A Class of Acceptance Sampling Plans for Variables

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1. Introduction

WHILE the Dodgé-Romig, the Columbia Statistical Research Group (SRG), and other attribute sampling plans have been (and still continue to be) popular in industrial practice, the scope and advantages of plans suitable for measurement or variables inspection, were not recognised presumably until Wald (1945) introduced sequential tests for measured quality characteristics. Since then, Wallis (1947), Curtiss (1950) and others have considered the problem of constructing acceptance sampling plans for variables. Bowker and Goode (1952) have recently published sampling inspection plans for variables similar to the *SRG* attribute plans. This is probably the only set of tables available for variables inspection, except for a brief table given earlier by Wallis (1947) who considered the case when p_1 (AQL), p_2 (LTPD), α (Producer risk) and β (Consumer risk) have been specified and gave single-sampling plans corresponding to certain combinations of p_1 and p_2 and for values of $\alpha = 5$ per cent. and $\beta = 10$ per cent. The plans by Bowker and Goode are AQL plans, *i.e.*, they provide for stipulations in terms of p_1 and α only, for suitably chosen sample sizes, and they cover single and double-sampling procedures both when process variability is known and when unknown.

2. Scope of the Present Plan

The sampling plans for variables given in this paper are single-sampling plans and are similar to the above-mentioned plans as regards the basic set-up. They, however, are different in many respects and possess the following features:—

(1) The plans given are for use when all the four quantities p_1, p_2, α and β are specified, *i.e.*, they are AQL–LTPD plans.

(2) The plans are specified by a quantity δ which is the same for an infinite number of combinations of p_1 and p_2 values.

(3) The criterion for acceptance or rejection differs in form from that of the other plans. A quantity t_{p_1} is introduced, which is to be obtained from Chart 1.

(4) A simple connection is established between known and unknown standard deviation plans and the two types as given here have identical OC. The sample size required for the unknown standard deviation plans is a multiple of that for the known standard deviation plans.

(5) The plans, although presented in a very compact form, are much more comprehensive than other variable plans available and cover a very wide range of stipulations. Plans are provided both for known and unknown product variability.

3. Sampling Plans when Product Standard Deviation is Known

Although from a theoretical view-point it would appear that the most general type of variables sampling plans should be those which assume the standard deviation of item quality to be not known, in actual industrial practice the standard deviation of item measurements for many types of products is found to be constant from lot to lot and can also be determined in advance. For such products known standard deviation plans should be used on account of the considerable saving in sample size and also in numerical computation that can be achieved.

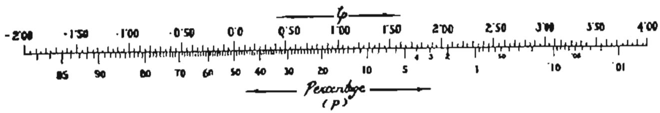

CHART 1. For obtaining the value of t_p (Normal Deviate) for given p (percentage)

However, for products where standard deviation is expected to vary from lot to lot and for products whose quality history is not known, unknown standard deviation plans have their importance. For products of the latter type, that is products with unknown quality history, it may be possible also to change over to known standard deviation plans as soon as evidence is available that standard deviation is constant and it can be estimated with the necessary precision.

3. 1. Operation Procedure for Known Standard Deviation Plans

I. Preliminary Steps

(*a*) Choose the Plan Number satisfying the given values of p_1 and p_2 from Chart 2.

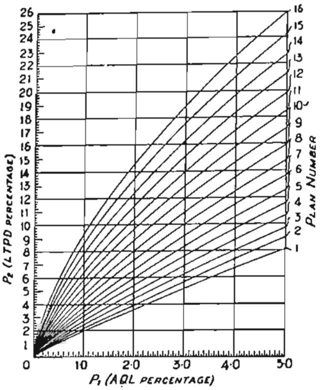

CHART 2. For obtaining plan number for given p_1 and p_2

(*b*) Enter Table I with the Plan Number and select n and k_δ.

(*c*) Obtain the value of t_{p_1} corresponding to p_1 from Chart 1 and calculate $k = (t_{p_1} - k_\delta)$.

II. Major Steps

1. Obtain measurements on n items selected (at random) from the lot.
2. Calculate the arithmetic mean $\bar{x}$ of the n measurements.
3. (i) when the specification limit is a lower limit L,
   reject the lot if $\bar{x} - k\sigma < L$
   accept the lot if $\bar{x} - k\sigma \geqslant L$

TABLE I

Single Sampling Acceptance Plans for Variables (for use when standard deviation is known)

$\alpha = 5\%;\ \beta = 10\%$

Plan No.	δ	n	k_δ
1	0·25	138	·1405
2	0·30	96	·1686
3	0·35	70	·1967
4	0·40	54	·2248
5	0·45	43	·2529
6	0·50	35	·2810
7	0·55	29	·3091
8	0·60	24	·3372
9	0·65	21	·3653
10	0·70	18	·3934
11	0·75	16	·4216
12	0·80	14	·4497
13	0·85	12	·4778
14	0·90	11	·5059
15	0·95	10	·5340
16	1·00	9	·5621

(ii) when the specification limit is an upper limit U,
reject the lot if $\bar{x} + k\sigma > U$
accept the lot if $\bar{x} + k\sigma \leqslant U$

(iii) when there are two specification limits, a lower L and an upper U,
reject the lot if *either* $\bar{x} - k\sigma < L$ or $\bar{x} + k\sigma > U$
accept the lot if $\bar{x} - k\sigma \geqslant L$ *and* $\bar{x} + k\sigma \leqslant U$

4. Sampling Plans when Product Standard Deviation is not known

When the standard deviation of item quality in lots is not known, we use the criterion $\bar{x} + ks > U$ instead of $\bar{x} + k\sigma > U$ in the previous case, where s is the sample standard deviation defined by $\sqrt{\frac{\Sigma(x - \bar{x})^2}{(n' - 1)}}$. This criterion will involve a larger sample size n' (which is a multiple of n the sample size for known standard deviation plans) in order to ensure the same specifications with regard to p_1, p_2, α, β. It will be seen that theoretically the same Table I can be used in this case also. But for the numerical accuracy necessary, a modified form of this table is given in Table II which is the one to be used for the unknown standard deviation plans.

TABLE II

Single Sampling Acceptance Plans for Variables (for use when standard deviation is not known)

$\alpha = 5\%;\ \beta = 10\%$

Plan No.	δ	n	k_δ
1	0·25	137·02	·1405
2	0·30	95·15	·1686
3	0·35	69·91	·1967
4	0·40	53·52	·2248
5	0·45	42·29	·2529
6	0·50	34·26	·2810
7	0·55	28·31	·3091
8	0·60	23·79	·3372
9	0·65	20·27	·3653
10	0·70	17·48	·3934
11	0·75	15·22	·4216
12	0·80	13·38	·4497
13	0·85	11·95	·4778
14	0·90	10·57	·5059
15	0·95	9·49	·5340
16	1·00	8·56	·5621

4. 1. Operation Procedure for Unknown Standard Deviation Plans

I. Preliminary Steps

(*a*) Choose the Plan Number for given values of p_1 and p_2 from Chart 2.

(*b*) Enter Table II with the Plan Number and select n and k_δ.

(*c*) Obtain the value of t_{p_1} corresponding to p_1 from Chart I and calculate

(i) $k = (t_{p_1} - k_\delta)$

(ii) $n' = n\dfrac{(k^2+2)}{2}$ (round n' to the nearest integer).

II. Major Steps

1. Obtain measurements on n' items selected (at random) from the lot.
2. Calculate the arithmetic mean $\bar{x}$ and standard deviation s from the n' measurements.
3. (i) when the specification limit is a lower limit L,
   reject the lot if $\bar{x} - ks < L$
   accept the lot if $\bar{x} - ks \geqslant L$

   (ii) when the specification limit is an upper limit U,
   reject the lot if $\bar{x} + k. > U$
   accept the lot if $\bar{x} + ks \leqslant U$

   (iii) When there are two specification limits, a lower L and an upper U,
   reject the lot i *either* $\bar{x} - ks < L$
   or $\bar{x} + ks > U$
   accept the lot if $\bar{x} - ks \geqslant L$
   and $\bar{x} + ks \leqslant U$

[*Note.*—Chart 2 could be extended to cover larger values of p_1 and p_2. So also the Tables I and II giving the plans could be expanded either for additional values of δ or for other α, β combinations such as $\alpha = 5\%$, $\beta = 5\%$, $\alpha = 10\%$, $\beta = 10\%$ and so on. More extensive plans than given here have been prepared and will be soon published elsewhere. But the charts and tables given here have in no way less coverage than many plans used in practice.]

5. Mathematical Note

In terms of the same mathematical set-up used by Wallis or Bowker and Goode, we have for the present plans the following results.

5. 1. Known Standard Deviation Plans

$$\sqrt{n}\,(t_{p_1} - k) = t_\alpha$$
$$\sqrt{n}\,(t_{p_2} - k) = t_{(1-\beta)} = -t_\beta$$

giving

$$n = \frac{(t_\alpha + t_\beta)^2}{\delta^2} \qquad (1)$$

where t_{p_1}, t_{p_2}, t_α, t_β are Normal deviates corresponding to percentages p_1, p_2, α, β and $\delta = (t_{p_1} - t_{p_2})$.

(Note that $\delta > 0$ as $p_2 > p_1$ and $t_{p_1} > t_{p_2}$)

and

$$k = (t_{p_1} - k_\delta) \qquad (2)$$

where $k_\delta = \dfrac{t_\alpha}{(t_\alpha + t_\beta)}\,\delta$

5. 2. Unknown Standard Deviation Plans

$$\frac{\sqrt{n'}\,(t_{p_1} - k)}{\sqrt{1 + \frac{k^2}{2}}} = t_\alpha$$

and

$$\frac{\sqrt{n'}\,(t_{p_2} - k)}{\sqrt{1 + \frac{k^2}{2}}} = -t_\beta$$

giving

$$n' = \frac{n\,(k^2 + 2)}{2} \qquad (3)$$

and

$$k = (t_{p_1} - k_\delta) \qquad (4)$$

[Note that k remains the same as in (2) and that there exists a very simple relation between n and n'.]

(*Continued on page* 30)