_i^S)^{-1}(x^*), (\omega_i^B)^{-1}(x^*)\Big]$ and for $i \in \{1, 2\}$, x^* must be such that the following holds for agent 1:

$$\int_{\underline{v}}^{(\omega_1^S)^{-1}(x^*/k)} (1+\eta)(1-r_1) f(v_2) dv_2 + \int_{(\omega_1^B)^{-1}(kx^*)}^{\overline{v}} (1+\eta\lambda)(0-r_1) f(v_2) dv_2 = 0$$



The explanation is as follows. For $v_1 \in \Big[(\omega_1^S)^{-1}(x^*), (\omega_1^B)^{-1}(x^*)\Big]$, the ironed virtual valuation of agent 1 is x^*. So agent 2's virtual valuation is to be compared with kx^* and x^*/k. Hence, we need to find the set of valuations of agent 2 for the following cases: (a) $\overline{\omega}(v_2|x, \alpha) < x^*/k$, (b) $x^*/k < \overline{\omega}(v_2|x, \alpha) < kx^*$ and (c) $\overline{\omega}(v_2|x, \alpha) > kx^*$. For $\overline{\omega}(v_2|x, \alpha) < x^*$, $\overline{\omega}(v_2|x, \alpha) = \omega^S(v_2|\alpha)$ and for $\overline{\omega}(v_2|x, \alpha) > x^*$, $\overline{\omega}(v_2|x, \alpha) = \omega^B(v_2|\alpha)$.

From the graph, it can be seen that for $v_2 < (\omega_1^S)^{-1}(x^*/k)$, $\omega^S(v_2|\alpha) < x^*/k$. According to allocation s^*, agent 1 will get the entire object. Since $(s_1^*, s_2^*) = (1, 0)$, $\mu_1 = 1$ and $\mu_2 = 0$. Therefore, $S_1^{ref}(v_1) - r_1 = \int_{\underline{v}}^{(\omega_1^S)^{-1}(x^*/k)} (1+\eta)(1-r_1) f(v_2) dv_2$ for $v_2 < (\omega_1^S)^{-1}(x^*/k)$. Similarly for $v_2 > (\omega_1^B)^{-1}(kx^*)$, $\omega^B(v_2|\alpha) > kx^*$ and the optimal mechanism allocates the entire object to agent 2. As a result, $\mu_1 = \lambda$ and $\mu_2 = 1$. Therefore, $S_1^{ref}(v_1) - r_1 = \int_{(\omega_1^B)^{-1}(kx^*)}^{\overline{v}} (1+\eta\lambda)(0-r_1)f(v_2)dv_2$ for $v_2 > (\omega_1^B)^{-1}(kx^*)$. For $(\omega_1^S)^{-1}(x^*/k) < v_2 < (\omega_1^B)^{-1}(kx^*)$, $x^*/k < \overline{\omega}(v_2|\alpha) < kx^*$ and implicitly, $\overline{\omega}(v_2|\alpha)/k < x^* < k\overline{\omega}(v_2|\alpha)$. Hence according to the optimal allocation, $s_1^* = r_1$ and $s_2^* = r_2$ and $S_i^{ref}(v_i) - r_i = 0$ for $(\omega_1^S)^{-1}(x^*/k) < v_2 < (\omega_1^B)^{-1}(kx^*)$. From all these cases, we get the equation.

So,

$$(1+\eta)(1-r_1)F\bigg((\omega_1^S)^{-1}(x^*/k)\bigg) - r_1(1+\eta\lambda)\bigg[1 - F\bigg((\omega_1^B)^{-1}(kx^*)\bigg)\bigg] = 0$$

This implies,

$$r_2F\bigg((\omega_1^S)^{-1}(x^*/k)\bigg) + kr_1F\bigg((\omega_1^B)^{-1}(kx^*)\bigg) = kr_1 \tag{3.17}$$

Similarly, for agent 2,

$$r_1F\bigg((\omega_2^S)^{-1}(x^*/k)\bigg) + kr_2F\bigg((\omega_2^B)^{-1}(kx^*)\bigg) = kr_2 \tag{3.18}$$

Adding (3.17) and (3.18) gives

$$F\bigg((\omega^S)^{-1}(x^*/k)\bigg) + kF\bigg((\omega^B)^{-1}(kx^*)\bigg) = k \tag{3.19}$$

because $\omega_1^A(v|\alpha) = \omega_2^A(v|\alpha)$ for $A \in \{B, S\}$

Notice that for $x^* = k\underline{v}$

$$\begin{aligned} F\bigg((\omega^S)^{-1}(x^*/k)\bigg) + kF\bigg((\omega^B)^{-1}(kx^*)\bigg) &= F\bigg((\omega^S)^{-1}(\underline{v})\bigg) + kF\bigg((\omega^B)^{-1}(k^2\underline{v})\bigg) \\ &< kF(\overline{v}) \\ &= k \end{aligned}$$

For $x^* = \dfrac{\overline{v}}{k}$

$$\begin{aligned} F\bigg((\omega^S)^{-1}(x^*/k)\bigg) + kF\bigg((\omega^B)^{-1}(kx^*)\bigg) &= F\bigg((\omega^S)^{-1}(\frac{\overline{v}}{k^2})\bigg) + kF\bigg((\omega^B)^{-1}(\overline{v})\bigg) \\ &> F(\underline{v}) + k \\ &= k \end{aligned}$$

Due to continuity and monotonicity of $F\bigg((\omega^S)^{-1}(v/k)\bigg) + kF\bigg((\omega^B)^{-1}(kv)\bigg)$, there will exist an $x^* \in \bigg[k\underline{v}, \frac{\overline{v}}{k}\bigg]$ that satisfies (3.19). ■

Chapter 4

Efficient Dissolution Mechanisms

4.1 Introduction

The classical work of Myerson and Satterthwaite (1983) laid the foundation for the bargaining models and showed that under extreme ownership shares where one agent has all the shares of the object (seller) and the other agent has none (buyer), efficient outcome can be implemented by an incentive compatible and individually rational mechanism if and only if an outside party provides a subsidy. Cramton et al. (1987) showed that efficient outcome is possible if the ownership structure is symmetric. Since then, literature has explored the possibility (impossibility) of reallocation of object to achieve ex-post efficiency by analyzing the conditions on the initial ownership shares (Makowski and Mezzetti (1993), McAfee (1991)).

In chapters 1 and 2, we have explored the literature on loss-averse preferences, and the evidence shows that the possibility of trade or dissolution is reduced due to the higher loss sensitivity associated with losing the share compared to the gains. We try to answer the following question: We try to answer the following question: Does the Cramton et al. (1987) possibility result hold with loss-averse agents? i.e., with an equal-share partnership, is it still possible to dissolve the partnership with an efficient, Bayesian incentive compatible and interim individually rational mechanism, or does it depend on the values of the loss aversion parameters? We find that with loss-averse agents, it is not always possible to dissolve an equal-share partnership efficiently. There exists a cut-off point for the loss aversion parameters such that the partnership cannot be dissolved by a Bayesian incentive compatible, individually rational, and efficient mechanism, even when the agents have equal shares, for any distribution. However, we cannot say that for parameter values less than the cutoff point, an equal share partnership is dissolvable by a Bayesian incentive compatible, interim individually rational, budget balanced and efficient mechanism. The particular values of loss aversion parameters such that the equal share partnership can be dissolved efficiently depend on the specific distribution. Therefore, without knowledge of the loss aversion parameters and distribution functions, it is not possible to decide whether efficient trade can take place. This is in contrast to Cramton et al. (1987) and our result generalises the result of Cramton et al. (1987) (when there is no loss aversion). The reason the possibility result breaks down is the following: As sensitivity to losses increases, the set of values at which trade could be implemented is reduced. This leads to a shrinking of the set of values at which dissolution takes place. A higher sensitivity to losses means it is less efficient to dissolve the partnership. Using the example of a unit interval uniform distribution, we provide a range for the loss aversion parameters at which the dissolution of a partnership is efficient.

We also consider a one owner partnership model (bilateral trade) and show that the impossibility result of Myerson and Satterthwaite (1983) still persists. The result is quite intuitive since the agents are less willing to participate in the trade due to the loss aversion with respect to losing the initial share. However, we find that the minimal subsidy required to implement the efficient outcome decreases as the loss sensitivity of agents increases. The possible reason for this is could be the following: As sensitivity to the losses increase, the set of values at which trade could be conducted is reduced. With less possibility of trade, the requirement for minimal subsidy also reduces. There could occur a possibility that the loss aversion parameters are so high that the agents do not participate in the trade and therefore, the minimal subsidy would be 0.

There are few papers in the literature that talk about the departure from quasi-linear preferences in the bilateral trade setting. Chatterjee and Samuelson (1983) showed that as agents become infinitely risk-averse, double auctions are efficient asymptotically. Garratt and Pycia (2023) relaxed the assumption that agents have quasilinear preferences in Myerson and Satterthwaite (1983) model. They showed that if the agents are risk-averse or the utility of the agents from the object is dependent on wealth, then there is a possibility that the trade among the agents is ex-post efficient. Under risk aversion or wealth effects, they give conditions to realize all gains from trade. Their results show that the impossibility of bilateral trade is due to the assumption of quasilinear preferences. Under quasilinear preferences, the reason for the impossibility is that the gains from trade are not sufficient to cover the information rents (due to private information) of the agents. On the other hand, additional efficiency gains are generated from risk aversion. Wolitzky (2016) examines efficiency within a bilateral trade model where both the buyer and seller know the expected valuation of each other. He demonstrates that efficient trade is feasible under certain parameter conditions and gives an exact characterization of that.

Benkert (2023) introduced loss aversion in the bilateral trade setting of Myerson and Satterthwaite (1983). He applies expectation-based loss averse preferences (Kőszegi and Rabin (2006, 2007)) by adapting the narrow bracketing model of Eisenhuth (2018). Eisenhuth

(2018) considers the problem of designing optimal auction for loss averse agents and he used two forms of utility functions: (a), gains and losses are evaluated in the good dimension and in the money dimension separately known as narrow bracketing and (b), gains and losses are evaluated over the entire risk neutral utility, known as wide bracketing. Benkert (2023) discusses that loss aversion decreases the buyer's information rent due to which there is a possibility that the gains from trade (which are also decreased due to reduction in agent's expected utility) can cover the information rent, depending on the parameters of loss aversion. Note that Benkert (2023) talks about implementing the materially efficient outcome. Our paper is different because of two reasons: 1) We consider a fixed reference point which is the initial share/endowment. 2) Benkert (2023) considers loss aversion with respect to transfers as well. Benkert (2023) also showed that a lower subsidy would be required to implement the efficient outcome.

Literature on the partnership dissolution focuses on the ownership structure that will implement efficient outcome. Fieseler et al. (2003) with positive interdependent valuation, showed that it may not be possible to achieve ex-post efficiency even with equal ownership. It is impossible to decide whether ex-post efficient reallocation can take place or not without the knowledge of distribution of private values. Schweizer (2006) showed that the possibility result holds for all prior distributions if the ex-post efficient surplus is sufficient to cover ex-post information rents and the value of outside option at the critical valuation for all type profiles. The impossibility result is true if the ex-post efficient surplus is lower than the ex-post information rents and the value of outside option at the critical valuation for all type profiles, irrespective of prior distribution. For the rest of the cases, the possibility or impossibility result depends on the prior distribution. In a partnership setting where agent's type is private information and types are drawn from different distributions, Figueroa and Skreta (2012) try to find the ownership structure to dissolve the partnership efficiently. They showed that if the agents' critical types (types at which the gains from trade are lowest) are equal, partnership can be dissolved efficiently. When types are drawn from symmetric distribution, equal property rights guarantee equal critical valuations for agents. In the case of asymmetric distributions, equal critical types hold for extremely unequal property rights. They also show that the agents with highest valuation must have a larger share of the object in the partnership. We could not find any paper in the literature that talks about departure from quasi-linear preferences in the partnership dissolution and hence ours is the first paper to study dissolution of partnership for the efficient outcome when preferences are non-standard.

The rest of the chapter is structured as follows: Section 4.2 introduces the formal framework. The impossibility result with loss averse preferences in bilateral trade and partnership setting is discussed in section 4.3. Section 4.4 derives optimal mechanisms that maximize expected total gains in bilateral trade. Section 4.5 concludes. All the proofs are relegated to an appendix 4.6 at the end .

4.2 Model

4.2.1 Types, Information and preferences

Two agents, denoted by $i \in \{1, 2\}$, hold the shares of an asset. Agent 1 owns a share r_1 of the asset and agent 2 owns r_2 where $r_1 + r_2 = 1$. Valuation for the entire asset by agent i is v_i which is a private information and $v_i \sim F[\underline{v}, \overline{v}]$ with positive continuous density f where $\underline{v} > 0$. Agents have loss averse preferences with respect to the share r_i, which acts as an endowment in this case.

Following Köszegi and Rabin (2006), preferences of the loss averse agent are represented using the following utility function.

$$\hat{u}_i(s_i, t_i|v_i, r_i) = \underbrace{s_i v_i + t_i}_{\text{material utility}} + \underbrace{\eta\mu_i\Big(s_i v_i - r_i v_i\Big)}_{\text{gain-loss utility in ownership}}$$

where

$$\mu_i(s_i) = \begin{cases} 1 & \text{if } s_i \geq r_i, \\ \lambda > 1 & \text{if } s_i < r_i \end{cases}.$$

$s_i \in [0, 1]$ is the actual consumption, $r_i \in [0, 1]$ is the reference level of endowment and $t_i \in \mathbb{R}$ is transfer (payment). Per unit valuation of the asset v_i is the private information of the agent, $s_i v_i$ is the intrinsic utility of the object and $r_i v_i$ is the reference utility. The term $s_i v_i + t_i$ is the material utility that incorporates the transfers. Gain-loss utilities are considered with respect to the loss in the reference utility. The loss aversion parameters are η and λ where $\eta > 0$ captures the importance of gain-loss utility relative to intrinsic utility, and $\lambda > 1$ captures the degree of loss aversion. We assume that both the agents have identical loss aversion parameters.

Note that μ_i is an indicator function: If $s_i \geq r_i$, then there is gain of endowment and if $s_i < r_i$, then there is loss. Greater weight to loss is reflected by the fact that $\lambda > 1$.

Value of μ can vary across the agents as it depends on the difference between allocation received and initial endowment, even though λ is same for all agents. Throughout the analysis, we assume that $k\underline{v} < \frac{\overline{v}}{k}$ where $k = \frac{1 + \eta\lambda}{1 + \eta}$.

4.2.2 Mechanism

A direct revelation mechanism is $(s,t) \equiv (s_1, s_2, t_1, t_2)$ where, $s_i : [\underline{v}, \overline{v}]^2 \longrightarrow [0,1]$ with $s_1(v_1,v_2) + s_2(v_1,v_2) \leq 1, \quad \forall v \in [\underline{v}, \overline{v}]^2$ and $t_i : [\underline{v}, \overline{v}]^2 \longrightarrow \mathbb{R}$. The agents directly report their valuations for the object, and then receive the share of the ownership $s_i(v_i, v_j)$ and the transfers $t_i(v_i, v_j)$.

Utility function of agent i from the mechanism when the other agent reports v_j is

$$\hat{u}_i(v_i, v_j) = s_i v_i + t_i + \eta\mu_i\bigg(s_i(v_i,v_j)v_i - r_i v_i\bigg) \quad \forall v_i \in [\underline{v}, \overline{v}]$$

Define $s_i^{ref}(v_i, v_j)$ as the modified allocation where

$$s_i^{ref}(v_i, v_j) = s_i(v_i, v_j) + \eta\mu_i\bigg(s_i(v_i,v_j) - r_i\bigg)$$

This allows us to compactly write $\hat{u}_i(v_i, v_j) = s_i^{ref}(v_i, v_j)v_i + t_i(v_i, v_j)$. Net payoff from the mechanism, denoted as $u_i(v_i, v_j)$, is defined as the difference between the utility from the trade $\hat{u}_i(v_i, v_j)$ and the reference utility i.e.

$$u_i(v_i, v_j) = \bigg(s_i^{ref}(v_i, v_j) - r_i\bigg)v_i + t_i(v_i, v_j)$$

The expected modified share and expected money transfer for player i when he announces v_i are $S_i^{ref}(v_i)$ and $T_i(v_i)$ where

$$\begin{aligned}
S_i^{ref}(v_i) - r_i &= \int_{v_j} (s_i(v_i,v_j) + \eta\mu_i(s_i(v_i,v_j) - r_i) - r_i) f(v_j) dv_j \\
&= \int_{v_j : s_i \geq r_i} (s_i(v_i,v_j) - r_i + \eta(s_i(v_i,v_j) - r_i)) f(v_j) dv_j \\
&+ \int_{v_j : s_i < r_i} (s_i(v_i,v_j) - r_i + \eta\lambda(s_i(v_i,v_j) - r_i)) f(v_j) dv_j \\
&= \int_{v_j} (1 + \eta\mu_i)(s_i(v_i,v_j) - r_i) f(v_j) dv_j
\end{aligned}$$

and

$$T_i(v_i) = \int_{v_j} t_i(v_i, v_j) f(v_j) dv_j$$

So the agent's expected net payoff is

$$U_i(v_i) = (S_i^{ref}(v_i) - r_i)v_i + T_i(v_i) \tag{4.1}$$

This model generalizes the bilateral trade setting considered by Myerson and Satterthwaite (1983). If agent 1 does not have any share of the object ($r_1 = 0$) and agent 2 has the entire ownership of the good ($r_2 = 1$), then the utility function of agent 1 is $(1+\eta)s_1(v_1, v_2)v_1 + t_1(v_1, v_2)$ where $(1+\eta)s_1(v_1, v_2)$ is the gain utility in the share of the object and the utility function of agent 2 is $(1+\eta\lambda)(s_2(v_1, v_2) - 1)v_2 + t_1(v_1, v_2)$ where $(1+\eta\lambda)(s_2(v_1, v_2) - 1)$ is loss in the share of the object due to loss aversion.

We define incentive compatibility and individual rationality of the mechanism.

DEFINITION **4.1** *The mechanism (s_1, s_2, t_1, t_2) is* ***Bayesian incentive compatible (BIC)*** *if for all i and for every $v_i, \hat{v}_i \in [\underline{v}, \overline{v}]$,*

$$U_i(v_i) \geq (S_i^{ref}(\hat{v}_i) - r_i)v_i + T_i(\hat{v}_i)$$

DEFINITION **4.2** *The mechanism (s_1, s_2, t_1, t_2) is* ***interim individually rational (IIR)*** *if for all i and for all $v_i \in [\underline{v}, \overline{v}]$,*

$$(S_i^{ref}(v_i) - r_i)v_i + T_i(v_i) \geq 0$$

We now give a necessary and sufficient condition for a mechanism to be incentive incompatible.

LEMMA **4.1** *The mechanism (s_1, s_2, t_1, t_2) is incentive compatible if and only if for agent i and j, $S_i^{ref}(v_i)$ is non-decreasing and*

$$U_i(v_i) = U_i(v_i^*) + \int_{v_i^*}^{v_i} (S_i^{ref}(x_i) - r_i)dx_i \tag{4.2}$$

For a given monotone allocation rule, payoff equivalence pins down interim expected payoffs U_i and payments T_i up to a constant. Equation (4.2) implies that expected net utility $U_i(v_i)$ is continuous and convex in v_i. The continuity of $U_i(v_i)$ implies it has a minimum at some $v_i^* \in [\underline{v}, \overline{v}]$ where v_i^* is defined in the following lemma.

LEMMA **4.2** *Given a Bayesian incentive-compatible mechanism (s, t), agent i's net utility is minimized at*

$$v_i^* \in \Omega(S_i^{ref}) = \{v_i : S_i^{ref}(z) - r_i \leq 0 \ \forall z < v_i; S_i^{ref}(z) - r_i \geq 0 \ \forall z > v_i\} \tag{4.3}$$

Note that $\Omega(S_i^{ref})$ is non-empty because $S_i^{ref}(v_i)$ is non-decreasing (Lemma 4.1)

LEMMA **4.3** *An incentive-compatible mechanism (s, t) is interim individually rational if and only if for all $i \in \{1, 2\}$*

$$U_i(v_i^*) \geq 0$$

For any allocation (s_1, s_2) such that $S_i^{ref}(v_i)$ is non-decreasing for all $i \in \{1,2\}$, $\Omega_i(S^{ref})$ is well-defined in (4.3) and is called the worst off types. Equations (4.2) and (4.3) imply that expected net utility $U_i(v_i)$ is continuous and convex in vi and is minimized at $v_i^* \in \Omega(S_i^{ref})$. The modified expected share $S_i^{ref}(v_i)$ is a continuous function with r_i in its range. The worst off v_i^* type satisfies $S_i^{ref}(v_i^*) - r_i = 0$ i.e. the worst-off type expects to receive a share equal to his initial ownership share r_i. As in Cramton et al. (1987) and Lu and Robert (2001), this means that, on an average, the worst-off type expects to be neither a buyer nor a seller of the asset. Therefore, he has no incentive to overstate or understate his valuation. Hence, he does not need to be compensated in order to induce him to report his valuation truthfully. It is no longer clear who is selling and who is buying prior to revelation of types, but on average agent i is a buyer if his type $v_i \geq \max_{v_i^* \in \Omega(S^{ref})} v_i^*$ and a seller if his type $v_i \leq \min_{v_i^* \in \Omega(S^{ref})} v_i^*$.

Define, for any $\alpha \geq 0$ and $v \in [\underline{v}, \overline{v}]$,

$$\omega_i^B(v_i|\alpha) = v_i - \alpha\frac{(1-F(v_i))}{f(v_i)} \quad \text{and} \quad \omega_i^S(v_i|\alpha) = v_i + \alpha\frac{F(v_i)}{f(v_i)} \tag{4.4}$$

where $\omega_i^B(v_i|\alpha)$ and $\omega_i^S(v_i|\alpha)$ are referred to as the α-virtual valuation of agents buyer-type and seller-type respectively. We will impose the regularity assumption that each agent's α-weighted valuation is strictly increasing, i.e.,

$$\frac{d}{dv}\omega_i^B(v|\alpha) \geq 0 \quad \text{and} \quad \frac{d}{dv}\omega_i^S(v|\alpha) \geq 0 \quad \forall v \in [\underline{v}, \overline{v}]$$

Given a Bayesian incentive compatible and interim individually rational mechanism, for $v_i^* \in \Omega(S_i^{ref})$, let

$$\omega_i(v_i|v_i^*, \alpha) = \begin{cases} \omega_i^B(v_i|\alpha) & \text{if} \quad v_i > v_i^*, \\ v_i^* & \text{if} \quad v_i = v_i^*, \\ \omega_i^S(v_i|\alpha) & \text{if} \quad v_i < v_i^*. \end{cases}$$

If $r_i = 0$, then $\underline{v} \in \Omega(S^{ref})$ for all implementable allocations s. For $v_i^* = \underline{v}$, $\omega_i(v_i|\underline{v}, \alpha) = v_i - \alpha\frac{(1-F(v_i))}{f(v_i)} = \omega_i^B(v_i|\alpha) \quad \forall v_i > \underline{v}$ implying agent i has buyer-type virtual valuations. If $r_i = 1$, then $\overline{v} \in \Omega(S^{ref})$ for all implementable allocations s. For $v_i^* = \overline{v}$, $\omega_i(v_i|\overline{v}, \alpha) = v_i + \alpha\frac{F(v_i)}{f(v_i)} = \omega_i^S(v_i|\alpha) \quad \forall v_i < \overline{v}$ implying agent i has seller-type virtual valuations. For $r_i \in (0,1)$, v_i^* is between $\underline{v}$ and $\overline{v}$ which means that agent i has virtual valuations of both, buyer-type and seller-type.

The above lemmas give a necessary and sufficient condition for a mechanism to be BIC, IIR and budget-balanced, specified below.

Lemma **4.4** *For any allocation s such that $S_i^{ref}(v_i)$ is non-decreasing for all $i \in \{1,2\}$, there exists a transfer function t where $\sum_{i=1}^{2} t_i = 0$ such that (s,t) is incentive compatible and* *individually rational if and only if*

$$\phi(r_1, r_2) = \sum_{i=1}^{2} \int_{\underline{v}}^{\overline{v}} \omega(v_i|v_i^*, 1)(S_i^{ref}(v_i, v_j) - r_i)f(v_i)dv_i \geq 0 \tag{4.5}$$

where $v_i^* \in \Omega(S_i^{ref})$.

4.3 Efficient outcome in Bilateral Trade and General Partnership

A partnership can be dissolved efficiently if there exists an ex post efficient trading mechanism (s, t) that is Bayesian incentive compatible and interim individually rational. One owner partnership is equivalent to bilateral trade of Myerson and Satterthwaite (1983). From proposition 2.1, the efficiency condition in the case of one owner partnership reduces to

$$s_1(v_1, v_2) = \begin{cases} 1 & \text{if} \quad v_1 > kv_2, \\ 0 & \text{if} \quad v_1 < \dfrac{v_2}{k}. \end{cases}$$

We check if the condition mentioned in lemma 4.4 is satisfied in buyer-seller setup. The result is stated below:

Proposition **4.1** *A one-owner partnership* $(r_i = 1, r_j = 0)$ *cannot be dissolved efficiently.*

The proof of the above proposition indicates that the minimum subsidy required to implement efficient outcome with loss averse agents is $(1+\eta)\int_{k\underline{v}}^{\overline{v}} \big(1 - F(x)\big)F(\frac{x}{k})dx$. Next we check the efficiency loss by comparing the minimum subsidy in case of loss aversion and standard case. In Myerson and Satterthwaite (1983), the minimal subsidy required was $\int_{\underline{v}}^{\overline{v}}(1-F(x))F(x)dx$.

Next we check the efficiency loss by comparing the minimum subsidy in case of loss aversion and standard case. In Myerson and Satterthwaite (1983), the minimal subsidy required was $\int_{\underline{v}}^{\overline{v}}(1 - F(x))F(x)dx$.

Let

$$S(k) = (1 + \eta) \int_{k\underline{v}}^{\overline{v}} \big(1 - F(x)\big)F(\frac{x}{k})dx$$

We want to check whether the subsidy increases if, for a given $\eta > 0$, λ increases.

We will use Leibniz rule to find $\frac{dS(k)}{d\lambda}$.

$$\begin{aligned}\frac{dS(k)}{d\lambda} &= (1+\eta)\bigg[\int_{k\underline{v}}^{\overline{v}}(1-F(x))f\left(\frac{x}{k}\right)\left(-\frac{x}{k^2}\right)\frac{dk}{d\lambda}dx\bigg] \\ &= -(1+\eta)\int_{k\underline{v}}^{\overline{v}}(1-F(x))f\left(\frac{x}{k}\right)\left(\frac{x}{k^2}\right)\frac{\eta}{1+\eta}dx \\ &= \eta\int_{k\underline{v}}^{\overline{v}}(1-F(x))f\left(\frac{x}{k}\right)\left(\frac{x}{k^2}\right)dx\end{aligned}$$

As λ increases, the minimal subsidy required decreases. Therefore, the loss in efficiency decreases with increase in the loss sensitivity parameter.

Now, we examine if Cramton et al. (1987) result remains applicable when preferences exhibit loss aversion for equal share partnerships or there exists an impossibility. The result is stated below.

THEOREM **4.1** *Equal share partnerships cannot be dissolved by a BIC, IIR, budget balanced and efficient mechanism for* $k \geq \sqrt{\frac{\overline{v}}{\underline{v}}}$.

We have found a cut-off point $k^* = \sqrt{\frac{\overline{v}}{\underline{v}}}$ for the loss aversion parameters beyond which the partnership cannot be dissolved by a Bayesian incentive compatible, individually rational, and efficient mechanism, even when the agents have equal shares, for any distribution. We cannot say that for $k < \sqrt{\frac{\overline{v}}{\underline{v}}}$ equal share partnership is dissolvable by a BIC, IIR, budget balanced and efficient mechanism. The particular values of loss aversion parameters such that the equal share partnership can be dissolved efficiently depend on the specific distribution. This is in contrast to Cramton et al. (1987) and our result generalizes the result of Cramton et al. (1987) (when there is no loss aversion). The impossibility of implementing efficient trade occurs due to the following reason. A higher sensitivity to losses leads to a reduction in the range of values at which trade can occur. Consequently, the set of values at which dissolution takes place is reduced. A heightened sensitivity to losses implies that dissolving the partnership becomes less efficient.

Garratt and Pycia (2023) showed that the impossibility result in bilateral trade setting hinges on the assumption that agents have quasilinear utility functions. Their results proved that ex-post efficient trade via BIC, IIR and budget balanced mechanism among privately informed parties is possible in situations where the trading parties exhibit risk aversion or where their utility derived from the traded object is contingent upon their wealth. Wolitzky (2016) introduced max-min preferences of Gilboa and Schmeidler (1989) in the bilateral trade setting and provided an exact characterization of when efficient trade is possible, while assuming that each agent knows the other agent's expected valuation for the good. These papers show that it is possible to have efficient outcome for different kind of behavioural preferences. Our result, on the other hand, breaks down the possibility result, when partners exhibit loss aversion.

It is difficult to derive a closed form solution of loss aversion parameters such that the efficient outcome is implementable for general distributions. However, we show an example to derive a closed form solution for uniform distribution.

Example **4.1** *Consider the case of uniformly distributed types on the unit interval* $[1, 2]$*. Then* $f(v) = 1$ *and* $F(v) = v - 1$*.*

First we find the worst off type v_i^* *using the condition* $S_i^{ref}(v_i^*) - r_i = 0$*.*

$$\frac{v_i^*}{k} - 1 + k(kv_i^* - 1) = k$$

which gives

$$v_i^* = \frac{2k^2 + k}{1 + k^3}$$

Then,

$$\phi = \int_{\overline{v}/k}^{\overline{v}} \left[v_i - \frac{1-F(v_i)}{f(v_i)}\right]\left((1+\eta)(1-r_i)F\left(\frac{v_i}{k}\right)\right) f(v_i)dv_i$$

$$+ \int_{v_i^*}^{\overline{v}/k} \left[v_i - \frac{1-F(v_i)}{f(v_i)}\right]\left((1+\eta)(1-r_i)F\left(\frac{v_i}{k}\right) - (1+\eta\lambda)r_i(1-F(kv_i))\right) f(v_i)dv_i$$

$$+ \int_{k\underline{v}}^{v_i^*} \left[v_i + \frac{F(v_i)}{f(v_i)}\right]\left((1+\eta)(1-r_i)F\left(\frac{v_i}{k}\right) - (1+\eta\lambda)r_i(1-F(kv_i))\right) f(v_i)dv_i$$

$$- \int_{\underline{v}}^{k\underline{v}} \left[v_i + \frac{F(v_i)}{f(v_i)}\right]\left(1+\eta\lambda)r_i(1-F(kv_i))\right) f(v_i)dv_i$$

$$= (1+\eta)\int_{2/k}^{2} \left[2v_i - 2\right]\left(\frac{v_i}{k} - 1\right)dv_i$$

$$+ \int_{v_i^*}^{2/k} \left[2v_i - 2\right]\left(\left(\frac{v_i}{k} - 1\right) - k\left(1 - \left(kv_i - 1\right)\right)\right)dv_i$$

$$+ \int_{k}^{v_i^*} \left[2v_i - 1\right]\left(\left(\frac{v_i}{k} - 1\right) - k\left(1 - \left(kv_i - 1\right)\right)\right)dv_i$$

$$- \int_{1}^{k} \left[2v_i - 1\right]\left(k\left(1 - \left(kv_i - 1\right)\right)\right)dv_i$$

$$= -\frac{4\,(k-1)\,(2k^2-k-4)}{3k^4} - \frac{(k^2-2)^2\,(3k^4-6k^3-k^2+3k-4)}{3k^4\cdot(k+1)^2\,(k^2-k+1)^2}$$

$$- \frac{k^3\cdot(k^2-2)^2\,(4k^4-3k^3+4k^2+6k-3)}{6\,(k+1)^2\,(k^2-k+1)^2} + \frac{(k-1)\,k^2\cdot(4k^2+k-11)}{6}.$$

For $v_i \sim U[1,2]$, $\phi \geq 0$ *for* $k \in [1, 1.1556]$ *and* $\phi < 0$ *for* $k > 1.1556$.

4.4 Maximizing expected gains in dissolution

For bilateral trade with loss averse preferences, ex post efficiency is unattainable. Therefore, we seek a mechanism that maximizes expected total gains from trade, subject to the incentive compatibility and individual-rationality constraints. Suppose agent 1 is a buyer and agent 2 is a seller. We have to choose s to maximize

$$W_0(s) = \int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}} \left(s_1^{ref}(v_1,v_2)v_1 + s_2^{ref}(v_1,v_2)v_2\right) f(v_1)f(v_2)dv_1dv_2$$

subject to

$$\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}} \left((1+\eta)s_1(v_1.v_2)\left[v_1 - \frac{(1-F(v_1))}{f(v_1)}\right] + (1+\eta\lambda)(s_2(v_1,v_2)-1)\left[v_2 + \frac{F(v_2)}{f(v_2)}\right]\right) f(v_1)f(v_2)dv_1dv_2 \geq 0$$

Consider the following allocation:

$$s_1^\alpha(v_1, v_2) = \begin{cases} 1 & \text{if } \omega_1^B(v_1|\alpha) \geq k\omega_2^S(v_2|\alpha), \\ 0 & \text{if } \omega_1^B(v_1|\alpha) < k\omega_2^S(v_2|\alpha). \end{cases} \tag{4.6}$$

and

$$s_2^\alpha(v_1, v_2) = \begin{cases} 1 & \text{if } \omega_2^S(v_2|\alpha) \geq \dfrac{\omega_1^B(v_2|\alpha)}{k}, \\ 0 & \text{if } \omega_2^S(v_2|\alpha) < \dfrac{\omega_1^B(v_2|\alpha)}{k}. \end{cases} \tag{4.7}$$

where $k = \dfrac{1+\eta\lambda}{1+\eta}$.

Using the constrained optimization, the Lagrange multiplier is γ and we get

$$\int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \Bigg((1+\eta)s_1(v_1.v_2)v_1 + \gamma\left[v_1 - \frac{(1-F(v_1))}{f(v_1)}\right](1+\eta)s_1(v_1.v_2)$$
$$+ (1+\eta\lambda)(s_2(v_1,v_2)-1) + \gamma\left[v_2 + \frac{F(v_2)}{f(v_2)}\right](1+\eta\lambda)(s_2(v_1,v_2)-1)\Bigg) f(v_1)f(v_2)dv_1dv_2$$
$$= \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \Bigg((1+\gamma)(1+\eta)s_1(v_1,v_2)v_1 - \gamma\frac{(1-F(v_1))}{f(v_1)}$$
$$+ (1+\gamma)(1+\eta\lambda)(s_2(v_1,v_2)-1) + \gamma\frac{F(v_2)}{f(v_2)}\Bigg) f(v_1)f(v_2)dv_1dv_2$$
$$= (1+\gamma)\int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \Bigg((1+\eta)s_1(v_1.v_2)\left[v_1 - \frac{\gamma}{1+\gamma}\frac{(1-F(v_1))}{f(v_1)}\right]$$
$$+ (1+\eta\lambda)(s_2(v_1,v_2)-1)\left[v_2 + \frac{\gamma}{1+\gamma}\frac{F(v_2)}{f(v_2)}\right]\Bigg) f(v_1)f(v_2)dv_1dv_2$$
$$= (1+\gamma)\int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \Bigg((1+\eta)s_1(v_1,v_2)\omega_1^B\Big(v_1|\gamma/1+\gamma\Big)$$
$$+ (1+\eta\lambda)(s_2(v_1,v_2)-1)\omega_2^S\Big(v_2|\gamma/1+\gamma\Big)\Bigg) f(v_1)f(v_2)dv_1dv_2$$

Any s that satisfies the constraint with equality and maximizes the Lagrangian for some $\gamma \geq 0$ must be a solution for our problem. The Lagrangian is maximized by $(s_1^\alpha(v_1,v_2), s_2^\alpha(v_1,v_2))$, when $\alpha = \dfrac{\gamma}{1+\gamma}$ and the constraint will be satisfied with equality.

Let

$$R(\alpha) = \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \left((1+\eta)\left[v_1 - \frac{(1-F(v_1))}{f(v_1)}\right] - (1+\eta\lambda)\left[v_2 + \frac{F(v_2)}{f(v_2)}\right]\right) s_1(v_1.v_2)f(v_1)f(v_2)dv_1dv_2$$

$$R(\alpha) = \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \Big((1+\eta)\omega_1^B(v_1|1) - (1+\eta\lambda)\omega_2^S(v_2|1)\Big) s_1^\alpha(v_1,v_2)f(v_1)f(v_2)dv_1dv_2$$

Now, $R(1) \geq 0$ because if $(1+\eta)\omega_1^B(v_1|1) \geq (1+\eta\lambda)\omega_2^S(v_2|1)$, then $s_1(v_1, v_2) = 1$ making $R(1) \geq 0$ and if $(1+\eta)\omega_1^B(v_1|1) < (1+\eta\lambda)\omega_2^S(v_2|1)$, then $s_1(v_1, v_2) = 0$ making $R(1) = 0$. Also, $R(0) < 0$ from theorem 4.1 (Impossibility result).

As α increases, $s_1^\alpha(v_1, v_2)$ decreases. Note that $(1+\eta)\omega_1^B(v_1|1) < (1+\eta\lambda)\omega_2^S(v_2|1)$. These two together make $R(\alpha)$ increasing.

It is obvious that $R(\alpha)$ is continuous. Thus, by Intermediate Value Theorem, there must be some $\alpha \in (0, 1]$ such that $R(\alpha) = 0$ and $(s_1^\alpha(v_1, v_2), s_2^\alpha(v_1, v_2))$ must satisfy (4.6) and (4.7). This gives the following result.

THEOREM **4.2** *There exists an incentive compatible and individually rational mechanism with* $(s_1^\alpha(v_1, v_2), s_2^\alpha(v_1, v_2))$ *defined in* (4.6) *and* (4.7) *for some* $\alpha \in [0, 1]$ *such that this mechanism maximizes the expected gains from trade among all incentive compatible and individually rational mechanisms.*

EXAMPLE **4.2** *Consider the same setup described in example 4.1. The α-virtual valuations of the agents are* $\omega_1(v_1|\alpha) = v_1 - \alpha(1-(v_1-1)) = (1+\alpha)v_1 - 2\alpha$ *and* $\omega_2(v_2|\alpha) = v_2 + \alpha(v_2 - 1) = (1+\alpha)v_2 - \alpha$. *The following is the optimal allocation :*

$$s_1^\alpha(v_1, v_2) = \begin{cases} 1 & \textit{if } (1+\alpha)v_1 - 2\alpha \geq k[(1+\alpha)v_2 - \alpha], \\ 0 & \textit{if } (1+\alpha)v_1 - 2\alpha < k[(1+\alpha)v_2 - \alpha]. \end{cases}$$

and

$$s_2^\alpha(v_1, v_2) = \begin{cases} 1 & \textit{if } (1+\alpha)v_1 - 2\alpha < k[(1+\alpha)v_2 - \alpha], \\ 0 & \textit{if } (1+\alpha)v_1 - 2\alpha \geq k[(1+\alpha)v_2 - \alpha]. \end{cases}$$

But we must have

$$\int_1^2 \int_1^2 ((1+\eta)(2v_1 - 2)s_1(v_1, v_2)) - (1+\eta\lambda)(2v_2 - 1)(1 - s_2(v_1, v_2)))dv_2 dv_1 = 0$$

So, the above equation is

$$\int_{k-[(k-2)\alpha/(1+\alpha)]}^2 \int_1^{(v_1/k)+((k-2)\alpha)/k(1+\alpha)} ((1+\eta)(2v_1 - 2)) - (1+\eta\lambda)(2v_2 - 1))dv_2 dv_1 = 0$$

This implies

$$\int_{k+(k-2)\alpha/(1+\alpha)}^2 \frac{(\alpha v_1 + v_1 - k - 2\alpha)\left((\alpha+1)v_1 - \alpha k - 2\right)}{(\alpha+1)^2 k} dv_1 = 0$$

Therefore,

$$-\frac{(3\alpha - 1)(k-2)^3}{6(\alpha+1)^3 k} = 0$$

So, $\alpha = \frac{1}{3}$ *and the optimal allocation is*

$$s_1(v_1, v_2) = \begin{cases} 1 & \textit{if } v_1 - kv_2 \geq (2-k)/4, \\ 0 & \textit{if } v_1 - kv_2 < (2-k)/4. \end{cases}$$

and

$$s_2(v_1, v_2) = \begin{cases} 1 & \textit{if } v_1 - kv_2 < (2-k)/4, \\ 0 & \textit{if } v_1 - kv_2 \geq (2-k)/4. \end{cases}$$

4.5 Conclusion and Future Direction

We provide a full characterization of incentive compatible and individually rational mechanisms when agents are loss averse. We showed how a particular type of preferences of agents can change the results. Although impossibility result in case of bilateral trade setting still persists but the reduction in the amount of subsidy required for the trade shows the possibility of no trade at all if agents are highly loss averse. Similar result holds even if the ownership structure of the object is symmetric. Cramton et al. (1987) result does not hold anymore if the loss sensitivity exceeds a cut off point.

Although we have found a sufficient condition on the parameters such that partnership cannot be dissolved efficiently for equal shares, it will be interesting to see if a similar cut off point can be estimated below which the equal share partnership can always be dissolved efficiently, for any distribution. Another take would be to consider the preferences of agents same as Benkert (2023) and how the result changes when the reference point is endogenous.

The idea of loss aversion could also be connected with expectations. Karle et al. (2015) investigated the effect of expectation-based loss aversion on purchasing decisions. They examine a scenario where consumers must choose between two similar products that vary in price and personal preferences. While consumers are aware of their preferences for both items, they only receive stochastic information regarding the prices, compelling them to form expectations about pricing. Ex-ante, there is an equal probability for each product to be priced lower. Once consumers learn the actual prices of both products, they make their decisions. The theoretical analysis involves agents who exhibit loss aversion, wherein they perceive losses or gains based on whether the actual price paid exceeds or falls short of their expected price. Therefore, the perceived loss associated with paying a high price is contingent on the ex-ante probability with which the consumer anticipates paying the lower price. Closely related papers to Karle et al. (2015) are Karle and Peitz (2014) and Heidhues and Köszegi (2008), where consumers form expectation-based reference points in a market characterized by oligopolistic firms. Consumers correctly anticipate the distribution of equilibrium prices in Karle and Peitz (2014), whereas consumers are uncertain about their tastes for low and high priced goods but observe posted prices in Heidhues and Köszegi (2008). It will be interesting to explore the impact of agents receiving stochastic information about the reference point on the bilateral trade and partnership model. Heidhues and Köszegi (2008).

An alternative approach to modeling a behavioral player would be to consider the exogenous share of the player as the status quo. It is likely that the status quo bias will break the possibility result when partners have equal shares (Samuelson and Zeckhauser (1988)). It will be a good exercise to compare the results of the initial shares being held as reference points with those of the status quo.

4.6 Appendix

4.6.1 Proof of Lemma 4.1

Proof: Necessity: Suppose that the mechanism (s, t) is Bayesian Incentive Compatible. Then,

$$U_i(v_i) \geq U_i(\hat{v}_i) + (S_i^{ref}(\hat{v}_i) - r_i)(v_i - \hat{v}_i) \tag{4.8}$$

which gives

$$U_i(v_i) - U_i(\hat{v}_i) \geq (S_i^{ref}(\hat{v}_i) - r_i)(v_i - \hat{v}_i)$$

Exchanging the roles of v_i and $\hat{v}_i$

$$U_i(\hat{v}_i) \geq U_i(v_i) + (S_i^{ref}(v_i) - r_i)(\hat{v}_i - v_i)$$

This implies

$$U_i(v_i) - U_i(\hat{v}_i) \leq (S_i^{ref}(v_i) - r_i)(v_i - \hat{v}_i) \tag{4.9}$$

(4.8) and (4.9) together imply that

$$(S_i^{ref}(v_i) - r_i)(v_i - \hat{v}_i) \geq U_i(v_i) - U_i(\hat{v}_i) \geq (S_i^{ref}(\hat{v}_i) - r_i)(v_i - \hat{v}_i) \tag{4.10}$$

This shows that if $v_i > \hat{v}_i$, $S_i^{ref}(v_i) \geq S_i^{ref}(\hat{v}_i)$. Therefore, $S_i^{ref}(\cdot)$ is non-decreasing.

Claim **4.1** *$U_i(\cdot)$ is Lipschitz continuous.*

Proof: Show that there exists $M > 0$, such that

$$|U_i(v_i) - U_i(\hat{v}_i)| \leq M|v_i - \hat{v}_i|$$

If $v_i > \hat{v}_i$,

$$U_i(v_i) - U_i(\hat{v}_i) \leq (S_i^{ref}(v_i) - r_i)(v_i - \hat{v}_i) \leq (1 + \eta)(1 - r_i)(v_i - \hat{v}_i)$$

If $v_i < \hat{v}_i$,

$$U_i(v_i) - U_i(\hat{v}_i) \geq (S_i^{ref}(\hat{v}_i) - r_i)(v_i - \hat{v}_i)$$

which can also be written as

$$\begin{aligned} -(U_i(v_i) - U_i(\hat{v}_i)) &\leq -(S_i^{ref}(\hat{v}_i) - r_i)(v_i - \hat{v}_i) \\ &\leq -(S_i^{ref}(v_i) - r_i)(v_i - \hat{v}_i) \end{aligned}$$

Therefore,

$$\begin{aligned} |U_i(v_i) - U_i(\hat{v}_i)| &\leq (S_i^{ref}(v_i) - r_i)|(v_i - \hat{v}_i)| \\ &\leq (1+\eta)(1-r_i)|(v_i - \hat{v}_i)| \end{aligned}$$

For $M = (1+\eta)(1-r_i)$, we have proved that $|U_i(v_i) - U_i(\hat{v}_i)| \leq M|(v_i - \hat{v}_i)|$. Therefore, $U_i(\cdot)$ is Lipschitz continuous. ■

This means that $U_i(\cdot)$ is differentiable almost everywhere. From equation (4.10), we have

$$S_i^{ref}(v_i) - r_i \geq \frac{U_i(v_i) - U_i(\hat{v}_i)}{v_i - \hat{v}_i} \geq S_i^{ref}(\hat{v}_i) - r_i$$

This implies

$$\frac{dU_i(v_i)}{dv_i} = S_i^{ref}(v_i) - r_i$$

and

$$U_i(v_i) = U_i(v_i^*) + \int_{v_i^*}^{v_i} (S_i^{ref}(x_i) - r_i)dx_i \tag{4.11}$$

$\forall v_i, v_i^* \in [\underline{v}, \overline{v}]$.

Substituting the expression for $U_i(v_i)$ in the above equation gives

$$(S_i^{ref}(v_i) - r_i)v_i + T_i(v_i) = U_i(v_i^*) + \int_{v_i^*}^{v_i} (S_i^{ref}(x_i) - r_i)dx_i$$

which can be rewritten as

$$T_i(v_i) = U_i(v_i^*) - (S_i^{ref}(v_i) - r_i)v_i + + \int_{v_i^*}^{v_i} (S_i^{ref}(x_i) - r_i)dx_i$$

Sufficiency: Suppose that the mechanism (s,t) is such that $S_i^{ref}(v_i)$ is non-decreasing and $U_i(v_i)$ satisfies (4.2)

$$\begin{aligned} U_i(v_i) - U_i(v_i^*) = \int_{v_i^*}^{v_i} (S_i^{ref}(u) - r_i)du &\geq \int_{v_i^*}^{v_i} (S_i^{ref}(v_i^*) - r_i)du \\ &= (S_i^{ref}(v_i^*) - r_i)(v_i - v_i^*) \end{aligned}$$

Therefore,

$$U_i(v_i) \geq U_i(v_i^*) + (S_i^{ref}(v_i^*) - r_i)(v_i - v_i^*)$$

Substituting the expression for $U_i(v_i^*)$ in the above equation gives

$$U_i(v_i) \geq (S_i^{ref}(v_i^*) - r_i)v_i + T_i(v_i^*)$$

Hence, (s, t) is Bayesian incentive compatible.

■

4.6.2 Proof of Lemma 4.2

Proof: By Lemma 4.1, the net utility function of trader i with valuation v_i is continuous and convex in v_i. Hence, $U_i(v_i)$ is minimized at the point where the derivative of net utility function is 0. Derivative of $U_i(v_i)$ is $S_i^{ref}(v_i) - r_i$ almost everywhere with $S_i^{ref}(v_i)$ is increasing in v_i. If $S_i^{ref}(v_i) - r_i > 0 \quad \forall v_i \in [\underline{v}, \overline{v}]$, then $U_i(v_i)$ is minimized at $v_i^* = \underline{v}$. Similarly, if $S_i^{ref}(v_i) - r_i < 0 \quad \forall v_i \in [\underline{v}, \overline{v}]$, then $U_i(v_i)$ is minimized at $v_i^* = \overline{v}$. On the other hand, if there exists p and q such that $S_i^{ref}(p) - r_i \leq 0$ and $S_i^{ref}(q) - r_i \geq 0$, then $U_i(v_i)$ is minimized at v_i^* where $S_i^{ref}(v_i^*) - r_i = 0$. The set of valuations at which $S_i^{ref}(v_i) - r_i = 0$ is denoted by

$$\Omega_i(S_i^{ref}) = \{v_i : S_i^{ref}(z) - r_i \leq 0 \ \forall z < v_i; S_i^{ref}(z) - r_i \geq 0 \ \forall z > v_i\}.$$

■

4.6.3 Proof of Lemma 4.3

Proof: A mechanism is IIR if

$$U_i(v_i) \geq 0 \quad \forall v_i \in [\underline{v}, \overline{v}]$$

Because of Lemma 4.1, $U_i(\cdot)$ is increasing. Therefore, we need to check individual rationality at the valuation v_i^* only.

$$U_i(v_i^*) \geq 0$$

■

4.6.4 Proof of Lemma 4.4

Proof: Necessary: If the mechanism is BIC, IIR and budget balanced, then s satisfies (4.5)

From Lemma 4.1, we know that

$$U_i(v_i) = U_i(v_i^*) + \int_{v_i^*}^{v_i} (S_i^{ref}(x_i) - r_i)dx_i$$

This gives,

$$T_i(v_i) = U_i(v_i^*) - (S_i^{ref}(v_i) - r_i)v_i + \int_{v_i^*}^{v_i} (S_i^{ref}(x_1) - r_i)dx_i$$

Taking expectation of $T_i(v_i)$ over v_i, we get

$$\begin{aligned}
\int_{\overline{v}}^{\overline{v}} T_i(v_i)f(v_i)dv_i &= \int_{\underline{v}}^{\overline{v}} \left(U_i(v_i^*) - (S_i^{ref}(v_i) - r_i)v_i + \int_{v_i^*}^{v_i} (S_i^{ref}(x_i) - r_i)dx_i\right) f(v_i)dv_i \\
&= U_i(v_i^*) - \int_{\underline{v}}^{\overline{v}} (S_i^{ref}(v_i) - r_i)v_i f(v_i)dv_i + \int_{\underline{v}}^{\overline{v}} \int_{v_i^*}^{v_i} (S_i^{ref}(x_i) - r_i)dx_i\Big) f(v_i)dv_i \\
&= U_i(v_i^*) - \int_{\underline{v}}^{\overline{v}} (S_i^{ref}(v_i) - r_i)v_i f(v_i)dv_i + \int_{v_i^*}^{\overline{v}} [1 - F(x_i)](S_i^{ref}(x_i) - r_i)dx_i \\
&- \int_{\underline{v}}^{v_i^*} F(x_i)(S_i^{ref}(x_i) - r_i)dx_i \\
&= U_i(v_i^*) - \int_{\underline{v}}^{v_i^*} (S_i^{ref}(v_i) - r_i)v_i f(v_i)dv_i - \int_{\underline{v}}^{v_i^*} F(x_i)(S_i^{ref}(x_i) - r_i)dx_i \\
&- \int_{v_i^*}^{\overline{v}} (S_i^{ref}(v_i) - r_i)v_i f(v_i)dv_i + \int_{v_i^*}^{\overline{v}} [1 - F(x_i)](S_i^{ref}(x_i) - r_i)dx_i \\
&= U_i(v_i^*) - \int_{v_i^*}^{\overline{v}} (S_i^{ref}(v_i) - r_i)\left(v_i - \frac{(1 - F(v_i))}{f(v_i)}\right) f(v_i)dv_i \\
&- \int_{\underline{v}}^{v_i^*} (S_i^{ref}(v_i) - r_i)\left(v_i + \frac{F(v_i)}{f(v_i)}\right) f(v_i)dv_i
\end{aligned}$$

The expected revenue function as

$$\begin{aligned}
R(s) &= -\int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \big(t_1(v_1, v_2) + t_1(v_1, v_2)\big) f(v_2)f(v_1)dv_2dv_1 \\
&= -\left(\int_{\underline{v}}^{\overline{v}} T_1(v_1)f(v_1)dv_1 + \int_{\underline{v}}^{\overline{v}} T_2(v_2)f(v_2)dv_2\right)
\end{aligned}$$

Substituting $\int_{\underline{v}}^{\overline{v}} T_i(v_i)f(v_i)dv_i$ in the revenue function gives

$$\begin{aligned}
\int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \big(t_1(v_1, v_2) + t_1(v_1, v_2)\big) f(v_2)f(v_1)dv_2dv_1 = \sum_{i=1}^{2} \Bigg(&\int_{v_i^*}^{\overline{v}} (S_i^{ref}(v_i) - r_i)\left(v_i - \frac{(1 - F(v_i))}{f(v_i)}\right) f(v_i)dv_i \\
&+ \int_{\underline{v}}^{v_i^*} (S_i^{ref}(v_i) - r_i)\left(v_i + \frac{F(v_i)}{f(v_i)}\right) f(v_i)dv_i - U_i(v_i^*)\Bigg)
\end{aligned}$$

$t_1(v_1, v_2) + t_1(v_1, v_2) = 0$ gives

$$\sum_{i=1}^{2}\left(\int_{v_i^*}^{\overline{v}}(S_i^{ref}(v_i)-r_i)\left(v_i-\frac{(1-F(v_i))}{f(v_i)}\right)f(v_i)dv_i+\int_{\underline{v}}^{v_i^*}(S_i^{ref}(v_i)-r_i)\left(v_i+\frac{F(v_i)}{f(v_i)}\right)f(v_i)dv_i\right) = U_i(v_i^*)$$

From lemma 4.3, we get,

$$\sum_{i=1}^{2}\left(\int_{v_i^*}^{\overline{v}}(S_i^{ref}(v_i)-r_i)\left(v_i-\frac{(1-F(v_i))}{f(v_i)}\right)f(v_i)dv_i+\int_{\underline{v}}^{v_i^*}(S_i^{ref}(v_i)-r_i)\left(v_i+\frac{F(v_i)}{f(v_i)}\right)f(v_i)dv_i\right) \geq 0$$

which means

$$\sum_{i=1}^{2}\int_{\underline{v}}^{\overline{v}}\omega(v_i|v_i^*)(S_i^{ref}(v_i, v_j) - r_i)f(v_i)dv_i \geq 0$$

Sufficiency: Suppose that s satisfies (4.5), then there exists a transfer such that the mechanism (s, t) is incentive compatible, individually rational and budget balanced.

Define

$$c_i = \frac{1}{2}\int_{\underline{v}}^{\overline{v}}\sum_{i=1}^{2}\omega_i(v_i|v_i^*, 1)(S_i^{ref}(v_i)-r_i)f(v_i)dv_i-\int_{v_i^*}^{\overline{v}}(S_i^{ref}(v_i)-r_i)-\int_{\underline{v}}^{\overline{v}}[uf(u)+F(u)-1](S_2^{ref}(u)-r_u)du$$

$$c_i = \frac{1}{2}\int_{\underline{v}}^{\overline{v}}\sum_{i=1}^{2}\omega_i(v_i|v_i^*, 1)(S_i^{ref}(v_i) - r_i)f(v_i)dv_i - \int_{v_i^*}^{\overline{v}}(S_i^{ref}(v_i) - r_i)dv_i$$
$$- \int_{\underline{v}}^{\overline{v}}[v_jf(v_j) + F(v_j) - 1](S_2^{ref}(v_j) - r_j)dv_j$$

This means

$$c_1 = \frac{1}{2}\int_{\underline{v}}^{\overline{v}}\sum_{i=1}^{2}\omega_i(v_i|v_i^*, 1)(S_i^{ref}(v_i) - r_i)f(v_i)dv_i - \int_{v_1^*}^{\overline{v}}(S_1^{ref}(v_1) - r_1)dv_1$$
$$- \int_{\underline{v}}^{\overline{v}}[v_2f(v_2) + F(v_2) - 1](S_2^{ref}(v_2) - r_2)dv_2$$

and

$$c_2 = \frac{1}{2}\int_{\underline{v}}^{\overline{v}}\sum_{i=1}^{2}\omega_i(v_i|v_i^*, 1)(S_i^{ref}(v_i) - r_i)f(v_i)dv_i - \int_{v_2^*}^{\overline{v}}(S_2^{ref}(v_2) - r_2)dv_2$$
$$- \int_{\underline{v}}^{\overline{v}}[v_1f(v_1) + F(v_1) - 1](S_1^{ref}(v_1) - r_1)dv_1$$

Adding c_1 and c_2 gives

$$
\begin{aligned}
c_1 + c_2 &= \int_{\underline{v}}^{\overline{v}} \sum_{i=1}^{2} \omega_i(v_i|v_i^*, 1)(S_i^{ref}(v_i) - r_i)f(v_i)dv_i - \int_{v_1^*}^{\overline{v}} (S_1^{ref}(v_1) - r_1)dv_1 - \int_{v_2^*}^{\overline{v}} (S_2^{ref}(v_2) - r_2)dv_2 \\
&- \int_{\underline{v}}^{\overline{v}} [v_1 f(v_1) + F(v_1) - 1](S_1^{ref}(v_1) - r_1)dv_1 - \int_{\underline{v}}^{\overline{v}} [v_2 f(v_2) + F(v_2) - 1](S_2^{ref}(v_2) - r_2)dv_2 \\
&= \int_{\underline{v}}^{\overline{v}} \sum_{i=1}^{2} \omega_i(v_i|v_i^*, 1)(S_i^{ref}(v_i) - r_i)f(v_i)dv_i - \int_{v_1^*}^{\overline{v}} (S_1^{ref}(v_1) - r_1)dv_1 - \int_{v_2^*}^{\overline{v}} (S_2^{ref}(v_2) - r_2)dv_2 \\
&- \int_{\underline{v}}^{v_1^*} [v_1 f(v_1) + F(v_1) - 1](S_1^{ref}(v_1) - r_1)dv_1 - \int_{\underline{v}}^{v_2^*} [v_2 f(v_2) + F(v_2) - 1](S_2^{ref}(v_2) - r_2)dv_2 \\
&- \int_{v_1^*}^{\overline{v}} [v_1 f(v_1) + F(v_1) - 1](S_1^{ref}(v_1) - r_1)dv_1 - \int_{v_2^*}^{\overline{v}} [v_2 f(v_2) + F(v_2) - 1](S_2^{ref}(v_2) - r_2)dv_2 \\
&= \int_{\underline{v}}^{\overline{v}} \sum_{i=1}^{2} \omega_i(v_i|v_i^*, 1)(S_i^{ref}(v_i) - r_i)f(v_i)dv_i - \int_{\underline{v}}^{v_1^*} [v_1 f(v_1) + F(v_1) - 1](S_1^{ref}(v_1) - r_1)dv_1 \\
&- \int_{\underline{v}}^{v_2^*} [v_2 f(v_2) + F(v_2) - 1](S_2^{ref}(v_2) - r_2)dv_2 - \int_{v_1^*}^{\overline{v}} [v_1 f(v_1) + F(v_1)](S_1^{ref}(v_1) - r_1)dv_1 \\
&- \int_{v_2^*}^{\overline{v}} [v_2 f(v_2) + F(v_2)](S_2^{ref}(v_2) - r_2)dv_2 \\
&= \int_{\underline{v}}^{\overline{v}} \sum_{i=1}^{2} \omega_i(v_i|v_i^*, 1)(S_i^{ref}(v_i) - r_i)f(v_i)dv_i - \sum_{i=1}^{2} \Bigg(\int_{v_i^*}^{\overline{v}} (S_i^{ref}(v_i) - r_i)\Bigg(v_i - \frac{(1 - F(v_i))}{f(v_i)}\Bigg) f(v_i)dv_i \\
&+ \int_{\underline{v}}^{v_i^*} (S_i^{ref}(v_i) - r_i)\Bigg(v_i + \frac{F(v_i)}{f(v_i)}\Bigg) f(v_i)dv_i \Bigg) \\
&= \int_{\underline{v}}^{\overline{v}} \sum_{i=1}^{2} \omega_i(v_i|v_i^*, 1)(S_i^{ref}(v_i) - r_i)f(v_i)dv_i - \int_{\underline{v}}^{\overline{v}} \sum_{i=1}^{2} \omega_i(v_i|v_i^*, 1)(S_i^{ref}(v_i) - r_i)f(v_i)dv_i \\
&= 0
\end{aligned}
$$

Now, we can define transfer function as

$$
t_i(v_i, v_j) = v_i(S_i^{ref}(v_i) - r_i) - \int_{\underline{v}}^{v_i} (S_i^{ref}(u) - r_i)du - [v_j(S_j^{ref}(v_j) - r_j) - \int_{\underline{v}}^{v_j} (S_j^{ref}(u) - r_j)du] - c_i
$$

This implies

$$
\begin{aligned}
t_1(v_1,v_2)+t_1(v_1,v_2) &= v_1(S_1^{ref}(v_1)-r_1)-\int_{\underline{v}}^{v_1}(S_1^{ref}(u)-r_1)du-[v_2(S_2^{ref}(v_2)-r_2)\\
&-\int_{\underline{v}}^{v_2}(S_2^{ref}(u)-r_2)du]-c_1+v_2(S_2^{ref}(v_2)-r_2)-\int_{\underline{v}}^{v_2}(S_2^{ref}(u)-r_2)du\\
&-[v_1(S_1^{ref}(v_1)-r_1)-\int_{\underline{v}}^{v_1}(S_1^{ref}(u)-r_1)du]-c_2\\
&=0 \quad (\because c_1+c_2=0)
\end{aligned}
$$

This transfer function gives

$$
\begin{aligned}
T_i(v_i) &= v_i(S_i^{ref}(v_i)-r_i)-\int_{\underline{v}}^{v_i}(S_i^{ref}(u)-r_i)du-\int_{\underline{v}}^{\overline{v}}(v_j(S_j^{ref}(v_j)-r_j))f(v_j)dv_j\\
&+\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{v_j}(S_j^{ref}(u)-r_j)du f(v_j)dv_j-c_i\\
&=v_i(S_i^{ref}(v_i)-r_i)-\int_{\underline{v}}^{v_i}(S_i^{ref}(u)-r_i)du-\int_{\underline{v}}^{\overline{v}}(v_j(S_j^{ref}(v_j)-r_j))f(v_j)dv_j\\
&+\int_{\underline{v}}^{\overline{v}}[1-F(u)](S_j^{ref}(u)-r_j)du-c_i\\
&=v_i(S_i^{ref}(v_i)-r_i)-\int_{\underline{v}}^{v_i}(S_i^{ref}(u)-r_i)du-\int_{\underline{v}}^{\overline{v}}[v_jf(v_j)+F(v_j)-1](S_j^{ref}(v_j)-r_j))dv_j-c_i\\
&=v_i(S_i^{ref}(v_i)-r_i)-\int_{v_i^*}^{v_i}(S_i^{ref}(u)-r_i)du-\frac{1}{2}\int_{\underline{v}}^{\overline{v}}\sum_{i=1}^{2}\omega_i(v_i|v_i^*,1)(S_i^{ref}(v_i)-r_i)f(v_i)dv_i
\end{aligned}
$$

From here we get,

$$
U_i(v_i)=v_i(S_i^{ref}(v_i)-r_i)+T_i(v_i)=\int_{v_i^*}^{v_i}(S_i^{ref}(u)-r_i)du+\sum_{i=1}^{2}\omega_i(v_i|v_i^*,1)(S_i^{ref}(v_i)-r_i)f(v_i)dv_i
$$

with

$$
U_i(v_i^*)=\sum_{i=1}^{2}\omega_i(v_i|v_i^*,1)(S_i^{ref}(v_i)-r_i)f(v_i)dv_i\geq 0
$$

So, the mechanism is BIC and IIR. ■

4.6.5 Proof of Proposition 4.1

Proof: Implementing the efficient allocation rule, we get

$$\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}}\Bigg((1+\eta)s_1(v_1.v_2)\left[v_1-\frac{(1-F(v_1))}{f(v_1)}\right]+(1+\eta\lambda)(s_2(v_1,v_2)-1)\left[v_2+\frac{F(v_2)}{f(v_2)}\right]\Bigg)f(v_1)f(v_2)dv_1dv_2$$
$$=(1+\eta)\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}}\Bigg(s_1(v_1.v_2)\left[v_1-\frac{(1-F(v_1))}{f(v_1)}\right]-k(1-s_2(v_1,v_2))\left[v_2+\frac{F(v_2)}{f(v_2)}\right]\Bigg)f(v_1)f(v_2)dv_1dv_2$$
$$=(1+\eta)\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}}\Bigg(s_1(v_1.v_2)\Big[v_1f(v_1)-1+F(v_1)\Big]f(v_2)dv_1dv_2$$
$$-k(1-s_2(v_1,v_2))\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}}\Big[v_2f(v_2)+F(v_2)\Big]\Bigg)f(v_1)dv_1dv_2$$
$$=(1+\eta)\Bigg(\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{v_1/k}\Big[v_1f(v_1)-1+F(v_1)\Big]f(v_2)dv_2dv_1-k\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{v_1/k}\Big[v_2f(v_2)+F(v_2)\Big]dv_2f(v_1)dv_1\Bigg)$$
$$=(1+\eta)\Bigg(\int_{\underline{v}}^{\overline{v}}\Big[v_1f(v_1)-1+F(v_1)\Big]F\left(\frac{v_i}{k}\right)dv_1-\int_{\underline{v}}^{\overline{v}}F\left(\frac{v_1}{k}\right)\frac{v_1}{k}\times kf(v_1)dv_1\Bigg)$$
$$=-(1+\eta)\int_{\underline{v}}^{\overline{v}}(1-F(v_1))F\left(\frac{v_1}{k}\right)dv_1$$
$$=-(1+\eta)\int_{k\underline{v}}^{\overline{v}}(1-F(v_1))F\left(\frac{v_1}{k}\right)dv_1$$
$$=-(1+\eta)\int_{k\underline{v}}^{\overline{v}}\big(1-F(x)\big)F\left(\frac{x}{k}\right)dx$$

which is a violation of the condition. This is the minimum subsidy required to implement efficient trade ■

4.6.6 Proof of Theorem 4.1

Proof: Ex-post efficiency from proposition 2.1 requires that

$$s_i(v_i,v_j)=\begin{cases}1 & \text{if} \quad v_i\geq kv_j,\\ r_i & \text{if} \quad \dfrac{v_j}{k}<v_i<kv_j,\\ 0 & \text{if} \quad v_i\leq\dfrac{v_j}{k}.\end{cases}$$

$$
\begin{aligned}
\phi(r_1,r_2) &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \sum_{i\in\{1,2\}} \omega_i(v_i|v_i^*,1)(1+\eta\mu_i)(s_i(v_i,v_j)-r_i)f(v_2)f(v_1)dv_2dv_1 \\
&= \sum_{i\in\{1,2\}} \int_{\underline{v}}^{\overline{v}} \omega_i(v_i|v_i^*,1)\left(\int_{\underline{v}}^{v_i/k}(1+\eta)(1-r_i)f(v_j)dv_j + \int_{kv_i}^{\overline{v}}(1+\eta\lambda)(0-r_i)f(v_j)dv_j\right)f(v_i)dv_i \\
&= \sum_{i\in\{1,2\}} \int_{\underline{v}}^{\overline{v}} \omega_i(v_i|v_i^*,1)\left((1+\eta)(1-r_i)F\left(\frac{v_i}{k}\right) - (1+\eta\lambda)r_i(1-F(kv_i))\right)f(v_i)dv_i \\
&= \sum_{i\in\{1,2\}} \left(\int_{v_i^*}^{\overline{v}}\left[v_i - \frac{1-F(v_i)}{f(v_i)}\right]\left((1+\eta)(1-r_i)F\left(\frac{v_i}{k}\right) - (1+\eta\lambda)r_i(1-F(kv_i))\right)f(v_i)dv_i\right. \\
&\left. + \int_{\underline{v}}^{v_i^*}\left[v_i + \frac{F(v_i)}{f(v_i)}\right]\left((1+\eta)(1-r_i)F\left(\frac{v_i}{k}\right) - (1+\eta\lambda)r_i(1-F(kv_i))\right)f(v_i)dv_i\right) \\
&= \sum_{i\in\{1,2\}} \left(\int_{\overline{v}/k}^{\overline{v}}\left[v_i - \frac{1-F(v_i)}{f(v_i)}\right]\left((1+\eta)(1-r_i)F\left(\frac{v_i}{k}\right)\right)f(v_i)dv_i\right. \\
&+ \int_{v_i^*}^{\overline{v}/k}\left[v_i - \frac{1-F(v_i)}{f(v_i)}\right]\left((1+\eta)(1-r_i)F\left(\frac{v_i}{k}\right) - (1+\eta\lambda)r_i(1-F(kv_i))\right)f(v_i)dv_i \\
&+ \int_{k\underline{v}}^{v_i^*}\left[v_i + \frac{F(v_i)}{f(v_i)}\right]\left((1+\eta)(1-r_i)F\left(\frac{v_i}{k}\right) - (1+\eta\lambda)r_i(1-F(kv_i))\right)f(v_i)dv_i \\
&\left. - \int_{\underline{v}}^{k\underline{v}}\left[v_i + \frac{F(v_i)}{f(v_i)}\right]\left((1+\eta)(1+\eta\lambda)r_i(1-F(kv_i))\right)f(v_i)dv_i\right) \\
&= \sum_{i\in\{1,2\}} \left(\psi + \int_{v_i^*}^{\overline{v}/k}\left[v_i - \frac{1-F(v_i)}{f(v_i)}\right]\left((1+\eta)(1-r_i)F\left(\frac{v_i}{k}\right) - (1+\eta\lambda)r_i(1-F(kv_i))\right)f(v_i)dv_i\right. \\
&\left. + \int_{k\underline{v}}^{v_i^*}\left[v_i + \frac{F(v_i)}{f(v_i)}\right]\left((1+\eta)(1-r_i)F\left(\frac{v_i}{k}\right) - (1+\eta\lambda)r_i(1-F(kv_i))\right)f(v_i)dv_i\right)
\end{aligned}
$$

where

$$
\begin{aligned}
\psi = &\int_{\overline{v}/k}^{\overline{v}}\left[v_i - \frac{1-F(v_i)}{f(v_i)}\right]\left((1+\eta)(1-r_i)F\left(\frac{v_i}{k}\right)\right)f(v_i)dv_i \\
&- \int_{\underline{v}}^{k\underline{v}}\left[v_i + \frac{F(v_i)}{f(v_i)}\right]\left((1+\eta\lambda)r_i(1-F(kv_i))\right)f(v_i)dv_i
\end{aligned}
$$

and

$$
(1+\eta)(1-r_i)F\left(\frac{v_i^*}{k}\right) - (1+\eta\lambda)r_i(1-F(kv_i^*)) = 0
$$

or

$$
(1-r_i)F\left(\frac{v_i^*}{k}\right) + kr_iF(kv_i^*) = kr_i
$$

Claim **4.2** *$\psi < 0$ when $r_i = 1/2$ for $k = \sqrt{\frac{\overline{v}}{\underline{v}}}$.*

Proof: Put $r_1 = 0$ and $r_2 = 1$. Then $v_1^* = \underline{v}$ and $v_2^* = \overline{v}$

$$\begin{aligned}\phi(0,1) &= \int_{\underline{v}}^{\overline{v}} \left[v_1 - \frac{1-F(v_1)}{f(v_1)}\right](1+\eta)F\left(\frac{v_1}{k}\right) - \int_{\underline{v}}^{\overline{v}} \left[v_2 + \frac{F(v_2)}{f(v_2)}\right](1+\eta\lambda)(1-F(kv_2))f(v_2)dv_2 \\ &= \int_{k\underline{v}}^{\overline{v}} \left[v_1 - \frac{1-F(v_1)}{f(v_1)}\right](1+\eta)F\left(\frac{v_1}{k}\right) - \int_{\underline{v}}^{\overline{v}/k} \left[v_2 + \frac{F(v_2)}{f(v_2)}\right](1+\eta\lambda)(1-F(kv_2))f(v_2)dv_2\end{aligned}$$

We know that a one owner partnership cannot be dissolved efficiently. This means that $\phi(0,1) < 0$. This means

$$\int_{k\underline{v}}^{\overline{v}} \left[v_1 - \frac{1-F(v_1)}{f(v_1)}\right](1+\eta)F\left(\frac{v_1}{k}\right) - \int_{\underline{v}}^{\overline{v}/k} \left[v_2 + \frac{F(v_2)}{f(v_2)}\right](1+\eta\lambda)(1-F(kv_2))f(v_2)dv_2 < 0$$

Simplifying,

$$\int_{\overline{v}/k}^{\overline{v}} \left[v_1 - \frac{1-F(v_1)}{f(v_1)}\right](1+\eta)F\left(\frac{v_1}{k}\right) + \int_{k\underline{v}}^{\overline{v}/k} \left[v_1 - \frac{1-F(v_1)}{f(v_1)}\right](1+\eta)F\left(\frac{v_1}{k}\right)$$
$$- \int_{k\underline{v}}^{\overline{v}/k} \left[v_2 + \frac{F(v_2)}{f(v_2)}\right](1+\eta\lambda)(1-F(kv_2))f(v_2)dv_2 - \int_{\underline{v}}^{k\underline{v}} \left[v_2 + \frac{F(v_2)}{f(v_2)}\right](1+\eta\lambda)(1-F(kv_2))f(v_2)dv_2$$
$$< 0$$

This implies

$$\psi + \int_{k\underline{v}}^{\overline{v}/k} \left[v_1 - \frac{1-F(v_1)}{f(v_1)}\right](1+\eta)F\left(\frac{v_1}{k}\right) - \int_{k\underline{v}}^{\overline{v}/k} \left[v_2 + \frac{F(v_2)}{f(v_2)}\right](1+\eta\lambda)(1-F(kv_2))f(v_2)dv_2 < 0$$

At $k = \sqrt{\frac{\overline{v}}{\underline{v}}}$, we have $k\underline{v} = \frac{\overline{v}}{k} = \sqrt{\underline{v}\overline{v}}$. Substituting in the above equation gives

$$\psi < 0$$

■

This means

$$(1+\eta)\left(\int_{\overline{v}/k}^{\overline{v}} \left[v_i - \frac{1-F(v_i)}{f(v_i)}\right]F\left(\frac{v_i}{k}\right)f(v_i)dv_i - \int_{\underline{v}}^{k\underline{v}} \left[v_i + \frac{F(v_i)}{f(v_i)}\right]\Big(kr_i(1-F(kv_i))\Big)f(v_i)dv_i\right) < 0$$

For $r_i = 1/2$, v_i^* satisfies

$$F\left(\frac{v_i^*}{k}\right) + kF(kv_i^*) = k$$

Claim **4.3** *When* $k = \sqrt{\frac{\overline{v}}{\underline{v}}}$, $v_i^* = \sqrt{\overline{v}\underline{v}}$ *satisfies the above equation because* $\frac{v_i^*}{k} = \underline{v}$ *and* $kv_i^* = \overline{v}$. *So,* $F\left(\frac{v_i^*}{k}\right) + kF(kv_i^*) = 0 + k(1) = k$.

Now, we prove that $\phi(1/2, 1/2) < 0$

$$\begin{aligned}\phi(1/2,1/2) = &\sum_{i\in\{1,2\}}\left(\int_{v_i^*}^{\overline{v}/k}\left[v_i - \frac{1-F(v_i)}{f(v_i)}\right]\left((1+\eta)(1-r_i)F\left(\frac{v_i}{k}\right) - (1+\eta\lambda)r_i(1-F(kv_i))\right)f(v_i)dv_i\right.\\ &\left.+\int_{k\underline{v}}^{v_i^*}\left[v_i + \frac{F(v_i)}{f(v_i)}\right]\left((1+\eta)(1-r_i)F\left(\frac{v_i}{k}\right) - (1+\eta\lambda)r_i(1-F(kv_i))\right)f(v_i)dv_i + \psi\right)\\ &= \sum_{i\in\{1,2\}}\psi < 0 \quad \text{from the claim}\end{aligned}$$

■

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