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A Simplified Method of Evaluation of Normal Burning Velocity of a Premixed Gaseous Mixture

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A simplified analytical method for finding out the burning velocity and temperature distribution of premixed laminar flame is reported. For analytical solution of the energy equation the heat release rate profile has been approximated by a triangle. The burning velocity and temperature profile thus computed are in good agreement with experimental data and those obtained by numerical solution of the energy equation.

THE normal burning velocity of flame is an important parameter in the design of combustion systems. The values of burning velocity for different combustible mixtures are normally determined experimentally in the absence of reliable analytical methods. Zeldovitch and Frank-Kamenetskii[1] proposed a theoretical analysis of the energy equation at the flame front leading to an expression for the evaluation of burning velocity in premixed gaseous flames. Recently, Bhaduri *et al.*[2] reported a numerical solution of the energy equation giving burning velocities which compared well with the experimental results. The numerical solution suggested, however, suffers from disadvantages inherent in the two-point boundary value problems and is also time-consuming. In the present communication, an analytical solution of the energy equation has been proposed by assuming an approximate triangular form for the heat release rate function, which is simpler and requires less time for solution. The theoretical values of burning velocity obtained by this method compare fairly well with experimental data.

Equation and Its Solution

The system of differential equations of heat and mass transfer due to similarity in the temperature and concentration fields in the reaction zone leads to a single equation of heat transfer (energy equation) at the flame front. Thus, for a one-dimensional system the equation for a stationary laminar flame in a uniform flow of mixture with velocity u is

$$\lambda\frac{d^2T}{dx^2}-C_p\rho u\frac{dT}{dx}+Q=0 \qquad ...(1)$$

where Q is the heat release rate

$$Q=qK_0C_fC_{O_2}\left(\frac{T_f-T}{T_f-T_i}\right)^2\left(\frac{273}{T}\right)^2e^{-E/RT}$$

Q is plotted against T in Fig. 1 and, as one can see, it can be approximated by a triangle as was suggested by Bhaduri and Basu[3]. This approximation yields linear expressions for Q for two zones, i.e.

$$Q=m_1(T-T_1) \quad 0\leqslant x\leqslant x_m \qquad ...(2)$$

$$Q=-m_2(T-T_f) \quad x_m\leqslant x \qquad ...(3)$$

where T_1 is the point of intersection of the linear approximation of the Q profile as given in Fig. 1. The temperature terms in the energy equation are made non-dimensional by substituting

$$\theta=\frac{T_f-T}{T_f-T_i}$$

The energy equation now reduces to

$$\frac{d^2\theta}{dx^2}-\frac{C_p\rho u}{\lambda}\frac{d\theta}{dx}-\frac{Q}{\lambda(T_f-T_i)}=0 \qquad ...(4)$$

The origin is so chosen that at $x=0$, $T=T_1$ and as

$$x\to-\infty, T=T_i \quad \text{and} \quad \frac{dT}{dx}=0, \text{ i.e. } \theta=1, \frac{d\theta}{dx}=0$$

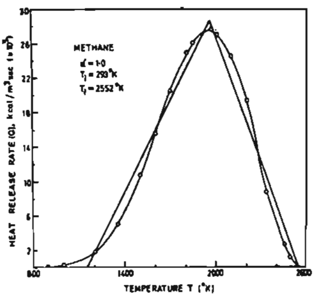

Fig. 1 — Actual and approximate heat release rate profiles

As

$$x \to +\infty, T = T_f \text{ and } \frac{dT}{dx} = 0, \text{ i.e. } \theta = 0, \frac{d\theta}{dx} = 0 \quad ...(5)$$

To solve Eq. (4) with the boundary condition (5), one can find solution for three different regions, depending on different values of Q and can match them. We call region I where $Q = 0$ and regions II and III where Q is given by Eqs. (2) and (3) respectively.

Solution for region I — For $x<0$: $Q = 0$, so Eq. (4) reduces to

$$\frac{d^2\theta}{dx^2} - K_1\frac{d\theta}{dx} = 0 \text{ where } K_1 = \frac{u\rho C_p}{\lambda}$$

The solution is

$$\theta = A' + B'e^{K_1 x}$$

The boundary conditions are

$$x \to -\infty,\ \theta = 1,\ \frac{d\theta}{dx} = 0$$

and $x = 0, \theta = \theta_1$

So,

$$\theta = 1-(1-\theta_1)e^{K_1 x} \quad ...(6)$$

Solution for region II — Substituting the approximate expression for Q given by Eq. (2) in Eq. (1), one gets

$$\frac{d^2\theta}{dx^2} - \frac{u\rho C_p}{\lambda}\frac{d\theta}{dx} - \frac{m_1(\theta_1-\theta)}{\lambda} = 0$$

or

$$\frac{d^2\theta}{dx^2} - K_1\frac{d\theta}{dx} - K_2(\theta_1-\theta) = 0 \quad ...(7)$$

where

$$K_1 = \frac{u\rho C_p}{\lambda},\ K_2 = \frac{m_1}{\lambda}$$

The solution of Eq. (7) may involve two roots, real or imaginary, depending on whether $K_1^2 >$ or $< 4K_2$. The solution is

$$\theta = \theta_1 + e^{\alpha' x}(A_1 \cos\beta' x + B_1 \sin\beta' x) \text{ if } K_1^2 < 4K_2 \quad ...(8)$$

where

$$\alpha' = \frac{K_1^2}{2};\ \beta' = \frac{\sqrt{4K_2 - K_1^2}}{2}$$

and A, B are constants.

Solution for region III — Substituting Eq. (3) for Q in Eq. (1), we have

$$\frac{d^2\theta}{dx^2} - K_1\frac{d\theta}{dx} - K_2'\theta = 0 \quad ...(9)$$

where

$$K_2' = \frac{m_2}{\lambda}$$

The solution is

$$\theta = A_2 e^{(\sqrt{(K_1^2+4K_2')/2}+K_1/2)x} + B_2 e^{(K_1/2-\sqrt{(K_1^2+4K_2)/2})x} \quad ...(10)$$

The boundary condition $x \to \infty$, when applied in Eq. (10) gives

$$A_2 = 0,\ \theta = B_2 e^{-\xi x} \quad ...(11)$$

where

$$\xi = \frac{\sqrt{K_1^2+4K_2'}-K_1}{2}$$

From the three solutions for the three regions it can be seen that there are two undetermined constants in the solution of region II and one undetermined constant in the solution of region III, which can be determined by matching the values of θ and $\frac{d\theta}{dx}$ at the common boundary of each region.

Matching the solutions in regions I and II at $x = 0$, one gets

$$A_1 = 0,\ B_1 = \frac{\theta_1 - 1}{\beta'}K_1$$

The solution in region II becomes

$$\theta = \theta_1 - \frac{1-\theta_1}{\beta'}K_1 \sin\beta' x e^{\alpha' x} \quad ...(12)$$

Next, equating the values of θ and $\frac{d\theta}{dx}$ for regions II and III and at $x = x_m$, where $\theta = \theta_m$, one gets

$$\theta_m = B_2 e^{-\xi x_m} = \theta_1 - \frac{1-\theta_1}{\beta'}K_1 \sin\beta' x_m e^{\alpha' x_m} \quad ...(13)$$

$$\frac{d\theta}{dx} = -B_2\xi e^{-\xi x_m} = -\frac{1-\theta_1}{\beta'}K_1[\alpha' e^{\alpha' x_m}\sin\beta' x_m + \beta' e^{\alpha' x_m}\cos\beta' x_m] = -\theta_m\xi \text{ (from Eq. 13)} \quad ...(14)$$

Combining Eqs. (13) and (14) and simplifying

$$\cot\beta' x_m = \frac{\theta_m\xi - \alpha'(\theta_1-\theta_m)}{\beta'(\theta_1-\theta_m)}$$

or

$$x_m = \frac{1}{\beta'}\cot^{-1}\left\{\frac{\theta_m\xi-\alpha'(\theta_1-\theta_m)}{\beta'(\theta-\theta_m)}\right\} \text{ where } K_1^2 < 4K_2 \quad ...(15)$$

For region I if $K_1^2 > 4K_2$

$$\theta = \theta_1 - \frac{1-\theta_1}{\beta''}K_1 e^{\alpha' x}\sinh\beta'' x \quad ...(16)$$

where

$$\beta'' = \frac{\sqrt{K_1^2-4K_2}}{2}$$

Carrying out the matching procedure as above, the final expression becomes

$$\tanh\beta'' x_m = \frac{\beta''(\theta_1-\theta_m)}{\theta_m\xi-\alpha'(\theta_1-\theta_m)}$$

or

$$x_m = \frac{1}{2\beta''}\ln\left(\frac{1+\varphi}{1-\varphi}\right) \text{ if } K_1^2 > 4K_2 \quad ...(17)$$

From Eq. (13)

$$\theta_1-\theta_m = \frac{1-\theta_1}{\beta'}K_1 e^{\alpha' x_m}\sin\beta' x_m \text{ for } K_1^2 < 4K_2 \quad ...(18)$$

Similarly,

$$\theta_1-\theta_m = \frac{1-\theta_1}{\beta''}K_1 e^{\alpha' x}\sinh\beta'' x \text{ for } K_1^2 > 4K_2 \quad ...(19)$$

These expressions are valid for a steady flame, since Eq. (1) has been deduced for a one-dimensional steady flame. The condition for a steady flame is that the flame velocity is equal to the flow velocity. Therefore, Eqs. (17), (18) and (19) are true only when $u_f = u$.

On eliminating x_m between Eqs. (15) and (18) or (17) and (19), one gets an expression for u which when solved should give the burning velocity. The procedure for the calculation can be summarized as follows.

(i) The values of m_1, m_2, θ_1 and θ_m are found out from the heat release curve.

(ii) The value of u is chosen and x_m is determined from Eqs. (16) and (17) or Eqs. (13) and (15).

(iii) The values of x_m and u are substituted in Eq. (18) or (19) according as $K_1^2 <$ or $> 4K_2$. The correct value of u is obtained by the method of tabulation.

Results and Discussion

A sample calculation of burning velocity was carried out by this method to test the accuracy of the method. Calculation was made for methane gas as fuel at different coefficients of excess air, and an initial temperature of 473°K. The results obtained by this method have been compared with the experimental data of Passauer[4]. The values of C_p and λ are taken at average temperature of the reaction zone. The results are presented in Table 1.

The temperature profile along the flame axis obtained according to the proposed method is shown in Fig. 2 along with the profile obtained by Bhaduri *et al.*[2]. It shows a good coincidence, except at large distances away from the origin. This may be due to the cumulative error in the numerical solution and also to the approximation on the heat release curve stipulated in the present analysis.

TABLE 1 — EFFECT OF COEFFICIENT OF EXCESS AIR ON THE NORMAL BURNING VELOCITY OF METHANE-AIR MIXTURE

Initial temp. °K	Final temp. °K	Coefficient of excess air	Burning velocity (m/sec)		
			Numerical method[3]	Present method	Experimental
300	2250	1·0	0·326	0·31	0·30
473	2893	1·0	0·959	0·78	0·65
473	2393	1·2	0·600	0·555	0·50
473	2083	1·5	0·300	0·320	0·30

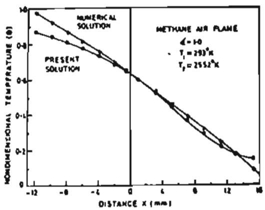

Fig. 2 — Flame temperature profiles obtained by the method of Bhaduri *et al.*[2] and the present method

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Nomenclature

T, T_i, T_f = running, initial and final flame temp.
x = coordinate
C_p = sp. heat of the mixture
ρ = density of the mixture
λ = thermal conductivity of the mixture
q = heat of reaction
K_0 = constant in Arrhenius equation
$\mathring{C}_f$, $\mathring{C}_{O_2}$ = initial conc of fuel and oxygen respectively
E = energy of activation
R = universal gas constant
m_1, m_2 = slopes of the heat release curve
θ_1, θ_m = non-dimensional temp. corresponding to the base and peak of the heat release curve

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