INT. J. SYSTEMS SCI., 1985, VOL. 16, NO. 1, 71–80

A hybrid edge detector and its properties

B. CHANDA†, B. B. CHAUDHURI† and D. DUTTA MAJUMDER†

A hybrid measure of picture gradient for a noisy situation is proposed. The definition of gradient has been initiated from the algorithm for finding the border in a binary picture. The merit of this operator is compared with those of other widely used operators. A measurement of error in extracting edges by thresholding the gradient is also suggested and it is shown that the optimum threshold can be chosen corresponding to the minima in the error function.

1. Introduction

Recognition or classification of a digital picture by computer is generally carried out using information obtained from the edges between different regions. Success of the recognition algorithm depends to some extent on the edges obtained in the previous stage of processing. Edge detection may be defined as the detection of abrupt changes in characteristic features such as brightness, colour or texture. The tasks are carried out either by frequency or spatial domain techniques (Rosenfeld and Kak 1976, Gonzalez and Wintz 1977). In this paper a spatial domain technique has been chosen for edge detection, which is simply the differentiation of the greylevel of pixels in the discrete domain. However, results obtained using this method are very much affected by the noise present in the picture. To solve this problem, the average value of the greylevels over a pre-defined neighbourhood is considered during the computation of spatial gradient. Methods proposed by Rosenfeld and Thurston (1971), Sobel (Duda and Hart 1973), Pewitt (1970) belong to this class and are considerably immune to spurious noise, but they also thicken the edges.

This paper suggests a measure for the gradient which has been initiated from an algorithm devised for finding the border of a binary image using parallel processing. The gradient at selected pixels of the image is modified using local properties, so that the effect due to noise is reduced and edges are kept reasonably thin. In § 2, the border-finding algorithm for a binary image and its evolution to the measure for the gradient in a multi-level image are described. The properties of the new operator are presented in § 3 and a comparison is made of the performance of this operator and that of other widely used operators, based on these properties. Another important problem in edge detection is the selection of a threshold which transforms the gradient picture to a two-level picture containing optimum edges of the regions. In § 4, selection of such a threshold is suggested, depending on measures of errors in thresholding. Section 5 describes the implementation of the algorithm and discusses the results.

2. Algorithms

An edge-detection technique segments the picture by finding the border between different regions, say, object and background. Binary pictures can be described as

---

Received 8 December 1983.

† Electronics and Communication Sciences Unit, Indian Statistical Institute, Calcutta 700035, India.

the optimum representation of such regions with least ambiguity. In this section, a parallel algorithm for finding the borders in a binary (having greylevel 1 for objects and 0 for background) picture is described and the same algorithm is generalized to find the gradient in a multi-level picture.

2.1. *Algorithm for finding a border*

By the 'border' of a binary picture subset (or object) S we mean the set S_b of pixels of S that have neighbours in $\bar{S}$ (the background). Given the overlay U_S representing S, it can be said that U_S is the entire binary picture. Hence the border-finding algorithm can be described as follows.

Algorithm I

Step 1. Compute the logical Exclusive-OR between U_S and $U_S(i-k, j-l)$, where $U_S(i-k, j-l)$ is U_S shifted by (k, l). If we consider all the eight neighbours, values of k and l will be $-1, 0, 1$. Let us call these pictures $U_S(k, l)$.

Step 2. Compute logical OR of $U_S(k, l)$ for all possible values of k and l to obtain U_{S_e}, that is, the overlay of S_e.

Step 3. To obtain U_{S_b}, that is, the overlay containing S_b, compute logical AND between U_{S_e} and U_S.

2.2. *Generalization to the measure of gradient*

Before generalizing the algorithm to make it applicable to multi-level pictures, some (natural) algebraic operations which are equivalent to boolean operations should be presented. Using a truth table it can be easily verified that

$$\begin{aligned} a \text{ AND } b &\equiv \min\{a, b\} \\ a \text{ OR } b &\equiv \max\{a, b\} \\ a \text{ Ex-OR } b &\equiv |a-b| \\ \bar{a} &\equiv M - a \end{aligned}$$

where M is the maximum value that a can attain. Hence, using these equivalent operations Algorithm I can be expressed for a multi-level picture $f(i, j)$ as given below.

Algorithm II

Step 1. Compute $|f(i,j) - f(i-k, j-l)|$ and denote it by $f_{k,l}(i, j)$, where $k = -1, 0, 1$ and $l = -1, 0, 1$.

Step 2. Compute $g(i, j) = \max_{k,l} \{f_{k,l}(i, j)\}$.

Step 3. Find $g'(i, j)$ analogous to U_{S_b} (in Algorithm I) by computing $\min\{g(i, j), f(i, j)\}$.

Since there exists some basic differences in the characteristics of binary and multi-level pictures, algorithms developed for them will reflect these. Algorithm I extracts a set of pixels, whereas Algorithm II finds the gradient of the greylevels. So, for convenience, Step 3 should be omitted from Algorithm II. Now the outcome of this algorithm will be equivalent to the outcome of Step 2 of Algorithm I, which gives the edge S_e of thickness two. S_e consists of the set of pixels S (border of $\bar{S}$) as well as the set of pixels $\bar{S}$ (border of S). Thresholding $g(i, j)$, similarly, will give edges of thickness two. To obtain edges of thickness one, any three consecutive shifts, only one of which is a diagonal shift, can be used instead of the eight shifts described in Steps 1.

However, for some noisy situations in multi-level pictures, extracted edges may become broken or discontinuous. Since in the next stage of processing a broken edge results in more problems than a thick one, all the eight shifts have been used to define the gradient. Hence, in short, the gradient $g(i, j)$ for a multi-level picture can be defined as

$$g(i, j) = \max_{k, l} \{ |f(i, j) - f(i-k, j-l)| \} \tag{1 a}$$

where $k, l \in \{-1, 0, 1\}$, or, in a more general way

$$g(i, j) = \max_{u, v} \{ |(i, j) - f(u, v)| \} \tag{1 b}$$

where (u, v) is the neighbouring pixel of (i, j).

2.3. *Modification of gradient*

Now, to modify the measure of gradient to make it immune to noise, some local properties are used in the following way.

Initially a coarse value of threshold T_c is chosen and, depending on this value, a large set of pixels can be discarded from the next step of consideration. That means, it is inferred that the pixels (i, j), for which $g(i, j) < T_c$, cannot lie on the edges.

For the next step, let us define a set A that contains the grey-levels of all the neighbours (u, v) of the candidate pixel (i, j) that is

$$A = \{ a_m | a_m = f(u, v), \text{ for } m = 1, 2, \ldots, M \} \tag{2}$$

Then compute the following values from the elements of A:

$$R_{ij} = \max_m \{a_m\} - \min_m \{a_m\}$$

$$\mu_{ij} = \frac{1}{M} \sum_{m=1}^{M} a_m$$

and

$$\sigma_{ij} = \frac{1}{M} \sum_{m=1}^{M} |a_m - \mu_{ij}|$$

Finally, the factor $W(i, j)$, which will be multiplied by $g(i, j)$ to make it immune to noise, takes the form

$$W(i, j) = \begin{cases} \dfrac{2\sigma_{ij}}{R_{ij}}, & \text{for } R_{ij} \neq 0 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

It is shown later that the value of $W(i, j)$ varies between 0·75 and 1·0 for a step edge and is much less than 0·75 when isolated spurious noise tries to induce false edge points. Hence, the gradient at pixel (i, j) is ultimately given by

$$G(i, j) = \begin{cases} g(i, j) W(i, j), & \text{for } g(i, j) \geq T_c \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

From the descriptions of the procedure to obtain $G(i,j)$ stated above, it is clear that the technique is hybrid in nature, so it will be called the *hybrid operator*.

3. Quantitative comparison of edge detectors

In this section, the performance of the hybrid edge detector will be compared with that of some other widely used edge detectors (Rosenfeld and Thurston (1971), Sobel (see Duda and Hart 1973) and Prewitt (1970)). All these operators are more or less immune to noise. Before finding differences, these popular edge detectors enhance the picture by smoothing. The Sobel and Prewitt operators smooth the picture over a 3×1 neighbourhood. The Rosenfeld–Thurston operator does the same over an $l \times l$ ($l = 3, 5, 7, \ldots$) neighbourhood. Increasing the mask size decreases the noise sensitivity because of the inherent noise averaging performed by the operator. However, use of a large size of neighbourhood is associated with the performance penalty which makes the edges thick. So we have taken here $l = 3$.

Now,

$$\left.\begin{aligned} G_1 &= \frac{1}{2+w}|f(i-1,j+1) + w\cdot f(i,j+1) + f(i+1,j+1) \\ &\qquad - f(i-1,j-1) - w\cdot f(i,j-1) - f(i+1,j-1)| \\ G_2 &= \frac{1}{2+w}|f(i+1,j-1) + w\cdot f(i+1,j) + f(i+1,j+1) \\ &\qquad - f(i-1,j-1) + w\cdot f(i-1,j) - f(i-1,j+1)| \end{aligned}\right\} \quad (5)$$

where for the Sobel operator $w = 2$ and for the Prewitt operator $w = 1$. For the operator proposed by Rosenfeld and Thurston (where $l = 3$)

$$\left.\begin{aligned} G_1 &= \frac{1}{9}\left|\sum_{l=1}^{3}\sum_{k=1}^{3} f(i-2+k, j-1+l) - \sum_{l=1}^{3}\sum_{m=1}^{3} f(i-2+k, j-3+l)\right| \\ G_2 &= \frac{1}{9}\left|\sum_{l=1}^{3}\sum_{k=1}^{3} f(i-1+k, j-2+l) - \sum_{l=1}^{3}\sum_{k=1}^{3} f(i-3+k, j-2+l)\right| \end{aligned}\right\} \quad (6)$$

Ultimately the gradient $G(i,j)$ is obtained by the root mean square, average or maximum of G_1 and G_2 as given in eqns. (5) and (6). Here the max operator is used, that is, $G(i,j) = \max\{G_1, G_2\}$, to achieve a meaningful comparison with the hybrid operator. Because of the nature of the point operator, G_1 and G_2 (in eqns. (5) and (6)) can be defined interchangeably without affecting the value of $G(i,j)$. An edge is deemed present if $G(i,j)$ exceeds a pre-defined threshold, say T.

3.1. *Edge orientation and displacement sensitivity*

Desirable properties of an edge detector are (i) amplitude response should be invariant to the choice of origin or reference axes, (ii) amplitude response should be invariant to edge orientation, and (iii) amplitude of the edge detector response should decay rapidly as the edge moves away from the centre of the mask. All the edge detectors considered here possess the first property. Regarding the other properties, the merits of edge detectors are compared with an ideal noise-free step edge. Figure 1 shows one such edge which passes through the centre of a 5 × 5 mask and is inclined to the vertical axis by an angle θ. Figure 2 shows the variation of response with the edge orientation (that is, θ) for different operators. It can be seen that the variation in amplitude for the hybrid detector is much less than for the other operators. Figure 3 presents models of vertical and diagonal edges displaced from the centre of the mask. Displacement sensitivity for different operators is shown in Fig. 4. This reveals that the amplitude response for the hybrid operator is maximum when the edge passes through the boundary of two adjacent pixels and decays more rapidly than the amplitude response curves for the other operators as the edge recedes in either direction. This characteristic is desirable for the extraction of the thin edge.

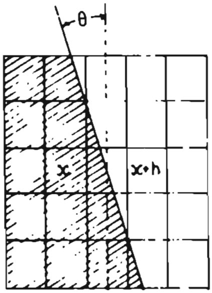

Figure 1. Model of an ideal step edge passing through the centre of the mask and having inclination θ with the vertical axis.

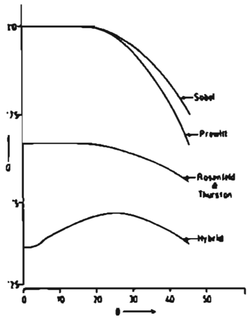

Figure 2. Amplitude response of different operators to inclination θ in the 'max' sense.

3.2. *Noise sensitivity*

Another important property that an edge detector should have is low sensitivity to random noise. The study of this property is complicated by the fact that the edges depend on the choice of an appropriate operator (root mean square, magnitude average or maximum of the magnitude) as well as the threshold. For noisy images,

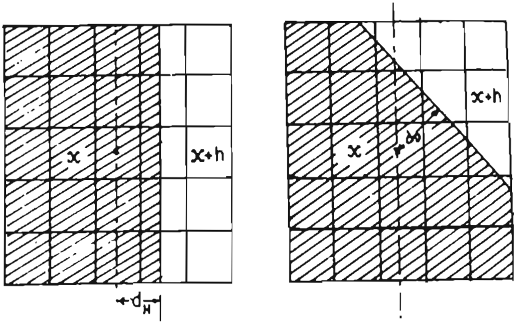

Figure 3. Models of a vertical and a diagonal step edge displaced from the centre of the mask.

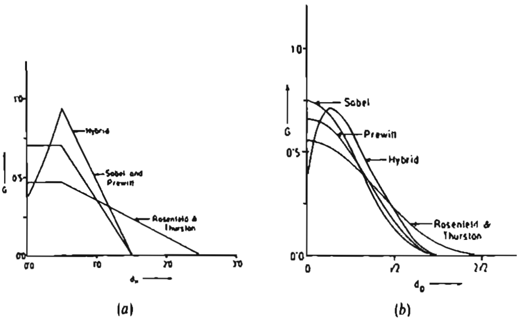

Figure 4. Displacement sensitivity of different operators for (*a*) vertical edge and (*b*) diagonal edge.

threshold selection becomes a trade-off between the missing of valid edges and creation of noise-induced false edges. Abdou and Pratt (1979) have studied the noise sensitivity of some widely used operators using statistical analysis. However, this type of analysis is not followed here, for the reason that the notion of valid edges is somewhat fuzzy and the conditional probability densities $p(G\,|\,\text{edge})$ and $p(G\,|\,\text{no edge})$ are to be modelled rather than estimated.

We have already said that the edge detectors proposed by Rosenfeld and Thurston (1971), Sobel (see Duda and Hart 1973), and Prewitt (1970) reduce the effects of spurious noises by smoothing over a pre-defined neighbourhood. So noise sensitivity of the hybrid operator will be considered in this section, based on the risk of missing of valid edge pixels and inclusion of false edge pixels induced by noise. Consider a mask which contains $(M+1)$ number of pixels within its area. Among these $(M+1)$ pixels, the candidate pixel and y number of other pixels have greylevel a and the rest $(M-y)$ number of pixels, have greylevel $a+h$. It can be readily assumed that if there exists any edge in the vicinity of the candidate pixel then

$$\frac{1}{3} \leqslant \frac{y}{M-y} \leqslant 3$$

Now the amplitude of the hybrid edge detector can be found as follows:

$$g(i,j) = |a+h-a| = |h|$$

$$\mu_{ij} = \frac{ya + (M-y)(a+h)}{M}$$

$$\sigma_{ij} = \frac{1}{M}[|\mu_{ij} - a|y + |\mu_{ij} - a - h|(M-y)]$$

$$= \frac{2y(M-y)}{M^2}|h|$$

$$R_{ij} = |h|$$

$$W(i,j) = \frac{2\sigma_{ij}}{R_{ij}} = \frac{4y(M-y)}{M^2}$$

Hence, finally, the gradient $G(i,j)$ can be given as

$$G(i,j) = \frac{4y(M-y)}{M^2}|h|$$

Figure 5 represents the variation of weighting factor $W(i,j)$ with y. It is seen that the value of $W(i,j)$ decays very quickly outside the interval $(1/3) \leqslant y/(M-y) \leqslant 3$. Hence it can be inferred that for isolated spurious noises, the value of $W(i,j)$ will be very low and consequently the effect of noise on $G(i,j)$, the amplitude of hybrid edge detector, will also be sufficiently reduced.

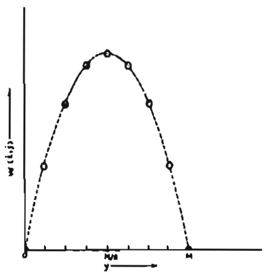

Figure 5. Plot of weighting factor $W(i,j)$ versus y (see text).

4. Selection of optimum threshold

Another major problem of edge detection in the case of a multi-level picture is the selection of the value of the threshold which transforms gradients of the picture onto another picture containing only the edges of the regions. It has been found from extensive experiments that an acceptable edge, in general, possesses the following properties:

(i) edges should be of thickness one;

(ii) at most of the pixels, an edge takes a turn of 0°, ±45°.

Now if the value of the threshold is decreased the edges will be connected and thick, and some protrusions may also appear along with a large number of isolated pixels which are not on the actual edge. On the other hand, if the value of the threshold is increased the edge becomes progressively thinner but some discontinuities also appear in the edges. So the threshold should be selected by compromising between these two situations. Based on the aforementioned properties, an error function has been suggested and the optimum threshold can be chosen corresponding in the minimum value of this function.

Let a pixel (i, j) and its neighbouring n pixels be selected as edge pixels due to thresholding. For a thin and connected edge (except in the case when (i) (i, j) is a terminal pixel, (ii) it lies on the boundary of the picture frame or (iii) a genuine bifurcation of edge occurs at (i, j)) the value of n will be 2. So a measurement of error can be given as $|n-2|$. Since discontinuity in an edge is a more severe error than the thickness, regarding the preservation of information, thus the measurement of error is modified to $E_1 = p^n|n-2|$, where p is a constant less than 1. Again the second property states that the distance between at least one pair of pixels of the set of n neighbouring pixels will be two. The notion of error measurement due to this property can be formulated as $E_2 = \max\{d_c[(k, l), (k', l')]\}$, where d_c the chequer-board distance, given by $d_c = \max\{|k-k'|, |l-l'|\}$, and pixels (k, l) and (k', l') are the elements of the set of n neighbouring pixels. Finally, the error function E takes the form $E = [\lambda E_1 + (1-\lambda)E_2] \times 100/n_t$, there n_t is the total number of edge pixels and $0 \leqslant \lambda \leqslant 1$. Now if the value of the threshold T is varied from $\max_{i,j}\{G(i,j)\}$ to $\min_{i,j}\{G(i,j)\}$ and corresponding values of the error function, E, are computed, then the plot of E versus T will take the shape of U approximately. The value of T that corresponds to the valley of the plot can be taken as the optimum threshold.

5. Results and discussions

To test the effectiveness of the hybrid operator as an edge detector, it has been applied to two artificial pictures having a vertical and a diagonal step edge, respectively. Other popular edge detectors are also applied to the same pictures for comparison of their merits. The algorithms have been simulated on a general purpose EC 1033 computer in FORTRAN. Both the input and output hard copies of the pictures are generated by overprinting of common characters in the computer line printer. Gradient pictures resulting from different operators are shown in Fig. 6. Edges obtained due to the operator proposed by Rosenfeld and Thurston are least sensitive to noise but the edges are certainly thicker than those obtained by other operators. Regarding noise sensitivity, the performance of the hybrid operator is superior to that of the Sobel operator and inferior to that of the Prewitt operator. Abdou and Pratt (1979) have shown that the merit of Prewitt's operator is better than that of Sobel's for a vertical edge and that they are comparable for a diagonal edge. However, the hybrid operator is quite insensitive to edge orientation,

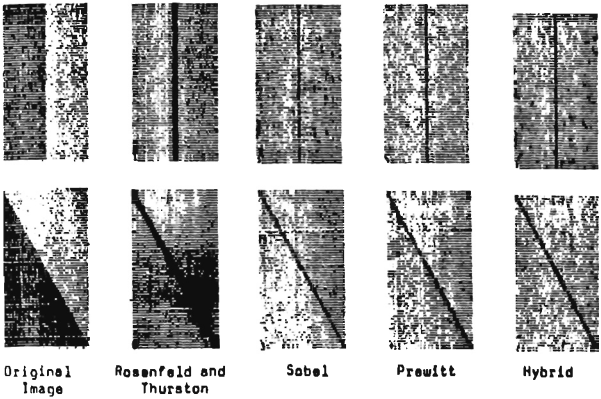

Figure 6. Gradient pictures obtained by using different operators for vertical and diagonal edges.

since actual measurement of edge strength, that is, $g(i, j)$ (eqn. (1)) is completely insensitive to edge orientation.

Improvement of the performance of the hybrid operator regarding the effect of spurious noise is shown in Fig. 7. Thresholding operations are applied to $g(i, j)$ and $G(i, j)$, i.e. the gradients before and after modification, respectively. Threshold values are chosen using the technique described in § 4. Figure 7 reveals that the hybrid measure of gradient is sufficiently immune to spurious noise.

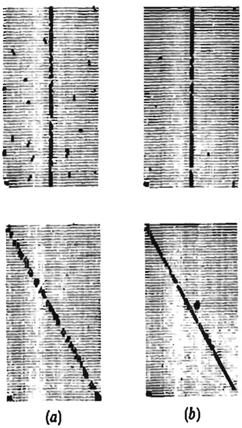

Figure 7. Results of optimum thresholding of (a) $g(i, j)$ (eqn. (1)) and (b) of $G(i, j)$ (eqn. (4)).

It seems that the number of computations required for the hybrid operator is considerably larger than for other operators. However, a coarse threshold T_c considerably reduces the number of pixels for which a weighting factor $W(i, j)$ is to be computed (see eqn. (4)). The threshold T_c can be selected easily from a wide range of values. Therefore, the efficiency of the hybrid operator regarding computer task complexity is comparable to other operators.

6. Conclusion

This paper has described a new edge detection scheme for noisy situations. The approach taken is conceptually different from other such edge detectors. Here the operations were implemented in the hierarchy: differentiation—enhancement—thresholding, whereas for other edge detectors the hierarchy is enhancement—differentiation—thresholding. The merit of this new operator, regarding some fundamental properties that an ideal edge detector should possess, is quite satisfactory, and the performance on a noisy picture is equally good. In addition, the scheme is not computationally expensive.

ACKNOWLEDGMENT

The authors would like to express their thanks to Mr. J. Gupta for typing the manuscript.

REFERENCES

ABDOU, I. E., and PRATT, W. K., 1979, *Proc. Inst. elect. electron. Engrs*, **67**, 753.

DUDA, R. D., and HART, P. E., 1973, *Pattern Classification and Scene Analysis* (New York: Wiley).

GONZALEZ, R. C., and WINTZ, P., 1977, *Digital Image Processing* (Massachusetts: Addison-Wesley).

PREWITT, J. M. S., 1970, *Picture Processing and Psychopictories*, edited by B. S. Lipskin and A. Rosenfeld (New York: Academic Press).

ROSENFELD, A., and KAK, A. C., 1976, *Digital Picture Processing* (New York: Academic Press).

ROSENFELD, A., and THURSTON, M., 1971. *I.E.E.E. Trans. Comput.*, **20**, 562.